UNIVERSITY OF Southampton

Economics Department
University of Southampton
Southampton SO17 1BJ, UK

Discussion Papers in
Economics and Econometrics

Joint Oligopoly-Oligopsony Model with Wage Markdown Power

Jian Tong (University of Southampton) and **Carmine Ornaghi** (University of Southampton)

No. 2101

This paper is available on our website
http://www.southampton.ac.uk/socsci/economics/research/papers

ISSN 0966-4246

Joint Oligopoly-Oligopsony Model with Wage Markdown Power

Jian Tong and Carmine Ornaghi*

October 2021, revised January 2022

Abstract

In imperfectly competitive markets, a producer-employer firm can be considered monopolist-monopsonist, facing downward sloping residual demand for product and upward sloping residual supply for labour. Firms can thereby exercise both product price markup and wage markdown powers. To study market outcomes in this setting, we define a Joint Oligopoly-Oligopsony Model – an extended Cournot oligopoly model with imperfectly competitive labour market – and investigate its welfare implications. We show that wage markdown power affects both firms' input mix – driving substitution of labour with non-labour variable inputs – and the scales of inputs (including employment) and output. By introducing Worker Surplus into our welfare analysis, our model clarifies the potentially efficiency-enhancing function of labour union and minimum wage regulation, aimed at curbing oligopsony wage markdown power. Furthermore, we show that competition policy and minimum wage regulation are complements, not substitutes: a lax merger control, which permits consolidation of market structure, can weaken worker power by way of reducing efficient minimum wage, as well as employment. Finally, we investigate the effects of "superstar firms" on market outcomes, allowing the superior efficiency of superstar firms to be a common cause of market concentration and pricing power. Our theory explains the rise of average labour productivity, the fall of labour income share and wage stagnation at industry level. It also shows that the superstar firm phenomenon produces an ambiguous welfare effect once worker welfare is taken into consideration.

Keywords: market power, monopsony, oligopsony, markdown, market structure, worker welfare, worker surplus, labor market, minimum wage

JEL Classification: D21, D33, D43, D6, E24, J2, J3, L13, L4

*Jian Tong: Department of Economics, University of Southampton, J.Tong@soton.ac.uk. Carmine Ornaghi: Department of Economics, University of Southampton, C.Ornaghi@soton.ac.uk. The authors like to thank Jośe Azar, Jan Eeckhout, John Van Reenen, Tommaso Valletti for broadening our engagement with the literature and discussion.

1 Introduction

The last four decades have witnessed a "tectonic drift" in the income distribution between labour and capital in most developed market economies, characterised by three naturally interwoven trends: *(i)* rising firms' pricing powers and profit margins (De Loecker *et al*, 2020); *(ii)* a decrease in the labour share of income (Elsby *et al*, 2013, Karabarbounis and Neiman, 2014, and Autor *et al*, 2020); and *(iii)* wage stagnation, associated with a weakening of the connection between labour productivity growth and real increases in hourly compensation.[1]

Economists have put forward alternative, not mutually exclusive, explanations for the origins of these trends. Some economists postulate that they are the outcome of an increase in competition, stemming from technological change and globalisation, which has favoured the rise of "superstar firms", a term coined by Autor *et al* (2020) to describe firms with superior efficiency that have above-average markups and below-average labour income shares.[2] These firms' large and growing product market shares and markup power can arguably have some merit in that they better serve consumers than their rivals. However, when producer-employer firms have product price markup power, standard oligopoly theories predict that the wage rate should equal the marginal revenue product of labour, i.e., the marginal product of labour multiplying the marginal revenue of output, which is below the product price. Accordingly, if the marginal product of labour increases in tandem with a rise of markup power, as postulated by the superstar firms hypothesis, wage growth can indeed trail behind the growth of marginal product of labour.

Other economists attribute rising concentration and profit margins to the weakening of antitrust enforcement in the product market which lessens competition, and enhances firms' pricing power, as well as exclusionary power, that results in persistent super-normal profits (Furman and Orszag, 2018, Barkai, 2020, Gutiérrez and Philippon, 2017, Shapiro, 2019).[3] Relatedly, Azar and Vives (2021) argue that firms' raising pricing power is driven by the increase of common ownership (in the form of pension, mutual and index funds) as this facilitates internalisation of rival firms' profitability, similar to collusion.

Another line of research has looked at changes in the labour market as possible causes of these trends. Notably, the weakening of unionisation and workers' collective wage bargaining power in the US over the last decades can be responsible for wage stagnation. As noted by Stansbury and Summers (2020), "a decline in worker power results

[1] Although the term wage stagnation often means a lack of growth in real wage rate (e.g., Acemoglu and Autor, 2011), it more specifically refers to the phenomenon that the growth rate of real wage falls below the potential indicated by the growth rate of labour productivity (see, e.g., Mishel, 2012; Bivens and Mishel, 2015). ILO and OECD (2015) went even further to suggest a causal connection: "A falling labour share often reflects more rapid growth in labour productivity than in average labour compensation, and an increase in returns to capital relative to labour."

[2] The superstar firm explanation is supported by De Loecker *et al* (2020), who find that the rises of average pricing power and profit margin in a sector were driven by a small number of superstar firms, while sector medians saw no rises.

The "superstar firm" hypothesis highlights the dispersion of firms' technical efficiencies within the same industries, and links the original causes of the efficiency differentials to efficiency-enhancing innovations and investments, particularly in intangible assets.

[3] Theoretical research also points out that consolidation of market structure may reduce investments and forestall innovation (see Motta and Tarantino, 2007).

in a redistribution of economic rents from labor to capital owners". This "declining worker power" hypothesis views worker power as a mechanism to redistribute economic rents created by markup power, with a potential negative impact on employment.[4] An alternative perspective on the role of the union is offered by the reviving and fast-growing literature on labour market monopsony power (see Manning, 2003, 2011 and 2020).[5] Starting from the premise that one source of economic rent is the "markdown" of wages below the marginal revenue product of labour by employers, the monopsony theory rationalises trade unions and minimum wage as not only a redistributive force, but also a potentially efficiency-enhancing allocative mechanism to curb employers' efficiency-reducing wage markdown power.[6]

The nature, cause and effect of wage markdown power are not merely of theoretical curiosity. Empirically, there has been a long-standing body of evidence that, contrary to the standard view of perfectly competitive labour markets, the labour supply functions faced by individual firms are less than perfectly elastic (Boal and Ransom, 1997, Ashenfelter *et al*, 2010, Manning, 2011). Recent cross-industry studies have also shown that firms operating in more concentrated markets exercise more wage markdown power, to the detriment of workers in terms of suppressed or stagnating wages (Azar, Marinescu and Steinbaum, 2019; Benmelech *et al*, 2018).[7] In-depth industry-specific studies have also taken off, with the aim to trace the root causes of wage markdown power to economic primitives, such as imperfectly elastic market-level or firm-level labour supply function (Azar, Berry and Marinescu 2019, Kroft *et al*, 2021). In two of the first empirical works that investigate both the price markup and wage markdown powers using micro panel data, Tortarolo and Zarate (2018), and Mertens (2021) show that both the product and labour markets exhibit imperfect competition, thus confirming that it is not sufficient to confine the empirical and theoretical analyses of market power and economic rent to product markets.[8]

Concerns about firms' wage markdown power have also been echoed by policy makers. Recognising the detriment to workers from weakening of merger control, a 2016 report produced by the Council of Economic Advisers of the White House highlighted that "antitrust laws apply to reductions in competition for employees as a result of mergers as readily as they do to reductions in product market competition. Yet few merger

[4] Note that the "declining worker power" hypothesis relies on the presence, but not an increase of product price markup power and its economic rent.

[5] The term "monopsony", which literally means "sole buyer", was coined by Joan Robinson (1933). The term is also used loosely to mean market power of a small number of buyers, although the more precise term should be oligopsony, the term that we decide to use in this paper, and covers the special case of monopsony.

[6] In words of Robinson (1933), "[the function of union] in removing exploitation lies not so much in the fact that it improves the bargaining strength of the workers as in the fact that by means of a 'common rule' it reproduces artificially the conditions of perfect elasticity of supply of labour to individual employers." Card and Krueger (1995) argued that the negative employment effects of minimum wage laws, predicted by neoclassical theory of wage determination, are minimal if not non-existent.

[7] In light of both new and classic work in the field of Industrial Organisation, Berry *et al* (2019) caution this recent literature on some of its limitations: "A main difficulty in this area is that most of the existing studies of monopsony and wages ... proceed to estimate regressions of wages on measures of concentration ... studies like this may provide some interesting descriptions of concentration and wages, but are not ultimately informative about whether monopsony power has grown and is depressing wages." These authors also call for more detailed industry-specific studies to establish the causal relation in imperfect competition in labour markets.

[8] De Loecker *et al* (2020) estimate market power using the production function approach pioneered by Hall (1988). They interpret the result as a measurement of markup. However, this interpretation is correct only in the absence of monopsony/oligopsony power.

complaints have cited employment monopsony concerns as a reason to challenge a transaction". Proposals for merger control reform to rectify this deficiency are emerging (Naidu *et al*, 2018; Marinescu and Hovenkamp, 2018). Strengthening the labour union and raising the federal minimum wage have also been policy goals of the current US administration.

This mounting consensus over the need for more vigorous antitrust enforcement that can protect not only consumers but also workers, as well as the call for strengthening unions and minimum wage regulation, has created new challenges for economists. Existing models of imperfect competition, including horizontal merger analysis, are not designed for dealing with the wage markdown power of firms as part of the overall market power. In addition, the interaction between antitrust enforcement and minimum wage regulation is relatively unknown.[9] Given that a firm is both a producer of products, and an employer of workers, its incentives when facing decisions on output levels, pricing, hiring and wages, depend on both product price markup power and wage markdown power, beyond each single component. When both forms of market power, as well as minimum wage regulation, are significant, ignoring any one of these factors may result in systematic error in predicting firm behaviour and assessing the effects of, e.g., horizontal mergers, decline of worker power, and erosion of real minimum wage.

Given the aforementioned challenges, the aim of this paper is to develop a partial equilibrium analysis that combines the oligopoly and oligopsony models to produce a holistic understanding of firms' overall market power, thus laying the foundation for a rational concerted use of antitrust enforcement and minimum wage regulation. We call our new analytical framework the Joint Oligopoly-Oligopsony Model with Wage Markdown Power, or simply JOOM.

The JOOM provides a unifying theoretical framework for the analysis of pricing power, consisting of both product price markup and wage markdown powers, their welfare effects and implications for competition policy, innovation policy and labour legislation. JOOM is also useful to guide empirical measurements of pricing powers in both product and labour markets[10], which are also of policy importance: for example, the pricing power measured by De Loecker *et al* (2020) may conflate both markup and markdown powers, and interpreting it as a measure of markup alone may lead to biased policy implication.

Main Findings. The main novelty of the JOOM framework consists in extending the classical Cournot model (Cournot 1838), where firms can have control over the product price but not the wages, by introducing the more general assumption that firms have wage markdown power in the labour market. Starting from this change in theoretical position, we derive the Cournot-Nash equilibrium and the relations between market structure, market power, producer surplus, price and wage determination, output and employment, and the welfare implications.

[9] For example, there are no models that have studied the impact of a horizontal merger on the effectiveness of existing minimum wage regulation or collective bargaining-determined wage and employment outcomes.

[10] See Tortarolo and Zarate (2018) and Mertens (2021) for examples of empirical research aimed at measuring both markup and markdown powers simultaneously.

At the centre of this conceptual work, we derive the fundamental equations of market power, and extend the measure of market power from the familiar Lerner index, whose application is restricted to product price markup power, to the more general index of overall market power, which also includes the wage markdown power.[11] We show that wage markdown power affects both firms' input mix – driving substitution of labour with non-labour variable inputs – and the scales of inputs (including employment) and output.

The JOOM framework also opens up and addresses novel normative questions by extending the partial equilibrium welfare analysis to include Worker Surplus. By introducing and elevating Worker Surplus on a par with Consumer Surplus as suitable welfare standards, our analysis makes apparent the potentially efficiency-enhancing function of labour union and minimum wage regulation, aimed at curbing oligopsony wage markdown power. Furthermore, we show, competition policy and minimum wage regulation are complements, not substitutes: a lax merger control, which permits consolidation of market structure, can weaken the worker power by way of reducing efficient minimum wage, as well as employment.

In addition, JOOM is flexible enough to accommodate a market structure with heterogeneous productivity distribution across firms, which is particularly suitable for studying the effect of superstar firms on market outcomes. We show that the "superstar firm hypothesis" can explain the rise of average labour productivity, the fall of labour share and wage stagnation at the industry level. Our analysis reveals that the welfare effects of the superstar firm phenomenon are ambiguous once we take worker welfare into consideration: while consumers benefit from lower prices, superstar firms are likely to disbenefit workers by lowering wages or causing wage stagnation. We therefore warn against the presumption that the superstar firm phenomenon is procompetitive because of consumer welfare improvement. Only if the dominance of a superstar firm is temporary, followed by catching up or even leapfrogging of rival firms, thus sustaining long run dynamism in innovation, the rise of superstar firms will benefit both consumers and workers. In contrast, if the superstar firms can persistently escape the competitive pressure of rivals by entrenching their dominance through consolidating exclusionary control over intangible assets, such as IP rights, the benefit to consumers will stagnate, and the worsening of worker welfare will perpetuate.

Contribution to Literature. We contribute theoretically to the literature on monopsony power – particularly the wage markdown power – broached by Robinson (1933). With a generic market structure of JOOM framework, we coin the term "price-taker marginal cost", upon which we define the "overall market power", extending Lerner index to a more general market power index. In these novel terms, we formulate the fundamental equation of market power, encompassing both product price markup and wage markdown powers. Our game-theoretic industrial organisation approach to the wage markdown power complements the search-and-matching

[11] The wage markdown power component, in turn, is a generalisation of the "rate of exploitation" from the classical monopsony theory *a la* Robinson (1933).

model approach commonly used in labour economics, as are surveyed by Manning (2003, 2011). Our extended partial equilibrium approach to wage markdown power also complements the general equilibrium approach to the effects of oligopoly power on labour market outcomes, used by Azar and Vives (2021), and De Loecker *et al* (2021).

Our theoretical analysis also contributes to the global debate on the causes of the secular fall of labour share of GDP. Karabarbounis and Neiman (2014) attribute it to substitution of labour with capital, induced by a secular decline of the prices of capital goods. A necessary condition for this argument is that the elasticity of substitution between capital and labour must exceed unity. Elsby *et al* (2013) identify offshoring of the labour-intensive component of the U.S. supply chain as a leading potential explanation of the decline in the U.S. labour share.[12] Doraszelski and Jaumandreu (2018) also see substitution of labour with non-labour inputs as a key factor, but propose that the substitution is caused by labour-augmenting (biased) technological change. In contrast to these aforementioned explanations which do not rely on rise of market power, De Loecker *et al* (2020) and Autor *et al* (2020) both argue that the rise of superstar firms' product market power are a main cause of the fall of labour share. The superstar firms theory is particularly appealing because it fits well with the growing evidence based on micro panel data, featuring substantial within industry dispersion of firms' productivity, and their persistence (Bartelsman and Doms, 2000, Syverson, 2011, De Loecker and Syverson, 2021). While labour market plays no or minimal role in the existing superstar firms theory, Stansbury and Summers (2020) point to the decline of workers' collective power as a cause of decline in labour share. JOOM provides a coherent framework to address all the aspects listed here: labour-capital substitution, rising market powers (including wage markdown power), and declining worker power. It shows that the rise of superstar firms' product price markup and wage markdown power can play important roles in causing the fall of labour share. Particularly, the rising wage markdown power can play this role through two mechanisms: first, it raises the share of gross profit and hence reduces the labour share of revenue; second, it induces substitution of labour with non-labour variable inputs. In addition, we show that the rise of superstar firms can also weaken worker power by way of reducing efficient minimum wage.

Our extended welfare analysis that includes worker surplus also provides theoretical support for the calls for reform of antitrust enforcement in order to protect labour market competition (between buyers), as exemplified by CEA (2016), Naidu *et al* (2018), and Marinescu and Hovenkamp (2018). We propose that a worker welfare standard should be introduced on a par with the prevalent consumer welfare standard.

The remainder of the paper is organised as follows: Section 2 explains the JOOM framework, derives a fundamental equation of market power, and explores its implications for input mix (i.e., substitution of labour with non-labour variable inputs) and scales of inputs and output. Sections 3 and 4 are devoted to the function of

[12] This view is reminiscent of the capital-labour substitutability hypothesis originated from Hicks (1932) and revived by Karabarbounis and Neiman (2014).

trade union and minimum wage in the tradition of Robinson (1933), and to the role of superstar firms, respectively. Section 5 concludes.

2 The Model

Conventional partial equilibrium analysis of imperfect competition focuses on one market, e.g., the product market in a Cournot oligopoly model, or the labour market in the classical monopsony model. This paper develops a joint oligopoly-oligopsony model (JOOM) that studies these two imperfectly competitive markets simultaneously. This extended partial equilibrium analysis cannot be decomposed into two isolated conventional partial equilibrium analyses without losing one of its defining characteristics, namely, the interaction of the product pricing power and wage setting power exercised by each firm.

Consider a pair of input-output markets with a joint oligopoly-oligopsony industry structure, dominated by a set of independent firms $\mathcal{N} \equiv \{1, \ldots, N\}$. Let $P(Q)$ be the downward-sloping demand function in the homogenous product market, where $Q \equiv \sum_{i \in \mathcal{N}} q_i$ is the total output, and q_i is the output of firm i. Each firm uses labour l_i and V other variable inputs $\mathbf{x}_i = (x_{vi})_{v=1}^{V}$, as well as (sunk) fixed input f_i for production. Let the production function be $q_i = F(\mathbf{x}_i, l_i; f_i)$ for each $i \in \mathcal{N}$. Let the supply of each variable input v be perfectly competitive, with constant price p_v. Let the supply of labour be characterised by the upward-sloping wage function $W(L)$, with $L = \sum_{i \in \mathcal{N}} l_i$ indicating the total employment of labour. The joint oligopoly-oligopsony industry structure refers to the product and labour markets.[13]

For each $i \in \mathcal{N}$, the conditional demand functions for labour and other variables are l_i^* and $\mathbf{x}_i^* = (x_{vi}^*)_{v=1}^{V}$, which are functions of $\left(q_i, (p_v)_{v=1}^{V}, W, L_{-i}\right)$, where $L_{-i} \equiv \sum_{j \in \mathcal{N} \setminus \{i\}} l_j$ is the aggregate employment by all firms other than i.[14] The conditional demand functions are solutions to cost minimisation problems faced by the firms, a general treatment of which is given in Appendix A. We treat f_i, for $i \in \mathcal{N}$, as exogenous parameters of the game. The game has the output q_i and input levels $\mathbf{x}_i$ and l_i as strategic variables, thus featuring a sort of quantity (or Cournot) competition, as opposed to price competition. Our preference for quantity competition to price competition is motivated by the fundamental fact that it takes time and effort to fill job vacancies: it is then more realistic to model each employer as committed to employment quantity/capacity than the prices, which are dictated by market clearing taking quantity/capacity as given.

Let $Q_{-i} \equiv \sum_{j \in \mathcal{N} \setminus \{i\}} q_j$ denote the aggregate output by all firms other than i. Then the profit (or more

[13] To ensure the generality of the theory, an extension of JOOM to allow oligopsony in other input markets is provided in Appendices A.1, A.2 and B.

[14] Given W and L_{-i}, the residual inverse labour supply function (or wage function) for firm i can be defined as $W_i(l_i) \equiv W(L_{-i} + l_i)$.

precisely, the producer surplus[15]) of firm i is given by

$$\pi_i\left(q_i, Q_{-i}, L_{-i}\right) = P\left(Q_{-i} + q_i\right) q_i - W\left(L_{-i} + l_i^*\right) l_i^* - \sum_{v=1}^{V} p_v x_{vi}^*. \tag{1}$$

When each firm plays its best response in a Nash equilibrium, the first order condition is

$$P' q_i + P - W' \frac{\partial l_i^*}{\partial q_i} l_i^* - W \frac{\partial l_i^*}{\partial q_i} - \sum_{v=1}^{V} p_v \frac{\partial x_{vi}^*}{\partial q_i} = 0. \tag{2}$$

By the standard definition, the marginal revenue for firm i is $MR_i = P + P' q_i$ and its marginal cost is $MC_i = \sum_{v=1}^{V} p_v \frac{\partial x_{vi}^*}{\partial q_i} + W\left(L\right) \frac{\partial l_i^*}{\partial q_i} + W'\left(L\right) \frac{\partial l_i^*}{\partial q_i} l_i^*$. If $W' = 0$, i.e., the labour supply is perfectly elastic, then the firms are price takers on the buyer side of the labour market. We therefore define a novel term – the **price taker marginal cost** – MC_i^{PT}, by

$$MC_i^{PT} \equiv \sum_{v=1}^{V} p_v \frac{\partial x_{vi}^*}{\partial q_i} + W\left(L\right) \frac{\partial l_i^*}{\partial q_i}, \tag{3}$$

where the relation between marginal cost MC_i and MC_i^{PT} is given by

$$MC_i = MC_i^{PT} + W'\left(L\right) \frac{\partial l_i^*}{\partial q_i} l_i^*, \tag{4}$$

$$MC_i = \begin{cases} MC_i^{PT} & \text{if } W'\left(L\right) = 0, \\ > MC_i^{PT} & \text{if } W'\left(L\right) > 0. \end{cases} \tag{5}$$

Appendix A.2 extends the definition of MC_i^{PT} to allow oligopsony markets for other variable inputs.

