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Discussion Papers in
Economics and Econometrics

Production Function, Market Power and Rent Sharing: Lessons from Hybrid Industrial-Labour Economics

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No. 2405

This paper is available on our website
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ISSN 0966-4246

Production Function, Market Power and Rent Sharing: Lessons from Hybrid Industrial-Labour Economics*

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March 26, 2024

Abstract

In this paper, we build a theoretical model of imperfect competition in both product and labour markets, featuring heterogeneous firm productivities, and use it to guide empirical identification of parameters of production and labour supply functions as well as market power indices. Our methodology offers novel treatments to three stringent assumptions of the cost share approach *à la* Solow (1957): (1) perfect competition in all markets, (2) constant returns to scale (CRS), and (3) Cobb-Douglas production function. To address (1), we show that the ratio between output elasticities of intermediate and labour inputs can be recovered from a subset consisting of competitive fringe firms. For (2) we provide theoretical and empirical evidence to support the notion of a short-run production function characterised by CRS. For (3), we augment the Cobb-Douglas production function with a correction term that captures the effects of firms' buyer power and workers' countervailing seller power in labour market. We validate our novel methodology with a panel of UK manufacturing firms, and show that our methodological innovations in (1)-(3) are all relevant in delivering new substantive findings.

Keywords: estimation of production function, multifactor productivity, market power, countervailing power, markdown, oligopsony, rent sharing, income distribution

JEL codes: D21, D33, D43, D6, E24, J2, J3, L4

*We thank Jan De Loecker, David Levine, and John Sutton for valuable suggestions and comments, and Susan Athey, Josѐ Azar, Jan Eeckhout, Alan Manning, Tommaso Valletti and John Van Reenen for broadening our engagement with the literature and discussion. We also thank seminar participants at the Competition and Market Authority (CMA) in London, at the CMA-Durham Workshop 2023, and at other institutions for their comments.

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1 Introduction

Obtaining reliable firm-level measures of market power and productivity, as well as returns to scale, is an essential pre-requisite for constructing credible metrics for the state of competition, technological progress and income distribution of a nation. The methodology most commonly used for joint estimation of firms' market power, productivity and returns to scale is the so-called production function approach, originated from the seminal paper by Hall (1988) and subsequently refined and popularised by the contribution of De Loecker and Warzynski (2012) and De Loecker, Eeckout and Unger (2020), among others.

Whereas this is an area of active research, there are still open challenges along all dimensions of theory, methodology and data. First, market power does not only have an overall level, but also a structure: its distribution between product and input markets. Pertinent to this, existing theoretical modelling has not adequately uncovered and unravelled multi-fold pair-wise entanglements, such as between returns to scale and overall level of market power,[1] between structure of market power and output elasticities of flexible inputs, and between imperfect competition in labour market and technological inputs substitution property. Second, the theoretical and methodological designs have to adapt to data limitation if we aim to achieve economy-wide applicability using broad-based accounting data. For instance, commonly used datasets report revenue from output and expenditure for inputs, but quantities for output and inputs (except for labour) are typically not available. These data constraints imply that output elasticities for flexible inputs are replaced by revenue elasticities, which, as demonstrated by Bond et al. (2021), are uninformative about firms' market power.[2]

A less data-demanding alternative for estimating production function parameters, introduced by Solow in his seminal 1957 article, relies on static first-order conditions of profit maximisation, instead of regression analysis. This methodology, known as the "factor cost share approach", has two important advantages. First, it avoids the "simultaneity bias", caused by unobservable firm productivity.[3, 4] Second, it reduces data requirement for empir-

[1]As our analysis will demonstrate, the implication of this entanglement is that a low estimate of short-run returns to scale inevitably results in a low estimate of market power. See Sections 2.2 and 2.3 for details.

[2]Recent work by De Ridder et al. (2024) shows that it is feasible to use revenue based estimates of output elasticity for computing dispersion and average level, as well as time trend of markup, whereas the estimation of levels of markups is biased in unpredictable ways.

[3]Overcoming the "simultaneity problem" is a main methodological focus for estimation of production function in general, and has motivated the "control function approach", also known as the "proxy variable approach", (Olley and Pakes 1996, Levinsohn and Petrin 2003, Ackerberg et al. 2015). Gandhi, Navarro and Rivers (2020), however, note that the proxy variable approach suffers from problems of weak identification, unless econometricians can use exceptionally large time-series variation in flexible input price data.

[4]The static first order condition, which captures the economic content of the theoretical model, helps overcome the simultaneity bias because its manipulation cancels out the unobserved firm productivity, the Hicks neutral productivity coefficient, which no longer needs to be identified simultaneously with the output

ical estimation so that standard accounting data suffice. However, the theoretical validity of this method relies on the assumption of firms being price-takers in all markets, which restricts its suitability for imperfectly competitive input or output markets. Furthermore, as noted by De Loecker and Syverson (2021), this approach assumes that the production function has constant returns to scale and is of Cobb-Douglas form: assumptions that impose seemingly stringent restrictions on the technology and inputs substitutability.

Gandhi, Navarro and Rivers (2020) extend the use of the static first order condition for flexible input in production function estimation. Their "share regression approach" uses regression of the revenue share of the intermediate input on all inputs to non-parametrically identify the output elasticity of intermediate input. Although their methodology relaxes the assumptions of constant returns to scale and Cobb-Douglas functional form, it does rely on the assumption of perfect competition in output market for identifying output elasticity of intermediate input. Consequently it is unsuitable for estimating markup power.

In this paper, we build a theoretical model of imperfect competition in both product and labour markets, featuring short-run production function[5] and heterogeneous firm productivities, and use it to guide empirical identification of parameters of production and labour supply functions as well as market power indices.[6] Our methodology offers novel treatments to three stringent assumptions of the cost share approach *à la* Solow (1957): (1) perfect competition in all markets, (2) constant returns to scale (CRS), and (3) Cobb-Douglas production function. Specifically, to address (1), we show that the ratio between output elasticities of intermediate and labour can be recovered from a subset consisting of competitive fringe firms. To address (2), we derive a general basic function of market powers, which informs our identification among three discrete alternatives: (approximate) short-run constant returns to scale (CRS), (strong) short-run decreasing returns to scale (DRS), and (strong) short-run increasing returns to scale (IRS).[7] We argue that the observed dispersions in firms' gross profit margin and value added per worker are only compatible with

elasticity of the flexible input.

[5]Short-run production function is our counterpart to the variable cost function for the short-run analysis. Unlike the latter, the analysis of the former is isolated from the influence of variation in input prices, which is an important advantage when we deal with imperfect competition in input markets. If the price of an input changes with its quantity traded, Shephard's duality theory (see Shephard, 1970) is no longer applicable. That is, there is no equivalence between using production and cost functions for estimating production function parameters.

[6]See Tortarolo and Zarate (2020), Mertens (2022) and Traina (2022) for examples of joint estimation of markups and wage markdowns. Previous work that has employed the production approach to investigate imperfect competition in both product and labour markets includes Dobbelaere (2004), Crépon et al. (2005) and Dobbelaere and Mairesse (2013). For an example of joint estimation of firms' seller power in product market and buyer power in material market, see Rubens (2023). For general equilibrium models that feature market power in both product and labour markets, see Azar and Vives (2021), and Deb et al. (2023).

[7]See Section 2.3 for definitions and details.

(approximate) short-run CRS. For example, we show that (strong) short-run DRS tend to offset the advantage of more efficient firms as they expand the scale of production, the same force that equalises marginal cost, eventually equalises gross profit margin as well as value added per worker across all firms. These predictions are inconsistent with the stylised fact of considerable dispersions in these two variables. To address (3), our theory disentangles the simple baseline technological inputs substitutability property of a Cobb-Douglas production function from a correction term, denoted by χ_L.[8] Our preferred interpretation, informed by both theoretical analysis and empirical evidence, is that it captures the effects of labour market imperfect competition and diverse rent-sharing mechanisms, and it can also be used to identify a key labour supply function parameter that measures the toughness of wage competition.

We validate our methodology using data of UK manufacturing firms retrieved from FAME, a dataset published by Bureau van Dijk, containing conventional accounting information that can be found in similar datasets. We show that our methodological innovations in (1)-(3) are all relevant in producing new substantive findings.

In addressing issue (1), we find that restricting the application of the cost share approach to competitive fringe firms raises the estimate of output elasticity of intermediate input from 0.75 (when using all firms in our sample) to 0.81, with an important knock-on effect on rent sharing discussed below.

While addressing issue (2), with our theoretical and empirical evidence to support the notion of (approximate) short-run CRS, we show that gross profit margin can be used as a comprehensive measure of firm market power. Furthermore, our theoretical and empirical analysis demonstrates that gross profit margin and value added per worker - two simple and directly measurable indices - are highly informative proxies for their common determinant: the short-run multi-factor productivity. These results also reinforce our support for the assumption that the short-run production function is characterised by CRS.

Our theoretical and evidential support for the assumption of the (approximate) short-run CRS contributes to addressing the (nearly) century-long controversy over short-run returns to scale since Sraffa's seminal (1926) paper, which is still relevant today.[9] We show that

[8] This correction term serves as a parameter to explain the dispersion in flexible inputs ratios across firm, akin to Raval's (2023) concept of "non-neutral productivity difference" although our preferred interpretation differs from his. See Online Appendix C for formal proof of equivalence between our correction term and Raval's (2023) firm-specific parameter for non-neutral productivity. In a sense, we provide one explicit theory to explain Raval's (2023) notion of labour-augmenting productivity difference.

[9] Whereas Sraffa (1926) described the debate in terms of diminishing, constant, or increasing returns (of non-fixed factors of production), we believe returns to scale is a more accurate term, and it is important to consider both long-run and short-run returns to scale (see Section 2.3). The significance of this debate, according to Sraffa (1926, p. 536), concerns "separating what is still alive from what is dead in the concept of the supply curve and of its effects on competitive price determination". To appreciate today's relevance of

the alternative hypothesis of strong DRS and strong IRS are inconsistent with the stylised fact of dispersions in firms' gross profit marginal and value added per worker. Our analysis has important efficiency and policy implications as it demonstrates that the dispersion in firm productivity and concentration of market power in the hands of dominant firms are a primary source of productive and allocative inefficiency (see Section 5).

In addressing issue (3), our method allows us to systematically quantify the magnitude of firms' buyer power and workers' countervailing seller power in the labour market for all manufacturing sectors in our dataset. The most striking empirical finding is that the number of firms exhibiting negative values of χ_L, is more than three times of those that have positive values. One contributing factor to this result is the above mentioned restriction of applying the cost share approach solely to competitive fringe firms. Without this restriction, the frequencies of negative and positive values of χ_L would be approximately equal.

The economic insight of this result depends on the interpretation of variable χ_L.[10] Our preferred interpretation is that a positive value of χ_L measures firm's buyer power in labour market, whereas a negative value of χ_L indicates the influence of the countervailing worker power, such as efficient bargaining between workers and firm owners. The large fraction of negative sign of χ_L then suggests that the majority of manufacturing firms in our data set are influenced by workers' countervailing seller power. The importance of seller countervailing power in labour market, first eloquently elucidated by Galbraith (1954),[11] has long been recognised by granting trade unions exemption from antitrust offence.[12] However, the magnitude of workers' countervailing power has never been systematically quantified before. Our analysis also sheds light on a key determinant of firms' rent-share type. Consistent with Galbraith's (1954) hypothesis that market power begets countervailing power, we show that workers' countervailing seller power does increase with firms' overall market power index. Workers in the firms with the highest gross profit margins are over ten times more likely to have seller power than their employers having buyer power in labour market in UK manufacturing industries. Besides identifying and explaining firms' labour market rent-sharing types, our structural estimation of workers' quasi-rent share coefficients for diverse rent-sharing types enables us to recover a baseline labour market oligopsony power parameter:

the controversy, note that Gandhi, Navarro and Rivers' (2020) approach to production function estimation is based on the premise of perfect competition, which in turn relies on the implicit assumption of decreasing returns to scale.

[10] One simple explanation for this result is that χ_L merely reflects an artefact of accounting measurement errors of flexible labour expenditure. We scrutinise and dismiss this explanation in Section 4.2.3.

[11] Galbraith (1954, p. 6) affirmed that: "[W]e must ... cherish the safeguards by which inherently weaker groups have found protection - labor from the perish-ability of its product and its unique compulsion to sell ... The economy is far more viable and its tensions are greatly alleviated because this protection exists."

[12] See Stansbury and Summers (2020) for a study on the relationship between workers' countervailing seller power in labour market and income distribution.

the toughness of wage competition. The structural estimation also shows that the efficient bargaining mechanism, by imposing floors to both wage and labour-intermediate input ratio, increases workers' quasi rent share relative to both oligopsony and the wage floor mechanism, confirming our theoretical predictions.

An alternative interpretation of χ_L is that it captures distortions caused by labour adjustment cost,[13] such as those caused by employment protection laws (see Hopenhayn and Rogerson, 1993). This explanation is also based on labour market institution, but unrelated to imperfect competition. In Section 4.2.2 we discuss our identification strategies for empirical discrimination among these competing interpretations, and the supports for our preferred interpretation. Finally, a third interpretation of the index χ_L is that it represents a generic correction term capturing what is not accounted for by the baseline inputs substitutability property of the Cobb-Douglas production function (see for example, Raval 2023). While some economists may prefer to interpret a production function as characterising broadly-defined technological property, and count measures like χ_L as part of that characterisation, we prefer to treat a production function as reflecting more narrowly defined technological property, and therefore interpret χ_L as imperfectly mirroring heterogeneous labour market rent-sharing mechanisms. We show that controlling for χ_L plays a key role in our consistent structural estimation of parameters of production and labour supply functions.

Our paper contributes to the literature on labour market imperfect competition and rent sharing, originated with the work of Robinson (1933) on labour market monopsony buyer power (see Manning (2011) and (2021) for comprehensive reviews). Our baseline model, built upon Card et al. (2018) and extended to labour market oligopsony with additional flexible intermediate input for production, shows that the broadly defined wage markdown power is a determinant of firms' flexible input ratio. Furthermore, our analysis of flexible input mix is compatible with various labour market rent-sharing mechanisms, ranging from oligopsony to collective bargaining (Robinson, 1933, Nickell and Andrews 1983, McDonald and Solow, 1981).

The remainder of the paper is organised as follows. Section 2 introduces the canonical Joint Oligopoly-Oligopsony Model, derives the basic equations of market power, and explores their implications for input mix and estimation of production function. Section 3 studies wage determination in three types of labour market rent-sharing mechanisms, ranging from oligopsony buyer power to two (collective bargaining) mechanisms supporting countervailing seller power. This is followed by a general theory of firm-level market power and income distribution, that captures imperfect competition in product and labour markets more gen-

[13] Treating labour as a non-flexible input and citing reason of adjustment cost is not unusual in the literature of production function estimation. For a recent influential example, see Gandhi, Navarro and Rivers (2020).

erally. Section 4 presents our empirical analyses. Section 5 discusses policy implications and Section 6 concludes. All proofs are given in the Appendix.

2 The Joint Oligopoly Oligopsony Model

A novelty of the Joint Oligopoly Oligopsony Model (JOOM) is to capture imperfect competition in both product and labour markets among firms with dispersed productivity, ranging from the competitive fringes to the dominant frontier (superstar) firms.[14] While the canonical model features Cournot quantity competition in the product market and wage posting competition in the labour market, we note that the main results (given in Theorem 2) generalise more widely (see Online Appendix A, and Theorem 6 in Section 2.6). When we further restrict the short-run production function of the canonical JOOM to the Cobb-Douglas form with constant returns to scale in Section 3.1, we call that version as our baseline model.

Let each firm $i \in \{1, \cdots, n\}$ face finitely elastic upward-sloping residual labour supply function $L_i(\mathbf{w})$, which depends on the posted wage vector $\mathbf{w} \equiv (w_1, \cdots, w_n)$. The firm specific supply elasticity $\epsilon_{Li} \equiv \frac{\partial L_i(\mathbf{w})}{\partial w_i}\frac{w_i}{L_i(\mathbf{w})} \in (0, \infty)$ is finite, implying imperfect competition in labour market, and satisfies: (i) $\frac{\partial \epsilon_{Li}}{\partial w_i} < 0$, (ii) $\frac{\partial \epsilon_{Li}}{\partial w_j} > 0$. Property (i) means a high-paying (and large) employer faces more inelastic labour supply and departs further from price-taker behavior. Property (ii) implies the firm specific labour supply becomes more elastic as a rival firm pays higher wage (and employs more workers). This underlies strategic interaction in labour market competition. These properties hold for some commonly used models, such as the wage posting model as well as the quantity competition model.

Let the product market demand system be described by $p_i = P_i(\mathbf{q})$, where p_i and $P_i(\mathbf{q})$ are the product price and residual inverse demand function for firm i, which depend on output vector $\mathbf{q} \equiv (q_1, \cdots, q_n)$. Each firm's residual demand elasticity, $\epsilon_i \equiv -\frac{1}{\frac{\partial P_i(\mathbf{q})}{\partial q_i}\frac{q_i}{P_i(\mathbf{q})}} \in (1, \infty)$, is finite, and has the following properties: (iii) $\frac{\partial \epsilon_i}{\partial q_i} < 0$, (iv) $\frac{\partial \epsilon_i}{\partial q_j} > 0$. Property (iii) implies that large firm faces more inelastic demand and departs further from price-taker behavior. Property (iv) means the residual demand becomes more elastic as a rival firm produces more. These properties are satisfied by many commonly used models, such as linear demand functions and homogeneous good market with constant price elasticity of demand.

Central to our analysis is the notion of short-run production function, denoted by $F_i(x_i, l_i)$, where x_i is the intermediate input, and l_i is flexible labour input. $F_i(x_i, l_i)$ depends on the fixed (tangible and intangible) capital, which is not explicitly modelled in the static setting, as it can only be changed in the long-run. Instead, we assume $F_i(x_i, l_i) = A_i f(x_i, l_i)$, where

[14]See Autor et al. (2020) for an influential study of the rise of superstar firms.

A_i is the predetermined Hicks technology coefficient, which is a simple way to capture the short-run multi-factor productivity (SMFP).[15]

Let the market for intermediate input be perfectly competitive with constant price p_X. The conditional short-run profit maximisation problem is:

$$\max_{q_i, x_i, w_i, q_i \leq F_i(x_i, L_i(\mathbf{w}))} \pi_i(\mathbf{q}, \mathbf{w}, x_i) = \underbrace{P_i(\mathbf{q})\, q_i}_{R_i} - \underbrace{[w_i L_i(\mathbf{w}) + p_X x_i]}_{C_i},$$

where π_i denotes gross profit, R_i and C_i respectively denote revenue and variable cost. The Nash equilibrium of the game is such that all firms conditionally maximise their gross profits.

2.1 Definitions of Marginal Cost and Market Power Indices

Whereas in the standard Cournot model, the definition of marginal cost can be trivially derived, in our canonical JOOM the definition of marginal cost has to be derived from the primitive short-run production function through Nash equilibrium analysis.

Denote the Lerner index, a standard measure of markup power, by $\rho_i \equiv \frac{p_i - MC_i}{p_i}$, and the wage markdown power index by $\chi_{Li} \equiv \frac{MRPL_i - w_i}{w_i}$, where MC_i and $MRPL_i$ denote marginal cost and marginal revenue product of labour, with their definitions derived from solving the following Lagrangian maximisation problem:

$$\max_{q_i, x_i, w_i, \lambda_i} \mathfrak{L}_i = \underbrace{P_i(\mathbf{q})\, q_i}_{R_i} - \underbrace{[w_i L_i(\mathbf{w}) + p_X x_i]}_{C_i} - \lambda_i [q_i - F_i(x_i, L_i(\mathbf{w}))], \tag{1}$$

as summarised by the lemma below.

