Advances in Homotopy Theory V

Titles and Abstracts

Samik Basu

Title: Spherical fibrations over highly connected manifolds

Abstract: A simply connected 4-manifold supports a circle bundle whose total space is a connected sum of copies of $S^2 \times S^3$. The proof of this relies on Smale's classification of spin 5-manifolds. We investigate the story from a homotopy theoretic point of view, which leads to generalizations to the case of $(n-1)$-connected $2n$-dimensional Poincare duality complexes. This is joint work with Aloke Kr. Ghosh.

Wanying Bi

Title: The magnitude homology of a hypergraph

Abstract: The magnitude homology, introduced by R. Hepworth and S. Willerton, offers a topological invariant that enables the study of graph properties. Hypergraphs, being a generalization of graphs, serve as popular mathematical models for data with higher-order structures. In this talk, we focus on describing the topological characteristics of hypergraphs by considering their magnitude homology. We begin by examining the distances between hyperedges in a hypergraph and establish the magnitude homology of hypergraphs. Additionally, we explore the relationship between the magnitude and the magnitude homology of hypergraphs. Furthermore, we derive several functorial properties of the magnitude homology for hypergraphs. Lastly, we present the Künneth theorem for the simple magnitude homology of hypergraphs.

Ayse Borat

Title: On the properties of higher discrete topological complexity

Abstract: Two discrete perspectives for topological complexity are introduced by Fernandez-Ternero, Macias-Virgos, Minuz and Vilches and by J. Gonzalez. In this talk, we will focus on the former viewpoint and introduce their higher analogues. Furthermore, we will introduce two methods for computing lower bounds of discrete topological complexity which include simplicial Lusternik-Schnirelmann category and geometric realisation.

This is joint work with H. Alabay, E. Cihangirli and E. Dirican Erdal.

Dan Cohen

Title: Supersolvable toric arrangements

Abstract: A toric arrangement is a finite collection of codimension one subtori in a complex torus. If the intersection pattern of these subtori satisfy the combinatorial condition of supersolvabiilty, the complement of the toric arrangement sits atop a tower of fiber bundles. We discuss the intimate relationship between these bundles and the classical Fadell-Neuwirth bundles of ordered configuration spaces.

Tyrone Cutler

Title: Homotopical Invariants of Frøyshov Type

Abstract: In the early 2000s Kim Frøyshov introduced an integral invariant of rational homology 3-spheres using Seiberg-Witten theory. This invariant was found to place restrictions on the possible intersection forms of bounding 4-manifolds and had immediate applications to the study of the 3-dimensional homology cobordism group θ_3^H. In the modern theory, Frøyshov's invariant is usually extracted from the monopole Floer homology of Kronheimer and Mrowka.

In this talk we will outline a homotopical aproach to defining a family of Frøyshov-type invariants for rational homology 3-spheres. This is done within the framework of Manolescu?s homotopy theoretic approach to Seiberg-Witten theory, which recovers monopole Floer homology from an S^1-equivariant stable homotopy type. The invariants we define are captured by certain equivariant Tate homology groups of this stable homotopy type and we will indicate the relationship between these objects. Sufficient background will be given to make this talk accessible. This is joint work with Stefan Behrens (Bielefeld).

Yuki Minowa

Title: On the cohomology of the classifying space of $SO(n)$-gauge groups over S^2

Abstract: The gauge group of a principal bundle P is the topological group of automorphisms of P. Gauge groups have been intensely studied, but not much is known about the cohomology of their classifying spaces. In this talk, I will talk about the cohomology of the classifying spaces of gauge groups of principal $SO(n)$-bundles over S^2. More precisely, we determine the cohomology of the classifying space of the gauge group of the non-trivial principal $SO(n)$-bundle over S^2 for $n = 3, 4$, and then I will show that the cohomology of the classifying space of the gauge group of the non-trivial principal $SO(n)$-bundle over S^2 for $n \geq 3$ is torsion free if and only if $n = 3, 4$. I will also talk about the classification of the homotopy types of the gauge groups of principal $SO(n)$-bundles over a Riemann surface for $n = 3, 4$.

Yuri Muranov

Title: Cubical and simplicial sets in the category of quivers

Abstract: A cubical set is a discrete object which is based on a union of cubes in various dimensions with a collection of special relations. This set is a natural analog of the simplicial set which is based on a union of simplexes. The notion of a cubical set was introduced by Kan as an algebraic model for the investigation of the singular cubical complex $S^{\square}(X)$ of a topological space X. Similarly to the simplicial case there is a weak homotopy equivalence $|S^{\square}(X)|_{Top} \sim X$ for any CW-complex X.

In this talk, we describe similar relationships between cubical and simplicial sets and their realizations in the categories of quivers and digraphs.

Lewis Stanton

Title: Loop space decompositions of moment-angle complexes associated to flag complexes

Abstract: Polyhedral products are natural subspaces of the Cartesian product of spaces, which have a diverse range of applications across mathematics. One particular special case, which appears in toric topology, is the moment-angle complex. Work of various authors has identified families of simplicial complexes for which their corresponding moment-angle complex is homotopy equivalent to a wedge of spheres. In particular, this implies that after looping, the moment-angle complex is homotopy equivalent to a finite type product of spheres and loops on spheres. However, there are simplicial complexes for which their corresponding moment-angle complex before looping is not a wedge of spheres, yet after looping, they still decompose as a product of spheres and loops on spheres. In this talk, I will survey the current progress in this direction, and then expand the family of simplicial complexes for which such a decomposition is known - namely to include simplicial complexes which are the k-skeleton of a flag complex.

Indira Zeinikesheva

Title: On dga models for equivariant cohomology rings of moment-angle complexes

Abstract: We consider a special case of polyhedral product, namely moment-angle complexes, and introduce two dga models for equivariant cohomology of moment-angle complexes with respect to the action of coordinate subtori.

We discuss the equivariant formality of moment-angle complexes, and give illustrative examples.