Accordingly, in the context of JOOM, the familiar Lerner index:

$$\rho_i \equiv \frac{P - MC_i}{P} \tag{6}$$

can be extended to

$$\tau_i \equiv \frac{P - MC_i^{PT}}{P}, \tag{7}$$

as a measure of overall market power, while ρ_i only measures the product price markup power. If the firm is a price-taker in each input market, then $MC_i = MC_i^{PT}$, and $\rho_i = \tau_i$. However, if $W' > 0$ then $\rho_i < \tau_i$, which means ρ_i fails to capture the full extent of market power exercised by firm i.

As will transpire, the overall market power index τ_i plays an important role in understanding pricing power in its entirety. Similarly, its measurement is crucial for empirical research. The following equation, in conjunction

[15] This is not the net profit because there is no deduction of the sunk fixed cost related to f_i. This can also be interpreted as the Marshaillian quasi-rent. It consists of the fixed cost and net profit.

with Appendix A.3, provides useful theoretical guidance for measuring τ_i:

$$\tau_i \equiv \frac{P - MC_i^{PT}}{P} = 1 - \sum_{v=1}^{V} \zeta_{vi}\phi_{vi} - \zeta_{Li}\phi_{Li}, \tag{8}$$

where $\zeta_{vi} \equiv \frac{\partial x_{vi}^*}{\partial q_i}\frac{q_i}{x_{vi}^*}$ and $\zeta_{Li} \equiv \frac{\partial l_i^*}{\partial q_i}\frac{q_i}{l_i^*}$ are respectively the conditional demand elasticities of input v and labour w.r.t. output; $\phi_{vi} \equiv \frac{p_v x_{vi}^*}{Pq_i}$ and $\phi_L \equiv \frac{Wl_i^*}{Pq_i}$ are respectively input v and labour shares of the firm's revenue. Empirically, the revenue shares of some variable inputs, including labour, can often be directly recovered from firms' financial data. If the conditional demand elasticities of these variable inputs can be estimated, then τ_i can be recovered according to equation (8).

2.1 Double Marginalisation in JOOM

In this section study the two components of the overall market power in high level abstraction: price markup and markdown of price-taker marginal cost.

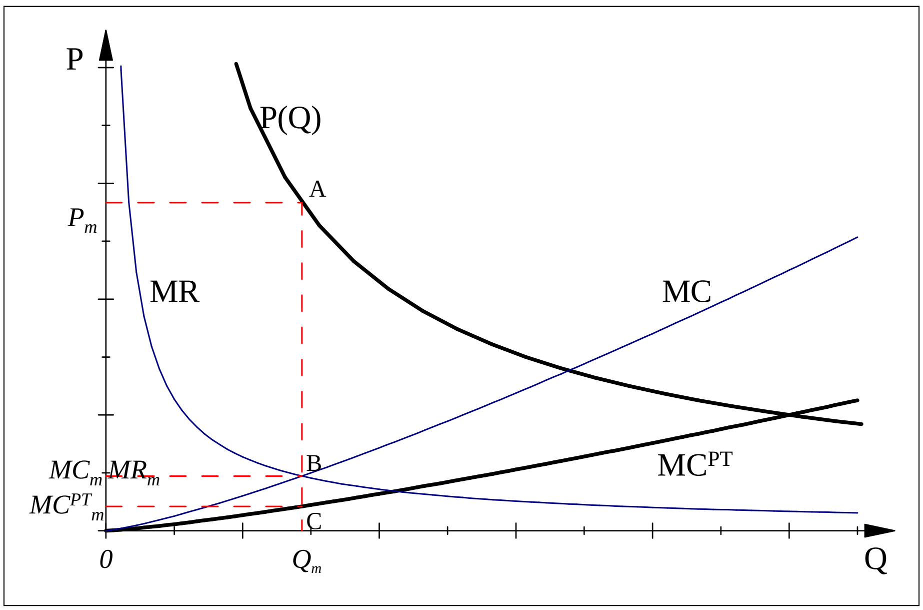

Figure 1: The joint monopoly-monopsony model

For this purpose, it is convenient to use the following related measure of the overall market power:

$$MP_i \equiv \frac{P}{MC_i^{PT}}, \tag{9}$$

with the one-to-one relation between MP_i and τ_i:

$$MP_i \equiv \frac{1}{1-\tau_i}. \tag{10}$$

The essence of the first order condition (2) is the familiar general equation of firm behaviour: $MR_i = MC_i$. For $W'(L) > 0$:

$$MC_i^{PT} < MC_i = MR_i, \tag{11}$$

which means the incremental inputs used to produce the marginal unit of output receive payments below the marginal revenue, reflecting an overall markdown power (against all inputs). Accordingly, we define the overall markdown as:

$$MD_i \equiv \frac{MC_i^{PT}}{MR_i}, \tag{12}$$

which measures the oligopsonist firm's ability to markdown the price-taker marginal cost MC_i^{PT} below its marginal revenue MR_i. This is the second source of a joint oligopolist-oligopsonist market power.

The familiar notion of markup (MU) is defined by $MU_i \equiv \frac{P}{MC_i}$. The following equations:

$$MP_i \equiv \frac{P}{MC_i^{PT}} = \frac{P}{MC_i}\frac{MC_i}{MC_i^{PT}} \equiv MU_i\frac{MC_i}{MC_i^{PT}} = MU_i\frac{MR_i}{MC_i^{PT}}$$

imply

$$MP_i = \frac{MU_i}{MD_i}. \tag{13}$$

The last equation reveals that a joint oligopolist-oligopsonist firm imposes double marginalisations[16] with respect to product price and price-taker marginal cost: a first marginalisation between price and marginal cost, measured by the markup MU_i, and a second wedge between price-taker marginal cost and marginal cost, measured by the inverse of overall markdown $\frac{1}{MD_i}$. The market power index MP_i measures the overall effect of the double marginalisation.

Figure 1 illustrates the relations among MP_i, MU_i and MD_i with the example of joint monopoly-monopsony model. Let subscript $_m$ indicate the monopoly-monopsony equilibrium. Graphically, this equilibrium is determined by the intersection of the marginal revenue (MR) and marginal cost (MC) curves (point B), with $MR_m = MC_m$. The wedges respectively between the inverse price curve $P(Q)$ and the marginal revenue curve MR, and between the marginal cost curve MC and the price-taker marginal cost curve MC^{PT}, are the causes

[16] The term "double marginalisation" in the joint oligopoly-oligopsony model here has a different sense from the more familiar meaning in the context of upstream (e.g., a manufacturer) monopoly and downstream (e.g., a retailer) monopoly, where each firm has a markup over its own marginal cost. In the JOOM context, each single firm makes double marginalisations in the form of respective markup and markdown with respect to product and input prices.

of the markup and overall markdown. The equilibrium product price markup is $MU_m = P_m/MC_m$. The overall markdown is $MD_m = MC_m^{PT}/MR_m$. The market power index is

$$MP_m = P_m/MC_m^{PT} = \frac{P_m/MC_m}{MC_m^{PT}/MR_m} = \frac{MU_m}{MD_m}.$$

2.2 The Fundamental Equation of Market Power

For firms which are price-takers in all input markets, the Lerner index can be used as a measure of overall market power. When $MC_i > MC_i^{PT}$, τ_i measures the overall departure of product price and wage rate from the perfectly competitive levels. Using the first order condition (2), we derive the fundamental equaiton of market power:

$$\frac{P - MC_i^{PT}}{P} = -\frac{P'Q}{P}\frac{q_i}{Q} + \frac{W'L}{W}\frac{l_i^*}{L}\left(\frac{\partial l_i^*}{\partial q_i}\frac{q_i}{l_i^*}\right)\frac{Wl_i^*}{Pq_i}$$

and equivalently

$$\tau_i \equiv \frac{P - MC_i^{PT}}{P} = \frac{s_i}{\epsilon} + \frac{s_{Li}}{\epsilon_L}\zeta_{Li}\phi_{Li}, \tag{14}$$

where $s_i \equiv \frac{q_i}{Q}$ is the product market share (and equivalently sales share) of firm i; $s_{Li} \equiv \frac{l_i^*}{L}$ is the labour input market share (and equivalently labour expenses share) of firm i.

To interpret the economic meaning of this fundamental equation of market power, it is essential to define the marginal revenue product of labour $MRPL_i \equiv MR_i \frac{\partial F}{\partial l_i}$ and marginal cost of labour $MC_{Li} \equiv W + W'(L)\, l_i^*$. Note that the first order conditions of profit maximisation entail both

$$MR_i = MC_i \tag{15}$$

and

$$MRPL_i = MC_{Li}. \tag{16}$$

It follows that for $W'(L) > 0$:

$$\frac{W(L)}{MRPL_i} = \frac{W(L)}{MC_{Li}} = \frac{W(L)}{W + W'(L)\, l_i^*} = \frac{1}{1 + \frac{s_{Li}}{\epsilon_L}} < 1, \tag{17}$$

which shows that the wage setting power component $\frac{s_{Li}}{\epsilon_L}$ of τ_i enables the firm to mark the wage $W(L)$ down below the marginal revenue product of labour $MRPL_i$, that is, labour is paid less than its marginal contribution to the firm's revenue. This is what the notion of exploitation of labour means in classical monopsony theory.

Following this tradition, we define the rate of exploitation of labour:

$$\chi_{Li} \equiv \frac{MRPL_i - W}{W}, \tag{18}$$

which measures the wage markdown power in terms of the extent to which the wage is below the marginal revenue product of labour.[17] Notably, our definition of rate of exploitation χ_{Li} is consistent with the one given by Manning (2003).

In the Cournot-Nash equilibrium, the solutions to ρ_i and χ_{Li} have the following elegant and intuitive results:

$$\rho_i = \frac{s_i}{\epsilon}, \tag{19}$$

$$\chi_{Li} = \frac{s_{Li}}{\epsilon_L}. \tag{20}$$

The fundamental equation (14) thereby implies:

$$\tau_i = \rho_i + \chi_{Li}\zeta_{Li}\phi_{Li}. \tag{21}$$

Eq. (21) shows that the rate of exploitation of labour χ_{Li} is a contributing factor of the overall market power τ_i.

Conceptually we can simply define the notion of **exploitation** as the behaviour of the advantaged party in an exchange relation to take advantage of the **inequality of bargaining power** against the disadvantaged party. In the context of joint oligopoly-oligopsony industry with wage markdown power, because the consumers and workers are respectively of larger numbers and in perfect competition within each group, their bargaining strength are weaker vis-a-vis the firms for $N < \infty$. In this sense, the Lerner index ρ_i is the rate of exploitation of the consumers by an oligopolist producer, just like χ_{Li} is the rate of exploitation of worker by an oligopsonist employer. The novel insight here is that $\chi_{Li} = \frac{s_{Li}}{\epsilon_L}$ shows the rate of exploitation of labour depends on the market structure, as indicated by market share $s_{Li} \in [0, 1]$, in addition to the labour supply elasticity $\epsilon_L < \infty$. Market concentration and market powers are determinants in the theory of exploitation.

2.3 Wage Markdown Power: A Driver for Substituting Labour with Capital

Each profit maximising firm is necessarily a cost minimiser. In general, cost minimisation entails that the technical rate of substitution between two variable inputs equals the ratio of marginal costs of them. For example, for firm

[17] The notion of "exploitation" was used by Pigou (1924), Hicks (1932) and Robinson (1933) to mean the gap between the marginal revenue product of labour $MRPL_i$ and wage W. Hicks (1932) also applied the notion of "exploitation" to consumers. In that sense, the Lerner index, which measures product price markup power, can be called the "rate of exploitation of consumers".

i, the technical rate of substitution between input v and labour is:

$$TRS_{vL,i} \equiv \frac{\frac{\partial F_i}{\partial x_{vi}}}{\frac{\partial F_i}{\partial l_i}} = \frac{MC_{vi}}{MC_{Li}} = \frac{p_v}{W + W' l_i^*} = \frac{p_v}{\left(1 + \frac{s_{Li}}{\epsilon_L}\right) W} = \frac{p_v}{\left(1 + \chi_{Li}\right) W}. \tag{22}$$

When the input markets (including labour market) are all perfectly competitive, cost saving is always efficiency improving. In the presence of oligopsony buyer power, e.g., in labour market, this is no longer a generally valid statement. It is worth noting that the rate of exploitation of labour $\chi_{Li} = \frac{s_{Li}}{\epsilon_L}$ affects the optimal substitution between labour and variable input v: The larger a firm's labour market share s_{Li}, the larger its wage markdown power measured by χ_{Li}, and the stronger its incentive to substitute labour with non-labour variable inputs.

This insight can be sharpened with the example of Cobb-Douglas production function with two variable inputs and constant returns to scale:

$$F\left(x_i, l_i; f_i\right) = F_i\left(x_i, l_i\right) = A_i x_i^{\alpha} l_i^{1-\alpha} \text{ for } i \in \mathcal{N},$$

where A_i is Hicks neutral productivity of firm i, which depends on the fixed asset f_i and does not affect its technical rate of substitution between input x and labour:

$$TRS_{xL,i} \equiv \frac{\frac{\partial F_i}{\partial x_i}}{\frac{\partial F_i}{\partial l_i}} = \frac{\alpha}{1-\alpha} \frac{l_i^*}{x_i^*} = \frac{p_x}{\left(1 + \frac{s_{Li}}{\epsilon_L}\right) W}. \tag{23}$$

Proposition 1 *Letting the production function of JOOM be $F_i\left(x_i, l_i\right) = A_i x_i^{\alpha} l_i^{1-\alpha}$ for $i \in \mathcal{N}$. The implied ratio of expenditures of input x and labour is*

$$\frac{p_x x_i^*}{W l_i^*} = \frac{\alpha}{1-\alpha} + \frac{\alpha}{1-\alpha} \frac{1}{\epsilon_L} s_{Li} = \frac{\alpha}{1-\alpha} + \frac{\alpha}{1-\alpha} \chi_{Li}. \tag{24}$$

Equation (24) predicts the ratio of expenditures of non-labour variable inputs and labour depends on labour market share s_{Li} and wage markdown power measured by χ_{Li}. If there is substantial dispersion in labour market share s_{Li} among firms in an industry, then equation (24) predicts substantial dispersion of variable input expenditure ratio $\frac{p_x x_i^*}{W l_i^*}$. This also provides a novel theoretical explanation and, in turn, a prediction that can be tested empirically for the dispersion in capital deepening, measured by $\frac{x_i^*}{l_i^*}$ (where the term "capital" includes non-labour variable inputs).

Equation (24) can also be used to guide empirical measurement of markdown power $\chi_{Li} = \frac{s_{Li}}{\epsilon_L}$, recoverable from firm level data on variable input expenditure ratios $\frac{p_x x_i^*}{W l_i^*}$ and labour market share s_{Li}.

Having treated the role of wage markdown power in firms' decision on substituting labour with non-labour variable inputs, for the rest of the paper, we abstract from this important issue by assuming labour is the sole variable input, thus highlighting the effects of markdown power on input and output scales, rather than input mix. This leads to a simple canonical model satisfying the following conditions: (i) the production function is $F(\mathbf{x}_i, l_i; f_i) = \frac{l_i}{a_i}$ for all $i \in \mathcal{N}$, with input elasticity $\zeta_{Li} = 1$. (ii) The elasticities of product demand and labour supply are constant, with $\epsilon > 1$ and $\epsilon_L \in (0, \infty)$. Thus we can write $P = P_0 Q^{-\frac{1}{\epsilon}}$ and $W(L) = W_0 L^{\frac{1}{\epsilon_L}}$.

This canonical model simplifies the analysis by (a) abstracting from the issue of substitution between multiple variable inputs; and by (b) allowing closed form solutions. The assumption $\epsilon > 1$ implies the industry revenue, denoted by $\Theta \equiv PQ = P_0 Q^{1-\frac{1}{\epsilon}}$, satisfies $\frac{\partial \Theta}{\partial Q} > 0$, $\frac{\partial \Theta}{\partial P} < 0$ and $\lim_{Q \to \infty} \Theta = \infty$, which means the size of the market can expand without bound if the price is reduced toward zero. This assumption serves the purpose of using a single industry to mimic boundless economic growth driven by technological progress. The restriction: $\zeta_{Li} = 1$ assumes away the possibility of $\zeta_{Li} < 1$, i.e., labour demand increases less than proportionally to output expansion because firm i substitutes labour with other variable inputs. An additional related possibility is that firms that have higher labour market shares and more wage markdown power tend to have smaller values of ζ_{Li}, that is, the tendency to substitute labour with non-labour variable inputs increases with labour market share and wage markdown power. Conditions for these possibilities are studied in Appendix A.3.

2.4 Market Structure, Markdown Power and Scales of Input and Output

In this section, we use the symmetric model with $a_i = a$. This makes each among the N to be a representative firm of the industry. Heterogeneity in firms' productivity, a robust stylised fact across most industries, is introduced in Section 4 when we study superstar firms and their laggard rivals.

For the symmetric model, $\frac{MC_i^{PT}}{P} = \frac{WL}{PQ}$ and $s_i = s_{Li} = \frac{1}{N}$, $\phi_i = \phi = \frac{WL}{PQ}$, $\rho_i = \rho$, $\chi_{Li} = \chi$, and $\tau_i = \tau$. Accordingly, the fundamental equation of market power (14) becomes:

$$\tau = 1 - \frac{WL}{PQ} = \left(\frac{1}{\epsilon} + \frac{1}{\epsilon_L}\frac{WL}{PQ}\right)\frac{1}{N}. \tag{25}$$

In the absence of real improvement of productivity in terms of reducing a, consolidation of market structure, say, through horizontal merger, implies a fall of the labour share and an increase of market power, that is $N \downarrow \Rightarrow \frac{WL}{PQ} \downarrow$ and $\tau \uparrow$. Formally, these results and their cause through the wage markdown channel are captured below.

Proposition 2 *For the symmetric canonical model, there exists a unique symmetric equilibrium where the labour* *share of revenue (at the firm and industry levels) and the overall market power are respectively given by*

$$\frac{WL}{PQ} = \frac{1-\frac{1}{\epsilon}\frac{1}{N}}{1+\frac{1}{\epsilon_L}\frac{1}{N}} = \frac{1-\rho}{1+\chi}, \tag{26}$$

$$\tau = \frac{\frac{1}{\epsilon}+\frac{1}{\epsilon_L}}{N+\frac{1}{\epsilon_L}} = \frac{\rho+\chi}{1+\chi}, \tag{27}$$

with

$$\rho = \frac{1}{\epsilon}\frac{1}{N}, \chi = \frac{1}{\epsilon_L}\frac{1}{N}, \tag{28}$$

and comparative statics:

$$\frac{\partial \chi}{\partial N} < 0, \frac{\partial\left(\frac{WL}{PQ}\right)}{\partial \chi}\frac{\partial \chi}{\partial N} > 0, \frac{\partial \tau}{\partial \chi}\frac{\partial \chi}{\partial N} < 0. \tag{29}$$

A consolidation of the market structure $N \downarrow$ increases not only firms' product price markup power, but also their wage markdown power, resulting in a decrease of labour share of income, wage stagnation[18] and higher overall market power. These effects can arise from the Cournot model alone. What JOOM adds is that there exists a second channel: the oligopsony wage markdown power channel, through which market concentration can cause these effects.

The inequality of bargaining power between the firms and consumers is measured by the equilibrium value of the Lerner index $\rho = \frac{1}{\epsilon}\frac{1}{N}$ and that between the firms and workers is measured by the rate of exploitation $\chi = \frac{1}{\epsilon_L}\frac{1}{N}$. Hereby, market structure – a measure of the extent to which the ownership of the capital assets of the industry is divided, independent and competitive – is a determinant of inequalities of bargaining powers in both product and labour markets. Ultimately, it is against these inequalities that corrective public policy interventions such as competition policy, trade union law, and minimum wage regulation are justified. A novel insight from JOOM is that competition policy can play a role in reducing the rate of exploitation of labour: merger control that reduces $\frac{1}{N}$ can decrease χ.

The symmetric canonical model delivers the following closed-form solutions to market outcomes, which reveal the effects of consolidation of market structure.

Lemma 3 *For the symmetric canonical model, there exists a unique symmetric equilibrium where employment*

[18] It is worth noting that a fall of the labour share $\phi = \frac{WL}{PQ}$ is closely related to wage stagnation in that

$$\phi = \frac{WL}{PQ} = \frac{W}{PQ/L},$$

i.e., labour share of revenue equals the wage to labour productivity ratio, and

$$\frac{d\ln\phi}{dt} = \frac{d\ln W}{dt} - \frac{d\ln\left(PQ/L\right)}{dt},$$

where $\frac{d\ln\phi}{dt}$, $\frac{d\ln W}{dt}$ and $\frac{d\ln(PQ/L)}{dt}$ are the growth rates of ϕ, W and PQ/L (average labour productivity). $\frac{d\ln\phi}{dt} < 0$ is equivalent to $\frac{d\ln W}{dt} - \frac{d\ln(PQ/L)}{dt} < 0$, i.e., the wage growth trailing behind the labour productivity growth.