Lemma 1 *At each Nash equilibrium point, the Lagrangian multiplier λ_i equals both the marginal revenue and marginal cost, i.e.,*

$$\lambda_i = MR_i \equiv \frac{\partial R_i}{\partial q_i} = MC_i \equiv \frac{\partial C_i}{\partial q_i}. \tag{2}$$

[15]We deliberately choose the term SMFP to differentiate from the familiar notion of total factor productivity (TFP) because fixed capital is not an argument of the short-run production function. SMFP is a residual of output unexplained by flexible labour and intermediate input, and it captures the contribution of all forms of fixed capital (including knowledge capital intangible or embodied in physical capital) some of which are notoriously difficult to measure directly. SMFP is also more relevant than TFP to a firm's short-run competitiveness.

The markup and markdown power indices ρ_i and χ_{Li} satisfy the following equations:

$$\rho_i = \frac{p_i - \lambda_i}{p_i}, \tag{3}$$

$$\chi_{Li} = \frac{\lambda_i \frac{\partial F_i(x_i, L_i(\mathbf{w}))}{\partial l_i} - w_i}{w_i}. \tag{4}$$

Eq. (2) - (4) underpin the definitions of marginal revenue, marginal cost, Lerner index and the markdown power index. The gross profit margin δ_i, one of our preferred candidates as measure of overall market power[16], is defined by:

$$\delta_i \equiv \frac{p_i - AVC_i}{p_i} \equiv \frac{p_i q_i - VC_i}{p_i q_i}, \tag{5}$$

where AVC_i and VC_i respectively denote average variable cost and variable cost. Note that in this paper we can use C_i and VC_i interchangeably, and that δ_i is also the producer surplus to revenue ratio.

2.2 Basic Equations of Market Powers

We aim to find an index of overall market power that, differently from the Lerner index, is robust to how the markup and markdown powers interact. For this purpose we analyse the coordination between markup and markdown powers within each firm , and how this affects the gross profit margin, as elucidated in the following theorem:

Theorem 2 *For the joint oligopoly-oligopsony model (JOOM), the markup and markdown power indices ρ_i and χ_{Li} satisfy the following (well known) equations:*

$$\frac{1}{1-\rho_i} = \frac{\theta_{Xi}}{\phi_{Xi}}, \tag{6}$$

$$\frac{1+\chi_{Li}}{1-\rho_i} = \frac{\theta_{Li}}{\phi_{Li}}, \tag{7}$$

where $\phi_{Xi} \equiv \frac{p_X x_i}{p_i F_i}$ and $\phi_{Li} \equiv \frac{w_i L_i}{p_i F_i}$ are respectively the revenue shares of intermediate input and labour, and $\theta_{Xi} \equiv \frac{\partial F_i(x_i, L_i(\mathbf{w}))}{\partial x_i} \frac{x_i}{F_i}$ and $\theta_{Li} \equiv \frac{\partial F_i(x_i, L_i(\mathbf{w}))}{\partial L_i} \frac{L_i}{F_i}$ are respectively the output elasticities of intermediate input and labour. Define $\Theta_i \equiv \theta_{Xi} + \theta_{Li}$ as the short-run elasticity

[16] The gross profit share of value added, which is highly positively correlated with gross profit margin, is an attractive alternative with two desirable features: (i) Its aggregation, using value added share as weight, is easier to implement and interpret. (ii) As we will show in Section 3.4, it is also a measure of income distribution, indicative of close link between market power and income distribution.

of scale. The effects of Θ_i, ρ_i and χ_{Li} on δ_i are captured by the basic equation below:

$$\delta_i = (1 - \Theta_i) + \Theta_i \rho_i + \frac{\theta_{Li} (1 - \rho_i) \chi_{Li}}{(1 + \chi_{Li})}, \tag{8}$$

implying δ_i is an increasing function of both ρ_i and χ_{Li}, with $\frac{\partial \delta_i}{\partial \rho_i} = \frac{\theta_{Li}}{1+\chi_{Li}} + \theta_{Xi} > 0$, $\frac{\partial \delta_i}{\partial \chi_{Li}} = \frac{\theta_{Li}(1-\rho_i)}{(1+\chi_{Li})^2} > 0$.

In the literature on production approach to estimation of market power, eq. (6) and (7) are standard. Our unique contribution is in eq. (8), which sheds light on the relationship - or entanglement - among firm's gross profit margin δ_i, short-run returns to scale, measured by Θ_i, and market power indices (ρ_i, χ_{Li}). For example, if perfect competition is assumed for all markets, then all variation in δ_i must be attributed to variation in firm specific short-run returns to scale Θ_i, which is an obviously stringent restriction. In contrast, if (approximate) CRS is assumed, i.e., $\Theta_i = \Theta \approx 1$, then the gross profit margin δ_i is a natural index of overall market power because δ_i is strictly increasing in both ρ_i and χ_i, and can therefore subsume the effects of both firm's seller power in product market and buyer power in labour market. From data and measurement point of view, an additional advantage of using δ_i over (ρ_i, χ_{Li}) is that it can be measured directly in "standard" accounting datasets.

Additionally, eq. (8) can shed light on the implications of alternative assumptions. For instance, several empirical studies claim that the short-run production function is characterised by DRS. To understand its implication, assume $\Theta_i = \Theta < 1$, i.e., the short-run elasticity of scale is uniform for an industry, and obeys DRS. Then eq. (8) implies that gross profit margin δ_i contains a uniform component $(1 - \Theta) > 0$, which captures the competitive rent, and is the same for all firms, irrespective of their productivity. For the canonical model where $\rho_i \geqslant 0$ and $\chi_{Li} \geqslant 0$, $(1 - \Theta)$ sets a positive lower bound to δ_i. This makes survival of inefficient firms easier because they can take advantage of short-run DRS by operating at small scale. In general, the possibility of DRS causes an entanglement: when a firm has a positive value of δ_i one has to ask whether it is caused by market power, or DRS. What is the main reason that prevents the most efficient firm from increasing the scale of production further? Is it the short-run DRS, or market power motive, as captured either by the concavity of its revenue function, or by the convexity of its labour expenditure function? If this entanglement is not addressed, the estimation of the short-run scale elasticity Θ_i and market power indices (ρ_i, χ_i) might be susceptible to biases that are not well understood.

2.3 A Theory of (Approximate) Short-run CRS

In this section we articulate our support for the (approximate) short-run CRS assumption. Most studies of returns to scale in the literature analyse long-run production function for which all factors of production are flexible, making the notion of fixed inputs irrelevant. The production approach to estimation of production function and market power pioneered by Hall (1988) is concerned with the implications of increasing returns to scale (IRS) for long-run production. In contrast, here we focus on the returns to scale for short-run production, which does not include fixed inputs as arguments. The short-run production is the counterpart to the variable cost function, but has the advantage of being immune from the effects of input prices that the latter function lacks. This approach finds its theoretical underpinnings - similarly for variable cost function - in the study of strategic interaction among competing firms, where the investment in the fixed inputs is sunk and has commitment value. To avoid confusion between the short-run and long-run increasing returns to scale, it is useful to note how the former affects the latter. For example, in the endogenous sunk cost literature (see Sutton, 1991, 1998), the unit variable cost (same as the marginal cost) is typically assumed to be invariant to output level, i.e. short-run CRS, and the fixed input is endogenous in the long-run. This shows that the short-run CRS assumption implies long-run IRS.[17]

At least since Sraffa's (1926) article, economists have recognised that the assumption of perfect competition is incompatible with the notion of long-run IRS. To support our assumption of short-run CRS, we need to define observationally non-equivalent hypotheses derived from two competing theories of competition, and validate or refute them with data. For this purpose it is useful to conceptualise some notions of returns to scale that connect the short-run returns to scale and the state of competition. We therefore define strong short-run DRS as sufficiently small value of $\Theta < 1$ such that long-run DRS with perfect competition is implied. Similarly, we define strong short-run IRS as sufficiently large value of $\Theta > 1$ such that long-run IRS with natural monopoly-monopsony is implied. We define the remainder scenario which is neither strong short-run DRS nor strong short-run IRS, as approximate CRS. Our reasoning in using this partition is that the goal of the empirical analysis of short-run returns to scale is not a precise estimate of parameter Θ but rather a reasonable discrimination among strong DRS with perfect competition, strong IRS, and approximate CRS. Different from the older industrial organisation literature which is more interested in

[17] For illustration, assume the long-run production is $F(K, x, l) = K^{\theta_K} f(x, l)$, where K is fixed capital, $\theta^K > 0$ is the output elasticity of K, and $Af(x, l) = A(x^\alpha l^{1-\alpha})^\Theta$ is the short-run production function, with short-run Hicks technology coefficient $A = K^{\theta_K}$ and short-run elasticity of scale Θ. The long-run elasticity is therefore $\Theta + \theta^K$. Note that the existence of the fixed input and the short-run CRS implies long-run IRS, i.e., $\Theta = 1$ implies $\Theta + \theta^K > 1$, but the converse is not true. The former condition is stronger than the latter.

the relationship between economies of scale and market concentration (see Sutton 1991),[18] our interest hereby lies in the relationship between returns to scale and the transmission mechanisms from heterogeneity in firm productivity to the dispersions in firm-level market power and value added per worker.

Hereafter we first explain why short-run DRS combined with perfect competition and (strong) IRS combined with natural monopoly-monopsony are untenable for typical manufacturing industries in our data set. We then qualify an upper bound of the possible approximation errors when (approximate) short-run CRS is adopted for empirical analysis.

2.3.1 Why Not (Strong) DRS?

We first discuss the theoretical implication of the alternative assumption of DRS when competing firms have heterogeneous multi-factor productivity A_i. Intuitively, DRS implies that the advantage of a high-productivity firm over a low-productivity rival is offset when the former produces at a larger scale than the latter because of dis-economies of scale. To sharpen the idea, assume that the number of competing firms in the market is sufficiently large and that the short-run production functions obey strong DRS to the extent that price-taking assumption is a good approximation of firms' behaviour.[19] An immediate implication is formalised below:

Corollary 3 *If the short-run production functions $F_i(x_i, l_i)$ have decreasing returns to scale, such that i.e., $\Theta_i = \Theta < 1$, and all firms are price takers in all markets, i.e., $\rho_i = \chi_{Li} = 0$, then*

$$\delta_i = (1 - \Theta) > 0. \tag{9}$$

This result shows that strong short-run DRS tends to equalise the gross profit margin across firms which are heterogeneous in Hicks technology coefficient A_i. The effects of this force is not confined to the gross profit margin. In Online Appendix B.1 we use CES production function to show that in the perfectly competitive equilibrium with short-run DRS, the marginal costs as well as the value added per worker and gross profit margin equalise among

[18]The older market structure-centred literature defines and measures economics of scale in terms of cost function and the minimum efficient scale of production (MES). In contrast, our approach to returns to scale focuses on (partial) identification of short-run returns to scale and its implication for state of competition or existence of market power.

[19]The reason we assume strong DRS is to analyse the possibility that DRS is the primary cause of positive value of gross profit margin and imperfect competition in output and labour markets are only a secondary cause, and refute it. There is no need for us to refute the alternative case that DRS is only a secondary cause of positive value of gross profit margin while imperfect competition in output and labour markets are the primary cause, because we argue for this case. That is, we treat weak DRS as approximate CRS. Our main goal is to get the first order approximation right, and to ensure that the approximation error or bias is understood and moderated.

all firms despite the dispersion in the distribution of firms' multi-factor productivity. This implication is inconsistent with the empirical fact that the distributions of firms' gross profit and value added per worker are highly dispersed for industries that include large number of firms, as shown in Online Appendix D.1 (Figure 6).

If for the sake of simplicity, we were to choose the primary reason, between short-run DRS and market power motive, to explain both the dispersion in δ_i distribution and the productivity-leader firms' restrain from further increasing production scale, the choice cannot be short-run DRS, since it is a force for equalisation of δ_i across firms in the absence of market power motive.

2.3.2 Why Not (Strong) IRS?

In the case of IRS, the most productive firm tends to scale up its production to take advantage of its superior productivity, and consequently amplifies the advantage to the extent that a natural monopoly-monopsony emerges. As a result, market concentration is extreme and there is no dispersion in surviving firms' gross profit margin or value added per worker. Again, this implication is inconsistent with observed high dispersion in firms' value added per worker and gross profit margin for most industries in our data (presented in Online Appendix D.1, Figure 6).

2.3.3 (Approximate) CRS and Approximation Errors

Having rejected both (strong) DRS and IRS, we turn to the remainder. Given the pivotal role that (approximate) short-run CRS hypothesis plays in our empirical analysis, the corollary below formalises some of its implications:

Corollary 4 *If the short-run production functions $F_i(x_i, l_i)$ have constant returns to scale, i.e., $\Theta_i = 1$, then the gross profit margin δ_i has the following properties:*

$$\begin{aligned}
\delta_i &= \rho_i + \frac{\theta_{Li}(1-\rho_i)\chi_{Li}}{1+\chi_{Li}}, && (10)\\
\delta_i &= 0 \text{ } \textit{if } \rho_i = \chi_{Li} = 0, && (11)\\
\delta_i &= \rho_i \text{ } \textit{if } \chi_{Li} = 0. && (12)
\end{aligned}$$

Remarkably, the variables $(\delta_i, \rho_i, \chi_{Li})$ in eq. (10) manifest that market power does not only have an overall level, but also a structure that reflects its distribution between product and labour markets.

It is important to note that results in eq. (10) - (12) apply not only to the canonical JOOM, but also to a much broader setting, as we demonstrate in Theorem 6. We also note that the CRS hypothesis used in Corollary 4 is a necessary and sufficient condition for the results in eq. (10) - (12). Since we only provide support for the assumption of approximate short-run CRS, we cannot rule out existence of approximation error caused by imposing the restriction: $\Theta_i = \Theta = 1$. Eq. (8) implies that, for all firms with $\chi_{Li} \geqslant 0$ in an industry, the inequalities $\delta_i \geqslant (1 - \Theta)$ hold, and imply that the minimum of δ_i for all firms with $\chi_{Li} \geqslant 0$ in the industry, δ_{min}, sets an upper bound to $(1 - \Theta)$. The bottom right panel of Figure 8 in Online Appendix D.2 presents the distribution of δ_{min}, for firms with $\chi_{Li} > 0$, across different industry-year. The mean value of δ_{min} in our preferred distribution (bottom right panel) is just above 0.10, indicating that the approximate CRS is a reasonable assumption. To put this assessment in context of the literature, Table 6 in Section 4.2.1 compares our empirical results based the (approximate) short-run CRS assumption, and counterpart results from Gandhi, Navarro and Rivers (2020), thereby reinforcing the validity of this approximation.

2.4 Markdown Power and Input Mix Decision

Building on the results above, the current and next sections lay the theoretical foundation for a novel method for estimating the short-run production function. By imposing the Cobb-Douglas restriction, the short-run production function parameters can be expressed in terms of A_i, α and Θ: $F_i(x_i, l_i) = A_i x_i^{\theta_X} l_i^{\theta_L} = A_i \left(x_i^{\alpha} l_i^{1-\alpha}\right)^{\Theta}$, where $\alpha \equiv \frac{\theta_X}{\Theta}$. Furthermore, by imposing the short-run CRS assumption $\Theta = 1$, α is the only technology parameter that needs to be estimated, with common output elasticities of inputs given by $\theta_X = \alpha$ and $\theta_L = 1 - \alpha$. The firm-specific parameters to be estimated are a proxy measure of A_i, and market power indices ρ_i and χ_{Li}.

Before delving further into our analysis, for the sake of clarity, we present in Table 1 the names and notations of the key variables used, and what they measure.

We note that the short-run production function captures the technological possibility of substitution of intermediate input for labour. The variable cost share of labour, $\psi_{Li} \equiv \frac{w_i l_i}{p_X x_i + w_i l_i}$, can be used to measure the state of the firm's input mix decision. In the JOOM, the wage markdown power of a firm, χ_{Li}, provides an incentive to substitute intermediate input for labour, which a price-taking firm in the labour market lacks. That is, in comparison with a wage-taking firm, a firm with wage markdown power is inclined to substitute intermediate input for labour. The following theorem captures this insight.

Theorem 5 *Let the short-run production function in the JOOM be Cobb-Douglas* $F_i(x_i, l_i) = A_i x_i^{\alpha} l_i^{1-\alpha}$ *for all* $i \in \{1, \cdots, n\}$, *and firms* i *and* i' *in the JOOM be such that* $\chi_{Li} \geqslant \chi_{Li'} = 0$, *then*

$$\psi_{Li} = \frac{1-\alpha}{1+\alpha\chi_{Li}} \leqslant \psi_{Li'} = 1 - \alpha. \tag{13}$$

Table 1: Key variables of interest

Name	Notation	Remark
Hicks technology coefficient (short-run)	A	Unobservable short-run multi-factor productivity
A proxy measure of A	Λ	Λ tracks A: $\Lambda = \frac{P}{(p_X)^\alpha}A$; can be estimated
Product price	P	Unobservable
Intermediate input price	p_X	Unobservable
Output elasticity of intermediate input	α	Can be estimated
Value added per worker	ω	A proxy for short-run productivity A
Gross profit margin	δ	An overall market power index
Marginal cost	MC	Unobservable
Markup power (Lerner) index	ρ	$\rho = \frac{P-MC}{P}$, $\frac{1}{1-\rho} = \frac{P}{MC}$ is markup; can be estimated
Marginal revenue product of labour	$MRPL$	Can be estimated
Wage	w	
Wage markdown power index	χ_L	$\chi_L = \frac{MRPL-w}{w}$; can be estimated
Labour share of variable cost	ψ_L	A measure of input mix

Note: The details about how Λ, ρ and χ_L are estimated are given in eq. (17), (16) and (15) respectively. $MRPL$ can be backed out from χ_L.

Furthermore, the expenditure ratio between intermediate input and labour is:

$$\frac{p_X x_i}{w_i l_i} = \frac{\alpha}{1-\alpha} + \frac{\alpha}{1-\alpha}\chi_{Li}, \tag{14}$$

which is equivalent to

$$\chi_{Li} = \frac{1-\alpha}{\alpha}\left(\frac{1}{\psi_{Li}} - \frac{1}{1-\alpha}\right) = \frac{1-\alpha}{\alpha}\frac{p_X x_i}{w_i l_i} - 1, \tag{15}$$

and the markup power index is given by

$$\rho_i = 1 - \frac{\phi_{Li}}{\alpha}\frac{p_X x_i}{w_i l_i} = 1 - \frac{\phi_{Xi}}{\alpha}. \tag{16}$$

Finally, for given α, there exists a proxy measure of A_i, defined by the following identities:

$$\Lambda_i \equiv \frac{(w_i)^{1-\alpha}}{\phi_{Li}\left(\frac{p_X x_i}{w_i l_i}\right)^\alpha} \equiv \frac{p_i}{(p_X)^\alpha}A_i. \tag{17}$$

In comparison with a rival who does not have wage markdown power, a firm tends to have lower labour share of variable cost. This implies that each firm's variable cost share of labour ψ_L can inform its wage markdown power χ_L. Eq. (14) predicts that the input expenditure ratio $\frac{p_X x_i}{w_i l_i}$ and ψ_{Li} depend on wage markdown power index χ_{Li} (given that $\frac{p_X x_i}{w_i l_i} \equiv \frac{1}{\psi_{Li}} - 1$). If there is substantial dispersion in χ_{Li} among firms in an industry, then eq. (14) and (15) predict substantial dispersion in $\frac{p_X x_i}{w_i l_i}$ and ψ_{Li}. Thus, $\frac{p_X x_i}{w_i l_i}$ (and equivalently ψ_{Li}) can be used for identifying χ_{Li} if the parameter α has been identified. Similarly, ϕ_{Li} and $\frac{p_X x_i}{w_i l_i}$ can be used to pin down ρ_i as well as Λ_i if α has been identified. The first identity in eq. (17) defines Λ_i. The second shows how Λ_i is related to and can serve as a proxy for A_i, as Λ_i tracks the unobservable A_i proportionally, with an unobservable coefficient $\frac{p_i}{(p_X)^\alpha}$. The usefulness of Λ_i for empirical analysis is demonstrated in Section 4.4.

