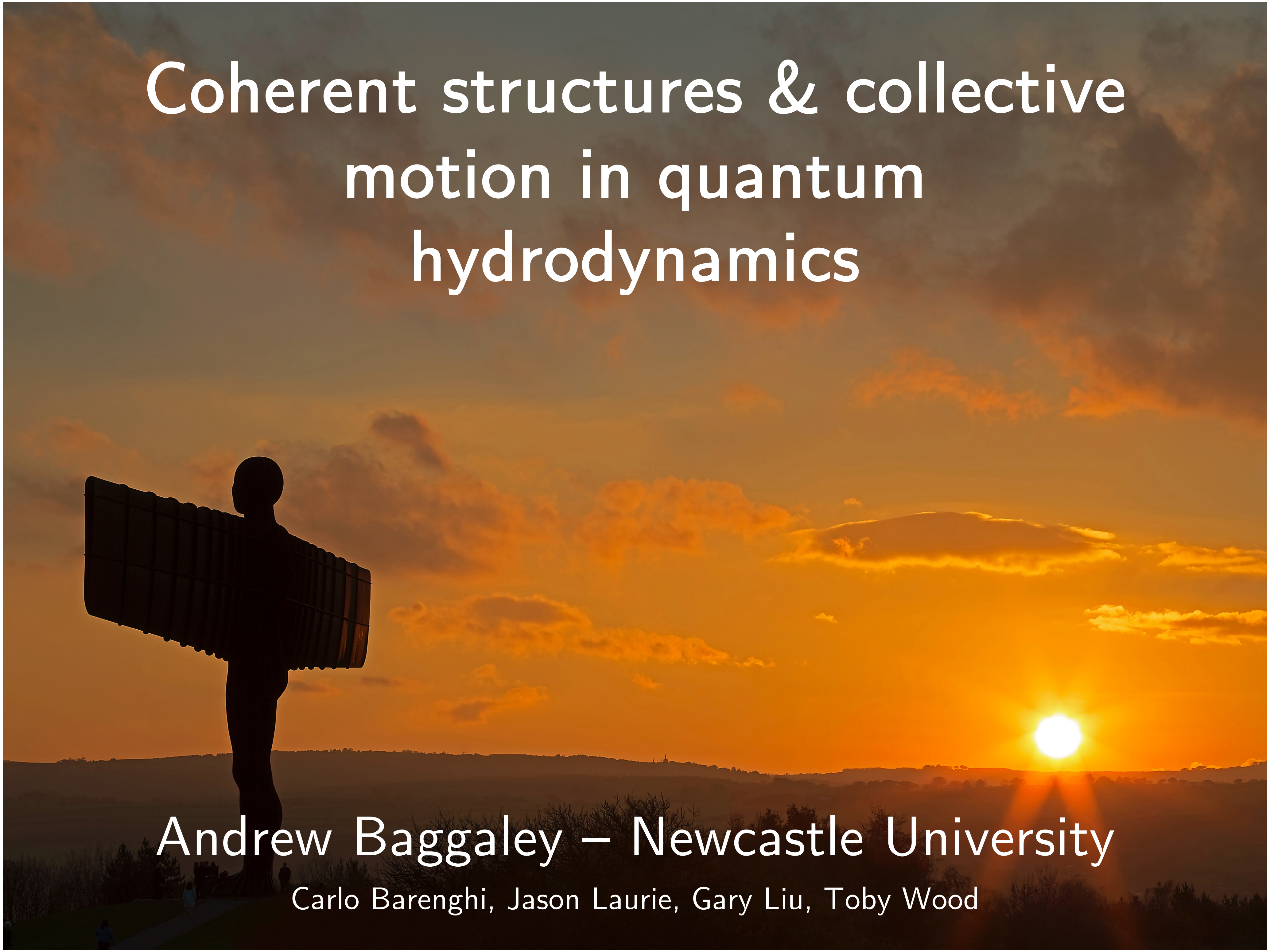