L, wage rate W, output Q, price P and industry labour productivity $\frac{PQ}{L}$ are:

$$\begin{aligned}
L &= L_s \left(\frac{1-\rho}{1+\chi}\right)^{\frac{1}{\frac{1}{\epsilon_L}+\frac{1}{\epsilon}}}, W = W_s \left(\frac{1-\rho}{1+\chi}\right)^{\frac{\frac{1}{\epsilon_L}}{\frac{1}{\epsilon_L}+\frac{1}{\epsilon}}}, \\
Q &= Q_s \left(\frac{1-\rho}{1+\chi}\right)^{\frac{1}{\frac{1}{\epsilon}+\frac{1}{\epsilon_L}}}, P = P_s \left(\frac{1+\chi}{1-\rho}\right)^{\frac{\frac{1}{\epsilon}}{\frac{1}{\epsilon}+\frac{1}{\epsilon_L}}}, \\
\frac{PQ}{L} &= \frac{P_s Q_s}{L_s} \left(\frac{1+\chi}{1-\rho}\right)^{\frac{\frac{1}{\epsilon}}{\frac{1}{\epsilon}+\frac{1}{\epsilon_L}}},
\end{aligned}$$

where $L_s \equiv \left(\frac{P_0}{a^{1-\frac{1}{\epsilon}} W_0}\right)^{\frac{1}{\frac{1}{\epsilon_L}+\frac{1}{\epsilon}}}$, $W_s \equiv \left(\frac{(P_0)^{\frac{1}{\epsilon_L}} (W_0)^{\frac{1}{\epsilon}}}{a^{\frac{1}{\epsilon_L}\left(1-\frac{1}{\epsilon}\right)}}\right)^{\frac{1}{\frac{1}{\epsilon_L}+\frac{1}{\epsilon}}}$, $Q_s \equiv \left(\frac{P_0}{a^{1+\frac{1}{\epsilon_L}} W_0}\right)^{\frac{1}{\frac{1}{\epsilon}+\frac{1}{\epsilon_L}}}$ *and* $P_s \equiv \left((P_0)^{\frac{\epsilon}{\epsilon_L}} a^{1+\frac{1}{\epsilon_L}} W_0\right)^{\frac{\frac{1}{\epsilon}}{\frac{1}{\epsilon}+\frac{1}{\epsilon_L}}}$ *are the limiting values of L, W, Q and P for $N \to \infty$.*

Proposition 4 *For the symmetric canonical model, there exists a unique symmetric equilibrium where the consolidation of market structure causes decreases of output, employment and wage rate through the channel of wage markdown power, i.e.,*

$$\frac{\partial Q}{\partial \chi}\frac{\partial \chi}{\partial N} > 0, \frac{\partial L}{\partial \chi}\frac{\partial \chi}{\partial N} > 0, \frac{\partial W}{\partial \chi}\frac{\partial \chi}{\partial N} > 0. \tag{30}$$

Proof. See appendix C. ■

The effect of market structure on wage rate is what JOOM delivers beyond the Cournot model. In Cournot model, i.e., imposing $\epsilon_L = \infty$, market structure does not affect wage rate. JOOM with $\epsilon_L < \infty$ shows it can, through the wage markdown mechanism. Also, through the wage markdown channel, consolidation of market structure can reduce the scales of input and output further than the product price markup mechanism *per se.*

Proposition 5 *For the symmetric canonical model, there exists a unique symmetric equilibrium where industry average labour productivity PQ/L can be increased by real improvement of marginal product of labour, or consolidation of market structure,*

$$\frac{\partial (PQ/L)}{\partial a} < 0, \frac{\partial \left(\frac{PQ}{L}\right)}{\partial N} < 0. \tag{31}$$

Proposition 5 distinguishes two distinct possible causes of increase in labour (revenue) productivity $\frac{PQ}{L}$. The first is technological progress that increases efficiency in the form of $a \downarrow$ with market concentration (and market structure) unchanged. The second is consolidation of market structure (say, because of lax merger control) while keeping a constant. The effects of these causes on income distribution are distinctively different because

$$\frac{\partial \left(\frac{LW}{PQ}\right)}{\partial a} = 0; \frac{\partial \left(\frac{LW}{PQ}\right)}{\partial N} > 0, \tag{32}$$

i.e., (symmetrical) technological progress does not reduce labour share of revenue or cause wage stagnation, while consolidation of market structure does.

The canonical model has labour as the only variable input. The technological progress represented by $a \downarrow$, is labour saving. Intuitively, on the one hand, this fact tends to reduce employment, and wage rate because labour supply is imperfectly elastic. On the other hand, reduced labour cost tends to induce higher output, which then tends to increase demand for labour. The net effect on employment can, in general, be ambiguous. Here the assumption: $\epsilon > 1$ does determine the net effects, as is shown below:

Proposition 6 *For the symmetric canonical model, there exists a unique symmetric equilibrium where an improvement of marginal product of labour, and a consolidation of market structure have the following effects on market outcomes:*

$$\frac{\partial W}{\partial a} < 0, \frac{\partial L}{\partial a} < 0 \tag{33}$$

and

$$\frac{\partial \Theta}{\partial a} < 0, \frac{\partial \Theta}{\partial N} > 0. \tag{34}$$

These results show that, controlling market structure, technological progress has desirable effects on labour market outcomes and product market size. The key to these results is that technological progress causes product market expansion, which then causes increase in demand for labour. These features of the canonical model thus provide an ideal framework to analyse the effects of rise of superstar firms, as they foster technological progress (in the form of improvement of marginal product of labour) as well as increase of market concentration. This is the object of Section 4.

2.5 Welfare Implications of Joint Oligopoly-Oligopsony Power

In the conventional partial equilibrium welfare analysis of market power, the stake holders include only the consumers and producers, but not the workers because the labour market is assumed to be perfectly competitive and there is no (non-trivial) worker surplus. This is not the case when the inverse labour supply function is upward sloping.[19] The upward-sloping inverse labour supply function is a measure of the marginal willingness to accept (WTA), or reservation wage of workers – this is the counterpart of interpreting a downward-sloping inverse demand function as quantifying the willingness to pay (WTP) or reservation price of consumers. Thus the worker surplus, WS, as defined by

$$WS \equiv \int_0^L \left(W\left(L \right) - W\left(y \right) \right) dy, \tag{35}$$

[19] Writing for the Handbook of Labour Economics, Alan Manning (2010) states: “It is increasingly recognized that labor markets are pervasively imperfectly competitive, that there are rents to the employment relationship for both worker and employer”. This means, as the author explains, “the loss of the current job makes the worker worse off”.

measures the gain of workers from their current employment relationship. For the canonical model, with constant elasticity of labour supply: $W(L) = W_0 L^{\frac{1}{\epsilon_L}}$,

$$\begin{aligned} WS &= W_0 \int_0^L \left(L^{\frac{1}{\epsilon_L}} - y^{\frac{1}{\epsilon_L}} \right) dy \\ &= W_0 \left[L^{\frac{1}{\epsilon_L}} y - \frac{1}{\frac{1}{\epsilon_L} + 1} y^{\frac{1}{\epsilon_L}+1} \right]_0^L \\ &= \frac{\frac{1}{\epsilon_L} W_0 L^{\frac{1}{\epsilon_L}+1}}{\frac{1}{\epsilon_L} + 1}. \end{aligned}$$

If $\epsilon_L = \infty$ (i.e., labour supply is perfectly elastic), then $WS = 0$ and consequently $\triangle WS = 0$. For $\epsilon_L < \infty$, labour market does matter for partial equilibrium welfare analysis and competition policy.

With the introduction of the worker welfare standard, anticompetitive harm (to workers) in the labour market, e.g., caused by a horizontal merger, can be defined by the condition: $\triangle WS < 0$. If $\triangle WS > 0$ then the merger is actually procompetitive in the labour market.

Similarly the standard definition of consumer surplus, CS, is:

$$CS \equiv \int_0^Q \left(P(x) - P(Q) \right) dx. \tag{36}$$

For canonical model, with constant elasticity of demand: $P(Q) = P_0 Q^{-\frac{1}{\epsilon}}$, we have

$$\begin{aligned} CS &\equiv \int_0^Q \left(P_0 x^{-\frac{1}{\epsilon}} - P_0 Q^{-\frac{1}{\epsilon}} \right) dx \\ &= P_0 \left[\frac{1}{1 - \frac{1}{\epsilon}} x^{1-\frac{1}{\epsilon}} - Q^{-\frac{1}{\epsilon}} x \right]_0^Q \\ &= \frac{\frac{1}{\epsilon} P_0 Q^{1-\frac{1}{\epsilon}}}{1 - \frac{1}{\epsilon}}. \end{aligned}$$

It is clear that the product market price P and total output Q, and the labour market wage W and employment L are the most welfare-relevant variables for defining competitive harms. If an increased market concentration and market power reduces CS, then by definition (based on welfare test) it causes consumer harm ($\triangle CS < 0$) in the product market. Similarly, if it reduces WS then by definition it causes worker harm in the labour input market. The indications of these harms can boil down, respectively, to the price effect $P \uparrow$ and output reduction $Q \downarrow$, and wage suppression $W \downarrow$ and employment reduction $L \downarrow$.

Conventional welfare analysis and antitrust policy have paid a lot attention to the price effect of market concentration and market power, but little to wage or employment effect. Traditionally, worker welfare has been the concern of labour and employment laws, relating to unionisation, collective wage bargaining and minimum wage regulation, instead of antitrust law. However, the protection of competition on the buyer side of the labour market is, in principle, as important as protection of competition in the product market in antitrust law. Hereby, we argue for extending partial equilibrium welfare analysis to include both the product price markup power and the wage markdown power.

Considering the producer surplus

$$PS \equiv \sum_{i \in \mathcal{N}} \pi_i. \tag{37}$$

For the canonical model

$$PS = PQ - WL = \Theta (1 - \phi).$$

The extended notion of total surplus is defined as

$$TS \equiv CS + PS + WS. \tag{38}$$

Let TS_s denote the total surplus under perfect competition in all markets.[20,21] Then the deadweight loss is given by

$$DWL = TS_s - TS. \tag{39}$$

The deadweight loss is a measure of inefficiency of resource allocation caused by the firms' overall market power. Intuitively, the deadweight loss includes the welfare losses to the consumers as well as the workers, which are not compensated by welfare transfers to the producers. The inefficiencies of resource allocation include underproduction and underconsumption of the product, $Q < Q_s$, and the underemployment of labour in this industry, $L < L_s$. The labour misallocation implication is absent in the convention partial equilibrium welfare analysis.

We can derive solutions to welfare metrics for the symmetric canonical model as follow:

$$\begin{aligned}
WS &= WS_s \phi^{\frac{\frac{1}{\epsilon_L}+1}{\frac{1}{\epsilon_L}+\frac{1}{\epsilon}}}, \\
CS &= CS_s \phi^{\frac{1-\frac{1}{\epsilon}}{\frac{1}{\epsilon}+\frac{1}{\epsilon_L}}}, \\
PS &= \Theta_s \phi^{\frac{1-\frac{1}{\epsilon}}{\frac{1}{\epsilon}+\frac{1}{\epsilon_L}}} (1-\phi), \\
TS &= \int_0^Q \left(P_0 x^{-\frac{1}{\epsilon}} - a W_0 (ax)^{\frac{1}{\epsilon_L}} \right) dx, \\
DWL &= \int_Q^{Q_s} \left(P_0 x^{-\frac{1}{\epsilon}} - a W_0 (ax)^{\frac{1}{\epsilon_L}} \right) dx,
\end{aligned}$$

[20] The subscript $_s$ indicates the perfectly competitive values of the relevant variables.

[21] When firms are heterogeneous, the corresponding perfect competition equilibrium may not be well defined. This problem, however, does not affect our main results.

where $WS_s \equiv \frac{\frac{1}{\epsilon_L} W_0 L_s^{\frac{1}{\epsilon_L}+1}}{\frac{1}{\epsilon_L}+1}$, $CS_s \equiv \frac{\frac{1}{\epsilon} P_0 Q_s^{1-\frac{1}{\epsilon}}}{1-\frac{1}{\epsilon}}$ and $\Theta_s \equiv P_s Q_s$.

Proposition 7 *For the symmetric canonical model, there exists a unique symmetric equilibrium where the welfare effects of consolidation of market structure are captured by the following comparative statics:*

$$
\begin{aligned}
\frac{\partial WS}{\partial N} &= \frac{\frac{1}{\epsilon_L}+1}{\frac{1}{\epsilon_L}+\frac{1}{\epsilon}} WS_s \phi^{\frac{\frac{1}{\epsilon_L}+1}{\frac{1}{\epsilon_L}+\frac{1}{\epsilon}}-1} \frac{\partial \phi}{\partial N} > 0, \\
\frac{\partial CS}{\partial N} &= \frac{1-\frac{1}{\epsilon}}{\frac{1}{\epsilon}+\frac{1}{\epsilon_L}} CS_s \phi^{\frac{1-\frac{1}{\epsilon}}{\frac{1}{\epsilon}+\frac{1}{\epsilon_L}}-1} \frac{\partial \phi}{\partial N} > 0, \\
\frac{\partial PS}{\partial N} &= \Theta_s \phi^{\frac{1-\frac{1}{\epsilon}}{\frac{1}{\epsilon}+\frac{1}{\epsilon_L}}} \frac{1-N}{N-\frac{1}{\epsilon}} \frac{\partial \phi}{\partial N} < 0, \\
\frac{\partial TS}{\partial N} &= \left(P_0 Q^{-\frac{1}{\epsilon}} - a W_0 (aQ)^{\frac{1}{\epsilon_L}} \right) \frac{\partial Q}{\partial N} > 0, \\
\frac{\partial DWL}{\partial N} &= -\left(P_0 Q^{-\frac{1}{\epsilon}} - a W_0 (aQ)^{\frac{1}{\epsilon_L}} \right) \frac{\partial Q}{\partial N} < 0.
\end{aligned}
$$

The conventionally-defined total surplus, $CTS \equiv CS + PS$, ignores workers surplus WS.

$$
\begin{aligned}
\frac{\partial CTS}{\partial N} &= \left(\frac{1-\frac{1}{\epsilon}}{\frac{1}{\epsilon}+\frac{1}{\epsilon_L}} CS_s \phi^{\frac{1-\frac{1}{\epsilon}}{\frac{1}{\epsilon}+\frac{1}{\epsilon_L}}-1} + \Theta_s \phi^{\frac{1-\frac{1}{\epsilon}}{\frac{1}{\epsilon}+\frac{1}{\epsilon_L}}} \frac{1-N}{N-\frac{1}{\epsilon}} \right) \frac{\partial \phi}{\partial N} < 0 \\
\text{if } \frac{CS_s}{\Theta_s} &< \frac{1-\frac{1}{N}}{1+\frac{1}{\epsilon_L}\frac{1}{N}} \frac{\frac{1}{\epsilon}+\frac{1}{\epsilon_L}}{1-\frac{1}{\epsilon}} \text{ or, equivalently, } N > \left(1+\frac{1}{\epsilon}\right)(1+\epsilon_L).
\end{aligned} \tag{40}
$$

That is, conditional on $N > \left(1+\frac{1}{\epsilon}\right)(1+\epsilon_L)$, conventional partial equilibrium welfare analysis could incorrectly treat consolidation of market structure (without real improvement of productive efficiency in terms of reducing a) as welfare improving or efficiency enhancing. This shows that the conventional partial equilibrium welfare analysis of events associated with increasing market concentration could be deeply flawed because of failing to take into account the wage markdown power.

2.5.1 Joint Monopoly-Monopsony Power: A Simple Illustration

To further illustrate the importance of including the worker surplus as part of the efficiency measurement, let's consider the case of $N = 1$ (i.e., the joint monopoly-monopsony model). The price-taker marginal cost is

$$MC^{PT}(Q) = aW(aQ).$$

The extended partial equilibrium analysis is shown in Figure 2. The intersection of the inverse demand curve $P(Q)$ and the MC^{PT} curve defines the perfectly competitive equilibrium, S, which determines the output Q_s and price P_s. Point B is the intersection of MR and MC curves, which determines the joint monopoly-monopsony output Q_m and price P_m. It is evident that

$$Q_m < Q_s, P_m > P_s$$

and implicitly

$$L_m < L_s, W_m < W_s,$$

where L_m, L_s, W_m and W_s are, respectively, the employments and wage rates.

Under perfect competition, the total surplus is TS_s, represented by the area enclosed by the $P(Q)$ and $MC^{PT}(Q)$ curves and the vertical axis, which equals the sum of CS_s and WS_s (i.e., the area of the shape ESO). That is,

$$TS_s = CS_s + WS_s, \text{ as } PS_s = 0.$$

Under the joint monopoly-monopsony industry structure, the total surplus is TS_m, represented by the area enclosed by the $P(Q)$ and $MC^{PT}(Q)$ curves, the vertical axis and the vertical line MABC, which equals the sum of CS_m, PS_m (area of FMCD) and WS_m (area of DCO). Thus

$$TS_m = CS_m + PS_m + WS_m.$$

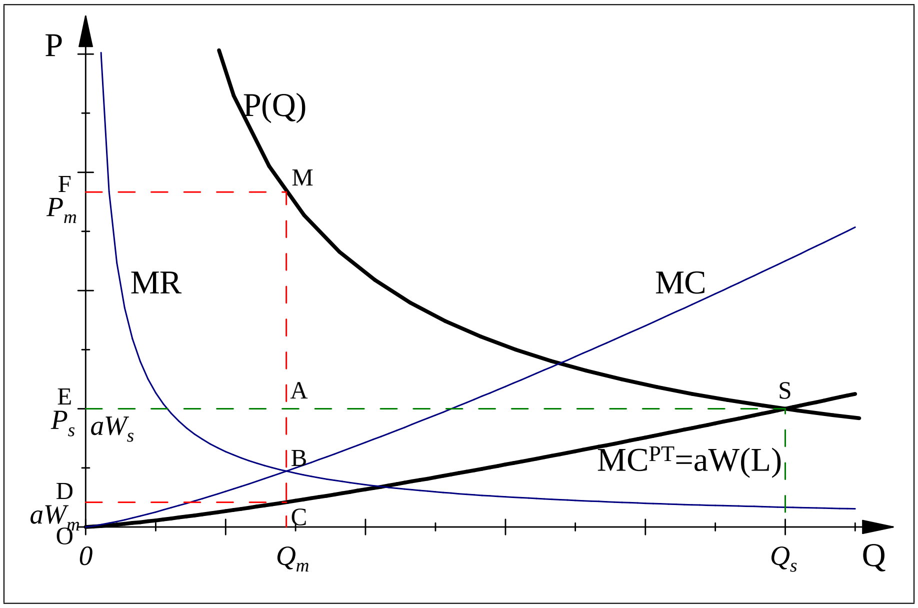

Figure 2: The joint monopoly-monopsony model

The welfare consequence of the monopolisation of the markets includes: (i) a reduction of consumer surplus, with the magnitude of $\triangle CS$ equals the area: FMSE, (ii) a reduction of worker surplus, with the magnitude of $\triangle WS$ equals the area: ESCD, and (iii) an increase of producer surplus, $\triangle PS$, equaling the area: FMCD.

In this illustrative example, there is no increase of productive efficiency. Thus, the gain in producer surplus is caused by pure transfer of surplus from the consumers and workers to the firm's owners. Meanwhile, there is a net welfare loss, i.e., the deadweight loss DWL_m, equaling the area of the shape: MSC, that is,

$$DWL_m = TS_s - TS_m > 0.$$

As a measure of inefficiencies in resource allocation, the deadweight loss DWL_m has two parts: the area of MSA, which is attributable to the product price markup power and the underproduction and underconsumption of the product, and the area of ASC, which is attributable to the wage markdown power, and measures the net efficiency loss caused by job destruction and underemployment of labour in this industry.

This insight can be generalised to JOOM with $N > 1$. To the extent that each firm is a monopolist-monopsonist facing the residual product demand and labour supply functions, these two components of inefficiencies attributable to product price markup and wage down powers are also present.

With the conventional partial equilibrium welfare analysis, worker surplus WS is ignored. Conventionally-defined total surplus under atomistical and monopoly-monopsony market structure are respectively

$$CTS_s = CS_s,$$

and

$$\begin{aligned} CTS_m &= CS_m + PS_m \\ &= CS_s \left(\frac{1-\frac{1}{\epsilon}}{1+\frac{1}{\epsilon_L}} \right)^{\frac{1-\frac{1}{\epsilon}}{\frac{1}{\epsilon}+\frac{1}{\epsilon_L}}} + \Theta_s \left(\frac{1-\frac{1}{\epsilon}}{1+\frac{1}{\epsilon_L}} \right)^{\frac{1-\frac{1}{\epsilon}}{\frac{1}{\epsilon}+\frac{1}{\epsilon_L}}} \left(1 - \frac{1-\frac{1}{\epsilon}}{1+\frac{1}{\epsilon_L}} \right). \end{aligned}$$

It can be shown that

$$CTS_m > CTS_s \text{ if } \frac{\left(\frac{1-\frac{1}{\epsilon}}{1+\frac{1}{\epsilon_L}} \right)^{\frac{1-\frac{1}{\epsilon}}{\frac{1}{\epsilon}+\frac{1}{\epsilon_L}}}}{1 - \left(\frac{1-\frac{1}{\epsilon}}{1+\frac{1}{\epsilon_L}} \right)^{\frac{1-\frac{1}{\epsilon}}{\frac{1}{\epsilon}+\frac{1}{\epsilon_L}}}} > \frac{\frac{1}{\epsilon}}{\frac{1}{\epsilon}+\frac{1}{\epsilon_L}} \frac{1+\frac{1}{\epsilon_L}}{1-\frac{1}{\epsilon}}. \tag{41}$$

The condition in (41) is satisfied for plausible values of the elasticities, for example $\epsilon = 1.2$ and $\epsilon_L = 0.5$.[22] In such

[22] For example, the estimates by Azar, Berry, and Marinescu (2019) are: market-level labour supply elasticity is about 0.6, while the firm-level labour supply elasticity is about 5.8. Using the Longitudinal Employer-Household Dynamics (LEHD) data from the United States Census Bureau, Webber (2015) estimate the average firm-specific elasticity of labour supply at 1.08, far from perfect competition.

cases, monopolisation of a previously atomistically competitive market is judged as efficiency enhancing because monopolisation reduces the average cost of labour and the cost saving dominates the consumer part of the dead weight loss. However, there is no real improvement of marginal product of labour $(1/a)$. The saving on labour cost is not a social welfare gain, but a transfer of surplus from workers to the producer. Here the conventional partial equilibrium welfare analysis is deeply flawed: it omits the loss of worker surplus, and misinterprets part of the loss as efficiency gain.