2.5 A Novel Approach for Estimating Production Function

Eq. (15) is useful to guide the measurement of both the technology parameter α and the markdown power index χ_{Li}. The key idea is to estimate α using the competitive fringe firms with no market power, and then compute χ_{Li} for the other firms in our dataset.

Theoretically, we define firm j as a competitive fringe firm in a given industry if $\chi_{Lj} = \rho_j = 0$. The notion of competitive fringe firm is useful for the estimation of parameter α because for such a firm $\alpha = 1 - \psi_{Lj}$ and $\frac{\alpha}{1-\alpha} = \frac{p_X x_j}{w_j l_j}$. By eq. (12) of Corollary 4, under short-run CRS, a competitive fringe firm necessarily has $\delta_j = 0$. For quantitative and empirical analysis, we use the weaker condition $\delta_j \approx 0$. Operationally, we identify as competitive firms those fringes with values of δ in the interval $[\delta_{\min}, \delta_{\min} + 0.05]$, where $\delta_{\min}$ denotes the minimum value of δ in the industry. We further identify as the "representative" fringe firm the one with the median[20] value of ψ_L among all those fringe firms, and then we estimate α based on this "representative"firm.[21] That is,

$$\alpha = 1 - median \left\{ \psi_{Lj} : \delta_{\min} \leq \delta_j \leq \delta_{\min} + 0.05 \right\}, \tag{18}$$

Once α is estimated, we can estimate χ_{Li}, ρ_i and Λ_i for each firm in the industry. This novel approach for estimating the production function parameter α, the markdown power index χ_L, the Lerner index ρ, and the proxy measure of short-run MFP Λ is implemented in Section 4.2.1 with a panel data set containing UK manufacturing firms. Here we make a general remark that the data requirement for this method is modest. Implementation does not require observation of prices and quantities of output and intermediate inputs. It is sufficient to observe revenue, total expenditure on flexible inputs, cost and employment of (ideally flexible) labour, which are usually available in standard accounting datasets.[22]

[20]The median is preferred to the mean as a measure of central location because only the median gives invariant results regardless whether we estimate α or $\frac{\alpha}{1-\alpha}$ as the basis. Even so, our results are similar using the specification with the mean.

[21]Since the procedure we use to identify competitive fringe firms is unaffected by the assumption of CRS, our estimations of α and χ_{Li} are not sensitive to the CRS assumption either. Accordingly, there is no circular reasoning fallacy in using χ_{Li} as a control to validate the CRS assumption, as we do in Section 2.6.

[22]In contrast, prevalent econometric methods of estimating production function, such as the control func-

Based on this method, the wage markdown power index χ_{Li} is calculated by

$$\chi_{Li} = \frac{\frac{p_X x_i}{w_i l_i}}{median\left\{\frac{p_X x_j}{w_j l_j} : \delta_{\min} \leq \delta_j \leq \delta_{\min} + 0.05\right\}} - 1. \tag{19}$$

Eq. (19) implies that χ_{Li} is positive (negative) when firm i has a labour cost share above (below) the median labour cost share of the fringe firms. This suggests that the firm pays its employees less (more) than their marginal revenue product of labour, and it is inclined toward substitution of intermediate input for labour (substitution of labour for intermediate input) in input-mix decision. The theoretical underpinning of the systematic differences in the signs of χ_{Li}, and their relation to labour market rent-sharing mechanisms are treated in Section 3.

2.6 Generalisation of Basic Equations and Interpretation of χ_L

Theorem 2 is based on the canonical JOOM, according to which the variable $\chi_{Li} \geqslant 0$ measures firms' buyer power in labour market. However, to accommodate a model where χ_{Li} can take negative values in the presence of workers' countervailing power, as developed in Section 3, a broadened interpretation of Theorem 2 is warranted. Furthermore, a broader interpretation of eq. (8) from Theorem 2 is required to allow for possible alternative explanations of ρ_i and χ_{Li}. For example, suppose the short-run production function obeys DRS, each firm is a price taker in all markets, and labour is no longer a flexible input because of adjustment cost (as in Hopenhayn and Rogerson (1993), and Gandhi, Navarro and Rivers (2020)), then ρ_i merely captures the effect of DRS (instead of markup), and χ_{Li} captures the effects of DRS and distortion caused by labour adjustment cost (instead of buyer or seller power in labour market). Taking another example, whereas labour is a flexible input and short-run CRS holds, but the true values of output elasticities θ_{Xi} and θ_{Li} are, however, heterogeneous among firms. In this case, ρ_i and χ_{Li} may capture measurement errors of respectively θ_{Xi} and θ_{Li}, and give rise to spurious patterns of imperfect competition.

To accommodate the above generalised interpretations of the basic equations from Theorem 2 we first extend the definition of marginal cost to

$$MC_i \equiv \frac{p_X}{\frac{\partial F_i}{\partial x_i}}. \tag{20}$$

This is the ratio between changes of cost and output caused by an infinitesimal change

tion approach (Olley and Pakes 1996, Levinsohn and Petrin 2003, Ackerberg et al. 2015), require data on input and output quantities, which are not available from typical accounting data.

of intermediate input dx_i, evaluated at the conditionally optimal level of x_i, taking other inputs as given. We then note that the extended definitions of ρ_i and χ_{Li} satisfy the following equations:

$$\rho_i \equiv \frac{p_i - MC_i}{p_i}, \tag{21}$$

$$\chi_{Li} \equiv \frac{MR_i \frac{\partial F_i}{\partial L_i} - w_i}{w_i}. \tag{22}$$

It is commonly assumed that the flexible intermediate input is optimally chosen. In this case marginal revenue and marginal cost must equalise. Consequently, the results stated in Theorem 2 extend to these scenarios as long as the marginal cost MC_i, Lerner index ρ_i and variable χ_{Li} are defined by eq. (20)-(22).

Theorem 6 (Extension Theorem) *Let marginal cost MC_i, Lerner index ρ_i and variable χ_{Li} be defined by eq. (20)-(22), and let equation $MR_i = MC_i$ hold. Then the three basic equations: eq. (6)-(8) in Theorem 2 hold, i.e.,*

$$\frac{1}{1-\rho_i} = \frac{\theta_{Xi}}{\phi_{Xi}},$$

$$\frac{1+\chi_{Li}}{1-\rho_i} = \frac{\theta_{Li}}{\phi_{Li}},$$

$$\delta_i = (1-\Theta_i) + \Theta_i \rho_i + \frac{\theta_{Li}(1-\rho_i)\chi_{Li}}{(1+\chi_{Li})}.$$

As alluded previously, the generalisation of eq. (6)-(8) also generalises the interpretation of our variable χ_{Li}, defined by eq. (19) in Section 2.5. Our preferred interpretation of χ_{Li} is that it measures either firms' buyer power in labour market or workers' countervailing seller power.

One sceptical view about this interpretation might point to the possibility that the variable χ_{Li} could be affected by modelling error caused by labour adjustment cost. For example, Gandhi, Navarro and Rivers (2020) assume labour choice is not flexible and product market is perfectly competitive, i.e., $\rho_i = 0$. In this case, by eq. (8) the variation in δ_i is (mainly) absorbed by either the variation in Θ_i, or the variation in χ_{Li}. If we further impose $\Theta_i = \Theta$ (and $\theta_{Li} = \theta_L$),[23] then the variation in δ_i is entirely caused by the variation in χ_{Li}.

But even with this alternative interpretation of the basic equations in Theorem 2, we can still use the dispersion in δ_i to support our empirical identification. Recall that in Section

[23] The assumption of $\Theta_i = \Theta$ is equivalent to the assuming homogeneity of degree Θ for short-run production function $F_i(x_i, l_i)$, that is, $F_i(x_i, l_i) = F_i(x_i/l_i, 1)(l_i)^{\Theta}$, for all i.

2.3.1 we argue that strong short-run DRS (resulting in perfect competition) is inconsistent with the dispersion in δ_i. The validity of this argument relies on the absence of any other distortions to optimal labour input choice. If such distortions were present, there could be an alternative explanation of the stylised fact of dispersions in δ_i and ω_i: the wedge caused by adjustment cost can transmit dispersion in firm productivity into dispersions in δ_i and ω_i (see Online Appendix B.2). This alternative theory, which is based on perfect competition in all markets, can be empirically discriminated from ours by controlling for χ_{Li} as illustrated formally by the following corollary.

Corollary 7 *For all i such that $\Theta_i = \Theta$, $\theta_{Li} = \theta_L$ and $\chi_{Li} = 0$,*

$$\delta_i = (1 - \Theta) + \Theta\rho_i, \tag{23}$$

$$\delta_i = (1 - \Theta) \;\; \textit{if } \rho_i = 0. \tag{24}$$

If we have a sub-sample of firms in the same industry, with the same short-run elasticity of scale Θ and output elasticity of labour θ_L, such that $\chi_{Li} = 0$, then the source of variation in δ_i is the variation in ρ_i (eq. (23)). The assumption of perfect competition in all markets implies $\rho_i = 0$, and hence the lack of dispersion in δ_i (eq. (24)). Because in this sub-sample the variation in the labour adjustment cost (or distortion to choice of labour input) is removed, the dispersion in ω_i should also disappear (see Section B.1). In Section 4.2.1 we provide empirical evidence in support of our theory and against this alternative interpretation by studying the dispersions in δ_i and ω_i among firms with $\chi_{Li} = 0$.

Another sceptical view about our preferred interpretation of χ_{Li} relates to the concern that variables χ_{Li} and ρ_i may capture the approximation errors caused by imposing the Cobb-Douglas functional form. In this case, as long as the short-run CRS assumption is maintained, the index δ_i remains a reasonable measure of overall market power measure, robust against the possible functional form approximation error. In Section 4.4 we show that the variables δ_i and χ_{Li} together are instrumental to reasonable structural estimation of parameters of production and labour supply functions, indicating that the variable χ_{Li} is usefully informative about wage determination, and that the Cobb-Douglas functional form is a reasonable parametric approximation despite its simplicity.

3 Rent-Sharing Mechanisms and Wage Determination

In this section, we augment the baseline JOOM - i.e. the canonical JOOM with the restriction of Cobb-Douglas short-run production with constant returns to scale - to include diverse rent-sharing mechanisms with the aim of developing a general theory of market power, labour market rent sharing and income distribution.

This theory presents our preferred models and its goal is to capture both firms' buyer power and workers' countervailing seller power in labour market. It therefore supports our preferred economic interpretation for the empirical estimate of χ_L, which will be subject to validation in Section 4.

Our theory consists of three component models that happen to partition the domain of variable χ_L. Specifically, the oligopsony model (OL), the wage floor model (WF) and the efficient bargaining model (EB) together form a partition of all possible relations between wage (W) and marginal revenue product of labour ($MRPL$), as they predict the following three mutually exclusive possibilities: $W < MRPL$ (OL) matching $\chi_L > 0$, $W = MRPL$ (WF) matching $\chi_L = 0$ and $W > MRPL$ (EB) matching $\chi_L < 0$.[24] Interestingly, competitive fringe firms lie in the common boundary of these three subsets, thus representing a point of continuity among these three models. Sections 3.1, 3.2 and 3.3 below analyse wage determination in, respectively, the baseline JOOM (OL), wage floor model (WF) and the efficient wage bargaining model (EB), which correspond to rent-sharing (RS) types I, II and III, empirically defined in Section 4. A common theme of these three models is to show how the diverse rent-sharing mechanisms determine the sign of the correction term χ_L, i.e., rent-sharing type, and how RS types and χ_L interact with firm productivity, measured by value added per worker, in wage determination. Finally, In section 3.4 we develop a general theory of market powers and income distribution, as well as investigate how and why the dispersion in firms' short-run productivity causes dispersions in firms' overall market power index, value added per worker, and rent sharing outcome.

3.1 Baseline JOOM

The baseline JOOM features a Cournot oligopoly product market, an oligopsony labour market, and Cobb-Douglas short-run production functions $F_i(x_i, l_i) = A_i x_i^{\alpha} l_i^{1-\alpha}$. We follow Card et al. (2018) to model a heterogeneous job market with wage posting, and firm-specific labour supply function: $L_i(\mathbf{w}) = \mathcal{L}\frac{(w_i-b)^{\beta} a_i}{\sum_{j=1}^{n}(w_j-b)^{\beta} a_j}$, where $\mathcal{L}$ is the aggregate labour supply, b is the outside option/benefit, a_i is the firm specific amenity parameter. We note that parameter $\beta \in [0, \infty)$ measures the toughness of wage competition, with $\beta \to \infty$ representing perfect competition, and $\beta = 0$ representing independent monopsonies. This parameter will be subject to structural estimation in Section 4.3. The term $\frac{(w_i-b)^{\beta} a_i}{\sum_{j=1}^{n}(w_j-b)^{\beta} a_j}$ is the logit probability for a worker to work for firm i given the wage vector $\mathbf{w}$. The elasticity

[24] This observation has been inspired by Dobbelaere and Mairesse (2013).

of firm-specific labour supply is:

$$\epsilon_{Li} \equiv \frac{\partial L_i(\mathbf{w})}{\partial w_i}\frac{w_i}{L_i(\mathbf{w})} = \frac{\beta(1-s_{Li})w_i}{w_i - b}, \tag{25}$$

where $s_{Li} \equiv \frac{L_i(\mathbf{w})}{\mathcal{L}}$ is the labour market share of firm i, with $\lim_{s_{Li}\to 1}\epsilon_{Li} = 0$. Notice that $\lim_{\beta\to\infty}\epsilon_{Li} = \infty$, consistent with that β is the toughness of wage competition parameter. The following partial derivatives (i) $\frac{\partial\epsilon_{Li}}{\partial w_i} = \frac{-\beta\frac{\partial s_{Li}}{\partial w_i}\left(1-\frac{b}{w_i}\right)-\beta(1-s_{Li})\frac{b}{w_i^2}}{\left(1-\frac{b}{w_i}\right)^2} < 0$, with $\lim_{w_i\to b}\epsilon_{Li} = \infty$, (ii) $\frac{\partial\epsilon_{Li}}{\partial w_j} > 0$, and (iii) $\frac{\partial\epsilon_{Li}}{\partial b} > 0$, indicate that (i) large and high wage employers face more inelastic residual labour supplies, (ii) a rise of a rival firm's wage raises the residual labour supply elasticity, and (iii) an increase in outside option/benefit raises every firm's labour supply elasticity.

In the baseline JOOM, profit maximisation implies: $\chi_{Li} \equiv \frac{MRPL_i - w_i}{w_i} = \frac{1}{\epsilon_{Li}} > 0$, and the existence of firm buyer power in the labour market. Substituting this into eq. (25) results in the following wage determination equation:

$$w_i = \frac{b}{1+\beta(1-s_{Li})} + \frac{\beta(1-s_{Li})}{1+\beta(1-s_{Li})}MRPL_i, \tag{26}$$

with $b \leq w_i \leq MRPL_i$. Eq. (26) shows that the profit maximising wage w_i is a weighted average of the outside option b and the marginal revenue product of labour $MRPL_i$, with respective weights $\frac{1}{1+\beta(1-s_{Li})}$ and $\frac{\beta(1-s_{Li})}{1+\beta(1-s_{Li})}$. For $s_{Li} \to 0$, the weights become constant, and w_i tracks $MRPL_i$ linearly. This limiting result is consistent with the monopsonistic competition labour market literature (see Card et al., 2018), indicating that the JOOM is more general than the monopsonistic competition model. The generality is clearly needed to capture the dominance of superstar firms, and the dispersion of market power between the superstar and the competitive fringe firms.

In empirical analysis, the variable $MRPL_i$ is typically replaced by the value added per worker $\omega_i \equiv \frac{p_i q_i - p_X x_i}{l_i}$. Accordingly, for Cobb-Douglas production function $F_i(x_i, l_i) = A_i x_i^{\alpha} l_i^{1-\alpha}$, w_i can be expressed as weighted average of b and ω_i:[25]

$$w_i = \frac{1}{1+\beta(1-s_{Li})(1-\alpha(1-\rho_i)(1+\chi_{Li}))}b + \frac{(1-\alpha)(1-\rho_i)\beta(1-s_{Li})}{1+\beta(1-s_{Li})(1-\alpha(1-\rho_i)(1+\chi_{Li}))}\omega_i, \tag{27}$$

with $\frac{\partial w_i}{\partial\omega_i} > 0$, $\frac{\partial^2 w_i}{\partial\omega_i\partial s_i} < 0$, $\frac{\partial^2 w_i}{\partial\omega_i\partial\rho_i} < 0$ and $\frac{\partial^2 w_i}{\partial\omega_i\partial\chi_i} < 0$. For competitive fringe firms, with

[25]Note the intermediate steps: $\omega_i = p_i A_i\left(\frac{x_i}{l_i}\right)^{\alpha} - p_X\left(\frac{x_i}{l_i}\right)$ and $MRPL_i = (1-\alpha)(1-\rho_i)p_i A_i\left(\frac{x_i}{l_i}\right)^{\alpha}$ imply: $MRPL_i = (1-\alpha)(1-\rho_i)\left(\omega_i + \frac{1-\psi_{Li}}{\psi_{Li}}w_i\right)$ and $MRPL_i = (1-\alpha)(1-\rho_i)\left(\omega_i + \frac{\alpha}{1-\alpha}(1+\chi_{Li})w_i\right)$.

$s_{Li} \to 0$, $\rho_i \to 0$ and $\chi_{Li} \to 0$, the coefficient on ω_i approaches to $\frac{\beta(1-\alpha)}{1+\beta(1-\alpha)}$. It is interesting to observe that this coefficient depends both on the technology parameter of output elasticity of labour $(1-\alpha)$, and on the toughness of wage competition parameter β. In Section 4.3 we structurally estimate β from estimation of $\frac{\beta(1-\alpha)}{1+\beta(1-\alpha)}$ and independently estimated parameter $(1-\alpha)$.

3.2 Wage Floor Model

We now consider the following wage floor model inspired by Robinson (1933) and Nickell and Andrews (1983). This is one of the mechanisms that support workers' countervailing seller power. Suppose the employee union sets a wage floor $\bar{w}_i$. The firm has the "right-to-manage" and maximises profit subject to the wage floor constraint, resulting in labour demand curve intersecting the labour supply curve L_i^S at the kink point $\left(\bar{w}_i, L_i^S(\bar{w}_i)\right)$, with $\chi_i = 0$ for $w_i = \bar{w}_i$ and $l_i \leq L_i^S(\bar{w}_i)$; and $\chi_i = \frac{1}{\epsilon_{Li}}$ for $w_i > \bar{w}_i$ and $l_i > L_i^S(\bar{w}_i)$. Let $L_i^D(\bar{w}_i)$ denote the conditional profit-maximising demand of labour by the firm. The model assumes that the union is committed to labour market clearing. Thus $\bar{w}_i$ equates $L_i^D(\bar{w}_i)$ and the firm specific labour supply $L_i^S(\bar{w}_i)$, with $\frac{\partial L_i^S(\bar{w}_i)}{\partial \bar{w}_i} > 0$. The labour market clearing equation implies

$$\frac{d\bar{w}_i}{dA_i} = \frac{\frac{\partial L_i^D(\bar{w}_i)}{\partial A_i}}{\frac{\partial L_i^S(\bar{w}_i)}{\partial \bar{w}_i} - \frac{\partial L_i^D(\bar{w}_i)}{\partial \bar{w}_i}} > 0 \text{ if } \frac{\partial L_i^D(\bar{w}_i)}{\partial \bar{w}_i} < 0 \text{ and } \frac{\partial L_i^D(\bar{w}_i)}{\partial A_i} > 0. \tag{28}$$

Eq (28) implies that, under the plausible assumptions that $\frac{\partial L_i^D(\bar{w}_i)}{\partial \bar{w}_i} < 0$ and $\frac{\partial L_i^D(\bar{w}_i)}{\partial A_i} > 0$, more productive firms tend to pay higher wages.