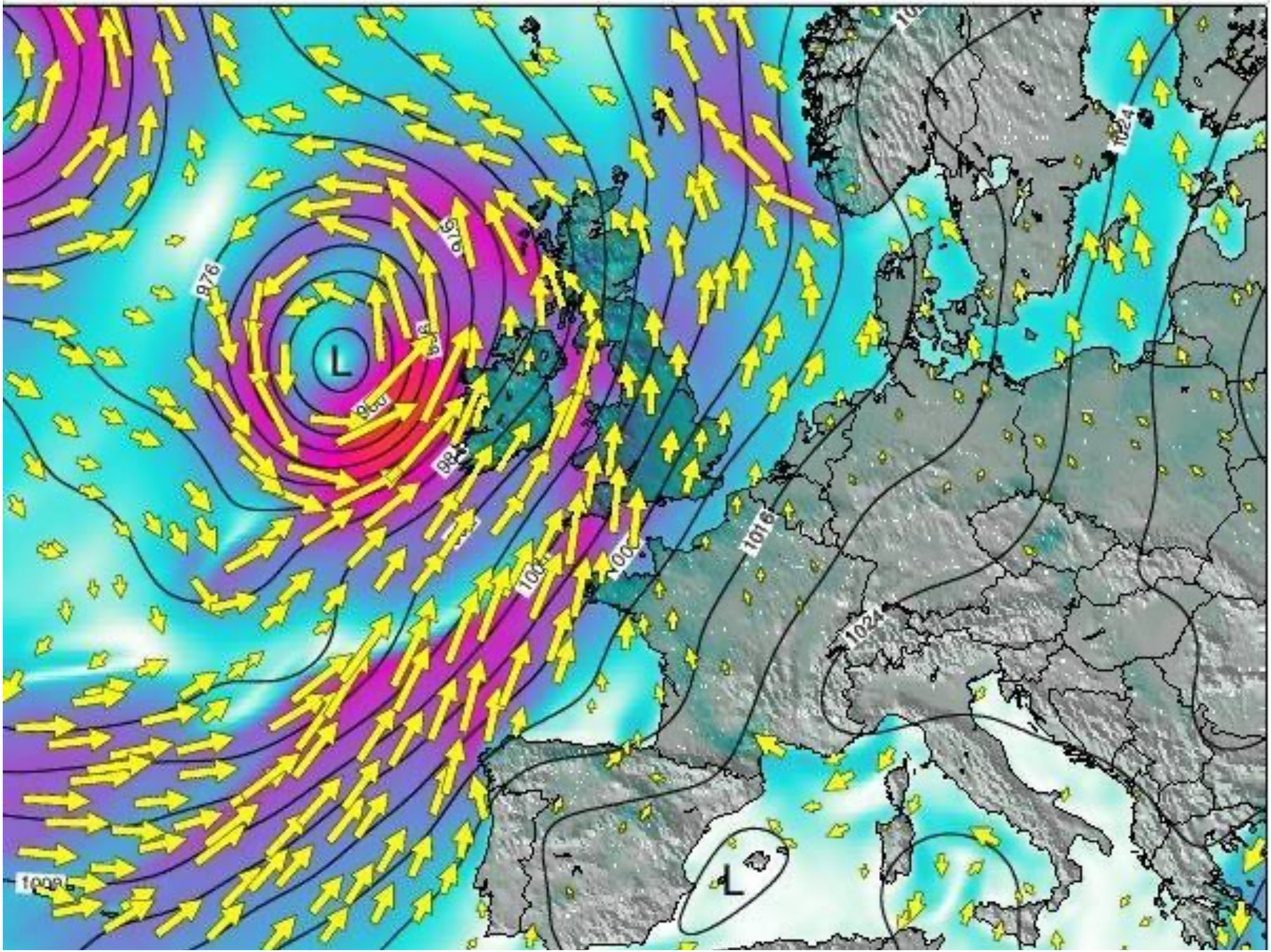