3 The Function of Trade Union and Minimum Wage Law

The classical monopsony model pioneered a surprising prediction (or explanation) of the effect of trade union or minimum wage law, which has been a sharp contrast with the neoclassical theory of wage. Given the fact that the classical monopsony model is a special case of the JOOM setting, it is natural for us to explore the predictions of JOOM on the effect of minimum wage regulation more generally.

For the canonical model with production function $F(\mathbf{x}_i, l_i; f_i) = \frac{l_i}{a_i}$, $MC_i^{PT} = Wa_i$ and $\zeta_{Li} = 1$, the fundamental equation of market power simplifies to:

$$\frac{P - W(L)\, a_i}{P} = \frac{s_i}{\epsilon} + \frac{s_{Li}}{\epsilon_L}\phi_{Li}, \tag{42}$$

which implies

$$W(L) = \frac{P\left(1 - \frac{s_i}{\epsilon}\right)}{a_i} - \frac{\frac{s_{Li}}{\epsilon_L}\phi_{Li}P}{a_i} < \frac{P\left(1 - \frac{s_i}{\epsilon}\right)}{a_i} = MRPL_i.$$

This extends the insight conveyed by the classical monopsony model that when the employer possesses wage markdown power, the wage rate is below the marginal revenue product of labour, that is, the worker is not paid for the full value the worker creates for the firm at the margin. The employer earns the markdown power rent from this employment relationship.

It has been well known from the classical monopsony model that the minimum wage regulation, targeted at appropriate level, can increase both wage rate and employment level. We now extend the analysis of the effect of minimum wage from the classical monopsony model to the JOOM setting. Let the minimum wage be set at the level of w. This alters the wage function from W to $\bar{W}$, satisfying:

$$\bar{W}(L) \equiv \begin{cases} W(L) & \text{for } L > \bar{L}_w \\ w & \text{for } L \leq \bar{L}_w \end{cases} \tag{43}$$

where $\bar{L}_w$ is a threshold of L satisfying $W\left(\bar{L}_w\right) = w$. Thus, the wage function $\bar{W}(L)$ has a kink at the point $\bar{L}_w$, where the marginal cost of labour faced by firm i: $\bar{W}(L) + \bar{W}'(L)\, l_i$ has a discontinuity at the point $\bar{L}_w$.

Figure 3 illustrates the intuition for the limiting case of a joint monopoly-monopsony market structure, i.e., $N = 1$, within the canonical model. To the extent that each firm in a joint oligopoly-oligopsony industry structure is like a joint monopolist-monopsonist facing its residual product demand and residual labour supply functions, the intuition carries over to the JOOM setting readily. The minimum wage regulation alters the price-taker marginal cost (MC^{PT}) curve: it replaces the OZ section with the horizontal GZ section. The point Z is a kink in the altered MC^{PT} curve, and it creates the discontinuity in the althered marginal cost (MC) curve, for which the OX section is replaced with the GZX section. Let the minimum wage be above the joint monopoly-monopsony wage, i.e., $w > W_m$. If the intersection of the MR curve and altered marginal cost curve is at a point like Y on the discontinuous (vertical) section of the altered MC curve, then it determines the output level Q_w and the product price P_w. Thus the minimum wage regulation leads to increased output $Q_w > Q_m$, lower product price, and increased employment $L_w = W^{-1}(w) > W^{-1}(W_m) = L_m$. As a result, it increases both consumer surplus and worker surplus, and also reduces the deadweight loss by the area of MWZC. Furthermore, if the minimum wage is slightly increased above w, the discontinuous (vertical) section of the altered MC curve will shift to the right, increasing the output and employment further as well as lowering the price. The minimum wage-employment relation, however, is not monotone: it has an inverted V shape, which has a peak of the employment linked to the point V, which is the intersection of the MR and MC^{PT} curves. If the minimum wage rises so that aw is above the point V, then the intersection of the new MC curve with the MR curve will be on the horizontal section of the new MC curve and to the left of the point of V, implying lower level of employment than point V.

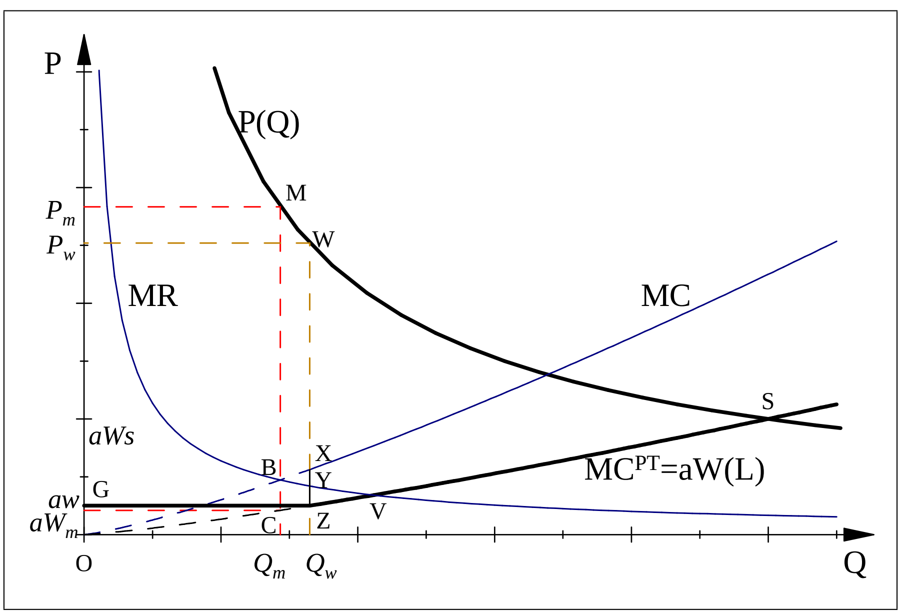

Figure 3: The joint monopoly-monopsony industry with minimum wage regulation

Now we treat the generaic market structure with $N \geq 1$ firms. Let $(q_i^{**}, l_i^{**})_{i\in\mathcal{N}}$ denote a Nash equilibrium of the canonical JOOM game without the minimum wage regulation. Let $Q^{**} \equiv \sum_{i\in\mathcal{N}} q_i^{**}$ and $L^{**} \equiv \sum_{i\in\mathcal{N}} l_i^{**}$. Let $(q_i^{***}, l_i^{***})_{i\in\mathcal{N}}$ denote a Nash equilibrium of the canonical JOOM game with the minimum wage regulation. Let $Q^{***} \equiv \sum_{i\in\mathcal{N}} q_i^{***}$ and $L^{***} \equiv \sum_{i\in\mathcal{N}} l_i^{***}$.

Proposition 8 *If there exist a Nash equilibrium of the canonical JOOM game without the minimum wage regulation, denoted by $(q_i^{**}, l_i^{**})_{i\in\mathcal{N}}$, and a Nash equilibrium of the canonical JOOM game with the minimum wage regulation, denoted by $(q_i^{***}, l_i^{***})_{i\in\mathcal{N}}$, with the binding minimum wage $w = W\left(\bar{L}_w\right)$ satisfying $w > W\left(L^{**}\right)$ and*

$$\frac{s_i^{**}}{\epsilon} < \frac{P^{**} - wa_i}{P^{**}} < \frac{s_i^{**}}{\epsilon} + \frac{s_{Li}^{**}}{\epsilon_L}\phi_{Li}^{**}, \tag{44}$$

for all $i \in \mathcal{N}$, where the superscript ** indicates the game without the minimum wage regulation, and

$$\frac{s_i^{***}}{\epsilon} < \frac{P^{***} - wa_i}{P^{***}}, \tag{45}$$

where the superscript *** indicates the game with the minimum wage regulation, then

$$L^{***} > L^{**}, Q^{***} > Q^{**}, W^{***} > W^{**}, P^{***} < P^{**}, \tag{46}$$

and

$$WS^{***} > WS^{**}, CS^{***} > CS^{**}. \tag{47}$$

Proof. The condition $w = W\left(\bar{L}_w\right) > W\left(L^{**}\right)$ implies $\bar{L}_w > L^{**}$. Condition (45) implies that in the Nash Equilibrium $(q_i^{***}, l_i^{***})_{i\in\mathcal{N}}$ each firm's best response is either a corner solution at $W^{***} = w$, as opposed to an interior solution at $W^{***} = w$, or $W^{***} > w$, and therefore $W^{***} \geq w > W\left(L^{**}\right) = W^{**}$, and $L^{***} \geq \bar{L}_w > L^{**}$. The rest of the proof is trivial. ■

The above proposition not only extends the aforementioned insight from the classical monopsony model to the canonical JOOM setting, but also shows that appropriately set minimum wage rate not only can raise the employment level, the wage rate and worker surplus, but also can increase the output level, lower the product price and increase consumer surplus. More results about minimum wages are presented in Sections 3.1 and 4.1.1.

3.1 Market Structure and Minimum Wage

For the canonical model, the minimum wage w alters the wage function (i.e., the inverse labour supply function) to:

$$\bar{W}\left(L\right) = \max\left(w, W_0 L^{\frac{1}{\epsilon_L}}\right). \tag{48}$$

This introduces a kink in the wage function at the point $\left(\left(\frac{w}{W_0}\right)^{\epsilon_L}, w\right)$. If the equilibrium employment is below or equal to $\left(\frac{w}{W_0}\right)^{\epsilon_L}$, the wage is equal to w; otherwise, the equilibrium wage is above w. By Proposition 8, it seems plausible that an appropriately chosen binding minimum wage can increase equilibrium employment. Denote by $\bar{w}$ the minimum wage that maximises the equilibrium employment, which we refer to as the efficient minimum wage. At $\bar{w}$, the labour supply is $L_{\bar{w}} = \left(\frac{\bar{w}}{W_0}\right)^{\epsilon_L}$. Firm i's (optimal) demand for labour l_i must equate its marginal revenue product of labour with the binding minimum wage $\bar{w}$, i.e.,

$$MRPL_i = \left(1 - \frac{s_i}{\epsilon}\right) P \frac{1}{a_i} = \bar{w}. \tag{49}$$

At $\bar{w}$ the labour market clears, i.e.,

$$\sum_{i \in \mathcal{N}} l_i = L_{\bar{w}} = \left(\frac{\bar{w}}{W_0}\right)^{\epsilon_L}.$$

Let w^{inf} and w^{sup} respectively denote the infimum and supremum of the sets of minimum wages that are employment-enhancing relative to the unregulated equilibrium. Obviously w^{inf} equals the unregulated equilibrium wage rate; the demands for labour at w^{inf} and w^{sup} should be equal; and

$$w^{\text{inf}} < \bar{w} < w^{\text{sup}}.$$

For $w \in \left(w^{\text{inf}}, \bar{w}\right)$, the demand for labour exceeds the supply, there is shortage of labour at the minimum wage w. For $w \in (\bar{w}, w^{\text{sup}})$, the supply of labour exceeds the demand, and there is involuntary unemployment. For $w = \bar{w}$ there is neither labour shortage nor involuntary unemployment. This is why we refer to $\bar{w}$ as the efficient minimum wage.

For the symmetric canonical model, we can derive the closed form solutions to w^{inf}, w^{sup} and $\bar{w}$ for the equilibrium. First,

$$w^{\text{inf}} \equiv W_s \left(\frac{1 - \frac{1}{\epsilon}\frac{1}{N}}{1 + \frac{1}{\epsilon_L}\frac{1}{N}}\right)^{\frac{\frac{1}{\epsilon_L}}{\frac{1}{\epsilon_L} + \frac{1}{\epsilon}}}, \tag{50}$$

where the right hand side of the above equation is the unregulated equilibrium wage rate, which gives an equivalent definition of w^{inf}.

Second, we solve for w^{sup}. To do this, we begin by characterising the regulated equilibrium with binding minimum wage $w \in (\bar{w}, w^{\text{sup}})$, then relate w^{sup} to its boundary case. Because the formulation of equilibrium outcomes provided by Lemma 3 is general, it can be applied to the regulated equilibrium with binding minimum wage $w \in (\bar{w}, w^{\text{sup}})$, if we substitute ϵ_L and W_0 with ∞ and w respectively. This case has a binding minimum wage constraint, and perfectly elastic labour supply. Lemma 3 then implies:

$$\begin{aligned} Q_w &= \left(\frac{P_0}{aw}\right)^{\epsilon}\left(1-\frac{1}{\epsilon}\frac{1}{N}\right)^{\epsilon}, \\ P_w &= aw\left(\frac{1}{1-\frac{1}{\epsilon}\frac{1}{N}}\right), \\ L_w &= \left(\frac{P_0}{a^{1-\frac{1}{\epsilon}}w}\right)^{\epsilon}\left(1-\frac{1}{\epsilon}\frac{1}{N}\right)^{\epsilon}, \\ W_w &= w, \end{aligned} \tag{51}$$

where the subscript $_w$ indicates binding minimum wage constraint.

For the minimum wage regulation to have an employment enhancing effect, it is necessary that

$$L_w = \left(\frac{P_0}{a^{1-\frac{1}{\epsilon}}w}\right)^{\epsilon}\left(1-\frac{1}{\epsilon}\frac{1}{N}\right)^{\epsilon} > L_s\left(\frac{1-\frac{1}{\epsilon}\frac{1}{N}}{1+\frac{1}{\epsilon_L}\frac{1}{N}}\right)^{\frac{1}{\frac{1}{\epsilon_L}+\frac{1}{\epsilon}}},$$

where the right hand side of the above inequality is the unregulated equilibrium employment level. This inequality is equivalent to:

$$w < w^{\text{sup}} \equiv W_s\left(1-\frac{1}{\epsilon}\frac{1}{N}\right)^{\frac{\frac{1}{\epsilon_L}}{\frac{1}{\epsilon}+\frac{1}{\epsilon_L}}}\left(1+\frac{1}{\epsilon_L}\frac{1}{N}\right)^{\frac{\frac{1}{\epsilon}}{\frac{1}{\epsilon}+\frac{1}{\epsilon_L}}}, \tag{52}$$

where the right hand side of "$\equiv$" gives an equivalent definition of w^{sup}. Overall, the minimum wage regulation has an employment enhancing effect relative to the unregulated equilibrium iff

$$w^{\text{inf}} < w < w^{\text{sup}}. \tag{53}$$

This condition is feasible iff

$$\frac{1}{\epsilon_L}\frac{1}{N} > 0. \tag{54}$$

For the symmetric canonical model with $N < \infty$ and $\epsilon_L < \infty$, the feasibility condition is satisfied.

Third, we solve for $\bar{w}$. Let $L_{\bar{w}}$ denote the employment level at $\bar{w}$. Because $\bar{w}$ is the boundary of interval $(\bar{w}, w^{\text{sup}})$, equation (51) must be satisfied, implying:

$$L_{\bar{w}} = \left(\frac{P_0}{a^{1-\frac{1}{\epsilon}}\bar{w}}\right)^{\epsilon}\left(1-\frac{1}{\epsilon}\frac{1}{N}\right)^{\epsilon}. \tag{55}$$

Furthermore, labour market clearing at the point $(L_{\bar{w}}, \bar{w})$ implies

$$\bar{w} = W_0\left(L_{\bar{w}}\right)^{\frac{1}{\epsilon_L}}. \tag{56}$$

Then equations (55) and (56) jointly determine:

$$\bar{w} \equiv W_s \left(1 - \frac{1}{\epsilon}\frac{1}{N}\right)^{\frac{\frac{1}{\epsilon_L}}{\frac{1}{\epsilon}+\frac{1}{\epsilon_L}}}, \tag{57}$$

which also gives an equivalent definition of $\bar{w}$. Because $\bar{w}$ not only maximises employment, but also maximises output and total surplus, it deserves to be called the "efficient minimum wage". The results above are summarised in Proposition 9.

Proposition 9 *For the symmetric canonical model with* $\frac{1}{\epsilon_L}\frac{1}{N} > 0$*, the minimum wage* w *has an employment enhancing effect iff*

$$W_s \left(\frac{1 - \frac{1}{\epsilon}\frac{1}{N}}{1 + \frac{1}{\epsilon_L}\frac{1}{N}}\right)^{\frac{\frac{1}{\epsilon_L}}{\frac{1}{\epsilon_L}+\frac{1}{\epsilon}}} \equiv w^{\inf} < w < w^{\sup} \equiv W_s \left(1 - \frac{1}{\epsilon}\frac{1}{N}\right)^{\frac{\frac{1}{\epsilon_L}}{\frac{1}{\epsilon}+\frac{1}{\epsilon_L}}} \left(1 + \frac{1}{\epsilon_L}\frac{1}{N}\right)^{\frac{\frac{1}{\epsilon}}{\frac{1}{\epsilon}+\frac{1}{\epsilon_L}}}. \tag{58}$$

(i) If $w < w^{\inf}$ *then the minimum wage constraint is not binding and therefore has no effect on the market outcome.*

(ii) If $w > w^{\sup}$ *then the minimum wage constraint is binding and reduces employment relative to the unregulated equilibrium outcomes.*

(iii) If $w^{\inf} < w < \bar{w} \equiv W_s \left(1 - \frac{1}{\epsilon}\frac{1}{N}\right)^{\frac{\frac{1}{\epsilon_L}}{\frac{1}{\epsilon}+\frac{1}{\epsilon_L}}}$ *then the minimum wage constraint is binding, and the regulated equilibrium outcomes feature labour shortage, satisfying:*

$$\begin{aligned}
L_w &= \left(\frac{w}{W_0}\right)^{\epsilon_L}, \\
Q_w &= \frac{1}{a}\left(\frac{w}{W_0}\right)^{\epsilon_L}, \\
P_w &= P_0 \left(\frac{1}{a}\left(\frac{w}{W_0}\right)^{\epsilon_L}\right)^{-\frac{1}{\epsilon}}, \\
\frac{\partial L_w}{\partial w} &> 0, \frac{\partial Q_w}{\partial w} > 0, \frac{\partial P_w}{\partial w} < 0.
\end{aligned}$$

(iv) If $w = \bar{w}$ *(the efficient minimum wage) the minimum wage constraint is binding, and the regulated equilibrium outcomes feature maximum employment and labour market clearing.*

(v) If $w > \bar{w}$ *then the minimum wage constraint is binding, and the regulated equilibrium outcomes feature*

involuntary unemployment, satisfying:

$$
\begin{aligned}
L_w &= \left(\frac{P_0}{a^{1-\frac{1}{\epsilon}}w}\right)^{\epsilon}\left(1-\frac{1}{\epsilon}\frac{1}{N}\right)^{\epsilon}, \\
Q_w &= \left(\frac{P_0}{aw}\right)^{\epsilon}\left(1-\frac{1}{\epsilon}\frac{1}{N}\right)^{\epsilon}, \\
P_w &= aw\left(\frac{1}{1-\frac{1}{\epsilon}\frac{1}{N}}\right), \\
\frac{\partial L_w}{\partial w} &< 0, \frac{\partial Q_w}{\partial w} < 0, \frac{\partial P_w}{\partial w} > 0.
\end{aligned}
$$

Corollary 10 $\frac{\partial \bar{w}}{\partial N} > 0$ *and* $\frac{\partial w^{\inf}}{\partial N} > 0$. $\lim_{N\to\infty} w^{\inf} = \lim_{N\to\infty} \bar{w} = \lim_{N\to\infty} w^{\sup} = W_s$.