In the current model $\chi_{Li} \equiv \frac{MRPL_i - w_i}{w_i} \leq \frac{1}{\epsilon_{Li}}$ holds. Substituting this inequality into (25), we obtain:

$$w_i \geq \frac{b}{1+\beta(1-s_{Li})} + \frac{\beta(1-s_{Li})}{1+\beta(1-s_{Li})} MRPL_i. \tag{29}$$

The profit maximising wage w_i weakly exceeds a weighted average of the outside option b and the marginal revenue product of labour $MRPL_i$, with respective weights being $\frac{1}{1+\beta(1-s_{Li})}$ and $\frac{\beta(1-s_{Li})}{1+\beta(1-s_{Li})}$.

For the current model with the Cobb-Douglas production function $F_i(x_i, l_i) = A_i x_i^{\alpha} l_i^{1-\alpha}$, inequality (29) and $\chi_i = 0$ imply:

$$w_i \geq \frac{1}{1+\beta(1-s_{Li})(1-\alpha(1-\rho_i))} b + \frac{(1-\alpha)(1-\rho_i)\beta(1-s_{Li})}{1+\beta(1-s_{Li})(1-\alpha(1-\rho_i))} \omega_i. \tag{30}$$

The weak inequality (30) holds with equality for $\rho_i = 0$ and $s_{Li} = 0$. Furthermore, for competitive fringe firms with $\delta_i \to 0$ and $s_{Li} \to 0$, the coefficient on ω_i approaches to

$\frac{\beta(1-\alpha)}{1+\beta(1-\alpha)}$, the same as for RS type I, thus showing the continuity of OL and WF at the point of $\delta_i \to 0$.

3.3 Efficient Bargaining Model

In this section we follow Crépon et al. (2005) to extend the single-input efficient bargaining model *à la* McDonald and Solow (1981) to joint-input production with labour and intermediate input. This is our workhorse model to capture workers' countervailing seller power in labour market.

Suppose the workers collectively bargain with the firm over both the level of employment l_i and wage w_i, with the preference to maximise worker surplus $l_i (w_i - b)$, where b is the reservation wage. The firm's preference is to maximise short-run profit $R_i - w_i l_i - p_X x_i$ (i.e., producer surplus), where $R_i = P_i(\mathbf{q})\, q_i$ is the firm revenue, and $q_i = F_i(x_i, l_i)$ is the output determined by the production function. Therefore the objective of *de facto* joint decision by the firm and workers departs from profit maximisation. The outcome is formalised by the Pareto efficient (extended) Nash bargaining solution, which solves the following maximisation problem: $\max_{w_i,l_i,x_i} [l_i (w_i - b)]^{\eta_i} [R_i - w_i l_i - p_X x_i]^{1-\eta_i}$, where $\eta_i \in [0, 1]$ is the workers' bargaining power coefficient. If η_i is treated as a free parameter then the locus of the solution to the above maximisation problem forms the contract curve, or the set of Pareto efficient outcomes, and hence the name 'efficient bargaining'. The first order conditions include:

$$\frac{\partial R_i}{\partial x_i} = p_X, \tag{31}$$

$$w_i = b + \frac{\eta_i}{1-\eta_i}\frac{R_i - w_i l_i - p_X x_i}{l_i}, \tag{32}$$

$$w_i = \frac{\partial R_i}{\partial l_i} + \frac{\eta_i}{1-\eta_i}\frac{R_i - w_i l_i - p_X x_i}{l_i}. \tag{33}$$

Proposition 8 *The efficient bargaining model satisfies the following equations:*

$$\frac{\partial R_i}{\partial l_i} = b, \tag{34}$$

$$w_i = (1-\eta_i)\, b + \eta_i \omega_i, \tag{35}$$

$$\chi_{Li} = \frac{\eta_i (b - \omega_i)}{\eta_i \omega_i + (1-\eta_i)\, b} \leqslant 0. \tag{36}$$

For the Cobb-Douglas production function $F_i(x_i, l_i) = A_i x_i^{\alpha} l_i^{1-\alpha}$, the input mix $\frac{l_i}{x_i}$ is a con-

stant given by

$$\frac{l_i}{x_i} = \frac{1-\alpha}{\alpha}\frac{p_X}{b} \geqslant \frac{1-\alpha}{\alpha}\frac{p_X}{w_i}. \tag{37}$$

An insightful interpretation of the extended Nash bargaining solution is that employee unions effectively impose two restrictions on wage and employment. The first is the wage floor constraint: $w_i \geq b + \eta_i (\omega_i - b)$. The second is the labour input floor constraint (a 'feather-bedding' or 'manning' rule): $\frac{l_i}{x_i} \geq \frac{1-\alpha}{\alpha}\frac{p_X}{b}$, which is invariant to A_i. This rent-sharing mechanism can then be reinterpreted as if the firm is a profit maximiser who faces these two constraints, both of which are binding in equilibrium and resulting in eq. (35) and (37). In comparison to the WF model, the union has one more instrument, which can be used to quote a higher wage floor without inducing the firm to substitute intermediate input for labour, leaving the input mix $\frac{l_i}{x_i}$ fixed and equal to that of the competitive fringe firms. The firm is only free to change the scale of production.[26, 27]

Recall that the (extended) Nash bargaining solution is Pareto efficient and therefore (ω_i, l_i) is on the contract curve. The precise location on the contract curve is determined by the bargaining power parameter η_i. The analysis so far does not tell us how η_i is determined, therefore leaves η_i as a free parameter. Hereafter, we use the Cobb-Douglas production function, the information contained in the labour supply function and the bargaining solution to show the determination of η_i in the close vicinity of $\delta_i = 0$, i.e., among approximately competitive fringe firms.[28]

For the current model, $\chi_{Li} = \frac{MRPL_i - w_i}{w_i} \leq \frac{1}{\epsilon_{Li}}$ for arbitrary value of $\beta > 0$. That is, the curvature of the firm-level labour supply function has no effect. Inequality (29) also holds, and we can apply the relations $1+\chi_{Li} = \frac{MRPL_i}{w_i} = \frac{b}{w_i}$ to it to derive the following inequalities:

$$\begin{aligned}
1 &\geq \frac{1}{1+\beta(1-s_{Li})\left(1-\alpha(1-\rho_i)\frac{b}{w_i}\right)}\frac{b}{w_i} + \frac{(1-\alpha)(1-\rho_i)\beta(1-s_{Li})}{1+\beta(1-s_{Li})\left(1-\alpha(1-\rho_i)\frac{b}{w_i}\right)}\frac{\omega_i}{w_i},\\
w_i &\geq \frac{1+\alpha(1-\rho_i)\beta(1-s_{Li})}{1+\beta(1-s_{Li})}b + \frac{(1-\alpha)(1-\rho_i)\beta(1-s_{Li})}{1+\beta(1-s_{Li})}\omega_i.
\end{aligned}$$

Since the second inequality above holds for arbitrary value of β, it also holds for the limit $\beta \to \infty$, implying

$$w_i \geq \alpha(1-\rho_i)b + (1-\alpha)(1-\rho_i)\omega_i, \tag{38}$$

[26] Although the firm is also free to increase the input ratio $\frac{l_i}{x_i}$, but that is suboptimal.

[27] This example shows that the estimation of production function is biased if the labour market rent-sharing institution is ignored and the observed input ratio is entirely attributed to features of the production function.

[28] In general, the determinants of η_i include the revenue function $R_i(q_i)$, the labour supply function $L_i(w_i)$, and firm short-run productivity parameter A_i.

with equality holding if $\rho_i = 0$.

For inequality (38), as $\rho_i \to 0$ the coefficient on ω_i converges to the output elasticity of labour $(1-\alpha)$.[29] Intuitively, the toughness of wage competition parameter β shapes firms' buyer power in labour market. Because the efficient bargaining mechanism generates seller power for workers to counterbalance the buyer power of firms, it effectively neutralises the effect of parameter β. Furthermore, the limit value of $(1-\alpha)$, which is the labour cost share of the competitive fringe firm, sets a maximal lower bond to this type of firms labour cost share ψ_i. These results imply an important difference between WF and EB: for the former the coefficient on ω_i converges to $\frac{\beta(1-\alpha)}{1+\beta(1-\alpha)}$, which is smaller than $(1-\alpha)$ if $\beta < \frac{1}{\alpha}$.

Table 2 summarises our diverse novel predictions about wage determination between RS type I and II (on one side) and type III (on the other side). The parameters α and β will be structurally estimated from the diverse wage regressions in Section 4.3, where the estimate of α will also be compared with the independent estimate based on eq. (18) in Section 2.5, to jointly validate our theory and methodology.

Table 2: Predictions of Models

RS type	Model of RS mechanism	Labour rent share for $\delta \to 0$
I	Oligopsony (OL)	$\frac{\beta(1-\alpha)}{1+\beta(1-\alpha)}$
II	Wage Floor (WF)	$\frac{\beta(1-\alpha)}{1+\beta(1-\alpha)}$
III	Efficient Bargaining (EB)	$(1-\alpha)$

Note: $(1-\alpha)$ is output elasticity of labour, and $\beta \in [0,\infty)$ measures toughness of wage competition.

3.4 Productivity, Market Power and Income Distribution

In this section we study the relationship between productivity, market power and income distribution (in terms of factor income shares). We will first show how market power and factor income shares are closely related. Then we show that under short-run CRS, firm productivity is a common determinant of both market power and income distribution. The theoretical results will underpin our empirical identification of the economic interpretation of variable χ_L as well as of the short-run returns to scale.

Given that δ is also a measure of the producer surplus ratio to revenue, we note that this index is also relevant for welfare analysis and income distribution. To see this, let's

[29] This characterises the limit of worker's quasi rent share for firms with market power as $\omega_i \to b$. For competitive fringe firms with $w_i = \omega_i = b$, $w_i = (1-c_i)\,b + c_i\omega_i$ for any $c_i \in [0,1]$, that is c_i is indeterminate.

define $\nu_{Li} \equiv \frac{\phi_{Li}}{\phi_{Li}+\delta_i}$ and $\nu_{Ki} \equiv \frac{\delta_i}{\phi_{Li}+\delta_i} \equiv \frac{\delta_i}{1-\phi_{Xi}}$ as the value added shares of labour and capital (gross profit) respectively. Obviously $\nu_{Li} \equiv \frac{\phi_{Li}}{1-\phi_{Xi}}, \nu_{Ki} \equiv \frac{1-\phi_{Xi}-\phi_{Li}}{1-\phi_{Xi}}, \nu_{Li} + \nu_{Ki} \equiv 1$. Define $\psi_{Li} \equiv \frac{\phi_{Li}}{\phi_{Xi}+\phi_{Li}}$ ($\psi_{Xi} \equiv \frac{\phi_{Xi}}{\phi_{Xi}+\phi_{Li}}$) as the variable cost share of labour (and intermediate input), and recall $\omega_i \equiv \frac{p_i q_i - p_X x_i}{l_i}$ as the value added per worker.[30]

Theorem 9 *The labour share of value added ν_{Li} can be expressed as functions of variables w_i and ω_i as well as δ_i and ψ_{Li} by the following identities:*

$$\nu_{Li} \equiv 1 - \nu_{Ki} \equiv \frac{w_i}{\omega_i} \equiv \frac{1}{1+\frac{\delta_i}{1-\delta_i}\frac{1}{\psi_{Li}}}, \text{ with } \frac{\partial \nu_{Li}}{\partial \delta_i} < 0, \frac{\partial \nu_{Li}}{\partial \psi_{Li}} > 0, \frac{\partial \nu_{Ki}}{\partial \psi_{Li}} < 0. \tag{39}$$

Additionally, the relationship between ψ_{Li} and χ_{Li} (see eq. (15)) implies

$$\nu_{Li} = \frac{1}{1+\frac{\alpha}{1-\alpha}\frac{\delta_i}{1-\delta_i}(1+\chi_{Li})}, \text{ with } \frac{\partial \nu_{Li}}{\partial \delta_i} < 0, \frac{\partial \nu_{Li}}{\partial \chi_{Li}} < 0. \tag{40}$$

Furthermore,

$$\nu_{Ki} \equiv 1 - \nu_{Li} = \frac{1}{1+\frac{1-\alpha}{\alpha}\frac{1-\delta_i}{\delta_i}\frac{1}{1+\chi_{Li}}}, \text{ with } \frac{\partial \nu_{Ki}}{\partial \delta_i} > 0, \frac{\partial \nu_{Ki}}{\partial \chi_{Li}} > 0, \frac{\partial \nu_{Ki}}{\partial \alpha} > 0. \tag{41}$$

Eq. (39) - (41) show that the gross profit margin δ is, by definition, a key determinant of labour share of value added ν_L and the gross profit share of value added share ν_K. Keeping χ_L constant, an increase of δ means lower value of ν_L and higher value of ν_K. The wage markdown power index χ_L and labour cost share ψ_L also affect ν_L and ν_K. Since χ_L and ψ_L reflect firm's exogenous rent-sharing mechanism, which is characterised either by firm's buyer power in labour market with $\chi_L > 0$, or by workers' countervailing seller power with $\chi_L \leqslant 0$, either of them is also a determinant of factor income share, independently of the overall market power. Specifically, firms with worker countervailing seller power tend to have higher labour income share than firms with positive buyer power.

To conclude this section, we revisit our argument for the (approximate) CRS assumption. In Section 2.3, we show why it is not reasonable to assume (strong) DRS or (strong) IRS for the short-run production function with labour and intermediate good as flexible inputs, because under these assumptions there lacks a transmission mechanism between the dispersion in firm productivity and dispersions in gross profit margin and value added per worker. The next proposition shows that dispersion in each of variables ω, ν_K and δ is caused by the dispersion in the unobservable productivity variable A under the hypothesis of short-run

[30] More precisely this is value added per unit of labour. Here we adopt the term value added per worker, which is commonly used in the empirical literature on rent sharing (Card et al., 2018).

CRS.

Proposition 10 *For Cobb-Douglas production function $F_i(x_i, l_i) = A_i x_i^{\alpha} l_i^{1-\alpha}$, the following equations and identities hold:*

$$\omega_i = A_i p_i \left[1 - \alpha\left(1 - \rho_i\right)\right] \left(\frac{x_i}{l_i}\right)^{\alpha}, \ \textit{with} \ \frac{\partial \omega_i}{\partial A_i} > 0, \tag{42}$$

$$\nu_{Ki} = 1 - \frac{p_X \frac{\psi_{Li}}{1-\psi_{Li}} \left(\frac{x_i}{l_i}\right)^{1-\alpha}}{A_i p_i \left[1 - \alpha\left(1 - \rho_i\right)\right]}, \ \textit{with} \ \frac{\partial \nu_{Ki}}{\partial A_i} > 0, \tag{43}$$

$$\delta_i = 1 - \frac{p_X \left(\frac{x_i}{l_i}\right)^{1-\alpha}}{A_i p_i \left(1 - \psi_{Li}\right)}, \ \textit{with} \ \frac{\partial \delta_i}{\partial A_i} > 0. \tag{44}$$

Eq. (42) indicates that controlling for prices p_i and p_X, markup power index ρ_i, the input mix $\frac{x_i}{l_i}$ and parameter α, the predetermined firm short-run productivity A_i is an exogenous determinant of ω_i. Similarly, eq. (43) and (44) indicate that with suitable controls, A_i is also an exogenous determinant of ν_{Ki} and δ_i. Overall, the dispersion in the distribution of A_i is a common cause of the dispersions in the distributions of ω_i, ν_{Ki} (and ν_{Li}) and δ_i. Also, for empirical analysis, ω_i is a highly informative proxy for the unobservable A_i. However, ω_i, a natural measure of firm-level labour productivity, is scaled by the effect of input mix $\frac{x_i}{l_i}$. In regression analysis, to mitigate the potential bias caused by the input mix $\frac{x_i}{l_i}$, ω_i should be used with χ_{Li} as a control variable.

Recall that in Theorem 5 we have introduced the observable variable Λ_i as a proxy for A_i. Identity (17) shows Λ_i is a positive linear function of A_i, and it can therefore be used to empirically validate the claim that A_i is a common determinant of ω_i, ν_{Ki} (and ν_{Li}) and δ_i (see Section 4.4).

It is important to emphasise that all results of Proposition 10 is based on the premise of short-run CRS, which supports our preferred explanation for the stylised fact of dispersions in δ and ω, as well as can be conversely supported by our preferred explanation of this stylised fact.

4 Empirical Analysis

This section starts with a description of the data used to estimate the general model that augments the baseline JOOM, and test its theoretical predictions. In Section 4.2, we first estimate the industry-level technology parameter α and firm-level markup and markdown power indices, and we then discuss the classification of firms into three rent-sharing (RS) types, corresponding to the oligopsony model (OL), the wage floor model (WF), and efficient bargaining model (EB). We also discuss the interpretations of the estimate of χ_L. Particularly, we provide empirical identification of our theoretical explanation of the dispersions in δ and ω against the alternative explanation based on assumption of short-run DRS with perfect competition in all markets and labour adjustment cost, which would imply that there is no dispersion in δ or ω for the sub-sample of firms with $\chi_{Li} = 0$. In Section 4.3 we structurally estimate parameters of production and labour supply functions from diverse wage regressions and validate the theoretical predictions about the effects of firms' buyer power in labour market and workers' countervailing seller power on rent sharing outcome and wage determination. Section 4.4 presents empirical evidence on the hypothesis that the dispersion in firms' short-run productivity is the common root cause of dispersions in firms' gross profit margin, value added per worker and factor income share.

4.1 Data and Variables

Our data are retrieved from FAME, a dataset published by Bureau van Dijk with comprehensive financial data of companies with 5 or more employees registered in the UK and Ireland. We examine companies operating within one of the 224 distinct 4-digit SIC industries in the UK manufacturing sector, covering the period from 2003 to 2019.[31] FAME is particularly suitable for our analysis because it provides detailed balance sheet data not only for most of the large companies, but also for thousands of smaller entities, which are more likely to include fringe firms with (almost) no market power. In this respect, the coverage offered by FAME is better than other well-known financial datasets, such as Compustat, as it includes private companies which are not required to file account. Importantly, we stress that our aim in this study is predominantly methodological, as we want to present a novel way to estimate production functions, markups and markdowns for a large number of industries as well as to illustrate the importance of controlling for rent-sharing mechanisms in estimating technology and labour market parameters.

The key variables retrieved from FAME dataset are listed in Table 3. Table 4 summarises the descriptive statistics and coefficients of variation (CV) at the industry level. The numbers show that there is substantial firm heterogeneity and large inequality in terms of CVs of δ, ω, ν_L and ν_K for 4-digit SIC-code industries in the UK manufacturing sector. Furthermore, the within-industry dispersion in the distributions of firms' market shares, in terms of revenue share s and value added share ς, is also very large.