Coherent structures & collective motion in quantum hydrodynamics

Andrew Baggaley – Newcastle University

Carlo Barenghi, Jason Laurie, Gary Liu, Toby Wood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976
984
1016
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Classical Vorticity

$$\boldsymbol{\omega} = \nabla \times \mathbf{v}$$

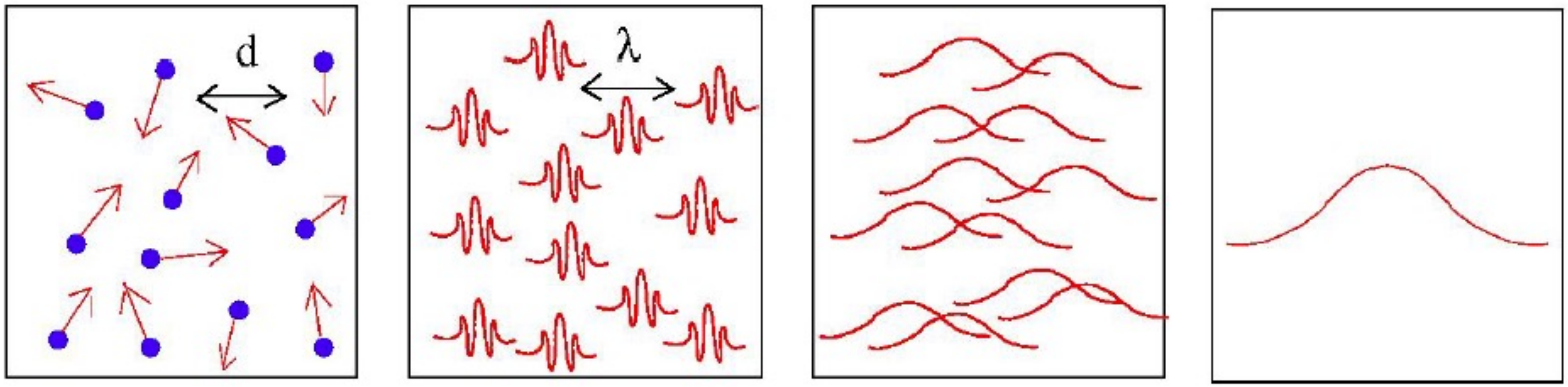
Bose - Einstein Condensation

Quantum Fluids

- Atom with momentum $p = mv$ has wavelength $\lambda = h/p$
- Average kinetic energy $mv^2/2 \approx k_B T$
- Wavelength increases with decreasing T:

$$\lambda \approx \frac{h}{\sqrt{mk_B T}}$$

- Compare λ against the average distance between atoms, d:

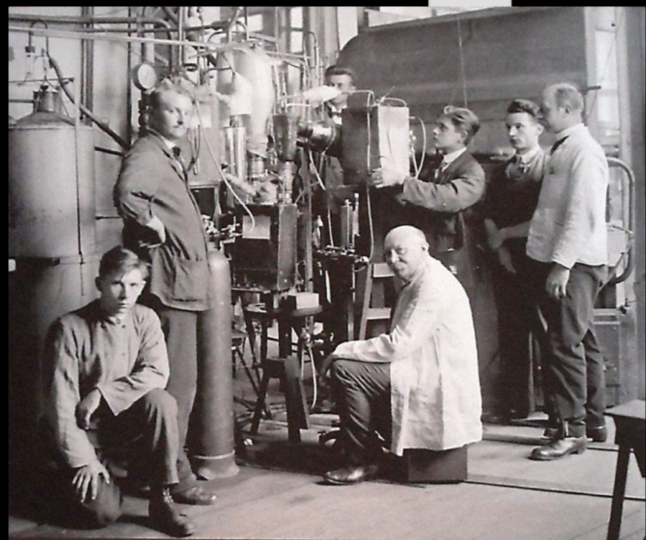


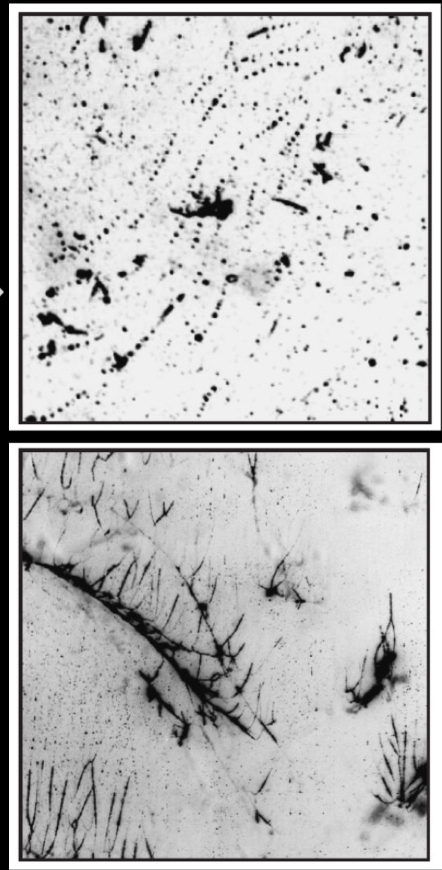
BEC occurs when $\lambda \approx d$

$$\Gamma = \oint_C \mathbf{v} \cdot d\mathbf{l} \in \mathbb{R}$$

Onsager, 1949

$$\Gamma = \oint_C \mathbf{v} \cdot d\mathbf{l} = \frac{2\pi\hbar}{m} n$$

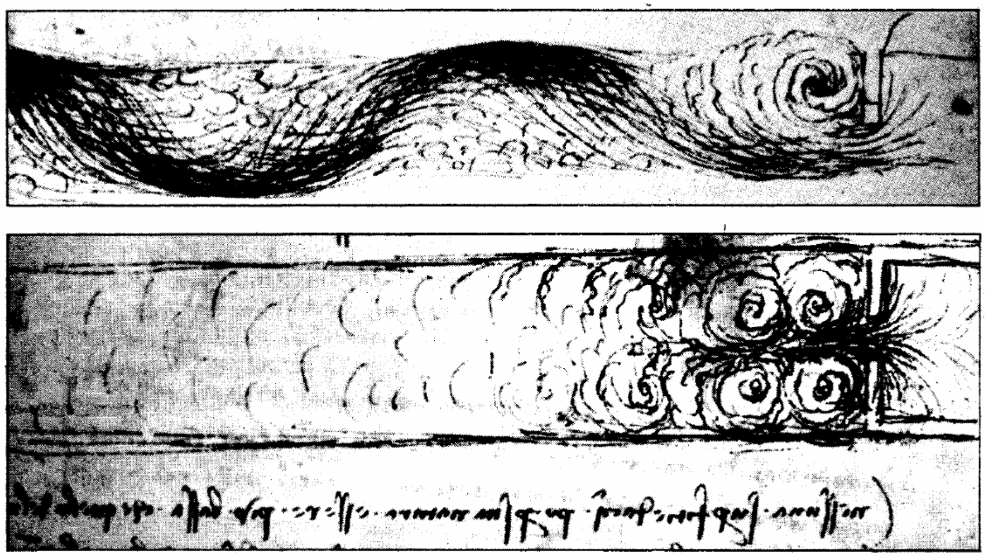




Paoletti *et al.*, 2008

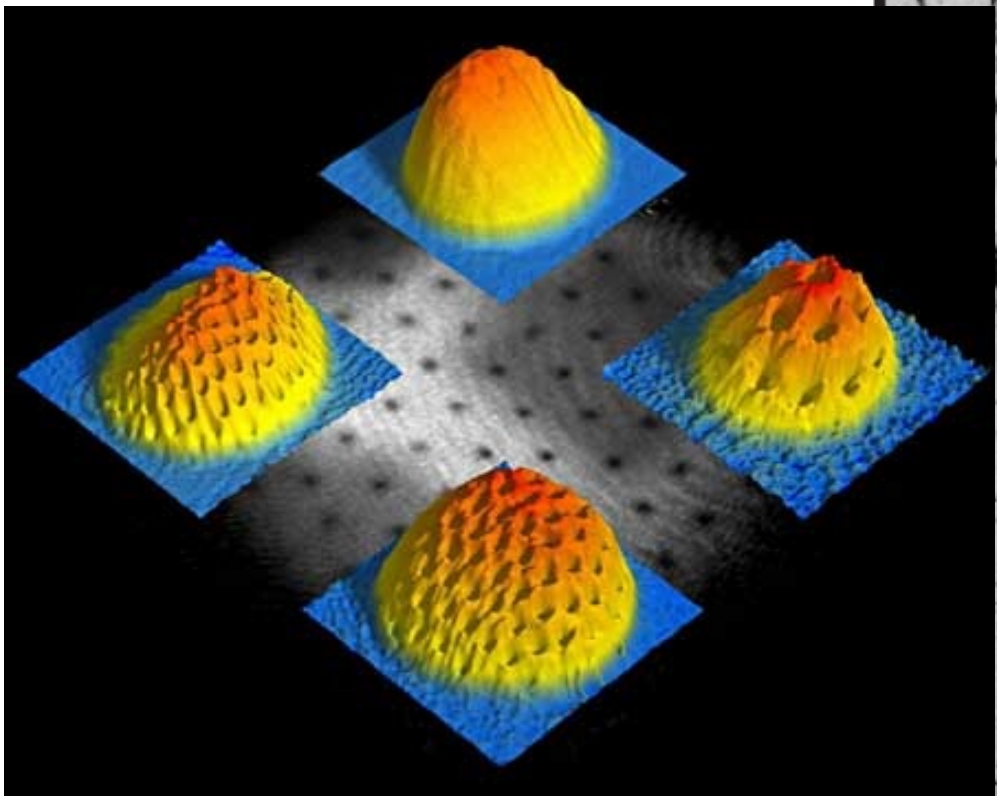
Kuchemann:

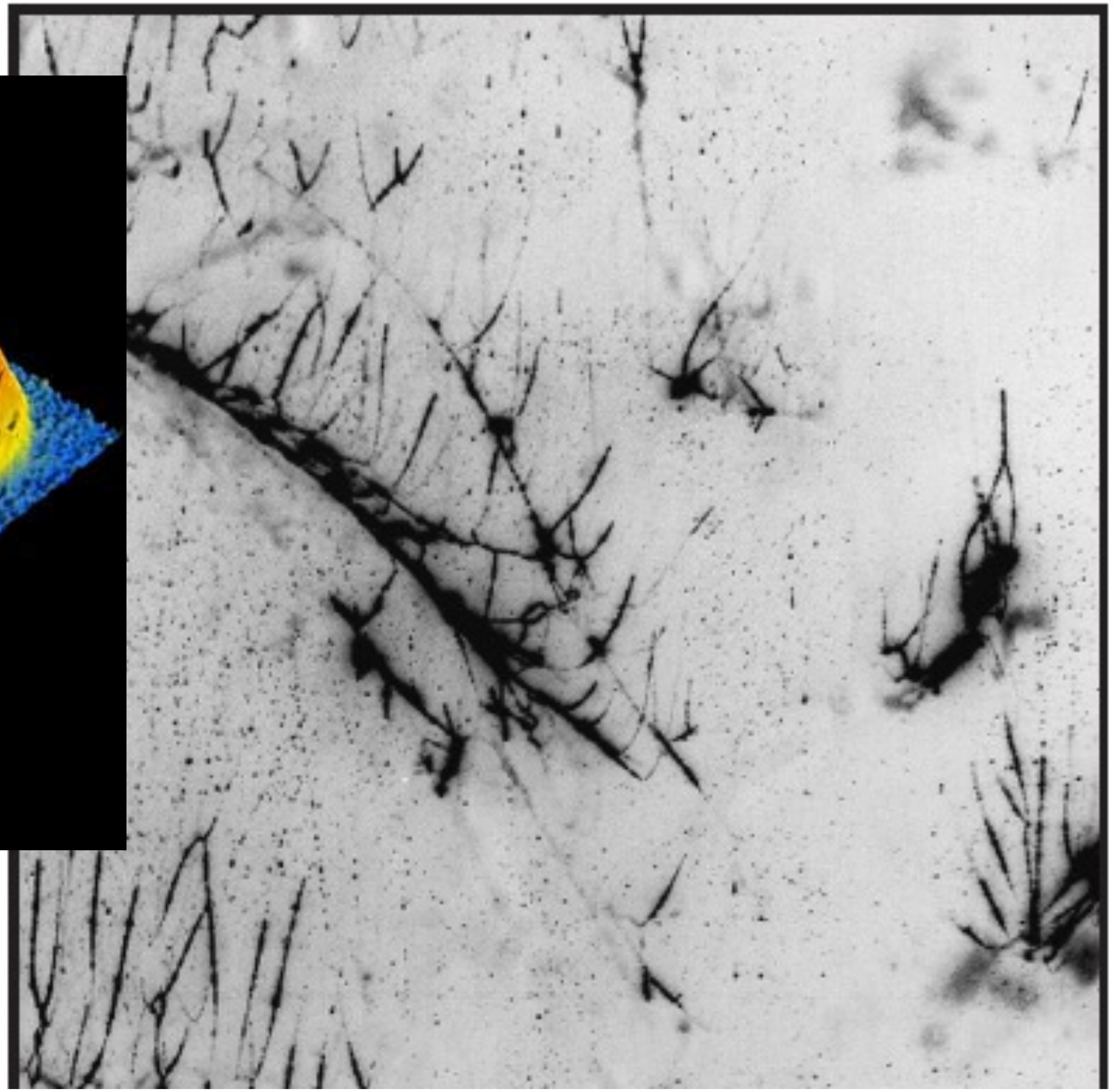
“vortices are the sinews and muscles of fluid motions”