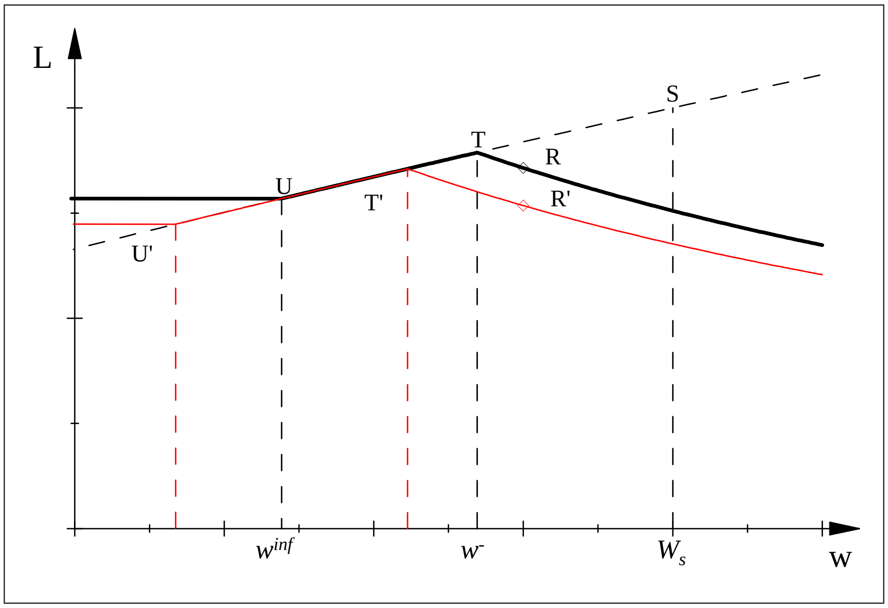

Figure 4: Market structure and mimimum wage-employment relation

The minimum wage-employment relationship is depicted by the solid non-monotonic black curve in Figure 4. When the minimum wage w is below $w^{\inf}$, it is not binding as the equilibrium wage rate stays at $w^{\inf}$, and w does not affect the employment level either (see the flat line ending with point **U**, with **U** depicting the unregulated equilibrium). When w is between $w^{\inf}$ and $\bar{w}$, the demand for labour exceeds the supply, and therefore an increase of minimum wage w raises the employment level (see upward-sloping line between points **U** and **T**). For $w > \bar{w}$, the supply for labour exceeds the demand and there is involuntary unempolyment, and therefore increasing the minimum wage further increases involuntary unemployment and reduces employment (see the downward-sloping line starting from point **T**). Interestingly, the effects of the minimum wage on employment depend on the market structure: a consolidation of market structure $N \downarrow \Rightarrow w^{\inf} \downarrow, \bar{w} \downarrow$, this is illustrated by the solid thin (red) curve, with the kink points **U** and **T** shifting to **U'** and **T'**. If w is initially slightly below $w^{\inf}$

(i.e., between **U'** and **U**) then as N decreases the new equilibrium wage rate and employment level both decline. If, in contrast, w is initially between $\bar{w}$ and W_s, as in point **R** (with **R** depicting the regulated equilibrium corresponding to w), then as N decreases, the equilibrium should shift from **R** to **R'**, reducing employment level. However, it is plausible that a weakening of trade union power or an erosion of real value of minimum wage rate occurs at the same time as N decreases. Then the new equilibrium can be at point **T'** instead of **R'**, resulting in simultaneous rise of market concentration, fall of wage rate and increase of employment. This last scenario is not only plausible but also relevant to the current debate about the cause of the secular trends of rising profitability, falling labour income share, and wage stagnation. Stansbury and Summers (2020) criticise the hypothesis of rising market concentration for not being able to explain the falling unemployment in the USA. Their alternative explanation is based on the hypothesis of declining worker power – a result of falling unionisation, that redistributes rents from labour to capital in a rent-sharing collective bargaining framework. They argue that the evidence of falling unemployment is not consistent with a rise of monopoly or monopsony power, which is inclined to decrease employment. Our JOOM is immune to the criticism of Stansbury and Summers (2020): It allows market concentration to rise and union power to fall simultaneously, and conditional on $\bar{w} < w < W_s$, it is plausible that $(N \downarrow, w \downarrow) \Rightarrow \left(\rho \uparrow, \chi \uparrow, \tau \uparrow, \frac{wL}{PQ} \downarrow, L \uparrow\right)$. Our JOOM theoretic explanation also allows us to see the function of trade union (and similarly for minimum wage) as protecting the workers from powerful firms' rent extraction – those rents belong to worker surplus in perfectly competitive markets. Trade union and minimum wage law can be employment enhancing relative to the unregulated equilibrium **U**, even though their effects are not necessarily employment maximising (i.e., point **R** is below **T**).

For $\epsilon_L < \infty$, employers have wage markdown power. The consolidation of market structure results in an increase of employers' wage markdown power. When the employers' wage markdown power is restricted by worker power in the form of the minimum wage constraint, a consolidation of market structure can reduce the demand for labour and cause involuntary unemployment if the minimum wage is not reduced. It can be argued that this puts pressure on the collective wage bargaining and can cause a weakening of worker power and result in lower minimum wage. In contrast, if $\epsilon_L = \infty$ and $W_0 = W_s$ then there cannot be wage markdown. In this case, the consolidation of market structure results in a fall of demand for labour, but not a change in wage rate (see W_s in Figure 5). This analysis implies that our call for the inclusion of worker welfare into the welfare standard of antitrust enforcement remains valid even when there exist trade union, collective wage bargaining or minimum wage regulation in the labour markets, aimed at curbing oligopsony wage markdown power.

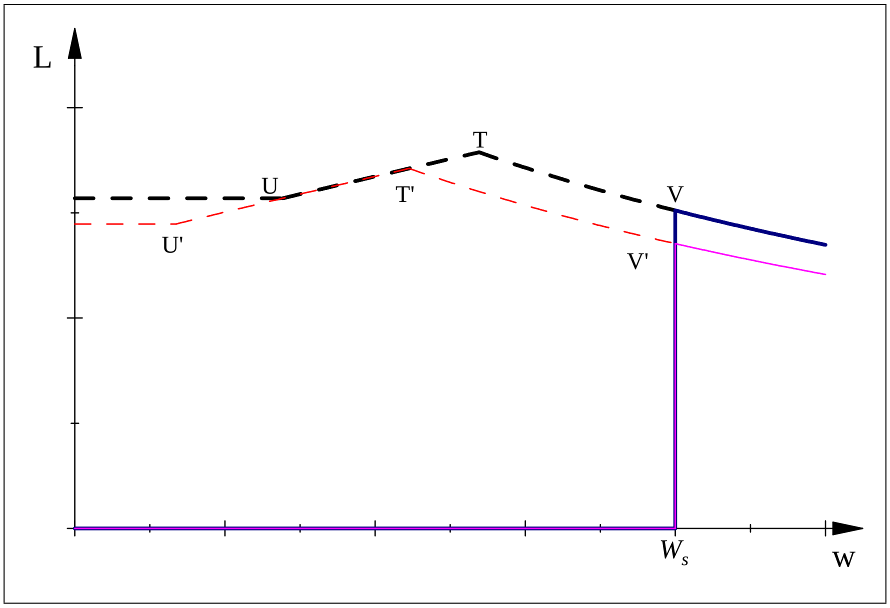

Figure 5: Market structure and mimimum wage-employment relation with perfectly competitive labour market (solid curves for $\tilde{\epsilon} = \infty$; dashed curves for $\tilde{\epsilon} < \infty$)

4 Superstar Firms

So far we have explored the symmetric canonical model within JOOM to study the effects of market concentration. The simplicity of the representative firm analysis enables us to derive many closed form results. This type of analysis is more useful for the case where exogenous increase of market concentration is the cause of rising pricing power. There is, however, a drawback: the representative firm approach does not fit the ubiquitous dispersion of firms' productivity within industries, and the fact that numerous industries are dominated by highly productive "superstar" firms.

To study the superstar firms phenomenon, we modify the canonical model with N symmetric firms by assuming that one of them, firm N, through some successful innovation achieves an improvement of marginal product of labour[23]. As a result, the $N-1$ rival firms remain symmetric, but firm N has superior marginal product of labour[24]. We can conveniently formulate the improvement in terms of labour input reduction rate $\gamma \in [0,1]$, defined as follows:

$$\gamma \equiv 1 - \frac{\hat{a}}{a}, \tag{59}$$

[23] For the canonical model with a single variable input, this is the same as an improvement of total factor productivity of equal magnitude.

[24] The simple canonical model we work with does not allow an exploration of changes in marginal products of other variable inputs. In a companion paper, Tong and Ornaghi (2020) use a tractable setting with linear demand and supply functions and Leontief production function to overcome this limitation.

where $\hat{a}$ is the reduced labour input per unit of output of firm N. As an implication

$$\hat{a} = a\left(1-\gamma\right).$$

A main cause of productivity improvement is division of tasks and the automation of some tasks previously undertaken by physical and mental labour, taken over by powered machineries, robotics and computer systems. All these cause labour saving per unit of output.[25] Superstar firms are typically the leaders in adopting these technological changes, in which they invest an endogenous large amount of their revenues, as measured by sunk fixed cost to sales ratios. In our static analysis[26], we take γ as predetermined, therefore treat it as common cause for changes in market concentration and pricing power in the joint oligopoly-oligopsony industry.

4.1 JOOM with a Superstar Firm

Let y stand respectively for a_i, q_i, l_i, s_i, s_{Li}, ϕ_i, τ_i, ρ_i of a representative non-superstar firm, and $\hat{y}$ denotes the superstar firm's counterpart of variable y. Note that

$$\begin{aligned} \hat{q} &= \hat{s}Q, \\ \hat{s}_L &= \hat{s}\frac{\left(1-\gamma\right)aQ}{L}. \end{aligned}$$

The fundamental equation of market power for "non-superstar" firms is given by

$$\tau \equiv \frac{P-aW}{P} = \frac{s}{\epsilon} + \frac{s}{\epsilon_L}\frac{aQ}{L}\frac{aW}{P}. \tag{60}$$

For the superstar firm, it is

$$\hat{\tau} \equiv \frac{P-\left(1-\gamma\right)aW}{P} = \frac{\hat{s}}{\epsilon} + \frac{\hat{s}}{\epsilon_L}\frac{\left(1-\gamma\right)aQ}{L}\frac{\left(1-\gamma\right)aW}{P}. \tag{61}$$

By definition, the following identity of market shares holds:

$$\hat{s} + \left(N-1\right)s = 1.$$

[25] This line of argument is also consistent with the case where there is substitution of labour with non-labour variable input in general, and outsourcing in particular. Automation and computerisation can play an important role in better managing and improving such substitution, and outsourcing.

[26] The foundation of the analysis lies in the endogenous sunk cost models *a la* Sutton (1991, 1998). Examples of endogenous sunk costs include expenses on activities of advertising and R&D, which shift the demand curve upward (in the context of quality competition) or the marginal cost curve downward. These factors make the total cost function to feature economies of scale, incompatible with perfectly competitive equilibrium.

Given the total output Q the demand for labour is:

$$L = (1-\gamma)\, a\hat{s}Q + (N-1)\, asQ.$$

The above six equations have the following implications:

$$\begin{aligned}
\frac{aQ}{L} &= \frac{1}{1-\gamma\hat{s}}, \\
\hat{s}_L &= \hat{s}\frac{1-\gamma}{1-\gamma\hat{s}}, \\
\hat{\tau} &\equiv 1-(1-\gamma)\,\phi = \frac{\hat{s}}{\epsilon} + \frac{\hat{s}}{\epsilon_L}\frac{1-\gamma}{1-\gamma\hat{s}}(1-\gamma)\,\phi, && (62) \\
\tau &\equiv 1-\phi = \frac{1-\hat{s}}{(N-1)\,\epsilon} + \frac{1-\hat{s}}{(N-1)\,\epsilon_L}\frac{1}{1-\gamma\hat{s}}\phi, && (63)
\end{aligned}$$

where $\phi \equiv \frac{aW}{P}$ is the labour share of revenue of a rival firm.

Equations (62) and (63) respectively imply

$$\begin{aligned}
\phi &= \frac{1-\frac{\hat{s}}{\epsilon}}{\left(1+\frac{\hat{s}}{\epsilon_L}\frac{1-\gamma}{1-\gamma\hat{s}}\right)(1-\gamma)}, && (64) \\
\phi &= \frac{1-\frac{1-\hat{s}}{(N-1)\epsilon}}{1+\frac{1-\hat{s}}{(N-1)\epsilon_L}\frac{1}{1-\gamma\hat{s}}}. && (65)
\end{aligned}$$

Proposition 11 *Consider the canonical model with a superstar firm and $N-1$ symmetric rivals. For $\gamma \in \left(0, \frac{1+\frac{\epsilon_L}{\epsilon}}{1+\epsilon_L}\right)$, there exists a Nash equilibrium such that $(\hat{s}, \phi)$ is uniquely determined by equations (64) and (65), and the aggregate market outcomes are given by*

$$\begin{aligned}
L &= \left(\frac{P_0 \frac{1-\frac{1-\hat{s}}{(N-1)\epsilon}}{1+\frac{1-\hat{s}}{(N-1)\epsilon_L}\frac{1}{1-\gamma\hat{s}}}(1-\gamma\hat{s})^{\frac{1}{\epsilon}}}{a^{1-\frac{1}{\epsilon}}W_0}\right)^{\frac{1}{\frac{1}{\epsilon}+\frac{1}{\epsilon_L}}}, W = \left(\frac{P_0\,(W_0)^{\frac{\epsilon_L}{\epsilon}} \frac{1-\frac{1-\hat{s}}{(N-1)\epsilon}}{1+\frac{1-\hat{s}}{(N-1)\epsilon_L}\frac{1}{1-\gamma\hat{s}}}(1-\gamma\hat{s})^{\frac{1}{\epsilon}}}{a^{1-\frac{1}{\epsilon}}}\right)^{\frac{\frac{1}{\epsilon_L}}{\frac{1}{\epsilon}+\frac{1}{\epsilon_L}}} \\
Q &= \left(\frac{P_0 \frac{1-\frac{1-\hat{s}}{(N-1)\epsilon}}{1+\frac{1-\hat{s}}{(N-1)\epsilon_L}\frac{1}{1-\gamma\hat{s}}}}{a^{1+\frac{1}{\epsilon_L}}W_0\,(1-\gamma\hat{s})^{\frac{1}{\epsilon_L}}}\right)^{\frac{1}{\frac{1}{\epsilon}+\frac{1}{\epsilon_L}}}, P = \left(\frac{a^{1+\frac{1}{\epsilon_L}}\,(P_0)^{\frac{\epsilon}{\epsilon_L}}\,W_0\,(1-\gamma\hat{s})^{\frac{1}{\epsilon_L}}}{\frac{1-\frac{1-\hat{s}}{(N-1)\epsilon}}{1+\frac{1-\hat{s}}{(N-1)\epsilon_L}\frac{1}{1-\gamma\hat{s}}}}\right)^{\frac{\frac{1}{\epsilon}}{\frac{1}{\epsilon}+\frac{1}{\epsilon_L}}}.
\end{aligned}$$

Proof. See Appendix C. ■

To measure the dispersion of firm level equilibrium outcomes and product market concentration, note the following relations:

$$\begin{aligned} s &= \frac{1-\hat{s}}{N-1}, HHI \equiv \hat{s}^2 + (N-1)\, s^2 = \hat{s}^2 + \frac{(1-\hat{s})^2}{N-1}, \\ \hat{\phi} &\equiv (1-\gamma)\,\phi, \bar{\phi} \equiv \hat{s}\hat{\phi} + (N-1)\, s\phi, \\ \hat{\tau} &= 1-\hat{\phi}, \tau = 1-\phi, \bar{\tau} \equiv \hat{s}\hat{\tau} + (N-1)\, s\tau = 1-\bar{\phi}, \end{aligned}$$

where HHI is the Herfindahl-Hirchman index of the product market; $\bar{\phi}$ is the product market share-weighted average labour revenue share; $\bar{\tau}$ is the product market share-weighted average of overall market power.

The intuition of the superstar firms theory can be sharpened by the degenerate case of the Cournot model, with $\epsilon_L = \infty$, for which analytical results for comparative statics can be derived.

Proposition 12 *For the limiting case: $\epsilon_L = \infty$, the equilibrium outcomes satisfy*

$$\begin{aligned} \hat{s} &= \frac{1+\gamma\,((N-1)\,\epsilon - 1)}{N-\gamma}, HHI = \hat{s}^2 + \frac{(1-\hat{s})^2}{N-1}, \\ \phi &= 1 - \frac{1-\hat{s}}{(N-1)\,\epsilon}, \hat{\phi} = \frac{(\epsilon N - 1)\,(1-\gamma)}{\epsilon\,(N-\gamma)}, \bar{\phi} = \frac{\epsilon N - 1}{\epsilon}\frac{1-\gamma\hat{s}}{N-\gamma}, \\ \hat{\tau} &= 1-\hat{\phi}, \tau = 1-\phi, \bar{\tau} = 1-\bar{\phi}. \end{aligned}$$

The following comparative statics hold for $\gamma \in \left(0, \frac{1}{\epsilon}\right)$:

$$\frac{\partial \hat{s}}{\partial \gamma} > 0, \frac{\partial s}{\partial \gamma} < 0, \frac{\partial HHI}{\partial \gamma} > 0, \frac{\partial \hat{\phi}}{\partial \gamma} < 0, \frac{\partial \phi}{\partial \gamma} > 0, \frac{\partial \bar{\phi}}{\partial \gamma} < 0, \frac{\partial \hat{\tau}}{\partial \gamma} > 0, \frac{\partial \tau}{\partial \gamma} < 0, \frac{\partial \bar{\tau}}{\partial \gamma} > 0. \tag{66}$$

Proof. See Appendix C. ■

Intuitively, the superior efficiency of the superstar firm, measured by γ, enables it to gain product market share, at the expense of its laggard rivals, thus resulting in a net increase in the product market HHI. Since the superstar firm has lower marginal cost than rivals, for the same market price, it has higher markup than rivals; in contrast, the heightened competitive pressure from the superior superstar firm reduces the rivals' markups. Thus, the superior efficiency of the superstar firm, measured by γ, is a common cause of rising product market concentration, average pricing power, and falling average labour revenue share.

For $\epsilon_L < \infty$, we are unable to analytically prove the comparative statics results given in (66) for $\gamma \in \left(0, \frac{1+\frac{\epsilon_L}{\epsilon}}{1+\epsilon_L}\right)$. Numerical simulations, however, can confirm they hold with plausible values of ϵ and ϵ_L.[27] Thus the same intuition goes through even after we add imperfect competition in the labour market. Crucially, the effects of the technological change, measured by γ, are so diverse on the superstar firm and its rivals, making the representative

[27] For illustration, we calibrate the canonical model with the following plausible baseline numerical values: $\epsilon = 1.2, P_0 = 1, \tilde{\epsilon} = 0.8, W_0 = 1, N = 4$, which are used for Figures 6-8.

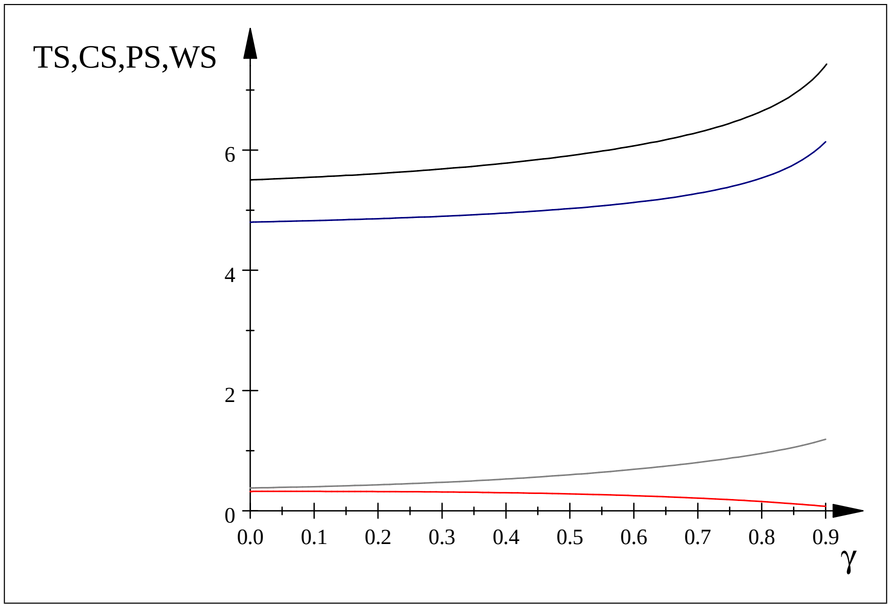

Figure 6: Total surplus TS (black), consumer surplus CS (blue), producer surplus PS (grey) and worker surplus (red) as functions of γ.

firm approach less meaningful for studying such industries.

It is important to note that the rise of superstar firms is not always "pro-competitive". Figure 6 shows the welfare measures TS, CS, PS and WS as functions of parameter γ under the baseline numerical values. The rise of the superstar firm does improve CS, PS and TS, but not WS. The benefit of increasing marginal product of labour is captured by producers as a group, and passed on to consumers, but not to workers.[28] Figures 7 and 8 can help explain why: while output goes up and price comes down, both employment and wages decrease. The rise of a superstar firm reduces demand for labour and hence worker welfare. In contrast, if the channel of markdown power is shut down, i.e., setting $\epsilon_L = \infty$, the rise of the superstar firm reduces employment, but wages are not affected, and worker surplus does not decrease as it remains zero.[29]

4.1.1 Superstar Firms and Effects of Trade Union or Minimum Wage

The alignment of the superstar firm theory with the stylised fact of firm productivity dispersion within industries makes it relevant to place the study of the function of trade union and minimum wage in the context of the

[28] According to NYT article: "Inside Amazon's Worst Human Resources Problem" (October 24, 2021) https://nytimes.blog/inside-amazons-worst-human-resources-problem/, "Amazon's workers routinely took a back seat to customers during the company's meteoric rise to retail dominance. Amazon built cutting-edge package processing facilities to cater to shoppers' appetite for fast delivery, far outpacing competitors. But the business did not devote enough resources and attention to how it served employees."

[29] The effect of the superstar firm on worker welfare is actually more subtle as it also depends on the product demand elasticity: for sufficiently large demand elasticity, the superstar may find it profitable to expand output to the extent that demand for labour increases, so that the benefit of high productivity are passed to workers as well. This possibility can be illustrated by the extreme case of shutting down the markup power channel, i.e., setting $\epsilon = \infty$. Thus, from a theoretical point of view, the effect of superstar firms on employment, wages and worker welfare is ambiguous: it can be a function of the relative strength of markup and markdown powers. Consequently, for practical welfare analysis and policy study it is important for empirical research to identify and quantify the markup and markdown components in estimation of market power in order to provide relevant evidence.