[31] Results are mostly consistent using 5-digit SIC-code industries.

Table 3: Data and Variables

Theoretical variable	Variable in FAME
Revenue R	Turnover
Variable cost VC	Cost of goods sold
Variable labour cost wl	Pay roll
Flexible labour input l	Employment
Wage w	$\frac{\text{Pay roll}}{\text{Employment}}$
Industry	4-digit SIC code
Cost of intermediate input $p_X x = VC - wl$	Cost of goods sold $-$ Pay roll
Gross profit $\pi = R - VC$	Turnover $-$ Cost of goods sold
Gross profit margin $\delta = \frac{\pi}{R}$	$\frac{\text{Gross profit}}{\text{Turnover}}$
Value added $\pi + wl$	Gross profit + Pay roll
Value added per worker $\omega = \frac{\pi+wl}{l}$	$\frac{\text{Gross profit + Pay roll}}{\text{Employment}}$
Labour share of value added $\nu_L = \frac{wl}{\pi+wl}$	$\frac{\text{Pay roll}}{\text{Gross profit + Pay roll}}$
Gross profit share of value added $\nu_K = \frac{\pi}{\pi+wl}$	$\frac{\text{Gross profit}}{\text{Gross profit + Pay roll}}$

Note: This table shows how the most relevant theoretical variables are measured using FAME data.

Table 4: Summary Statistics (1)

	Obs	Mean	Std dev		Obs	Mean	Std dev
ω	60367	68.72	88.98	ν_K	60367	0.5860	0.1466
δ	60367	0.2842	0.1370	s	60367	0.0578	0.1406
ν_L	60367	0.4140	0.1466	ς	60367	0.0583	0.1484
CV_ω	3156	0.5196	0.3153	CV_{ν_K}	3156	0.2272	0.1317
CV_δ	3156	0.4467	0.1867	CV_s	3156	1.307	0.8043
CV_{ν_L}	3156	0.3181	0.1380	CV_ς	3156	1.249	0.8148

Note: See Table 3 for definitions of ω, δ, ν_L and ν_K. s and ς are firms' market shares in terms of, respectively, revenue and value added in the corresponding SIC4 industry. CV are the coefficient of variation of the variables in a SIC4 industry and year.

4.2 Estimation Results

4.2.1 Production Function, Market Power, and Rent-sharing Types

The theoretical underpinning for the identification of the short-run production function parameters rests on the notion of competitive fringe firm, discussed in Section 2.5. The empirical estimation of parameter α and χ_L are given by eq. (18) and (19). Concretely, for each year t and industry k, a firm i is classified as a "competitive" fringe firm if its gross profit is in the bottom 5% of the distribution. Accordingly, we estimate α_{kt} by:

$$\tilde{\alpha}_{kt} = median\left\{1 - \psi_{Likt} : \delta_{\min kt} \leq \delta_{ikt} \leq \delta_{\min kt} + 0.05\right\},$$

where $\delta_{\min kt}$ is the minimum of δ_{ikt} across all i-firms in industry k at time t. Once $\tilde{\alpha}_{kt}$ is obtained, the estimator $\tilde{\chi}_{Likt}$ is calculated according to:

$$\tilde{\chi}_{Likt} = \frac{1-\tilde{\alpha}_{kt}}{\tilde{\alpha}_{kt}} \frac{p_{Xkt}x_{ikt}}{w_{ikt}l_{ikt}} - 1,$$

where $p_{Xkt}x_{ijt}$ and $w_{ikt}l_{ikt}$ are available from our dataset. The estimator $\tilde{\rho}_{ikt}$ is then computed as

$$\tilde{\rho}_{ikt} = 1 - \left(1 + \tilde{\chi}_{Likt}\right) \frac{\phi_{Likt}}{1 - \tilde{\alpha}_{kt}},$$

where ϕ_{Likt} is the revenue share of labour for firm i in industry k in period t.

After obtaining $\tilde{\alpha}_{kt}$, the proxy measure for the unobservable variable A can also be computed according to:

$$\tilde{\Lambda}_{ikt} \equiv \frac{\left(w_{ikt}\right)^{1-\tilde{\alpha}_{kt}}}{\left(\frac{p_{Xkt}x_{ikt}}{w_{ikt}l_{ikt}}\right)^{\tilde{\alpha}_{kt}} \phi_{Likt}}.$$

For ease of notation, in the remainder of the paper, we use $(\alpha_{kt}, \chi_{Likt}, \rho_{ikt})$ unless specifically stated otherwise. For the empirical analysis, they should be interpreted as $(\tilde{\alpha}_{kt}, \tilde{\chi}_{Likt}, \tilde{\rho}_{ikt})$.

Table 5 shows that the mean value of output elasticity of intermediate input α is 0.81 with standard deviation 0.13. To contextualise this within existing literature, in Table 6 we compare our results with the results reported in Gandhi, Navarro and Rivers (2020) for manufacturing industries in Columbia and Chile.[32] The mean values of OLS estimates of α range from 0.73 to 0.79, very close to our result. This is accompanied by similarity in the estimates of (θ_X, θ_L) to ours. Remarkably, their OLS estimates of the short-run returns

[32] We note that Gandhi, Navarro and Rivers (2020) estimate the long-run production function which includes fixed capital as an argument. In contrast, in our analysis, the contribution of fixed capital is subsumed into the short-run multi-factor productivity. For comparison, we calculate the short-run elasticity of scale Θ as the sum of output elasticities of intermediate and labour inputs from Gandhi, Navarro and Rivers' (2020) estimates, i.e., $\Theta = \theta_X + \theta_L$. The comparison is then essentially confined to $(\theta_X, \theta_L, \Theta)$.

Table 5: Summary Statistics (2)

	Obs	Mean	Std dev
α	3517	0.8098	0.1254
χ_L	60367	-0.1347	2.965
ρ	60367	0.3657	0.2344
Λ	60367	5.984	32.66
CV_Λ	3156	0.4106	0.5015

Note: α is calculated for each SIC4 industry and year. CV_Λ is the coefficient of variation of Λ in a SIC4 industry and year. The number of observations for CV_Λ is smaller than for α because in some years there is only one firm in some industries, thus preventing calculation of CV_Λ.

to scale Θ range from 0.97 to 0.98, hardly different from the approximate CRS which we impose. In contrast, their estimates of Θ based on the GNR approach range from 0.89 to 0.93. Similarly, the estimates of α based on the GNR approach range from 0.59 to 0.61. These lower estimates can be explained by the fact that the GNR approach is predicated on the restriction of perfect competition in product market, i.e., $\rho = 0$. Eq. (8) suggests that achieving a lower value of ρ requires a lower value of Θ, all else being equal. That is, the restriction $\rho = 0$ causes a downward bias to the estimate of Θ. Furthermore, eq. (6) implies that the restriction: $\rho = 0$ also forces a downward bias to the estimate of θ_X, as well as a downward bias to the estimate of $\alpha = \theta_X/(\theta_X + \theta_L)$.

Table 5 also shows that the distribution of Λ is highly dispersed, indicating that the distribution of A must also be highly dispersed. Similarly, Table 5 indicates that there is also considerable dispersion in χ_L, which is visually confirmed by the top-left panel of Figure 1. Since this dispersion captures the deviation of firms' inputs substitutability from the Cobb-Douglas baseline, our findings are consistent with those of Raval (2023), which also show dispersion in the substitutability of firm-specific inputs. However, Raval's (2023) explanation focuses on non-neutral productivity differences across firms, whereas we relate this pattern to labour market rent-sharing mechanisms. In Online Appendix C we prove formally an equivalence between χ_L and the firm-specific parameter for non-neutral productivity. In a sense, we provide an explicit theoretical explanation to Raval's (2023) notion of labour-augmenting productivity difference.

To this aim, we now define rent-sharing (RS) types using the distribution of χ_L and δ. Specifically, for given industry k in year t, we divide the sample of firms into three subsets using the following partition of the $(\delta_{ikt}, \chi_{Likt})$ plane:

Table 6: Comparison with GNR (2020)

	θ_X	θ_L	$\Theta = \theta_X + \theta_L$	$\alpha = \frac{\theta_X}{\Theta}$	$\rho = 1 - \frac{\phi_X}{\theta_X}$
TO/UK	0.81	0.19	1	0.81	0.37
OLS/Columbia	0.72	0.26	0.98	0.73	0.25*
GNR/Columbia	0.54	0.35	0.89	0.61	0
OLS/Chile	0.77	0.20	0.97	0.79	0.29*
GNR/Chile	0.55	0.38	0.93	0.59	0

Note: The results in this table are mean values of estimates unless otherwise stated. Our own results are presented in the row: TO/UK. The rest are imported or inferred from Gandhi, Navarro and Rivers (2020, Table 2). Row: OLS/Columbia reports results from GNR (2020) based on simple OLS approach and Columbia data. Row: GNR/Columbia shows results using GNR (2020) approach and Columbia manufacturing data. The rows: OLS/Chile and GNR/Chile and have similar descriptions but substitute Chile data for Columbia data. * indicates that the results are calculated using ratio of mean values instead of mean value of ratio, and mean value of ϕ_X is the mean value of θ_X reported for corresponding GNR rows.

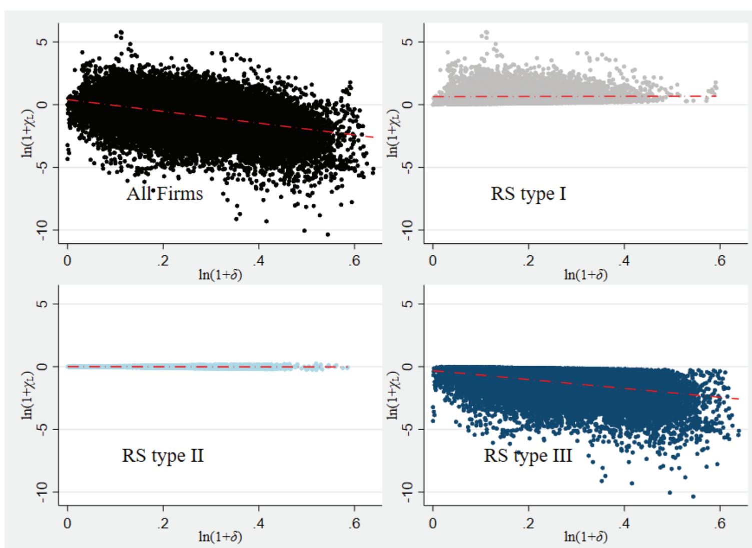

Figure 1: Scatter plots of $(\ln(1+\delta_{ikt}), \ln(1+\chi_{Likt}))$ and the subsets of partition. The full set (of all firms) is partitioned into three mutually exclusive subsets, labelled RS Types I, II and III.

$$\begin{aligned}
\text{RS type I} &\equiv \left\{(\delta_{ikt}, \chi_{Likt}) : \ln\left(1+\chi_{Likt}\right) \geq 0.5 \ln\left(1+\delta_{ikt}\right) > 0\right\}, \\
\text{RS type II} &\equiv \left\{(\delta_{ikt,Likt}) : \left|\ln\left(1+\chi_{Likt}\right)\right| < 0.5 \ln\left(1+\delta_{ikt}\right)\right\}, \\
\text{RS type III} &\equiv \left\{(\delta_{ikt,Likt}) : \ln\left(1+\chi_{Likt}\right) \leq -0.5 \ln\left(1+\delta_{ikt}\right) < 0\right\}.
\end{aligned}$$

As the point $(\delta_{ikt}, \chi_{Likt}) = (0,0)$, which theoretically represents the competitive fringe firms, is on the common boundary of the three subsets, the proposed partition has the desirable property of having the fringe firms as the limit points for the three RS types. At the same time, this partition clearly separates RS types I and III (by having RS type II in between) except for the limit point $\delta_{ikt} = 0$. The use of function $\ln(1+\chi_{Li})$ ensures that we have a symmetry around $\chi_{Li} = 0$, since the range of $\ln(1+\chi_{Li})$ is $(-\infty, \infty)$, whereas the parameter value choice of 0.5 allows to have a reasonable sample size of RS type II firms. Furthermore, the function $\ln(1+\delta_{ikt})$ allows for a more clear separation between RS types I and III for higher values of δ_{ikt}. Figure 1 shows the boundaries between the RS types on the $(\ln(1+\delta_{ikt}), \ln(1+\chi_{Likt}))$ plane. It highlights the dispersion in δ and χ_L dimensions, as well as how RS types relate to the broadly defined wage-markdown power χ_L.

Table 7: RS Types Distribution

	RS type			
	I	II	III	Total
Obs.	11863	6533	41971	60367
%	19.7	10.8	69.5	100

Table 7 shows the firm-year frequencies by RS types over the sample period. RS types I, II and III comprise 20%, 11% and 70% of observations among UK manufacturing firms, respectively. Such a large proportion of RS type III is a surprising result. Table 10 of Online Appendix D.3, which reports the changes of RS type distribution between 2005 and 2015, reveals a considerable degree of persistence of RS types.

To demonstrate the impact of restricting the application of the cost share approach to competitive fringe firms, we can compare the results above with those obtained using the full sample of firms, as would be admissible under the assumption of perfect competition. Table 11 in Appendix D.3 shows that the distribution of RS types I, II and III, based on the perfect competition model, would be respectively, 42%, 17% and 41%. In comparison with Table 7 the proportion of RS type III reduces from 70% to 41%.

4.2.2 Interpretations of Estimate of χ_L

The estimated value of χ_{Li} has two interpretations: a mechanical one and a theory-based one. Its direct interpretation is the extent to which a firm's input mix departs from the optimal choice by a representative fringe firm (or the benchmark firm). Parameter α is the benchmark indicating the baseline inputs substitutability property, whereas the variable χ_{Li} is a firm-specific correction term. For $\chi_{Li} = 0$, the inputs substitutability property is the same as the representative fringe firm; for $\chi_{Li} > 0$, it is inclined to substitute intermediate input for labour, displaying "labour-augmenting technological bias", while $\chi_{Li} < 0$ indicates the opposite substitution pattern, manifesting "intermediate-input-augmenting technological bias". Apart from increasing the degree of freedom for the more flexible "production function", what meaningful interpretation can there be for this correction term χ_{Li}?

Our theory-based interpretation of χ_{Li} is not technological. The baseline JOOM theory shows that it represents the wage markdown power index. Our general theory explicitly connects the signs of χ_{Li} with different types of labour market rent sharing. Besides documenting their remarkable heterogeneity, hereby we advance a novel theory of the determination of RS type based on Galbraith's (1954) hypothesis that "market power begets countervailing power": the probability of workers having countervailing seller power in the labour market (the fraction of RS type III firms) increases in the magnitude of firms' overall market power (measure by δ_i). The left and right panels of Figure 2 show how the ratio of firm-year frequencies of RS type III to RS type I, N_{III}/N_I, changes with, respectively, $ln(1+\delta)$ and $ln(\omega)$. Whereas the firm-year frequency of RS type III is more than three time larger than that of RS type I in the full sample, this ratio is not invariant across δ or ω. N_{III}/N_I increases in $ln(1+\delta)$, from the magnitude near 1 to over 30. Apart from the bottom left bin in the figure, N_{III}/N_I decreases in $ln(\omega)$, from 6 to slightly over 1. The left panel of Figure 2 strongly confirms our explanation of the determination of workers' countervailing seller power. This empirical regularity lends support to the interpretation of χ_{Li} as capturing firms' buyer power or workers' countervailing seller power in labour market, versus alternative interpretations.

According to Corollary 7 and Online Appendix B.2, under the assumption of short-run DRS with perfect competition, the dispersions in δ and ω should disappear if the labour adjustment cost is removed by imposing $\chi_{Li} = 0$. Figure 7 in Online Appendix D.1 shows the relationship between the dispersion, respectively, in δ and ω, and the number of firms for the RS type II sub-sample of our dataset. It demonstrates that dispersion in δ and ω remain large for this sample when the number of firms is above certain threshold, such as 10. These results contradict the alternative explanation of the dispersions in δ and ω based on the assumption of short-run (strong) DRS and labour adjustment cost, in favour of our theory.

To further strengthen our preferred interpretation, the next section is devoted to discussing likely measurement errors in the accounting data we use, and scrutinise their potential to cause spurious correlation involving χ_{Li}.

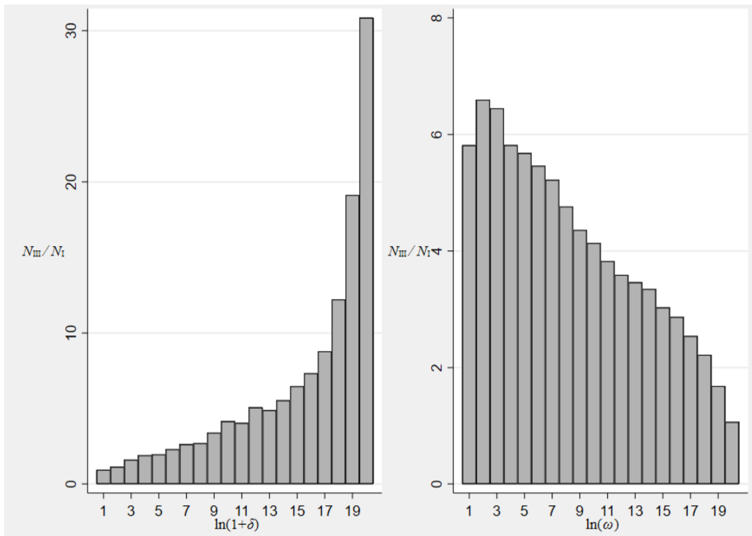

Figure 2: Ratio of firm-year frequencies of RS type III firms to RS type I firms. In the left panel, the sample is divided into 20 equal-distant bins along the $ln(1+\delta)$ dimension. In the right panel, the sample is divided into 20 equal-distant bins along the $ln(\omega)$ dimension. The vertical axis is the ratio of type III over type I firm-year frequencies N_{III}/N_I.

4.2.3 Discussion on Measurement Errors and Control for Their Effects

One likely source of error for the measurement of flexible labour expenditure is that the variable payroll in FAME may overlap with overhead. For example, if a firm hires scientists and engineers for R&D, their wages enter the payroll but do not belong to the expenditure on flexible labour employed for short-run production. In this scenario, the measure of labour cost reported in FAME, wl, is an upward bias of the true flexible labour expenditure w^*l^*: $wl > w^*l^*$. At the same time, the labour input reported in FAME creates a positive error to the measurement of flexible labour input: $l > l^*$. On the contrary, the reported "cost of good sold" can be considered a good approximation of the variable cost of production, so that: $(wl + p_X x) \approx (w^*l^* + p_X^* x^*)$. This implies a negative bias to the measurement of flexible intermediate input expenditure: $p_X x < p_X^* x^*$. Accordingly, the computed flexible labour cost share ψ_L is positively biased and the flexible intermediate input cost share ψ_X is negatively biased by the same magnitude. For the same reason, the estimation of α is negatively biased.

The effect of this type of measurement error on the estimation of χ_L is ambiguous, depending on the relative magnitudes of the biases on α and ψ_X. If one is willing to assume that the magnitude of the negative bias on ψ_X is increasing in the short-run productivity ω because firms that do in-house R&D also tend to be more productive, then the magnitude of downward bias on χ_L should also increase in ω. As an implication, the ratio of the firm-year frequency of RS type III firms to RS type I firms should increase in ω. And everything else being equal, there should be a negative correlation between χ_L and ω. These two implications, are inconsistent with, respectively, Figure 2 left panel, which shows that the ratio NIII/NI is decreasing in ω, and Figure 3, which shows that χ_L and value added per worker are positively correlated in the pooled sample including all RS types, and in the RS type I sub-sample.