These two drawings from Leonardo's studies in hydrodynamics, now in the collection of the Library of the Institut de France, represent vortex formation with the flow of water around an obstacle or through an opening in a partition within a trough. The second figure of symmetric counter-rotating vortices brings to mind Theodore von Karman's vortex street of asymmetric counter-rotating vortices formed in the wake of a circular cylinder moving through a field. In his 1954 **Aerodynamics,** *von Karmann wrote: "I do not claim to have discovered these vortices: they were known long before I was born. The earliest picture in which I have seen them is one in a church in Bologna, Italy, where St. Christopher is shown carrying the child Jesus across a flowing stream. Behind the saint's naked foot the painter indicated alternating vortices."*

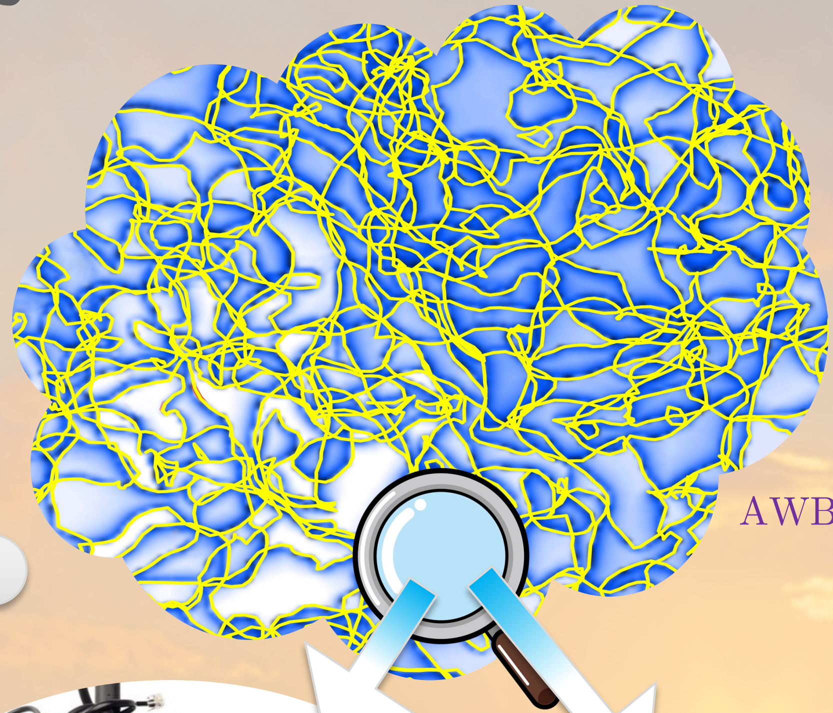
If this is true then Quantum Turbulence represents the 'skeleton'





Complex structures in quantum hydrodynamics?

n.b. Term "Quantum Turbulence" due to Donnelly & Swanson (1986)