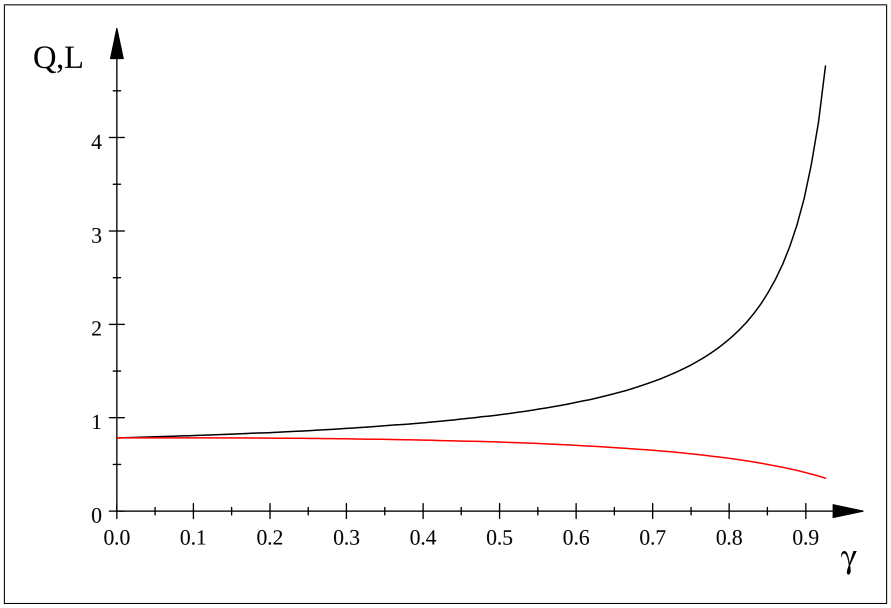

Figure 7: Output Q and employment L as functions of γ. Q (black) and L (red).

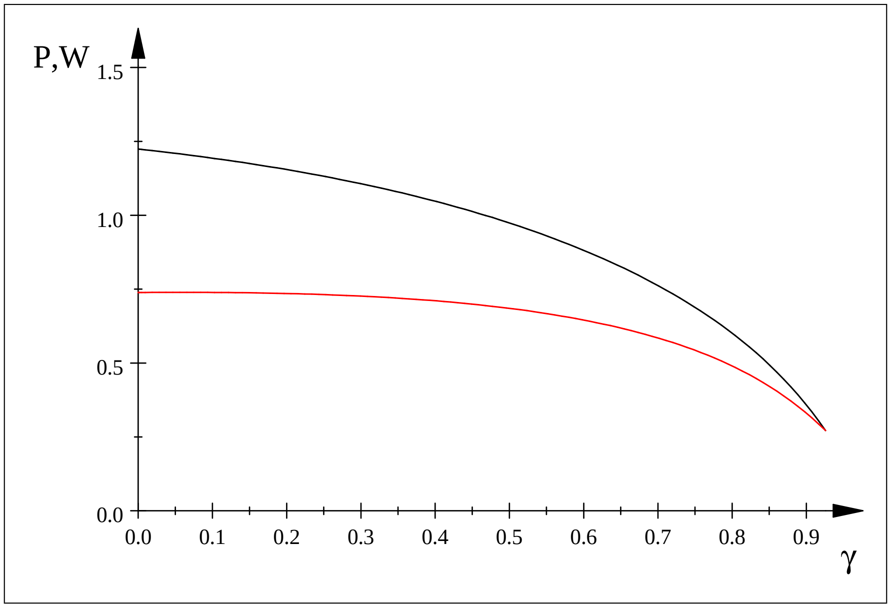

Figure 8: Price P and wage rate W as functions of γ. P (black) and W (red).

superstar firm phenomenon.

Recall the wage function altered by regulatory minimum wage w, given by eq. (48):

$$\bar{W}(L) = \max\left(w, W_0 L^{\frac{1}{\epsilon_L}}\right),$$

which has a kink at the point $\left(\left(\frac{w}{W_0}\right)^{\epsilon_L}, w\right)$.

Let the subscript $_w$ indicate the Nash equilibrium outcome under binding minimum wage constraint. In such context, the product and labour market outcomes are characterised by

$$\frac{aQ_w}{L_w} = \frac{1}{1-\gamma \hat{s}_w}, \tag{67}$$

where $\hat{s}_w$ is the product market share of the superstar firm under minimum wage regulation.

Consistent with Section 3.1, let $\bar{w}$ denote the the minimum wage that maximises the equilibrium employment, which we refer to as the efficient minimum wage. Then $L_{\bar{w}} = \left(\frac{\bar{w}}{W_0}\right)^{\epsilon_L}$ is the maximised employment.

If the minimum wage constraint is binding with $w < \bar{w}$ and the equilibrium features the kink point on the regulated labour supply function, then the employment satisfies $L_w = \left(\frac{w}{W_0}\right)^{\epsilon_L} < L_{\bar{w}}$. In this case, the demand for labour exceeds the supply, causing labour shortage. There can exist multiple Nash equilibrium points in this case. Without imposing more structure on how equilibrium is selected, one cannot determine the market share distributions among the firms, or the total output. The only endogenous variable that is determinable given the wage rate w is the employment L_w.

Proposition 13 *Consider the canonical model with a superstar firm and $N-1$ symmetric rivals, and binding minimum wage $w < \bar{w}$ such that $L_w = \left(\frac{w}{W_0}\right)^{\epsilon_L} < L_{\bar{w}}$ and there is labour shortage. Then*

$$\frac{\partial L_w}{\partial w} > 0, \frac{\partial L_w}{\partial \gamma} = 0. \tag{68}$$

If the minimum wage constraint is binding with $w > \bar{w}$ and the employment L_w satisfies $L_w < L_{\bar{w}} < \left(\frac{w}{W_0}\right)^{\epsilon_L}$, then there is insufficient demand for labour relative to the supply, resulting in involuntary unemployment. In this case the markdown powers of all firms are removed. The non-superstar firms' fundamental equation becomes

$$1 - \phi_w = \frac{1-\hat{s}_w}{(N-1)\,\epsilon}, \tag{69}$$

where ϕ_w denotes the labour income share of a non-superstar firm. The superstar firm's fundamental equation is

$$1 - (1-\gamma)\,\phi_w = \frac{\hat{s}_w}{\epsilon}. \tag{70}$$

In this case, the closed form solution of the equilibrium is available, including:

$$\hat{s}_w = \frac{1 + ((N-1)\epsilon - 1)\gamma}{N-\gamma}, \tag{71}$$

$$\phi_w = \frac{N - \frac{1}{\epsilon}}{N-\gamma}. \tag{72}$$

Proposition 14 *Consider the canonical model with a superstar firm and $N-1$ symmetric rivals, and binding minimum wage $w > \bar{w}$ such that $L_w < \left(\frac{w}{W_0}\right)^{\epsilon_L}$, i.e., there is involuntary unemployment. For $N \geq 2$, $\gamma \geq 0$, $\epsilon \to 1_+$:*

$$\begin{aligned}
\frac{\partial \hat{s}_w}{\partial w} &= 0, \frac{\partial \hat{s}_w}{\partial \gamma} > 0, \\
\frac{\partial \phi_w}{\partial w} &= 0, \frac{\partial \phi_w}{\partial \gamma} > 0, \\
\frac{\partial P_w}{\partial w} &> 0, \frac{\partial P_w}{\partial \gamma} < 0, \\
\frac{\partial Q_w}{\partial w} &< 0, \frac{\partial Q_w}{\partial \gamma} > 0, \\
\frac{\partial L_w}{\partial w} &< 0, \frac{\partial L_w}{\partial \gamma} < 0.
\end{aligned} \tag{73}$$

Proof. See Appendix C. ■

For the comparative statics, we use the condition $\epsilon \to 1_+$ to capture the idea that ϵ is not sufficiently large to allow L_w to increase in γ. That is, when γ increases, the output expansion effect which tends to increase demand for labour cannot offset the reallocation effect which tends to reduce demand for labour because the superstar firm hires less labour per unit of output.

The next proposition establishes a sufficient condition for the existence and uniqueness of the efficient minimum wage $\bar{w}$, and its comparative statics w.r.t. γ.

Proposition 15 *Consider the canonical model with a superstar firm and $N-1$ symmetric rivals. For $N \geq 2$, $\gamma \geq 0$, $\epsilon \to 1_+$: There exists a unique efficient minimum wage $\bar{w}$ such that for $w = \bar{w}$ the minimum wage constraint is binding, and the regulated equilibrium outcomes feature maximum employment and labour market clearing, and*

$$\frac{\partial \bar{w}}{\partial \gamma} < 0. \tag{74}$$

Proof. See Appendix C. ■

The point $\left(\bar{w}, \left(\frac{\bar{w}}{W_0}\right)^{\epsilon_L}\right)$ is the single peak in the minimum wage-employment relation, as is shown in Figure 9. The rise of a superstar firm shifts the demand curve for labour as well as the minimum wage-employment relation curve downward. The peak point $\left(\bar{w}, \left(\frac{\bar{w}}{W_0}\right)^{\epsilon_L}\right)$ therefore shifts downward and to the left from **T** to **T'**, that is, both efficient minimum wage and employment are decreased. This shows that the rise of the superstar firm weakens the worker power in terms of efficient minimum wage, and consequently reduces workers' welfare in terms of worker surplus.[30]

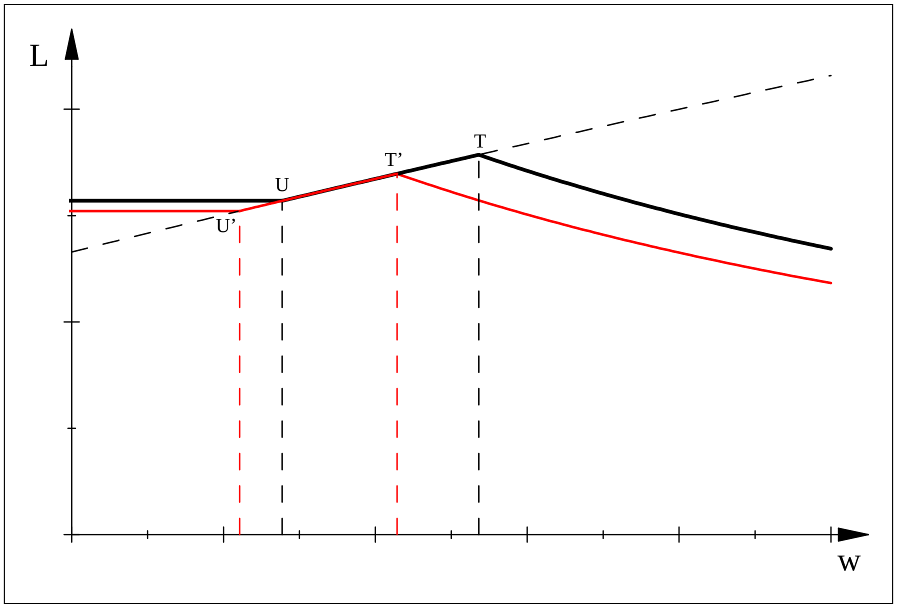

Figure 9: Effect of γ on the minimum wage-employment relation: an increase of γ shifts the relation from the black to the red curve

4.1.2 Economic Dynamism and Shared Prosperity

The roles of superstar firms in market competition are both of great importance and complicated. First, the rise of superstar firms is a result of competition. In their infancy, the "would be" superstar firms are innovators who strive for superior total factor productivity or marginal product of labour. Successful superstar firms gain in sales and product market shares, and they exert more competitive pressure on their rivals. As argued by Demsetz (1973), firms that have large product market shares which cause high market concentration are superior competitors, and market concentration can be an outcome of competition. The anticipated weaker competition from rivals and rising market power of their own are the incentive for their innovation in the first place. This has been recognised as an important driver of innovation and economic development by economists since Schumpeter (1942).

If the dominance of a superstar firm is temporary, followed by catching up or even leapfrogging of rival firms, the rise of superstar firms has the potential to benefit workers as well as consumers. Figure 10 shows that in

[30] Notice that this result does depend on the assumption that the demand elasticity ϵ cannot exceed 1 too much. Whether this assumption is reasonable is for empirical research to assess.

a market structure with $n \leq N$ equally efficient firms, the benefit of labour productivity improvement can be passed on to workers in the form of higher wages (with employment following a similar pattern). Everything else being equal, a symmetric improvement of marginal product of labour among all firms may benefit workers by raising employment and wage rate if the elasticity of demand for the product is lower-bounded away from one.

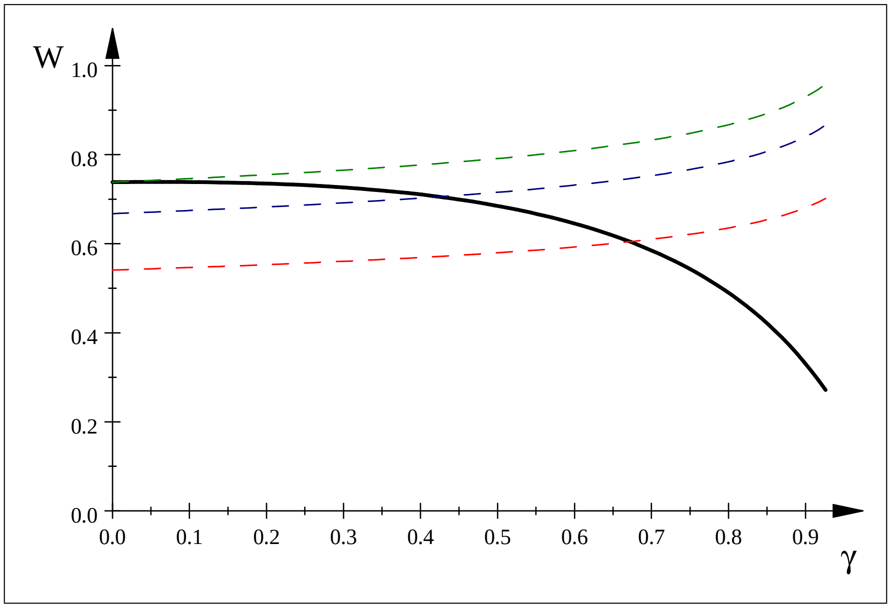

Figure 10: Wage rate W as function of γ, and market structure: (1) One superstar firm with 3 inferior rivals (black, solid); (2) Two superstar firms with 0 inferior rivals (red, dash); (3) Three superstar firms with 0 inferior rivals (blue, dash); (4) Four superstar firms with 0 inferior rivals (green, dash)

However, to sustain their leadership, superstar firms have strong incentives to prevent their superior productivity from being eroded by technological diffusion. In this respect, the fact that supertar firms have perpetuated their dominance too easily for too long[31] may be an indication that antitrust enforcement has failed to prevent the superstar firms from consolidating the control of intangible assets, particularly, IP rights, with the effect of raising and sustaining barriers to entry.

We conclude this section by noting that the superstar firm phenomenon has an interesting implication for the growth of labour productivity at the industry level. The (labour market share-weighted) average marginal product of labour is given by

$$\begin{aligned} & \frac{1}{a}\left(1-\hat{s}_L\right)+\frac{1}{a\left(1-\gamma\right)}\hat{s}_L \\ = \; & \frac{1}{a}+\frac{1}{a}\frac{\gamma}{1-\gamma}\hat{s}_L \\ = \; & \frac{1}{a}\frac{1}{1-\gamma\hat{s}}<\frac{1}{a}\frac{1}{1-\gamma} \text{ for } \gamma\in\left(0,\frac{1+\frac{\epsilon_L}{\epsilon}}{1+\epsilon_L}\right). \end{aligned} \tag{75}$$

[31] For evidence of persistent super-normal profits, see Furman and Orszag (2018), Barkai (2020) and Gutiérrez and Philippon (2017).

The inequality above means that the average marginal product of labour trails behind that of the superstar firm. This point is also illustrated by Figure 11. The reason for this lies in the fact that the superstar firm has higher than average marginal product of labour, and thus employs less labour than proportionate to output. As a result, its labour market share $\hat{s}_L$ is smaller than its product market share $\hat{s}$:

$$\hat{s}_L = \hat{s}\frac{1-\gamma}{1-\gamma\hat{s}} < \hat{s} \text{ for } \gamma \in \left(0, \frac{1+\frac{\epsilon_L}{\epsilon}}{1+\epsilon_L}\right). \tag{76}$$

Consequently, the rise of superstar firms cannot raise average labour productivity to the extent it raises average market power at the industry level. The slowing down of labour productivity growth rate in recent decades is a well documented stylised fact. Even when the superstar firms pursue improvement of their own labour productivity, they contribute less to the growth of average labour productivity. When the superstar firms stall innovation because of complacency protected by barriers to entry, labour productivity growth at the industry level only gets slower. This suggests that relying solely on superstar firms to foster productivity growth may not be a winning strategy.

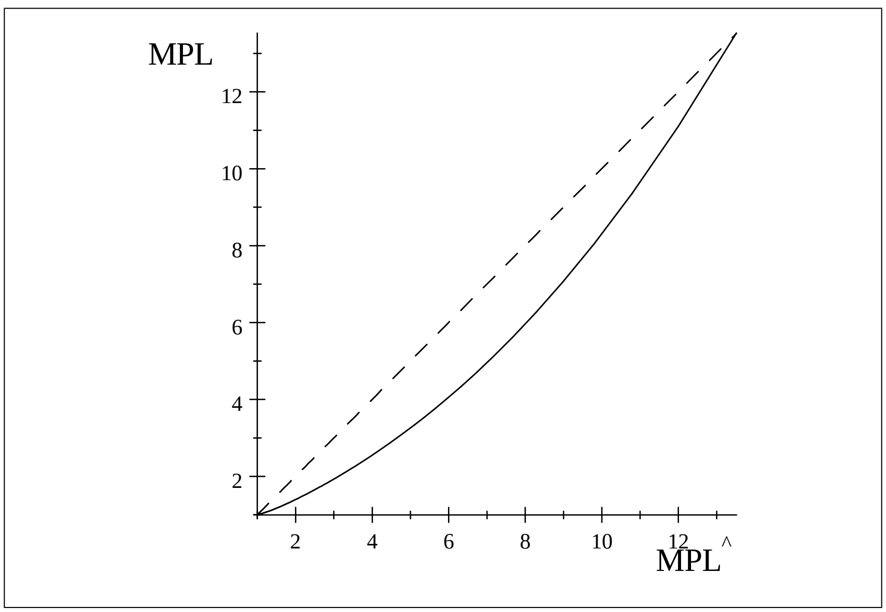

Figure 11: The relationship between MPL of the superstar firm and the industry average (solid curve), which is below the 45° line (dashed).

5 Conclusion

In 1933, Joan Robinson divided her pioneering new book on Economics of Imperfect Competition into two parts: "Monopoly, the principles of selling; and Monopsony, the principles of buying", thus effectively treating both markup and markdown powers of sellers and buyers in isolation. In this paper, aiming at investigating the documented (secular) trends of rising market power, falling labour income share and wage stagnation, we treat both markup and wage markdown powers of producer-employer firms as integral parts of the theory of market power. By extending Cournot oligopoly model to allow imperfectly competitive labour market, we construct a joint oligopoly-oligopsony model (JOOM) to investigate the producer-employer firms' behaviour, and its welfare implications for consumers and workers.

Our analysis extends the notion of exploitation of labour – wage below the marginal revenue product of labour – from the context of monopsony to general oligopsony, and shows that the rate of exploitation of labour increases with market concentration. Our analysis calls for the inclusion of worker welfare into the welfare standard of antitrust enforcement. We show this call remains valid even in the presence of trade union, collective wage bargaining or minimum wage regulation, aimed at curbing oligopsony wage markdown power. Our analysis also sheds new light on the superstar firm phenomenon. It warns against the presumption that the superstar firm phenomenon is procompetitive on the basis of consumer welfare improvement; and it emphasises the importance of preventing superstar firms from entrenching barriers to entry through consolidating exclusionary control over intangible assets, such as IP rights.

While the superstar firms theory can explain the secular trends of rising concentration, decreasing labour share of income and wage stagnation, the theory in itself does not take a stance on how relevant the markdown power component is with respect to the overall market power of producer-employer firms. JOOM makes clear that empirical research is needed to quantify the relative importance of wage markdown and product price markup powers at firm level in various sectors, since the effectiveness of policy interventions to tackle inequality in income distribution will depend on the relative weight of the two. Moreover, given that existing empirical works on market power have been plagued with problems of measurement errors, functional form misspecification and endogeneity[32], we believe that JOOM can be useful not only to make sense of observed stylised facts, but also to pave the way for a novel approach to the measurement of overall market power and its components.

References

[1] Acemoglu, D. and D. Autor (2011), "Skills, Tasks and Technologies: Implications for Employment and Earnings," Chap. 12 in Handbook of Labor Economics, vol. 4B, edited by David Card and Orley Ashenfelter, 1043-1171, Amsterdam: Elsevier.

[2] Ashenfelter, O. C. and H. Farber and M. R. Ransom (2010), "Labor Market Monopsony", Journal of Labor

[32] For instance, in a recent paper, Bond *et al* (2021) warn against drawing inferences about markup estimates obtained with the production (the most popular method to estimate markups) when firm-level output prices are unobserved.

Economics, 28 (2): 203–210.

[3] Autor, D. and L. F. Katz and C. Patterson and J. Van Reenen (2020), "The Fall of the Labor Share and the Rise of Superstar Firms", The Quarterly Journal of Economics (2020), 645-709. doi:10.1093/qje/qjaa004.

[4] Azar, J. and S. Berry and I. Marinescu (2019), "Estimating Labor Market Power". Available at https://papers.ssrn.com/sol3/papers.cfm?abstract_id=3456277.

[5] Azar, J. and I. Marinescu and M. Steinbaum (2019), "Labor Market Concentration", NBER Working Paper 24147, http://www.nber.org/papers/w24147.