Another possible confounding factor is that some firms achieve high levels of ω through in-house R&D (and therefore incur high overhead labour cost), but some other firms achieve high levels of ω through out-sourced R&D (without overhead of labour cost). Under this scenario low values of χ_L indicate that firms engage in in-house R&D, while higher values suggest reliance on outsourced R&D. We cannot dismiss this possibility as our data show a negative correlation between the variable R&D intensity and χ_L. For this reason, we proceed to verify the robustness of our findings by controlling for the effects of such measurement error in two ways. First, we exclude all firms that report positive R&D expenditure from our data sample and re-run the results reported in Table 7, Table 10 of Online Appendix D.3, and Figures 1 to 3. Only minor differences between these two groups are found, which confirms the robustness of our findings. Second, we check the robustness of the results in Section 4.3 by controlling for R&D intensity, and by including firm fixed effect in our wage regression to control for unobserved heterogeneity in firm-specific measurement errors.

4.3 Wage Regressions and Rent-Sharing Mechanisms

In this section we show that controlling for rent-sharing types is important for structural estimation of production and labour supply function parameters from wage regressions, since the theory predicts that labour market rent-sharing mechanisms capture the influences of firms' buyer power in labour market or workers' countervailing seller power, thus affecting wage determination.

Consider the following regression equation:

$$w_i = \xi_0 + \xi_1\omega_i + \xi_2\omega_i \ln(1-\delta_i) + \xi_3\omega_i \ln(1+\chi_{Li}) + \xi_4 \ln(1-\delta_i) + \xi_5 \ln(1+\chi_{Li}) + \varepsilon_i, \quad (45)$$

Eq. (45) is used for structural estimation of model parameters α and β, to be backed out

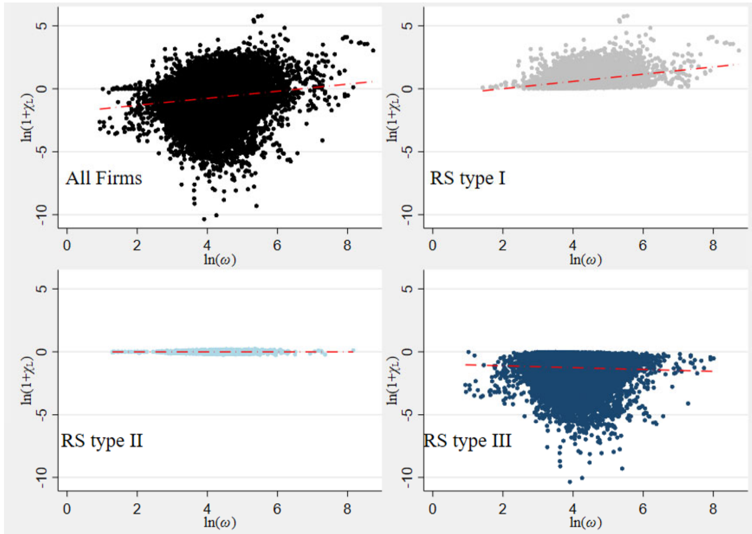

Figure 3: Scatter plots of $(\ln \omega_{ikt}, \ln(1+\chi_{Likt}))$ and the subsets of partition. The horizontal axis reflects the magnitude of value added per worker ω_{ikt}.

from the diverse values of coefficient $\xi_1 > 0$ for various RS types, with variables $\omega_i \ln(1-\delta_i)$, $\omega_i \ln(1+\chi_{Li})$, $\ln(1-\delta_i)$ and $\ln(1+\chi_{Li})$ as relevant controls. Based on theoretical predictions summarised in Table 2. the following two hypotheses can be tested:

(H1) The coefficient ξ_1 is the same for RS types I and II.

(H2) The value of ξ_1 for RS type III should exceed that for RS types I and II.

Table 8 reports the wage regression results based on eq. (45). In our theoretical model each firm's short-run productivity A_i is predetermined, and should be treated as exogenous for analysing the determination short-run market outcome. Following the theoretical underpinnings, for eq. (45), the explanatory variable ω_i, a proxy of A_i,[33] is also treated as exogenous. In addition to reporting the OLS regression, we also include fixed effect to control for measurement error discussed in Section 4.2.3 as well as unobserved heterogeneity due to labour and managerial quality and other firm-level institutional differences that are (almost) time invariant. The regression results with inclusion of R&D intensity to control for measurement error are very similar to those reported in Table 8.[34]

The estimated values of ξ_1 are positive and statistically significant at 1% level for all RS types. The estimated coefficients for RS types I and II are very similar: a result consistent with (H1). In light of the theoretical predictions summarised in Table 2, the estimated coefficients for RS type III imply that the value of the production function parameter α is in the range of $[0.80, 0.82]$, which is consistent with the mean value 0.81 re-

[33] We empirically validate the claim that ω_i is a proxy measure of A_i in Section 4.4, and report evidence of correlation between ω_i and Λ_i (a partial measure of A_i) in Table 9.

[34] Further robustness check using the instrument variable approach is reported in Table 12 of Online Appendix D.4, which also shows comparable results.

Table 8: Regressions of w on ω

	RS type I		RS type II		RS type III	
	(1)	(2)	(3)	(4)	(5)	(6)
	OLS	Firm FE	OLS	Firm FE	OLS	Firm FE
ω	0.1290***	0.0980***	0.1223***	0.1161***	0.1807***	0.2014***
	(0.0248)	(0.0317)	(0.0330)	(0.0314)	(0.0248)	(0.0227)
$\omega \ln(1-\delta)$	0.0559***	0.0381***	0.0690***	0.0641***	0.1106***	0.1009***
	(0.0106)	(0.0144)	(0.0244)	(0.0211)	(0.0155)	(0.0128)
$\omega \ln(1+\chi_L)$	-0.0262***	-0.0192***	-0.0469	-0.0071	-0.0311***	-0.0175***
	(0.0056)	(0.0073)	(0.0652)	(0.0232)	(0.0076)	(0.0053)
$\ln(1-\delta)$	9.710***	10.41**	6.096***	4.982**	6.934***	7.037***
	(2.999)	(4.603)	(1.384)	(2.234)	(0.8699)	(1.141)
$\ln(1+\chi_L)$	.5088**	1.051*	1.076	-1.293	-0.4975	-0.1333
	(0.2390)	(0.5790)	(5.974)	(2.244)	(0.4158)	(0.2700)
Ind. FE	Yes	No	Yes	No	Yes	No
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
R^2 adj	0.519	0.779	0.597	0.852	0.616	0.884
Obs	11863	11863	6533	6533	41971	41971

Note: The table shows the estimated coefficients of eq. (45). Standard errors are clustered at industry level. ***, ** and * respectively indicates statistical significance at 1%, 5% and 10% level.

ported in Table 5. If we set $\alpha = 0.81$ and consider that the range of values of ξ_1 for RS types I and II are $[0.10, 0.13]$, we infer that the value of parameter β is in the range of $\left[\frac{0.1}{(1-0.1)(1-0.81)}, \frac{0.13}{(1-0.13)(1-0.81)}\right] = [0.58, 0.79]$. This indicates moderate toughness of wage competition. By eq. (25) $\epsilon_{Li} = \frac{\beta(1-s_{Li})w_i}{w_i - b}$. If we set $\dfrac{w_i}{b} = 1.3$ (as in Card et al. (2018, S51)) and set s_{Li} from 0.01 to 0.1, then ϵ_{Li} is in the range between 2.3 and 3.4. This is consistent with many empirical studies that find that the elasticity of labour supply to the firm is lower than 4 (Manning, 2011). Importantly, these results are consistent with hypothesis (H2), indicating that the efficient bargaining mechanism supports workers' countervailing seller power, which counteracts firms' buyer power in labour market, and therefore increases workers' quasi rent share in the neighbourhood of $\delta = 0$ relative to oligopsony and the wage floor mechanism.

Estimated eq. (45) can be rewritten as:

$$w_i = \hat{c} + \left[\hat{\xi}_1 + \hat{\xi}_2 \ln(1-\delta_i) + \hat{\xi}_3 \ln(1+\chi_{Li})\right]\omega_i + \hat{\varepsilon}_i,$$

where $\hat{c}$ summarises the terms that do not directly depend on ω_i, and the terms in the square bracket measure the net effect of ω_i on w_i. Table 8 shows that $\hat{\xi}_2 > 0$ at 1% level of statistical significance for all RS types. This implies that firms with higher overall market power, measured by δ_i, tend to share with their employees a smaller fraction of their quasi rents. This can be one of the microeconomic mechanisms that causes firm level wage stagnation. This tendency can be either reinforced or mitigated depending on the sign of χ_{Li}, as $\hat{\xi}_3 < 0$ for all RS types, with statistical significance of 1% level for RS types I and III. For RS type I, $\chi_{Li} > 0$ implies that $\hat{\xi}_3 \ln(1 + \chi_{Li}) < 0$: a reinforcement of wage stagnation.[35] For RS type III, $\chi_{Li} < 0$ implies $\hat{\xi}_3 \ln(1 + \chi_{Li}) > 0$: a mitigation of wage stagnation.[36] Overall, these provide further evidence that collective bargaining (influencing RS types II and III firms) raises workers' countervailing seller power to counteract firms' potential buyer power in labour market (influencing RS type I firms) and that worker's quasi rent share increases more with efficient bargaining (RS type III firms).

Recalling that 11% and 70% of all UK manufacturing firms in our dataset can be classified as RS types II and III, respectively, the importance of the role played by collective bargaining in wage determination and income distribution cannot be understated. Importantly, the fact that hypothesis (H2) is confirmed in our data shows that the mitigation is quantitatively significant.[37] The fact that cross-firm wage differentials only partially track the cross-firm differences in labour productivity (also measured by ω) shows that collective bargaining can mitigate but not eliminate firm-level wage stagnation.

4.4 Correlation between Dispersed Distributions of Productivity, Market Power and Factor Income Share

Our data of UK manufacturing firms reveal large heterogeneity within a typical four (or five-digit) SIC-code industry along the following key dimensions: (1) value added per worker ω, (2) gross profit margin δ, (3) value added share of labour or capital, ν_L or ν_K. (4) proxy measure of short-run productivity, Λ. Figures 4 and 5 visualise the dispersion in dimensions (1) - (4) in two different years.

The dispersions in dimensions (1) - (3) are calculated directly from data and are therefore free from debatable assumptions. We refer to them as stylised facts (1) - (3). In Section 2.3 we have alluded that stylised facts (1) - (2) can be used to discriminate the three competing hypotheses of short-run DRS, IRS and CRS, in favour of CRS. The theoretical justification

[35] Using the results in column (1) of Table 8, the maximum value of the square-bracketed term for RS type I firms equals $\hat{\xi}_1 = 0.129$, and the 1 percentile value is reduced to 0.045.

[36] Using the results in column (5) of Table 8, the maximum value of the square-bracketed term for RS type III firms is 0.404, exceeding $\hat{\xi}_1 = 0.181$, and the 1 percentile value is to 0.064. These results indicate that the effects of χ_{Li} for RS type III can mitigate, but not always eliminate, the wage stagnation effects of δ_i.

[37] This point is further supported by regression of $\ln w$ on $\ln \omega$, reported in Table 13 in Online Appendix D.5.

for our line of reasoning is the fact that the exogenous dispersion in firms' short-run multi-factor productivity A causes dispersions in ω and δ under CRS, but not (necessarily) under DRS or IRS.

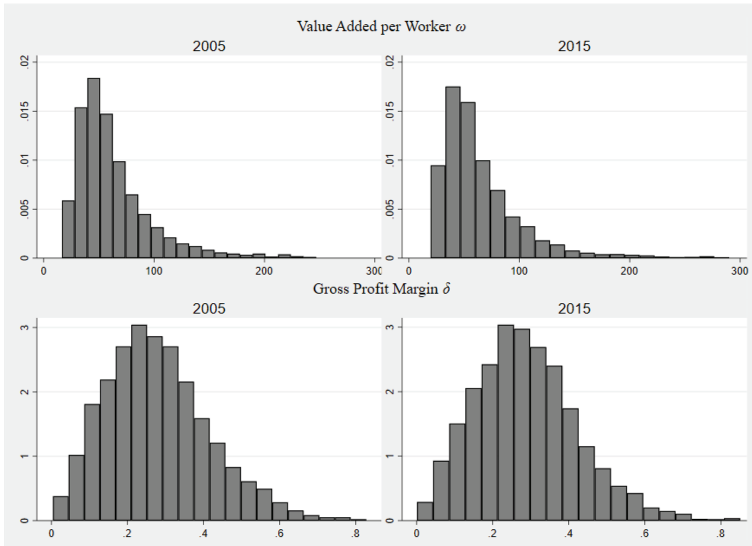

Figure 4: Histograms of Value Added per Worker ω, and Gross Profit Margin δ

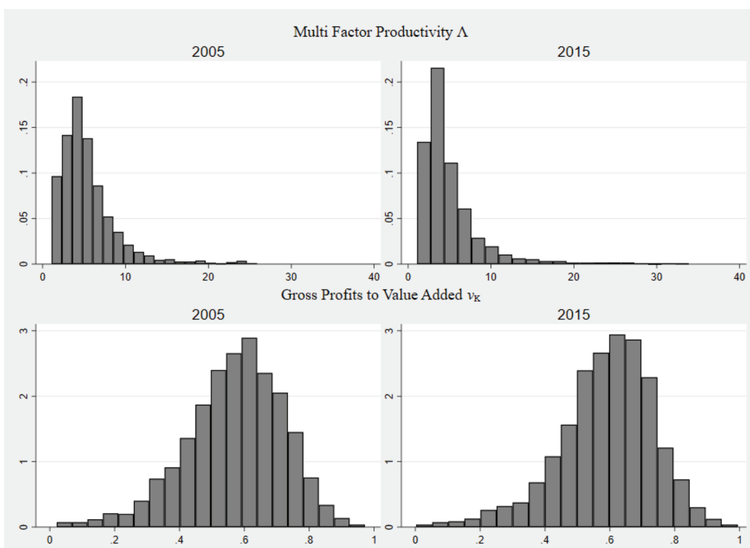

Figure 5: Histograms of Λ (a proxy Short-run MFP), and Gross Profit Share of Value Added ν_K

In this section, we provide evidence on correlation between the distributions of short-run productivity, market power and factor income shares. Our theoretical argument (Proposition 10 in Section 3.4) is that the dispersion in firms' short-run productivity A is a common root cause of dispersions in ω, δ and ν_K (or ν_L). Since A is unobservable, we use Λ as its empirical counterpart, as discussed in Theorem 5. We therefore regress respectively $\ln \omega$, δ and ν_K on

$\ln \Lambda$. The results are reported in Table 9. There is overwhelming evidence of strong positive correlations between Λ and, respectively, ω, δ and ν_K. These correlations hold across all RS types. They give strong support to our approach and theoretical prediction. By implication, the theoretical analysis and empirical evidence also reinforce the assumption of short-run CRS, and our empirical identification strategy relying on the fact of dispersions in ω and δ. As has been shown (Section 2.3 and Online Appendix B), DRS in the short-run tend to cause equalisation of ω and δ, not dispersions (in the absence of market power and other distortions). Such prediction is inconsistent with the stylised fact of dispersions in ω and δ, illustrated by Figure 6 in Online Appendix D.1.

Table 9: Regression of $\ln \omega$, δ and ν_K on $\ln \Lambda$

	RS type I			RS type II			RS type III		
	$\ln \omega$	δ	ν_K	$\ln \omega$	δ	ν_K	$\ln \omega$	δ	ν_K
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
$\ln \Lambda$	2.510***	0.3441***	0.4748***	2.173***	0.5141***	0.6184***	0.6030***	0.2301***	0.1689***
	(0.1052)	(0.0207)	(0.0275)	(0.0820)	(0.0114)	(0.0262)	(0.0485)	(0.0096)	(0.0151)
α	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
χ_L	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
R^2 adj	0.765	0.666	0.555	0.797	0.868	0.689	0.392	0.695	0.354
Obs	11863	11863	11863	6533	6533	6533	41971	41971	41971

Note: The variable Λ is a partial measure of the unobservable variable A, which is a common determinant of variables ω, δ and ν_K. These variables also depend on technology parameter α, labour market imperfect competition χ_L, and rent-sharing type. All regressions include industry and year fixed effects. Standard errors are clustered at industry level. *** indicates statistical significance at 1% level.

5 Welfare and Policy Implications

In this section we explore the welfare and policy implications of our analysis. It is well known that markup power has a static inefficiency implication in the form deadweight loss: the part of loss of consumer surplus, relative to perfectly competitive equilibrium, that is not transferred to producer surplus. When the productivity of competing firms is dispersed, imperfect competition causes another form of static inefficiency in addition to deadweight loss. To see this, consider a static Cournot oligopoly model in which firms' Cobb-Douglas production functions are $F_i(x_i, l_i) = A_i x_i^{\alpha} l_i^{1-\alpha}$, i.e., they are the same except for the differential productivity A_i. Under perfect competition only the most efficient firm(s) would produce and the market price would equal the lowest marginal cost among all firms. In the Nash equilibrium, a form of productive inefficiency arises because the most efficient firms produce less than socially optimal, and thus some of their sales or value added is reallocated to the less productively efficient firms. This line of argument can trace its origin to Banerjee and Duflo (2005), and Hsieh and Klenow (2009), who treat the dispersion in firm productivity distribution as a measure of productive inefficiency. If imperfect competition in labour market is added to the analysis through the baseline joint oligopoly-oligopsony model, then a new form of deadweight loss is introduced. That is the part of loss of worker surplus (a counterpart of consumer surplus) that is not transferred to producer surplus. Furthermore, the productive inefficiency now also includes the part that arises because some of the less efficient firms employ labour that would only belong to the most efficient firms under perfect competition.

We note that the validity of the analysis above relies on existence of market power. If the standard neoclassical theory's assumption of strong short-run DRS holds true, then market power cannot be the source of productive and allocative inefficiency. Instead, there must be other distortions, such as labour adjustment cost due to employment protection law (see Hopenhayn and Rogerson, 1993), which causes sustained dispersion in firm productivity. In this scenario, the market system has an self-correction mechanism driven by the technological force of strong short-run DRS to sustain perfect competition. Enforcement of competition policy, apart from the policy area of cartel, is hardly necessary. The efficiency-enhancing policy has to be deregulation, for example, of labour protection law that causes labour adjustment cost. However, our theoretical and evidential support for the assumption of the (approximate) short-run CRS shows that the hypothesis of strong DRS associated with perfect competition is inconsistent with the stylised fact of dispersions in firms' gross profit marginal and value added per worker (see Section 2.3.1). Similarly, our analysis rejects the hypothesis of strong short-run IRS with natural monopoly-monopsony, which implies a defeatist view of the effectiveness of competition policy.

Overall, our results imply that the dispersion in firm productivity and (non-transitory) concentration of market power in the hands of dominant firms is a primary source of productive and allocative inefficiency. The policy recommendation from this insight is to prevent entrenchment of concentrated market power, and to promote knowledge diffusion by striking a right balance between strict enforcement of competition law and intellectual property protection.

6 Conclusion

This paper develops a unifying hybrid industrial-labour economics model with imperfect competition in both product and labour markets, which also allows for diverse rent-sharing mechanisms. Our theoretical framework provides the economic foundations for a novel way of estimating short-run production function and market power indices, based on applying the cost share approach only to the competitive fringe firms.