[6] Azar, J. and X. Vives (2021), "General Equilibrium Oligopoly and Ownership Structure", Econometrica, 89 (3): 999-1048.

[7] Baker, J. B. (2007), "Beyond Schumpeter vs. Arrow: How Antitrust Fosters Innovation", Antitrust Law Journal, 74 (3): 575-602.

[8] Baker, J. B. (2019), The Antitrust Paradigm: Restoring a Competitive Economy, Cambridge, MA: Harvard University Press.

[9] Baker, J. B. and S. C. Salop (2015), "Antitrust, Competition Policy, and Inequality", The Georgetown Law Journal Online, 104, 1. Available at https://georgetownlawjournal.org/articles/164/ antitrust-competition- policy- inequality/pdf.

[10] Baker, J. B. and C. Shapiro (2008), "Reinvigorating Horizontal Merger Enforcement", Chapter 15, How the Chicago School Overshot the Mark: The Effect of Conservative Economic Analysis on U.S. Antitrust, ed. by Robert Pitofsky, Oxford: Oxford University Press.

[11] Barkai, S. (2020),"Declining Labor and Capital Shares", Journal of Finance, 75 (5): 2421-2463.

[12] Benmelech, E. and N. Bergman and H. Kim (2018), "Strong Employers and Weak Employees: How Does Employer Concentration Affect Wages?" NBER Working Paper No. 24307.

[13] Berry, S. and M. Gaynor and F. S. Morton (2019), "Do Increasing Markups Matter? Lessons from Empirical Industrial Organization". Journal of Economic Perspectives, 33 (3): 44-68.

[14] Bartelsman, E. J. and M. Doms (2000), "Understanding Productivity: Lessons from Longitudinal Microdata", Journal of Economic Literature, 38 (3): 569–594.

[15] Bivens, J. and L. Mishel (2015), "Understanding the Historic Divergence between Productivity and a Typical Worker's Pay," EPI Briefing Papers 406.

[16] Boal, W. M. and M. R. Ransom (1997), "Monopsony in the Labor Market", Journal of Economic Literature, 35: 86-112.

[17] Bond, S. and A. Hashemi and G. Kaplan and P. Zoch (2021), "Some Unpleasant Markup Arithmetic: Production Function Elasticities and Their Estimation from Production Data", Journal of Monetary Economics, 121: 1–14.

[18] Card, D. and A. B. Krueger (1995), Myth and Measurement: the New Economics of the Minimum Wage, Princeton University Press.

[19] Council of Economic Advisers (2016), "Labor Market Monopsony: Trends, Consequences, and Policy Responses". Available at https://obamawhitehouse.archives.gov/sites/default/files/page/files/20161025_monopsony_labor_mrkt_cea.pdf

[20] Cournot, A. A. (1897), Researches into the Mathematical Principles of the Theory of Wealth. 1838. Trans. N. T. Bacon. New York: Macmillan.

[21] De Loecker, J. and J. Eeckhout and G. Unger (2020), "The Rise of Market Power and the Macroeconomic Implications," Quarterly Journal of Economics, 135: 561-644.

[22] De Loecker, J. and J. Eeckhout and S. Mongey (2021), "Quantifying Market Power and Business Dynamism in the Macroeconomy". University of Chicago, Becker Friedman Institute for Economics Working Paper No. 2021-54, Available at SSRN: https://ssrn.com/abstract=3837433

[23] De Loecker, J. and C. Syverson (2021), "An Industrial Organization Perspective on Productivity", NBER Working Paper 29229.

[24] De Loecker, J. and F. Warzynski (2012), "Markups and Firm-Level Export Status", American Economic Review, 102: 2437-2471.

[25] Demsetz, H. (1973), "Industry Structure, Market Rivalry, and Public Policy", Journal of Law and Economics, 16 (1): 1-9.

[26] Doraszelski, U. and J. Jaumandreu (2018), "Measuring the Bias of Technological Change", Journal of Political Economy, 126 (3): 1027-1084.

[27] Elsby, M. and B. Hobijn and A. Sahin (2013), "The Decline of the U.S. Labor Share", Brookings Papers on Economic Activity 2013: 1-63.

[28] Farrell, J. and C. Shapiro (1990), "Horizontal Mergers: An Equilibrium Analysis," American Economic Review, 100: 107-126.

[29] Furman, J. and P. Orszag (2018), "A Firm-Level Perspective on the Role of Rents in the Rise of Inequality", in Toward a Just Society: Joseph Stiglitz and Twenty-First Century Economics, edited by Martin Guzman, 19-47. New York: Columbia University Press.

[30] Froeb, L. M. and G. J. Werden (1998), “A Robust Test for Consumer Welfare Enhancing Mergers among Sellers of a Homogeneous Product”, Economics Letters 58: 367–369.

[31] Gutiérrez, G. and T. Philippon (2017), “Declining Competition and Investment in the US,” NBER Working Paper 23583.

[32] Hall, R. E. (1988), “The Relation between Price and Marginal Cost in U.S. Industry,” Journal of Political Economy, 96 (5): 921-947.

[33] Hall, R. E. (2018), “New Evidence on the Markup of Prices over Marginal Costs and the Role of Mega-frms in the US Economy”, NBER Working Paper No. 24574.

[34] Hemphill, C. S. and N. L. Rose. (2018), “Mergers that Harm Sellers”, 127 Yale Law Journal, 127: 2078-2109.

[35] Hicks, J. R. (1932), The Theory of Wages, London: Macmillan.

[36] Hovenkamp, H. J. (2017), “Appraising Merger Efficiencies”, Faculty Scholarship at Penn Law, 1762. Available at https://scholarship.law.upenn.edu/faculty_scholarship/1762.

[37] Hovenkamp, H. J. (2019), “Is Antitrust’s Consumer Welfare Principle Imperiled?”, Faculty Scholarship at Penn Law, 1985. Available at https://scholarship.law.upenn.edu/faculty_scholarship/1985

[38] Hovenkamp, H. J, and C. Shapiro (2018), “Horizontal Mergers, Market Structure, and Burdens of Proof”, Yale Law Journal, 127(7): 1996–2025.

[39] International Labour Organization and Organisation for Economic Co-operation and Development (2015), “The Labour Share in G20 Economies”, Report prepared for the G20 Employment Working Group, with contributions from International Monetary Fund and World Bank Group.

[40] Karabarbounis, L. and B. Neiman (2014), “The Global Decline of the Labor Share”, Quarterly Journal of Economics, 129: 61-103.

[41] Kroft, K. and Y. Luo and M. Mogstad and B. Setzler (2021), “Imperfect Competition and Rents in Labor and Product Markets: The Case of the Construction Industry”, NBER Working Paper 27325.

[42] Kwoka, J. E. (2017), “The Structural Presumption and Safe Harbor in Merger Review: False Positives or Unwarranted Concerns?”, Antitrust Law Journal 81: 837-72.

[43] Lerner, A. P. (1934), “The Concept of Monopoly and the Measurement of Monopoly Power”, The Review of Economic Studies, 1 (3): 157-175.

[44] Manning, A. (2003), Monopsony in Motion: Imperfect Competition in Labor Markets, Princeton University Press, Princeton and Oxford.

[45] Manning, A. (2011), "Imperfect Competition in the Labor Market." Chap. 11 in Handbook of Labor Economics, vol. 4b, edited by Orley Ashenfelter and David Card, 973-1041. Amsterdam: Elsevier.

[46] Manning, A. (2020), "Monopsony in Labor Markets: A Review", Industrial and Labor Relations Review, XX(X): 1-24.

[47] Marinescu, I. E. and H. Hovenkamp (2018), "Anticompetitive Mergers in Labor Markets", Indiana Law Journal, 94, pp. 1031-1063.

[48] Mertens, M. (2021), "Micro-mechanisms behind Declining Labor Shares: Rising Market Power and Changing Models of Production", International Journal of Industrial Organization.

[49] Mishel, L. (2012), "The Wedges between Productivity and Median Compensation Growth," Issue Brief, Economic Policy Institute.

[50] Motta, M. and E. Tarantino (2017), "The Effect of Horizontal Mergers, when Firms Compete in Prices and Investments". Universitat Pompeu Fabra, Economics Working Paper Series, Working Paper No. 1579.

[51] Naidu, S. and E. A. Posner and G. Weyl (2018), "Antitrust Remedies for Labor Market Power," Harvard Law Review,132 (2): 536-601.

[52] Novshek, W. (1985), "On the Existence of Cournot equilibrium", Review of Economic Studies, 52: 85-98.

[53] Pigou, A. C. (1924), The Economics of Welfare (2nd edition), London: Macmillan.

[54] Robinson, J. (1933), The Economics of Imperfect Competition, London: Macmillan.

[55] Robinson, J. (1978), Contributions to Modern Economics, Academic Press.

[56] Salop, S. C. (2015), "The Evolution and Vitality of Merger Presumptions: Decision-Theoretic Approach", Antitrust Law Journal, 80 (2): 269-306.

[57] Segal, Ilya and M. D. Whinston (2007), "Antitrust in Innovative Industries", American Economic Review, 97 (5): 1703-1730.

[58] Schumpeter, J. (1942), Capitalism, Socialism and Democracy. New York: Harper & Brothers.

[59] Shapiro, C. (2010), "The 2010 Horizontal Merger Guidelines: From Hedgehog to Fox in Forty Years," Antitrust Law Journal, 77: 49-108.

[60] Shapiro, C. (2012), "Competition and Innovation: Did Arrow Hit the Bull's Eye?", The Rate and Direction of Inventive Activity Revisited, University of Chicago Press.

[61] Shapiro, C. (2019), "Protecting Competition in the American Economy: Merger Control, Tech Titans, Labor Markets", *Journal of Economic Perspectives*, 33 (3): 69-93.

[62] Sharpe, A. and J. Uguccioni (2016), "Decomposing the Productivity-Wage Nexus in Selected OECD Countries, 1986–2013", CSLS Research Report, 2016-16.

[63] Smith, A. (2005), The Wealth of Nations. A Penn State Electronic Classics Seriers Publication.

[64] Stansbury, A. and L. H. Summers (2020), "The Declining Worker Power Hypothesis: An Explanation for the Recent Evolution of the American Economy".

[65] Sutton, J. (1991), Sunk Costs and Market Structure, MIT Press.

[66] Sutton, J. (1998), Technology and Market Structure, MIT Press.

[67] Syverson, C. (2011), "What Determines Productivity?" Journal of Economic Literature 49 (2): 326–365.

[68] Tirole, J. (1988), The Theory of Industrial Organization, MIT Press.

[69] Tong, J. and C. Ornaghi (2020), "The Oligopoly-oligopsony Structure and Hidden Market Power – Part 2: Application", Mimeo, University of Southampton.

[70] Tortarolo, D. and R. D. Zarate (2018), "Measuring Imperfect Competition in Product and Labor Markets: An Empirical Analysis Using Firm-level Production Data". Accessed at SSRN: https://ssrn.com/abstract=3098141 or http://dx.doi.org/10.2139/ssrn.3098141.

[71] Webber, D. A. (2015), "Firm Market Power and the Earnings Distribution", Labor Economics, 35: 123-134.

[72] Werden, G. J. (2017), "Cross-market Balancing of Competitive Effects: What Is the Law, and What Should It Be", Journal of Corporation Law, 43 (1), 119-142.

[73] Williamson, O. E. (1968), "Economies as an Antitrust Defense: The Welfare Trade-Offs", American Economic Review, 58: 18-36.

A Variable and Marginal Costs with Oligopsony Power

In Appendix A, we derive analytical results about the variable and marginal costs, as well as the overall market power, of an oligopolist-oligopsonist firm, for a general production function, and in the case of Cobb-Douglas production function. This provides useful theoretical guidance for empirically measuring the different components of pricing powers.

A.1 General Formulation

Consider the following oligopsonist cost minimisation problem:

$$\min_{F(\mathbf{x},l)\geq q} \left\{ \sum_{v=1}^{V} P_v\left(x_v\right) x_v + W\left(l\right) l \right\} \tag{77}$$

where $F(\mathbf{x}, l)$ is the short-run production function with $\mathbf{x} = (x_v)_{v=1}^{V}$, $W(l)$ is the oligopsonist employer's residual inverse labour supply function (or wage function) with $W'(\cdot) \geq 0$, $P_v(x_v)$ is the oligopsonist buyer's residual inverse supply function for variable input v, with $P_v'(\cdot) \geq 0$. The firm level residual functions P_v, W and output q are taken as given.[33] Let the Lagrangian objective function be:

$$\mathcal{L}(\mathbf{x}, l, \lambda; q) = \sum_{v=1}^{V} P_v(x_v) x_v + W(l) l - \lambda (F(\mathbf{x}, l) - q). \tag{78}$$

The first order conditions (FOC) are:

$$\frac{\partial \mathcal{L}}{\partial x_v} = P_v' x_v + P_v - \lambda \frac{\partial F}{\partial x_v} = 0, \tag{79}$$

i.e.,

$$\lambda \frac{\partial F}{\partial x_v} = MC_v \equiv P_v + P_v' x_v', \tag{80}$$

where $MC_v \equiv P_v + P_v' x_v'$ is the marginal cost of input v,

$$\frac{\partial \mathcal{L}}{\partial l} = W' l + W - \lambda \frac{\partial F}{\partial l} = 0, \tag{81}$$

i.e.,

$$\lambda \frac{\partial F}{\partial l} = MC_L \equiv W' l + W, \tag{82}$$

where $MC_L \equiv W' l + W$ is the marginal cost of labour,

$$\frac{\partial \mathcal{L}}{\partial \lambda} = F(\mathbf{x}, l) - q = 0. \tag{83}$$

Let $(\mathbf{x}^*, l^*, \lambda^*)$ denote the solution to the FOC equation system. The solution is a function of parameter q, and the residual functions P_v for all v and W. It satisfies the equation:

$$F(\mathbf{x}^*, l^*) = q, \tag{84}$$

and the technical rate of substitution equations:

$$TRS_{vL} \equiv \frac{\frac{\partial F}{\partial x_v}}{\frac{\partial F}{\partial l}} = \frac{MC_v}{MC_L} \equiv \frac{P_v(x_v^*) + P_v'(x_v^*) x_v^*}{W(l^*) + W'(l^*) l^*}. \tag{85}$$

[33] It should be noted that the firm level residual inverse demand and supply functions P_v and W depend on rival firms' variable inputs (which are also taken as given) and market level inverse demand and supply functions.

The oligopsonist variable cost function is given by

$$VC\left(q,(P_v)_{v=1}^V,W\right)\equiv\mathcal{L}\left(\mathbf{x}^*,l^*,\lambda^*;q\right)=\sum_{v=1}^V P_v\left(x_v^*\right)x_v^*+W\left(l^*\right)l^*. \tag{86}$$

The envelope theorem implies that the oligopolist marginal cost is given by

$$\begin{aligned}&MC\left(q,(P_v)_{v=1}^V,W\right)\equiv\tfrac{dVC}{dq}=\tfrac{\partial\mathcal{L}(\mathbf{x}^*,l^*,\lambda^*;q)}{\partial q}=\lambda^*\\&=\textstyle\sum_{v=1}^V\left(P_v\left(x_v^*\right)+P_v'\left(x_v^*\right)x_v^*\right)\frac{\partial x_v^*}{\partial q}+\left(W\left(l^*\right)+W'\left(l^*\right)l^*\right)\frac{\partial l^*}{\partial q}.\end{aligned} \tag{87}$$

A.2 Extension of Price Taker Marginal Cost

Equation (87) can be rewritten as

$$MC\left(q,(P_v)_{v=1}^V,W\right)=MC^{PT}\left(q,(P_v)_{v=1}^V,W\right)+\sum_{v=1}^V P_v'\left(x_v^*\right)x_v^*\frac{\partial x_v^*}{\partial q}+W'\left(l^*\right)l^*\frac{\partial l^*}{\partial q}, \tag{88}$$

where

$$MC^{PT}\left(q,(P_v)_{v=1}^V,W\right)\equiv\sum_{v=1}^V P_v\left(x_v^*\right)\frac{\partial x_v^*}{\partial q}+W\left(l^*\right)\frac{\partial l^*}{\partial q}, \tag{89}$$

is the extended definition of Price Taker Marginal Cost. Thus,

$$MC^{PT}\left(q,(P_v)_{v=1}^V,W\right)=MC\left(q,(P_v)_{v=1}^V,W\right)-\sum_{v=1}^V P_v'\left(x_v^*\right)x_v^*\frac{\partial x_v^*}{\partial q}-W'\left(l^*\right)l^*\frac{\partial l^*}{\partial q}, \tag{90}$$

which is oligopsonist marginal cost MC net of its input price change effects $\sum_{v=1}^V P_v'x_v^*\frac{\partial x_v^*}{\partial q}$ and $W'l^*\frac{\partial l^*}{\partial q}$ – the extra cost caused by increased input prices following an increment of output at the margin. Note

$$\begin{aligned}MC^{PT}\left(q,(P_v)_{v=1}^V,W\right)&=\sum_{v=1}^V\left(\frac{\partial x_v^*}{\partial q}\frac{q}{x_v^*}\right)\frac{P_v\left(x_v^*\right)x_v^*}{q}+\left(\frac{\partial l^*}{\partial q}\frac{q}{l^*}\right)\frac{W\left(l^*\right)l^*}{q}\\&=\sum_{v=1}^V\zeta_v\frac{P_v\left(x_v^*\right)x_v^*}{q}+\zeta_L\frac{W\left(l^*\right)l^*}{q},\end{aligned}$$

which implies

$$\tau\equiv\frac{P-MC^{PT}\left(q,(P_v)_{v=1}^V,W\right)}{P}=1-\sum_{v=1}^V\zeta_v\phi_v-\zeta_L\phi_L. \tag{91}$$

This provides useful theoretical guidance for measuring the overall market power index τ. The revenue shares of some inputs v and labour can be recovered from firms' financial data; if the input elasticities ζ_v and ζ_L can be estimated (see Section A.3 for example), then τ can be estimated as well according to equation (91).

In the case of a Leontief production function, we have $\zeta_v=\zeta_L=1$, i.e., the cost minimising input bundle simply scale up with the output level. Then the overall market power equals the producer surplus share of revenue, i.e.,

$$\tau \equiv \frac{P - MC^{PT}\left(q, (P_v)_{v=1}^{V}, W\right)}{P} = 1 - \sum_{v=1}^{V} \phi_v - \phi_L. \tag{92}$$

Additionally,

$$MC^{PT} = AVC, \tag{93}$$

$$\tau = 1 - \frac{AVC}{P} = 1 - \frac{VC}{Pq}, \tag{94}$$

where $AVC \equiv \frac{VC}{q}$ is the average variable cost.

Another setting where $\zeta_v = \zeta_L = 1$ hold is with constant returns to scale (CRS) technology and perfect competition in all input markets. Thus,

$$MC^{PT} = MC = AVC. \tag{95}$$

A.3 Example with Cobb-Douglas Production Function

Consider the example with Cobb-Douglas production function:

$$F_i(x_i, l_i) = A_i x_i^{\alpha} l_i^{1-\alpha} \text{ for } i \in \mathcal{N}, \tag{96}$$

where variable input x_i can be interpreted either as material, fuel or intermediate input, which we assume to have perfectly elastic supply. Let the market level labour supply elasticity be constant $\epsilon_L < \infty$.

From

$$TRS_{xL,i} \equiv \frac{\frac{\partial F_i}{\partial x_i}}{\frac{\partial F_i}{\partial l_i}} = \frac{\alpha}{1-\alpha}\frac{l_i^*}{x_i^*} = \frac{MC_{xi}}{MC_{Li}} \equiv \frac{p_x}{W + W' l_i^*}, \tag{97}$$

we derive

$$\frac{p_x x_i^*}{W l_i^*} = \frac{\alpha}{1-\alpha} + \frac{\alpha}{1-\alpha}\frac{1}{\epsilon_L} s_{Li}, \tag{98}$$

where $s_{Li} \equiv \frac{l_i^*}{L} = \frac{l_i^*}{L_{-i} + l_i^*}$ is labour market share of firm i.[34]

Empirically, the values of $\frac{p_x x_i^*}{W l_i^*}$ and s_{Li} are often available from micro panel data. By regressing $\frac{p_x x_i^*}{W l_i^*}$ on s_{Li}, the primitive parameters α and ϵ_L can be inferred from the regression coefficients. So equation (98) provides theoretical guidance for measuring firm level wage markdown power $\frac{s_{Li}}{\epsilon_L}$.

[34] The derivation of eq. (98) uses the fact that $L = L_{-i} + l_i^*$ and the market wage function and the residual wage function of a firm has the same derivative. Therefore the market level labour supply elasticity $\epsilon_L = \frac{W}{W' L}$ and $W + W' l_i^* = \left(1 + \frac{s_{Li}}{\epsilon_L}\right) W$, even though W is the residual wage function for a firm, not the market wage function.

Equation (97) also implies

$$\ln x_i^* = \ln\left(\frac{\alpha}{1-\alpha}\frac{1}{p_x}\right) + \ln W + \ln\left(1+\frac{s_{Li}}{\epsilon_L}\right) + \ln l_i^*, \tag{99}$$

and

$$\begin{aligned}
\frac{\partial \ln x_i^*}{\partial \ln l_i^*} &= \frac{\partial \ln W}{\partial \ln l_i^*} + \frac{\partial \ln\left(1+\frac{s_{Li}}{\epsilon_L}\right)}{\partial \ln l_i^*} + 1 && (100)\\
&= 1 + \frac{s_{Li}}{\epsilon_L} + \frac{\partial \ln\left(1+\frac{s_{Li}}{\epsilon_L}\right)}{\partial \ln l_i^*}. && (101)
\end{aligned}$$

Note the following results:

$$\frac{\partial \ln\left(1+\frac{s_{Li}}{\epsilon_L}\right)}{\partial \ln l_i^*} = \frac{\partial \ln\left(1+\frac{1}{\epsilon_L}\frac{l_i^*}{L_{-i}+l_i^*}\right)}{\partial \ln l_i^*} = \frac{\frac{\frac{1}{\epsilon_L}\frac{(L_{-i}+l_i^*)-l_i^*}{(L_{-i}+l_i^*)^2}}{1+\frac{1}{\epsilon_L}\frac{l_i^*}{L_{-i}+l_i^*}}}{\frac{1}{l_i^*}} = \frac{\frac{\frac{1}{\epsilon_L}\frac{L_{-i}}{L^2}}{1+\frac{1}{\epsilon_L}\frac{l_i^*}{L}}}{\frac{1}{l_i^*}} = \frac{\frac{L_{-i}}{L}}{1+\frac{1}{\epsilon_L}\frac{l_i^*}{L}}\frac{1}{\epsilon_L}\frac{l_i^*}{L} = \frac{(1-s_{Li})\frac{s_{Li}}{\epsilon_L}}{1+\frac{s_{Li}}{\epsilon_L}} \in \left[0, \frac{\frac{s_{Li}}{\epsilon_L}}{1+\frac{s_{Li}}{\epsilon_L}}\right] \tag{102}$$

which can be used to derive:

$$\frac{\partial \ln x_i^*}{\partial \ln l_i^*} = 1 + \frac{s_{Li}}{\epsilon_L} + \frac{(1-s_{Li})\frac{s_{Li}}{\epsilon_L}}{1+\frac{s_{Li}}{\epsilon_L}} = 1 + \frac{2-s_{Li}+\frac{s_{Li}}{\epsilon_L}}{1+\frac{s_{Li}}{\epsilon_L}}\frac{s_{Li}}{\epsilon_L} \in \left[1+\frac{s_{Li}}{\epsilon_L}, 1+\frac{2+\frac{s_{Li}}{\epsilon_L}}{1+\frac{s_{Li}}{\epsilon_L}}\frac{s_{Li}}{\epsilon_L}\right] \tag{103}$$

The production funtion (96) then implies

$$\begin{aligned}
\frac{\partial \ln q_i}{\partial \ln l_i^*} &= \alpha\frac{\partial \ln x_i^*}{\partial \ln l_i^*} + 1 - \alpha && (104)\\
&= \alpha\left(1+\frac{2-s_{Li}+\frac{s_{Li}}{\epsilon_L}}{1+\frac{s_{Li}}{\epsilon_L}}\frac{s_{Li}}{\epsilon_L}\right) + 1 - \alpha && (105)\\
&= 1 + \alpha\left(\frac{2-s_{Li}+\frac{s_{Li}}{\epsilon_L}}{1+\frac{s_{Li}}{\epsilon_L}}\frac{s_{Li}}{\epsilon_L}\right), && (106)
\end{aligned}$$

and

$$\zeta_{Li} \equiv \frac{\partial \ln l_i^*}{\partial \ln q_i} = \frac{1}{\frac{\partial \ln q_i}{\partial \ln l_i^*}} = \frac{1}{1+\alpha\left(\frac{2-s_{Li}+\frac{s_{Li}}{\epsilon_L}}{1+\frac{s_{Li}}{\epsilon_L}}\frac{s_{Li}}{\epsilon_L}\right)} \leq 1. \tag{107}$$

Note that eq. (96) also implies

$$\frac{\partial F_i(x_i, l_i)}{\partial x_i}\frac{x_i}{F_i(x_i, l_i)} = \alpha, \tag{108}$$

and

$$\frac{\partial F_i(x_i, l_i)}{\partial l_i}\frac{l_i}{F_i(x_i, l_i)} = 1 - \alpha. \tag{109}$$

Equation:

$$F_i\left(x_i^*, l_i^*\right) = A_i\left(x_i^*\right)^{\alpha}\left(l_i^*\right)^{1-\alpha} = q_i \tag{110}$$

implies

$$\frac{\partial F_i\left(x_i^*, l_i^*\right)}{\partial x_i^*}\frac{\partial x_i^*}{\partial q_i} + \frac{\partial F_i\left(x_i^*, l_i^*\right)}{\partial l_i^*}\frac{\partial l_i^*}{\partial q_i} = 1, \tag{111}$$

and equivalently

$$\alpha\zeta_{xi} + (1-\alpha)\zeta_{Li} = 1. \tag{112}$$

Eq. (112) then implies

$$\zeta_{xi} \equiv \frac{\partial \ln x_i^*}{\partial \ln q_i} = \frac{1-(1-\alpha)\zeta_{Li}}{\alpha} = \frac{1+\left(\frac{2-s_{Li}+\frac{s_{Li}}{\epsilon_L}}{1+\frac{s_{Li}}{\epsilon_L}}\frac{s_{Li}}{\epsilon_L}\right)}{1+\alpha\left(\frac{2-s_{Li}+\frac{s_{Li}}{\epsilon_L}}{1+\frac{s_{Li}}{\epsilon_L}}\frac{s_{Li}}{\epsilon_L}\right)} \geq 1. \tag{113}$$

As the parameter and variables: α, $\frac{s_{Li}}{\epsilon_L}$ and s_{Li} can be recovered from micro panel data, the input elasticities ζ_{Li} and ζ_{xi} can also be recovered according to (107) and (113).

The following two numerical examples show how the input elasticities change with market structure.

Example 1: symmetric model, $\alpha = \frac{1}{3}$, $\epsilon_L = 0.8$, $s_{Li} = \frac{1}{N}$.

$$\zeta_{Li} = \frac{1}{1+\alpha\left(\frac{2-\frac{1}{N}+\frac{1}{N\epsilon_L}}{1+\frac{1}{N\epsilon_L}}\frac{1}{N\epsilon_L}\right)}, \tag{114}$$

$$\zeta_{xi} = \frac{1+\left(\frac{2-\frac{1}{N}+\frac{1}{N\epsilon_L}}{1+\frac{1}{N\epsilon_L}}\frac{1}{N\epsilon_L}\right)}{1+\alpha\left(\frac{2-\frac{1}{N}+\frac{1}{N\epsilon_L}}{1+\frac{1}{N\epsilon_L}}\frac{1}{N\epsilon_L}\right)}. \tag{115}$$

Figure 12 shows that consolidation of market structure of the symmetric model, i.e., reducing N, causes the labour input elasticity to decrease and the non-labour variable input elasticity to increase. This is because as the labour market becomes more concentrated, each firm has more wage markdown power, and its marginal cost of labour increases, inducing it to substitute labour with non-labour variable input.

Example 2: Asymmetric model with a superstar firm and $N-1$ symmetric laggard rivals: $\alpha = \frac{1}{3}$, $\epsilon_L = 0.8$, $s_{Li} = \frac{1-\hat{s}_L}{N-1}$ for $i \in \{1, 2, \ldots, N-1\}$, $s_{LN} = \hat{s}_L$, $N = 4$, where $\hat{s}_L$ is the labour market share of the superstar firm.

For $i \in \{1, 2, \ldots, N-1\}$,

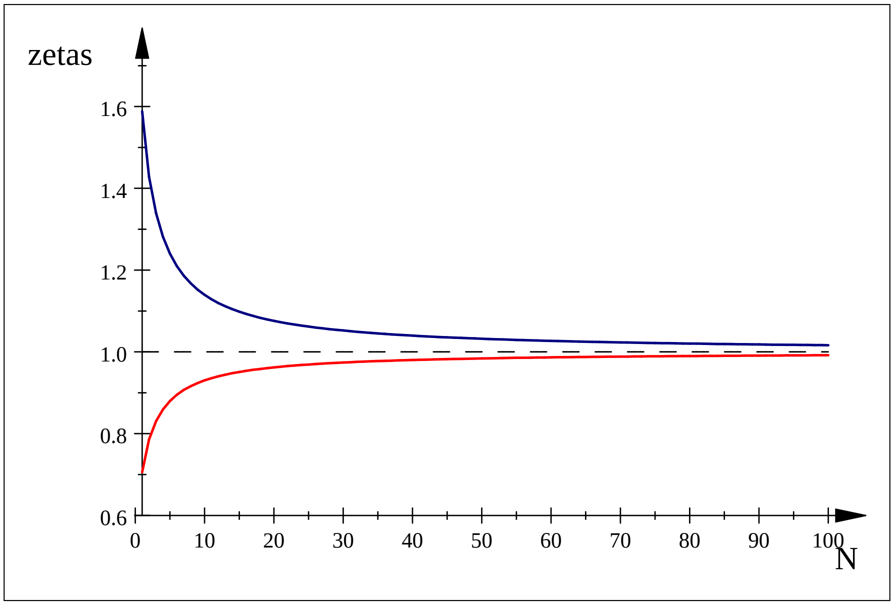

Figure 12: Example 1, symmtetric model. ζ_{Li} (red) and ζ_{xi} (blue) as functions of N.

$$\zeta_{Li} = \frac{1}{1+\alpha\left(\frac{2-\frac{1-\hat{s}_L}{N-1}+\frac{\frac{1-\hat{s}_L}{N-1}}{\epsilon_L}}{1+\frac{\frac{1-\hat{s}_L}{N-1}}{\epsilon_L}}\,\frac{\frac{1-\hat{s}_L}{N-1}}{\epsilon_L}\right)}, \tag{116}$$

$$\zeta_{xi} \equiv \frac{1+\left(\frac{2-\frac{1-\hat{s}_L}{N-1}+\frac{\frac{1-\hat{s}_L}{N-1}}{\epsilon_L}}{1+\frac{\frac{1-\hat{s}_L}{N-1}}{\epsilon_L}}\,\frac{\frac{1-\hat{s}_L}{N-1}}{\epsilon_L}\right)}{1+\alpha\left(\frac{2-\frac{1-\hat{s}_L}{N-1}+\frac{\frac{1-\hat{s}_L}{N-1}}{\epsilon_L}}{1+\frac{\frac{1-\hat{s}_L}{N-1}}{\epsilon_L}}\,\frac{\frac{1-\hat{s}_L}{N-1}}{\epsilon_L}\right)}; \tag{117}$$

$$\zeta_{LN} = \frac{1}{1+\alpha\left(\frac{2-\hat{s}_L+\frac{\hat{s}_L}{\epsilon_L}}{1+\frac{\hat{s}_L}{\epsilon_L}}\,\frac{\hat{s}_L}{\epsilon_L}\right)}, \tag{118}$$

$$\zeta_{xN} = \frac{1+\left(\frac{2-\hat{s}_L+\frac{\hat{s}_L}{\epsilon_L}}{1+\frac{\hat{s}_L}{\epsilon_L}}\,\frac{\hat{s}_L}{\epsilon_L}\right)}{1+\alpha\left(\frac{2-\hat{s}_L+\frac{\hat{s}_L}{\epsilon_L}}{1+\frac{\hat{s}_L}{\epsilon_L}}\,\frac{\hat{s}_L}{\epsilon_L}\right)}. \tag{119}$$

Figure 13 shows that an increase of the labour market share of the superstar firm, $\hat{s}_L$, which can be caused by technical progress of the superstar firm, i.e., $A_N\uparrow$, causes the superstar firm's labour input elasticity to decrease and the non-labour variable input elasticity to increase. For the laggard rivals, the opposite is true. This also increases the dispersion of the input mix $\frac{x_i^*}{l_i^*}$ and the input expenditure ratio $\frac{p_x x_i^*}{W l_i^*}$ among the firms. The last prediction is empirically testable.

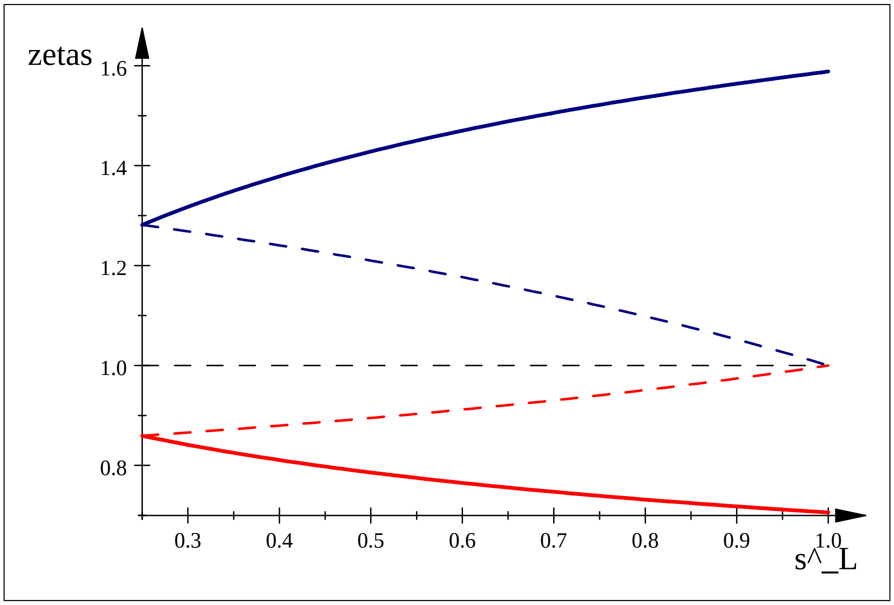

Figure 13: Example 2, asymmtetric model with a superstar firm and lagard rivals. Superstar firm: ζ_{LN} (red, solid) and ζ_{xN} (blue, solid); rivals: ζ_{Li} (red, dashed) and ζ_{xi} (blue, dashed), as functions of $\hat{s}_L$.

B Generalisation of the Fundamental Equation of Market Power

The fundamental equaiton of market power (14) can be extended to:

$$\frac{P - MC_i^{PT}}{P} = \frac{s_i}{\epsilon} + \sum_{v=1}^{V} \frac{s_{vi}}{\epsilon_v} \zeta_{vi} \phi_{vi} + \frac{s_{Li}}{\epsilon_L} \zeta_{Li} \phi_{Li}. \tag{120}$$

Similarly, equation (21) can be extended to:

$$\tau_i = \rho_i + \sum_{v=1}^{V} \chi_{vi} \zeta_{vi} \phi_{vi} + \chi_{Li} \zeta_{Li} \phi_{Li}, \tag{121}$$

where χ_{vi} is the rate of exploitation of input v by firm i, defined similarly with the rate of exploitation of labour χ_{Li}.

C Proofs

Proof of Proposition 11. Eq. (64) and (65) jointly determine the following equation with $\hat{s}$ the unique unknown variable:

$$1 = \frac{\hat{s}}{\epsilon} + \left(1 + \frac{\hat{s}}{\epsilon_L} \frac{1-\gamma}{1-\gamma\hat{s}}\right)(1-\gamma) \frac{1 - \frac{1-\hat{s}}{(N-1)\epsilon}}{1 + \frac{1-\hat{s}}{(N-1)\epsilon_L} \frac{1}{1-\gamma\hat{s}}}, \tag{122}$$

which simplifies to

$$A\hat{s}^2 + B\hat{s} + C = 0, \tag{123}$$

where

$$\begin{aligned}
A &\equiv (1 + (N-1)\,\epsilon_L\gamma - (1-\gamma)\,(1-\gamma-\epsilon_L\gamma)), &(124)\\
B &\equiv -((((N-1)\,\epsilon_L + 1) + ((N-1)\,\epsilon_L\gamma + 1)\,\epsilon) + (1-\gamma)\,(((N-1)\,\epsilon - 1)\,(1-\gamma-\epsilon_L\gamma) + \epsilon_L)), &(125)\\
C &\equiv (\epsilon + \epsilon_L + ((N-1)\,\epsilon - 1)\,\epsilon_L\gamma). &(126)
\end{aligned}$$

The unique solution that satifies

$$\lim_{\gamma\to 0} \hat{s} = \frac{1}{N} \tag{127}$$

is

$$\hat{s} = \frac{-(B) - \sqrt{(B)^2 - 4AC}}{2A}. \tag{128}$$

For $\hat{s} = 1$: equation (122) implies

$$\gamma = \frac{1 + \frac{\epsilon_L}{\epsilon}}{1 + \epsilon_L}, \tag{129}$$

which is the threshold value of γ for monopolisation of the joint oligopoly-oligopsony industry.

Proof of Proposition 12. It follows from the proof of Proposition 11, with $\epsilon_L = \infty$.

Proof of Proposition 14. The complete closed-form solutions and some comparative statics are the following:

$$\begin{aligned}
\hat{s}_w &= \frac{1 + ((N-1)\,\epsilon - 1)\,\gamma}{N-\gamma}, \frac{\partial \hat{s}_w}{\partial w} = 0, \frac{\partial \hat{s}_w}{\partial \gamma} > 0, \frac{\partial \hat{s}_w}{\partial N} = \frac{-(1-\gamma)}{(N-\gamma)^2} < 0 &(130)\\
\phi_w &= \frac{N - \frac{1}{\epsilon}}{N-\gamma}, \frac{\partial \phi_w}{\partial w} = 0, \frac{\partial \phi_w}{\partial \gamma} > 0; \text{for } \epsilon \to 1_+ : \frac{\partial \phi_w}{\partial N} = \frac{\frac{1}{\epsilon} - \gamma}{(N-\gamma)^2} > 0 &(131)\\
P_w &= \frac{(N-\gamma)\,aw}{N - \frac{1}{\epsilon}}, \frac{\partial P_w}{\partial w} > 0, \frac{\partial P_w}{\partial \gamma} < 0; \text{for } \epsilon \to 1_+ : \frac{\partial \phi_w}{\partial N} = \frac{\left(\gamma - \frac{1}{\epsilon}\right) aw}{\left(N - \frac{1}{\epsilon}\right)^2} < 0 &(132)\\
Q_w &= \left(\frac{P_0\left(N - \frac{1}{\epsilon}\right)}{(N-\gamma)\,aw}\right)^{\epsilon}, \frac{\partial Q_w}{\partial w} < 0, \frac{\partial Q_w}{\partial \gamma} > 0; \text{for } \epsilon \to 1_+ : \frac{\partial Q_w}{\partial N} > 0 &(133)\\
L_w &= a\left(\frac{P_0\left(N - \frac{1}{\epsilon}\right)}{(N-\gamma)\,aw}\right)^{\epsilon}\left(1 - \gamma\frac{1 + ((N-1)\,\epsilon - 1)\,\gamma}{N-\gamma}\right), \frac{\partial L_w}{\partial w} < 0 &(134)
\end{aligned}$$

For $N \geq 2$, $\gamma \geq 0$, $\epsilon \to 1_+$:

$$\begin{aligned}
\frac{1}{L_w}\frac{\partial L_w}{\partial \gamma} &= \frac{\partial \ln L_w}{\partial \gamma} && (135)\\
&= \frac{\epsilon+1}{N-\gamma} - \frac{2+2\left(\left(N-1\right)\epsilon-1\right)\gamma}{N-2\gamma-\left(\left(N-1\right)\epsilon-1\right)\gamma^2} && (136)\\
&< \frac{\epsilon+1}{N-\gamma} - \frac{2+2\left(\left(N-1\right)\epsilon-1\right)\gamma}{N-2\gamma} && (137)\\
&< \frac{\epsilon+1}{N-\gamma} - \frac{2+2\left(\left(N-1\right)\epsilon-1\right)\gamma}{N-\gamma} && (138)\\
&= \frac{\epsilon-1}{N-\gamma} - \frac{2\left(\left(N-1\right)\epsilon-1\right)\gamma}{N-\gamma} && (139)\\
&< 0_+. && (140)
\end{aligned}$$

Proof of Proposition 15. Equating the expressions of L_w given in (134) and $L_w = \left(\frac{w}{W_0}\right)^{\epsilon_L}$, yields a unique solution:

$$w = \bar{w} \equiv a^{\frac{1-\epsilon}{\epsilon+\epsilon_L}} \left(W_0\right)^{\frac{\epsilon_L}{\epsilon+\epsilon_L}} \left(\frac{P_0\left(N-\frac{1}{\epsilon}\right)}{N-\gamma}\right)^{\frac{\epsilon}{\epsilon+\epsilon_L}} \left(1-\gamma\frac{1+\left(\left(N-1\right)\epsilon-1\right)\gamma}{N-\gamma}\right)^{\frac{1}{\epsilon+\epsilon_L}}. \tag{141}$$

The equation also implies

$$\frac{\partial L_w}{\partial w}dw + \frac{\partial L_w}{\partial \lambda}d\lambda = \frac{\epsilon_L w^{\epsilon_L-1}}{\left(W_0\right)^{\epsilon_L}}dw. \tag{142}$$

For $w = \bar{w}$ and $N \geq 2$, $\gamma \geq 0$, $\epsilon \to 1_+$:

$$\frac{\partial \bar{w}}{\partial \lambda} = \frac{dw}{d\lambda} = \frac{\frac{\partial L_w}{\partial \lambda}}{\frac{\epsilon_L w^{\epsilon_L-1}}{\left(W_0\right)^{\epsilon_L}} - \frac{\partial L_w}{\partial w}} < 0. \tag{143}$$

The inequality above holds because, by Proposition 14, $\frac{\partial L_w}{\partial \lambda} < 0$ and $\frac{\partial L_w}{\partial w} < 0$.

For binding minimum wage $w < \bar{w}$: $\frac{\partial L_w}{\partial w} = \frac{\partial\left(\frac{w}{W_0}\right)^{\epsilon_L}}{\partial w} > 0$; for $w > \bar{w}$: $\frac{\partial L_w}{\partial w} < 0$. Therefore $w = \bar{w}$ uniquely maximises L_w.