Using data of UK manufacturing firms, we find evidence of three types of rent-sharing mechanisms in labour market: oligopsony, wage floor and efficient bargaining, each accounting for respectively around 20%, 11% and 70% of firm-year observations in our data set. We show that collective bargaining raises workers' countervailing seller power, which results in higher shares of quasi rents accruing to workers. This helps alleviate, but does not eliminate, the wage stagnation problem at the firm level. We also show that the dispersed firm-level short-run productivity distribution is a common root cause of dispersed distributions of firm-level overall market power index, and factor income share.

A key insight from our theoretical and empirical analysis is that the existing literature has over-emphasised the importance and estimation of product market markup power, and has under-emphasised significance and measurement of the degree of short-run returns to scale, as well as the overall market power. Particularly, insufficient attention has been given to assessing the validity of the approximate short-run CRS hypothesis and evaluating how well the firm's gross profit margin serves as an indicator of overall market power. Such methodological improvements in measuring market power in product and labour market can provide a firmer foundation for policy interventions. For instance, they can reveal whether market power concentration in the hands of a few dominant firms tends to entrench, or is merely transitory.

One important advantage of our methodology is that it can be implemented using "standard" accounting data, sidestepping estimation problems due to unobserved prices and measurement errors of fixed capital. Accordingly, its application can be extended to multiple countries and to other sectors, including services, for which available evidence is scant despite being the most important part of the economy in developed nations.

The resulting micro-founded aggregate-able metrics to measure productivity, market powers, and rent sharing are also essential input for a comprehensive analysis of economy-wide productivity growth, aiming at better understanding its drivers and drags. As an example of future research, in a companion paper, Tong and Ornaghi (2024), we empirically investigate how market power variables affect the response of reallocation of resources across firms to productivity shocks, as well as how the overall market power affects firm innovation and productivity growth.

Appendix

Proof of Lemma 1: The first-order conditions for the maximisation problem (1) are:

$$\begin{aligned}
\frac{\partial \mathfrak{L}_i}{\partial q_i} &= P_i(\mathbf{q}) + \frac{\partial P_i(\mathbf{q})}{\partial q_i} q_i - \lambda_i = 0, && (46)\\
\frac{\partial \mathfrak{L}_i}{\partial x_i} &= -p_X + \lambda_i \frac{\partial F_i(x_i, L_i(\mathbf{w}))}{\partial x_i} = 0, && (47)\\
\frac{\partial \mathfrak{L}_i}{\partial w_i} &= -L_i(\mathbf{w}) - w_i \frac{\partial L_i(\mathbf{w})}{\partial w_i} + \lambda_i \frac{\partial F_i(x_i, L_i(\mathbf{w}))}{\partial L_i} \frac{\partial L_i(\mathbf{w})}{\partial w_i} = 0, && (48)\\
\frac{\partial \mathfrak{L}_i}{\partial \lambda_i} &= q_i - F_i(x_i, L_i(\mathbf{w})) = 0. && (49)
\end{aligned}$$

Let $\mathfrak{L}_i^*$, R_i^* and C_i^* respectively denote maximised profit, and its (optimal) revenue and cost components. The following equation: $\frac{\partial \mathfrak{L}_i^*}{\partial q_i} = \frac{\partial R_i^*}{\partial q_i} - \frac{\partial C_i^*}{\partial q_i} = 0$, and eq. (46) imply (2). Based on these results, we can define or interpret λ_i in eq. (47) and (48) as either marginal revenue or marginal cost. We can then use λ_i in the definition of marginal revenue product of labour, as $MRPL_i = \lambda_i \frac{\partial F_i(x_i, L_i(\mathbf{w}))}{\partial x_i}$.

Proof of Theorem 2: Eq. (47) and (3) imply (6). Eq. (48), (3) and (4) imply

$$\frac{\lambda_i}{p_i} = 1 - \rho_i = \frac{\left(1 + \frac{1}{\epsilon_{Li}}\right) \phi_{Li}}{\theta_{Li}}, \chi_{Li} = \frac{1}{\epsilon_{Li}}, \tag{50}$$

where $\epsilon_{Li} \equiv \frac{1}{\frac{\partial L_i(\mathbf{w})}{\partial w_i} \frac{w_i}{L_i(\mathbf{w})}}$. Eq. (50) implies (7). Eq. (7) and (6) imply $\phi_{Li} = \frac{\theta_{Li}(1-\rho_i)}{1+\chi_{Li}}$, $\phi_{Xi} = \theta_{Xi}(1-\rho_i)$ and

$$1 - (\phi_{Li} + \phi_{Xi}) = \rho_i + \frac{\theta_{Li}(1-\rho_i)\chi_{Li}}{(1+\chi_{Li})} + (1 - \theta_{Li} - \theta_{Xi})(1-\rho_i). \tag{51}$$

Identities in (5) and eq. (51) imply

$$\delta_i \equiv 1 - \phi_{Li} - \phi_{Xi}, \tag{52}$$

and (8).

Proof of Theorem 5: Eq. (47), (48) and (50) imply

$$\frac{\theta_{Li}}{\theta_{Xi}} = \frac{w_i(1+\chi_{Li}) l_i}{p_X x_i}. \tag{53}$$

For the Cob-Douglas production function $F_i(x_i, l_i) = A_i x_i^{\alpha} l_i^{1-\alpha}$: $\theta_{Xi} = \alpha$ and $\theta_{Li} = 1 - \alpha$, eq. (53) implies

$$\psi_{Li} = \frac{1-\alpha}{1+\alpha\chi_{Li}}, \tag{54}$$

and (13). Eq. (14) and (15) are immediate implications of (54). (16) is implied by (6).

Proof of Theorem 6: Eq. (20) and (21) imply (6). Eq. $MR_i = MC_i$, (21) and (22) imply (7). Eq. (6), (7) and (52) imply (8).

Proof of Proposition 8: Eq. (32) and (33) imply $\frac{\partial R_i}{\partial l_i} \equiv MRPL_i = b$. $\omega_i \equiv \frac{R_i - p_X x_i}{l_i}$ allows eq. (32) to be rewritten as (35). $\chi_{Li} \equiv \frac{MRPL_i - w_i}{w_i} = \frac{b-w_i}{w_i}$ and (35) imply (36). For the Cobb-Douglas production function $F_i(x_i, l_i) = A_i x_i^{\alpha} l_i^{1-\alpha}$, $\frac{\partial R_i}{\partial l_i} \equiv MRPL_i = \frac{\partial R_i}{\partial q_i}(1-\alpha) A_i \left(\frac{x_i}{l_i}\right)^{\alpha} = b$, $\frac{\partial R_i}{\partial l_i} = \frac{\partial R_i}{\partial q_i} \alpha A_i \left(\frac{x_i}{l_i}\right)^{\alpha-1} = p_X$. These two equations imply (37).

Proof of Theorem 9: Trivial.

Proof of Proposition 10: Eq. (42) is implied by the Cobb-Douglas production function and eq. (6). Eq. (43) is implied by the Cobb-Douglas production function, eq. (6) and the definition of ψ_{Li}. Eq. (44) is implied by the Cobb-Douglas production function and the definition of ψ_{Li}.

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A Extensions

A.1 JOOM with Price Competition and Wage Posting

In this section we study a variant of the JOOM presented in Section 2, by replacing quantity competition in the product market with price competition. Let the firm-specific demand be $D_i(\mathbf{p})$, and the short-run profit maximisation problem be:

$$\max_{p_i,x_i,w_i,D_i(\mathbf{p})\le F_i(x_i,L_i(\mathbf{w}))} \pi_i(\mathbf{p},\mathbf{w},x_i) = p_i D_i(\mathbf{p}) - w_i L_i(\mathbf{w}) - p_X x_i. \tag{55}$$

The Lagrangian multiplier method is given by:

$$\max_{p_i,x_i,w_i,\lambda_i} \mathfrak{L}_i = p_i D_i(\mathbf{p}) - w_i L_i(\mathbf{w}) - p_X x_i - \lambda_i \left(D_i(\mathbf{p}) - F_i(x_i, L_i(\mathbf{w}))\right). \tag{56}$$

The first order conditions are:

$$\frac{\partial \mathfrak{L}_i}{\partial p_i} = D_i(\mathbf{p}) + p_i \frac{\partial D_i(\mathbf{p})}{\partial p_i} - \lambda_i \frac{\partial D_i(\mathbf{p})}{\partial p_i} = 0, \tag{57}$$

$\frac{\partial \mathfrak{L}_i}{\partial x_i} = 0$ and $\frac{\partial \mathfrak{L}_i}{\partial w_i} = 0$, which are identical to (47) and (48). The optimal Lagrangian multiplier λ_i remains to be interpreted as both the marginal cost and marginal revenue, and therefore $\frac{\lambda_i}{p_i} = 1 - \rho_i$ holds, where ρ_i continues to denote the Lerner index. Let

$$\frac{\partial \mathfrak{L}_i}{\partial \lambda_i} = D_i(\mathbf{p}) - F_i(x_i, L_i(\mathbf{w})) = 0. \tag{58}$$

The marginal revenue is given by

$$\lambda_i = p_i + \frac{D_i(\mathbf{p})}{\frac{\partial D_i(\mathbf{p})}{\partial p_i}} = \left(1 - \frac{1}{\epsilon_i}\right) p_i, \tag{59}$$

where $\epsilon_i \equiv -\frac{\partial D_i(\mathbf{p})}{\partial p_i} \frac{p_i}{D_i(\mathbf{p})}$ is the residual demand elasticity of firm i.

Eq. (6) and (50) remain valid for this variant of JOOM. Consequently, eq. (6) - (8) can be extended hereto.

A.2 JOOM with Quantity and Employment Competition

Let the product market demand system be described by $P_i(\mathbf{q})$ for all i, and the labour market supply by wage function $W(L)$, where $L = \sum_{j=1}^{n} l_j$ is the total aggregate labour input and l_j is labour input of firm j. For this variant, the short-run profit maximisation is given by:

$$\max_{q_i, x_i, l_i, q_i \le F_i(x_i, l_i)} \pi_i(\mathbf{q}, \mathbf{l}, x_i) = P_i(\mathbf{q}) q_i - W(L) l_i - p_X x_i, \tag{60}$$

with the Lagrange multiplier method:

$$\max_{q_i, x_i, l_i, \lambda_i} \mathfrak{L}_i = P_i(\mathbf{q}) q_i - W(L) l_i - p_X x_i - \lambda_i (q_i - f_i(x_i, l_i)). \tag{61}$$

The first order conditions that need slightly new treatment are:

$$\frac{\partial \mathfrak{L}_i}{\partial x_i} = -p_X + \lambda_i \frac{\partial F_i(x_i, l_i)}{\partial x_i} = 0, \tag{62}$$

$$\frac{\partial \mathfrak{L}_i}{\partial l_i} = -W(L) - W'(L) l_i + \lambda_i \frac{\partial F_i(x_i, l_i)}{\partial l_i} = 0, \tag{63}$$

$$\frac{\partial \mathfrak{L}_i}{\partial \lambda_i} = q_i - F_i(x_i, l_i) = 0. \tag{64}$$

The optimal Lagrangian multiplier λ_i remains to be interpreted as both the marginal cost and marginal revenue, and therefore $\frac{\lambda_i}{p_i} = 1 - \rho_i$ holds, where ρ_i continues to denote the Lerner index. Define the residual labour supply elasticity for this variant by $\epsilon_{Li} \equiv \frac{1}{\frac{\partial W}{\partial l_i} \frac{l_i}{W}}$ and note that $\epsilon_{Li} = \frac{\epsilon_L}{s_{Li}}$, where $\epsilon_L \equiv \frac{1}{\frac{\partial W}{\partial L} \frac{L}{W}}$ is the market level labour supply elasticity and $s_{Li} \equiv \frac{l_i}{L}$ is the firm's labour market share. With these minor adjustments in place, eq. (6) and (50) remain valid for this variant of JOOM. Consequently, eq. (6) - (8) can be extended to the current variant of JOOM.

A.3 JOOM with Price and Employment Competition

Let the product market demand system be described by $D_i(\mathbf{p})$ for all i, and the labour market supply by wage function $W(L)$, where $L = \sum_{j=1}^{n} l_j$ is the total aggregate labour input and l_j is labour input of firm j. The modified short-run profit maximisation problems are given by:

$$\max_{p_i, x_i, l_i, D_i(\mathbf{p}) \le F_i(x_i, l_i)} \pi_i(\mathbf{p}, \mathbf{l}, x_i) = p_i D_i(\mathbf{p}) - W(L) l_i - p_X x_i. \tag{65}$$

$$\max_{p_i,x_i,l_i,\lambda_i} \mathfrak{L}_i = p_i D_i(\mathbf{p}) - W(L)\, l_i - p_X x_i - \lambda_i \left(D_i(\mathbf{p}) - F_i(x_i, l_i)\right). \quad (66)$$

The first order conditions are the same as eq. (57), (62) and (63) and

$$\frac{\partial \mathfrak{L}_i}{\partial \lambda_i} = D_i(\mathbf{p}) - F_i(x_i, l_i) = 0. \quad (67)$$

For this setting, we need to redefine: $\epsilon_i \equiv -\frac{\partial D_i(\mathbf{p})}{\partial p_i}\frac{p_i}{D_i(\mathbf{p})}$ and $\epsilon_{Li} \equiv \frac{1}{\frac{\partial W}{\partial l_i}\frac{l_i}{W}}$ with $\epsilon_{Li} = \frac{\epsilon_L}{s_{Li}}$ and $\epsilon_L \equiv \frac{1}{\frac{\partial W}{\partial L}\frac{L}{W}}$. Then eq. (6) and (50) remain valid for this variant of JOOM. Consequently, eq. (6) - (8) can be extended to the current variant of JOOM.

B DRS with Perfect Competition

B.1 Model without Distortions

Let each firm's short-run production function have the CES functional form:

$$q_i = A_i \left[\alpha x_i^{1-\frac{1}{\sigma}} + (1-\alpha)\, l_i^{1-\frac{1}{\sigma}}\right]^{\frac{\Theta}{1-\frac{1}{\sigma}}},$$

where Θ is the elasticity of scale, σ is the elasticity of substitution. Note that the CES form degenerates to the Cobb-Douglas form in the limit $\sigma \to 1$. Without imposing this restriction, the results are not dependent on it. We assume perfect competition in all markets, and $\Theta < 1$, i.e., with decreasing returns to scale.

The marginal rate of technical substitution has the following property:

$$\frac{\partial F_i/\partial x_i}{\partial F_i/\partial l_i} = \frac{\alpha}{1-\alpha}\left(\frac{l_i}{x_i}\right)^{\frac{1}{\sigma}} = \frac{p_X}{w},$$

which implies

$$x_i = \left(\frac{\alpha}{1-\alpha}\frac{w}{p_X}\right)^{\sigma} l_i,$$

and the derived demand functions for inputs, and the variable cost function:

$$
\begin{aligned}
x_i &= \left[\frac{\alpha^{\sigma-1} w^{\sigma-1}}{\alpha^{\sigma} w^{\sigma-1} + (1-\alpha)^{\sigma} p_X^{\sigma-1}}\right]^{\frac{\sigma}{\sigma-1}} \left(\frac{q_i}{A_i}\right)^{\frac{1}{\Theta}}, &(68)\\
l_i &= \left[\frac{(1-\alpha)^{\sigma-1} p_X^{\sigma-1}}{\alpha^{\sigma} w^{\sigma-1} + (1-\alpha)^{\sigma} p_X^{\sigma-1}}\right]^{\frac{\sigma}{\sigma-1}} \left(\frac{q_i}{A_i}\right)^{\frac{1}{\Theta}}, &(69)\\
C_i &= \frac{\alpha^{\sigma} p_X w^{\sigma} + (1-\alpha)^{\sigma} p_X^{\sigma} w}{\left[\alpha^{\sigma} w^{\sigma-1} + (1-\alpha)^{\sigma} p_X^{\sigma-1}\right]^{\frac{\sigma}{\sigma-1}}} \left(\frac{q_i}{A_i}\right)^{\frac{1}{\Theta}}. &(70)
\end{aligned}
$$

The assumption of perfect competitin in the product market implies $\partial C_i/\partial q_i = p$, and

$$
q_i = \frac{\left[\alpha^{\sigma} w^{\sigma-1} + (1-\alpha)^{\sigma} p_X^{\sigma-1}\right]^{\frac{\sigma}{\sigma-1}\frac{\Theta}{1-\Theta}}}{\left[\alpha^{\sigma} p_X w^{\sigma} + (1-\alpha)^{\sigma} p_X^{\sigma} w\right]^{\frac{\Theta}{1-\Theta}}} A_i^{\frac{1}{1-\Theta}} (p\Theta)^{\frac{\Theta}{1-\Theta}}. \tag{71}
$$

Now we can show that firm activities are highly responsive to productivity using the elasticities of output and inputs with respect to productivity:

$$
\begin{aligned}
\frac{d\ln q_i}{d\ln A_i} &= \frac{1}{1-\Theta} > 1,\\
\frac{d\ln x_i}{d\ln A_i} &= \frac{\partial \ln x_i}{\partial \ln A_i} + \frac{\partial \ln x_i}{\partial \ln q_i} + \frac{d\ln q_i}{d\ln A_i} = \frac{1}{1-\Theta} > 1,\\
\frac{\partial \ln l_i}{\partial \ln A_i} &= \frac{\partial \ln l_i}{\partial \ln A_i} + \frac{\partial \ln l_i}{\partial \ln q_i} + \frac{d\ln q_i}{d\ln A_i} = \frac{1}{1-\Theta} > 1.
\end{aligned}
$$

Each firm's value added is defined by $VA_i = pq_i - p_X x_i$. It is routine to verify:

$$
\frac{\partial \ln VA_i}{\partial \ln A_i} = \frac{1}{1-\Theta} > 1.
$$

The value added per worker is defined by:

$$
\omega_i \equiv \frac{VA_i}{l_i} = \frac{pq_i - p_X x_i}{l_i}.
$$

Using eq. (68), (69) and (71), we derive:

$$
\omega_i = \frac{(1-\Theta)\left(\frac{\alpha}{1-\alpha}\right)^{\sigma}\left(\frac{w}{p_X}\right)^{\sigma-1} + 1}{\Theta} w, \tag{72}
$$

which shows that value added per worker is invariant to productivity, resulting in equalisation of value added per worker, and hence

$$\frac{\partial \ln \omega_i}{\partial \ln A_i} = 0.$$

The marginal cost function of each firm obeys $MC_i = p$, it is also invariant to productivity, i.e.,

$$\frac{\partial \ln MC_i}{\partial \ln A_i} = 0.$$

The gross profit margin is

$$\delta_i \equiv \frac{pq_i - p_X x_i - wl_i}{pq_i} = 1 - \Theta, \tag{73}$$

which is invariant to productivity.

B.2 Model with Distortions

To reconcile the theoretical predictions with the observation of dispersions in value added per worker and gross profit margin, some form of distortion will have to be added. Mirroring the resources mis-allocation literature, we explore the distortion caused by labour adjustment cost. This distortion is equivalent to a firm-specific labour income "tax/subsidy" τ_i^L imposed onto the market wage rate w, then the wage rate faced by firm i becomes $\tilde{\omega}_i = (1 + \tau_i^L)w$ with $(1 + \tau_i^L) > 0$, and eq. (72) should be replaced by

$$\tilde{\omega}_i = \frac{(1-\Theta)\left(\frac{\alpha}{1-\alpha}\right)^{\sigma}\left[\frac{(1+\tau_i^L)w}{p_X}\right]^{\sigma-1} + 1}{\Theta}(1+\tau_i^L)w, \tag{74}$$

with $\frac{\partial \tilde{\omega}_i}{\partial \tau_i^L} > 0$. In this case,

$$\frac{\partial \tilde{\omega}_i}{\partial A_i} = \frac{\partial \tilde{\omega}_i}{\partial \tau_i^L}\frac{\partial \tau_i^L}{\partial A_i} > 0 \text{ if } \frac{\partial \tau_i^L}{\partial A_i} > 0, \tag{75}$$

that is, a dispersion in $\tilde{\omega}_i$ can be generated by a dispersion in the distortion τ_i^L, if τ_i^L is positively correlated with A_i (a standard assumption in the resources mis-allocation literature). At the same time, the externally observable gross profit margin becomes

$$\begin{aligned}\tilde{\delta}_i &\equiv \frac{p\tilde{q}_i - p_X\tilde{x}_i - w\tilde{l}_i}{p\tilde{q}_i} = \frac{p\tilde{q}_i - p_X\tilde{x}_i - \tilde{\omega}_i\tilde{l}_i}{p\tilde{q}_i} + \frac{\tau_i^L w\tilde{l}_i}{p\tilde{q}_i} \\ &= 1 - \Theta + \frac{(1-\alpha)^{\sigma} p_X^{\sigma}\Theta\tau_i^L}{\alpha^{\sigma} p_X(1+\tau_i^L)^{\sigma} w^{\sigma} + (1-\alpha)^{\sigma} p_X^{\sigma}(1+\tau_i^L)w},\end{aligned} \tag{76}$$

where $\tau_i^L w\tilde{l}_i$ is the labour "tax/subsidy" not included in variable cost of the firm. The relationship between $\tilde{\delta}_i$ and A_i is given by

$$\frac{\partial\tilde{\delta}_i}{\partial A_i} = \frac{\alpha^\sigma \left(1-\alpha\right)^\sigma \Theta^{\sigma-1} p_X^{\sigma+1} w^\sigma (1+\tau_i^L) + \left(1-\alpha\right)^{2\sigma} \Theta p_X^{2\sigma} w}{\left[\alpha^\sigma p_X (1+\tau_i^L)^\sigma w^\sigma + \left(1-\alpha\right)^\sigma p_X^\sigma (1+\tau_i^L) w\right]^2} \frac{\partial \tau_i^L}{\partial A_i} > 0 \text{ if } \frac{\partial \tau_i^L}{\partial A_i} > 0, \tag{77}$$

implying that the dispersion in A_i causes a dispersion in $\tilde{\delta}_i$, if τ_i^L is positively correlated with A_i.

C CES Function and Non-neutral Productivity

In this section we first show that, for the CES production function, having a free firm-specific distribution parameter is equivalent to having a free firm-specific labour-augmenting productivity parameter. We then show that the combination of Cobb-Douglas production and a firm-specific correction term described below have certain equivalence to using the CES production function.

Let the short-run production function be of CES form:

$$F_i\left(x_i, l_i\right) = A_i \left[\alpha_i x_i^{1-\frac{1}{\sigma}} + \left(1-\alpha_i\right) l_i^{1-\frac{1}{\sigma}}\right]^{\frac{\Theta}{1-\frac{1}{\sigma}}}, \tag{78}$$

where A_i is Hicks neutral productivity parameter, σ is the elasticity of substitution, Θ is the elasticity of short-run returns to scale, and α_i is the firm-specific distribution parameter. Note that the firm specific-parameter α_i is essentially a free parameter that is useful to explain the observed dispersion in inputs ratio $\varkappa_i \equiv \frac{x_i}{l_i}$ across firms. To see this, taking $\varkappa_i$, price of intermediate input p_X and wage rate w as given. Under the premise of perfect competition in flexible input markets, the standard relationship $\varkappa_i \equiv \frac{x_i}{l_i} = \left(\frac{\alpha_i}{1-\alpha_i}\frac{w}{p_X}\right)^\sigma$ implies

$$\alpha_i = \frac{p_X \varkappa_i^{\frac{1}{\sigma}}}{p_X \varkappa_i^{\frac{1}{\sigma}} + w}, \tag{79}$$

that is, the free parameter α_i can explain firm-specific observation of $\varkappa_i$.

Now we show that this free parameter α_i can be equivalently formulated as a non-neutral productivity parameter. To see that, note that eq. (78) can rewritten as

$$F_i\left(x_i, l_i\right) = \tilde{A}_i \left[\alpha x_i^{1-\frac{1}{\sigma}} + \left(1-\alpha\right)\left(\tilde{B}_i l_i\right)^{1-\frac{1}{\sigma}}\right]^{\frac{\Theta}{1-\frac{1}{\sigma}}}, \tag{80}$$

which has a firm-specific labour-augmenting productivity parameter:

$$\tilde{B}_i = \left[\frac{\alpha\,(1-\alpha_i)}{\alpha_i\,(1-\alpha)}\right]^{\frac{1}{1-\frac{1}{\sigma}}}, \tag{81}$$

where α is the industry common distribution parameter, and

$$\tilde{A}_i = A_i \left(\frac{\alpha_i}{\alpha}\right)^{\frac{\Theta}{1-\frac{1}{\sigma}}} \tag{82}$$

is the modified Hicks neutral productivity parameter. The formulation (80) is akin to Raval (2023).

Formulation (78) has one advantage over eq. (80) in that it is insensitive to the elasticity of substitution parameter σ. To illustrate this, note that for the limit $\sigma \to 1$, the production function (78) converges to the Cobb-Douglas form:

$$F_i\,(x_i, l_i) = A_i \left(x_i^{\alpha_i} l_i^{1-\alpha_i}\right)^{\Theta}, \tag{83}$$

which inherits the free firm-specific distribution parameter α_i, useful for explaining firm specific input ratio $\varkappa_i$ under the premise of perfect competition in flexible input markets.

Now let production function (78) be approximated by

$$F_i\,(x_i, l_i) = A_i \left(x_i^{\alpha} l_i^{1-\alpha}\right)^{\Theta}, \tag{84}$$

which results from restrictions: $\sigma \to 1$ and $\alpha_i = \alpha$. Since the distribution parameter α is no longer firm-specific, this production can no longer explain the dispersion in inputs ratio $\varkappa_i$ across firms under the premise of perfect competition in flexible input markets. Our alternative theoretical explanation based on diverse rent-sharing mechanisms has the free non-technological parameter

$$\chi_{Li} = \frac{1-\alpha}{\alpha}\frac{p_X}{w}\varkappa_i - 1. \tag{85}$$

In case the assumption of perfect competition in flexible input markets is true, then the relationship between the free parameters χ_{Li} and α_i is described by:

$$\chi_{Li} = \frac{1-\alpha}{\alpha}\left(\frac{\alpha_i}{1-\alpha_i}\right)^{\sigma}\left(\frac{w}{p_X}\right)^{\sigma-1} - 1, \tag{86}$$

which shows that χ_{Li} and α_i have certain equivalence for any non-degenerate CES production function with $\sigma \neq 1$. The Cobb-Douglas approximation $A_i \left(x_i^{\alpha} l_i^{1-\alpha}\right)^{\Theta}$ combined with the free parameter χ_{Li} proves to be reasonably flexible. The dispersions in α_i and χ_{Li} are equally good in explaining the dispersion in $\varkappa_i$. These results show that imposing the Cobb-Douglas functional form (84) combined with the firm-specific correction term χ_{Li} is not a more stringent restriction than using the CES production function with labour augmenting productivity parameter.

D Supplementary Results

D.1 Dispersed Distributions

Figure 6 presents scatter plots of the number of firms for different values of the coefficients of variation (CV) in δ (left panel) and ω (right panel) for the full sample of SIC4 manufacturing industries. It shows sizeable dispersions in the distributions of δ and ω as long as the number of firms exceeds certain threshold, say, 20.

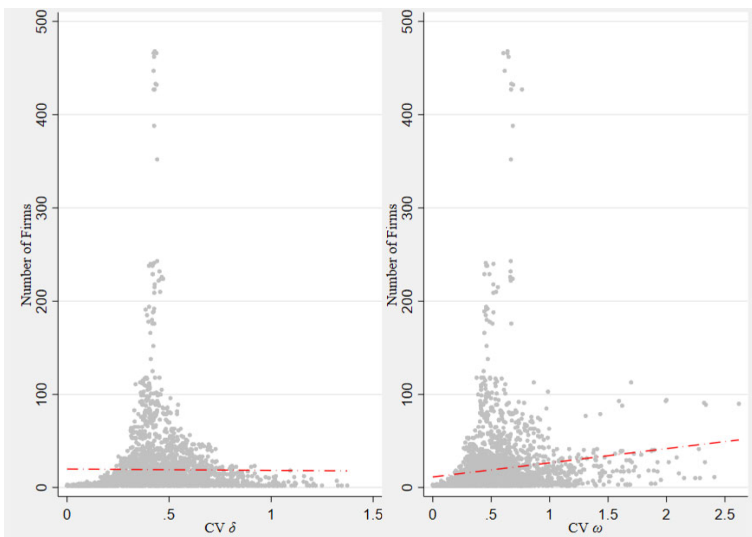

Figure 6: Dispersions and Number of Firms (full sample)

Figure 7 presents the same scatter plot but only for the sub-sample of RS type II firms. This also shows sizeable dispersions in the distributions of δ and ω as long as the number of firms exceeds certain threshold, say, 10.

D.2 Distributions of δ_{min}

Figure 8 shows the distributions of δ_{min} for various samples. Our interest is on RS type I firms (bottom panels), because it is only for these firms that δ_i represents an upper bound to the scale parameter $(1 - \Theta)$. However, for comparison, we also report the equivalent distributions for all RS types (top two panels).

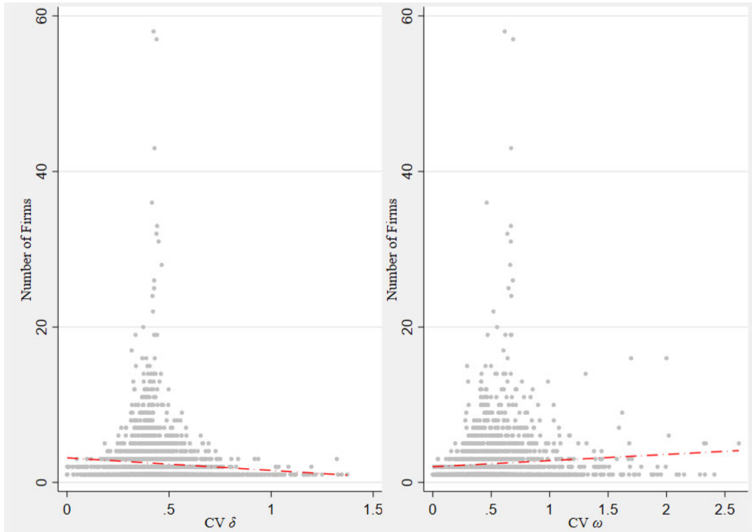

Figure 7: Dispersions and Number of Firms (RS type II)

For RS type I firms the panel on the right restricts the sample to industry-year observations with at least five RS type I firms. The bottom right panel is our preferred distribution because it avoids negative value of χ_{Li} and the case where the number of firms is so small that the accuracy of approximation of competitive fringe firms is limited. For this distribution, the mean value of δ_{min} is just above 0.1. By eq. (8) δ_{min} forms an upper bound to $(1-\Theta)$, which is also an upper bound to the approximation error caused by imposing the restriction: $\Theta = 1$.

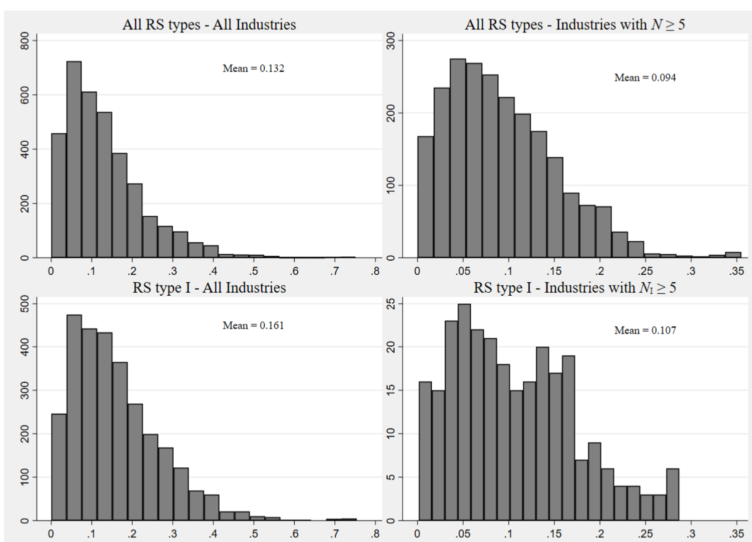

Figure 8: Distributions of δ_{min}

D.3 More on RS Types Distribution

Table 10 reports the Markov transition matrices of firms in the sample between 2005 and 2010 (top panel) and between 2005 and 2015 (bottom panel). For instance, the probability of transition from RS type I to RS type II after 5 (respectively 10) years is 14.5% (respectively 12.4%). The fact that the percentages on the diagonal are large suggests that there is persistency in the RS type classification, in particular for RS types I and III.

Table 10: RS Types Transition Matrix

	RS type 2010			
RS type 2005	I	II	III	Total
I	4,435 (54.09)	1,190 (14.51)	2,575 (31.40)	8,200 (100.00)
II	1,147 (23.65)	1,377 (28.39)	2,326 (47.96)	4,850 (100.00)
III	2,027 (8.66)	1,779 (7.60)	19,600 (83.74)	23,406 (100.00)
Total	7,609 (20.87)	4,346 (11.92)	24,501 (67.21)	36,456 (100.00)
	RS type 2015			
RS type 2005	I	II	III	Total
I	3,848 (47.55)	1,002 (12.38)	3,242 (40.06)	8,092 (100.00)
II	1,179 (24.77)	1,056 (22.18)	2,525 (53.05)	4,760 (100.00)
III	2,056 (8.93)	1,632 (7.09)	19,329 (83.98)	23,017 (100.00)
Total	7,083 (19.75)	3,690 (10.29)	25,096 (69.97)	35,869 (100.00)

Table 11: RS Types Distribution Based on Perfect Competition Model

	RS type			
	I	II	III	Total
Obs.	25091	10461	24812	60367
%	41.6	17.3	41.1	100

To show the effect of restricting the application of cost share approach to competitive fringe firms, we can compute the distribution of RS types when we use all firms in the sample to calculate the parameter α, based on the assumption of perfect competition in product and labour markets (results reported in Table 11). In comparison with Table 7 the proportion of RS type III reduces from 70% to 41%.

D.4 IV Wage Regressions

Our theoretical analysis underpins the argument that firms' short-run productivity A is predetermined and its proxy measurement ω should be treated as exogenous in wage regressions. For robustness check, we may entertain an alternative analysis that allows for a reverse causality: higher wages w causing higher value added per worker ω. Specifically, since w measures the average wage rate among varying skill levels, higher w may indicate that a firm employs a higher proportion of high-skilled workers, which may cause higher ω, and thus an endogeneity problem. If firms' differences in employing high- or low-skilled workers are constant over time, then the fixed effect regression can duly control for this unobserved heterogeneity. Hereafter, we check the robustness of our results using two different instrumental variables: lag values of ω and fixed capital per worker. The use of lags of ω can soften any concern that change in labour quality in t (and, in turn, in average wage) may drive changes in productivity in the same period. We use fixed capital per worker as an alternative IV since in our theoretical framework fixed capital is a determinant of short-run productivity. Tables 12 reports the results of the two IV regressions for RS types I and III, together with OLS regression results for ease of comparison. We do not report the estimates for RS type II because we obtain very low value of the first-stage F statistics, most likely due to the fact of small sample size of RS type II. The main features of the OLS (our preferred specification) results are by and large confirmed in the IV regressions, specifically the support for hypothesis (H2).

D.5 Further Wage Regressions

The equation below:

$$\ln w_i = \xi_6 + \xi_7 \ln \omega_i + \varepsilon_i.$$

is a reduced-form regression, useful to test the following two hypothesis:

(H3) $\xi_7 \in (0, 1)$ for all RS types.

(H4) The value of ξ_7 for RS type III exceeds that for RS types I.[38]

Table 13 reports OLS and FE regression results based on this equation. The coefficient on $\ln \omega$ is known in the literature as the 'elasticity of rent sharing'. Its estimates for RS types I is in the ranges of $[0.34, 0.38]$ and $[0.39, 0.47]$, the value for RS type III is in the range of $[0.47, 0.51]$, all consistent with hypothesis (H3), which captures cross-firm wage differential: $\xi_7 > 0$, as well as firm-level wage stagnation: $\xi_7 < 1$. All relevant regressions are

[38]Because the reduced form regression does not match the theoretical prediction very tightly, this hypothesis focuses on the sharper difference between oligopsony labour market rent-sharing mechanism and efficient bargaining mechanism only.

Table 12: Regressions of w on ω (2)

	RS type I			RS type III		
	(1) OLS	(2) IV	(3) IV	(4) OLS	(5) IV	(6) IV
ω	0.1290***	0.1414***	0.1655***	0.1807***	0.1847***	0.3466***
	(0.0248)	(0.0202)	(0.0624)	(0.0248)	(0.0234)	(0.0315)
$\omega \ln(1-\delta)$	0.0559***	0.0552***	0.0605**	0.1106***	0.1263***	0.2045***
	(0.0106)	(0.0077)	(0.0272)	(0.0155)	(0.0152)	(0.0196)
$\omega \ln(1+\chi_L)$	-0.0262***	-0.0310***	-0.0685***	-0.0311***	-0.0479***	-0.0101
	(0.0056)	(0.0048)	(0.0236)	(0.0076)	(0.0131)	(0.0074)
$\ln(1-\delta)$	9.710***	15.17***	0.9944	6.934***	8.257***	7.340***
	(2.999)	(3.801)	(13.22)	(0.8699)	(0.9917)	(1.417)
$\ln(1+\chi_L)$	0.5088**	1.309***	8.898*	-0.4975	0.3607	-3.320***
	(0.2390)	(0.4155)	(5.150)	(0.4158)	(0.7371)	(0.5006)
Ind. FE	Yes	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
R^2 adj	0.519			0.616		
F-test (1st Stage)		43.04	1.951		9.209	17.40
Obs	11863	7075	10940	41971	33514	39013

Note: The table shows the estimated coefficients of eq. (45). Columns (2), (3), (5) and (6) are IV regressions. The IVs used for (2) and (5) are lag value of ω, and its respective interaction terms with $\ln(1-\delta)$ and $\ln(1+\chi_L)$. The IVs used for (3) and (6) are logarithm of fixed capital per worker, and the respective interaction terms of fixed capital per worker with $\ln(1-\delta)$ and $\ln(1+\chi_L)$. Kleibergen-Paap Wald F-stat is reported for the first stage. Standard errors are clustered at industry level. ***, ** and * respectively indicates statistical significance at 1%, 5% and 10% level.

also consistent with (H4), thus indicating that the efficient bargaining mechanism supports workers' countervailing seller power which counteracts firms' buyer power in labour market.

Table 13: Regressions of $\ln w$ on $\ln \omega$

	RS type I		RS type II		RS type III	
	(1)	(2)	(3)	(4)	(5)	(6)
	OLS	Firm FE	OLS	Firm FE	OLS	Firm FE
$\ln \omega$	0.3832***	0.3443***	0.3925***	0.4680***	0.5072***	0.4690***
	(0.0206)	(0.0317)	(0.0190)	(0.0765)	(0.0254)	(0.0170)
Ind. FE	Yes	No	Yes	No	Yes	No
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
R^2 adj	0.522	0.852	0.648	0.857	0.645	0.903
Obs	11863	11863	6533	6533	41971	41971

Note: The table shows the estimated coefficients of $\ln \omega$. Standard errors are clustered at industry level. *** indicates statistical significance at 1% level.