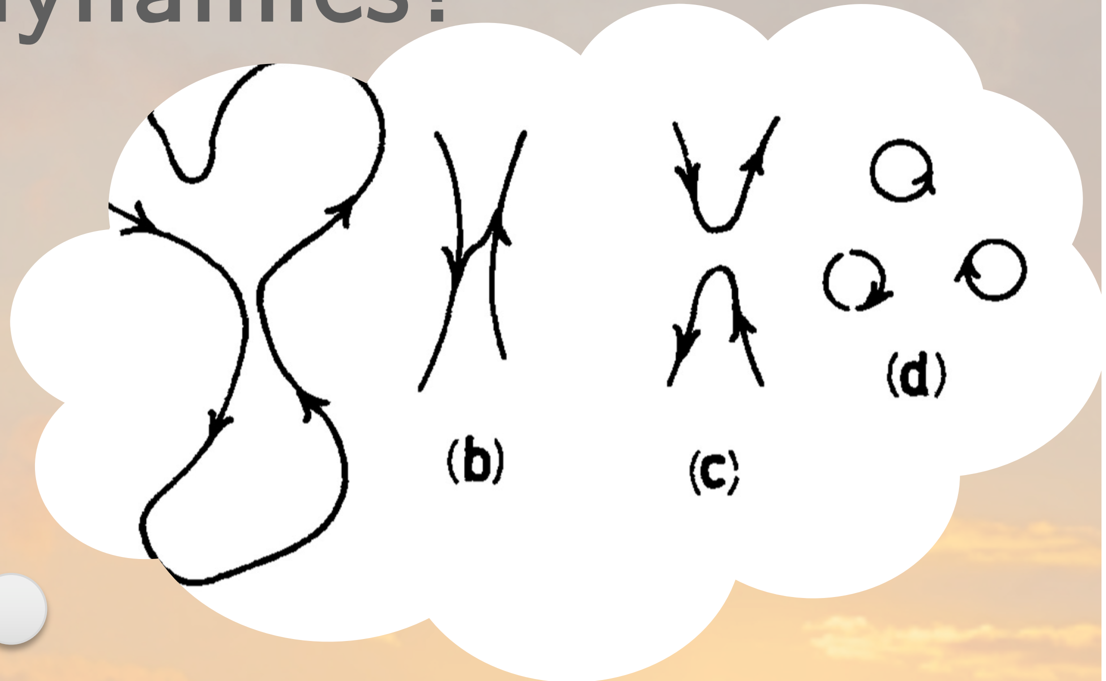


AWB *et al*, 2015

Feynman, 1955

Complex structures in quantum hydrodynamics?

n.b. Term "Quantum Turbulence"
due to Donnelly & Swanson (1986)



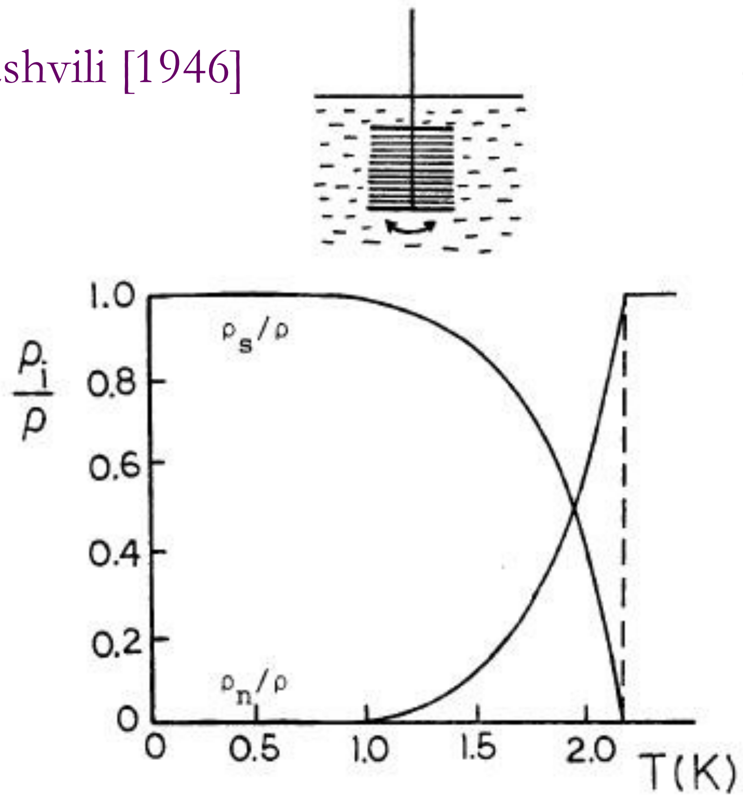
Feynman, 1955

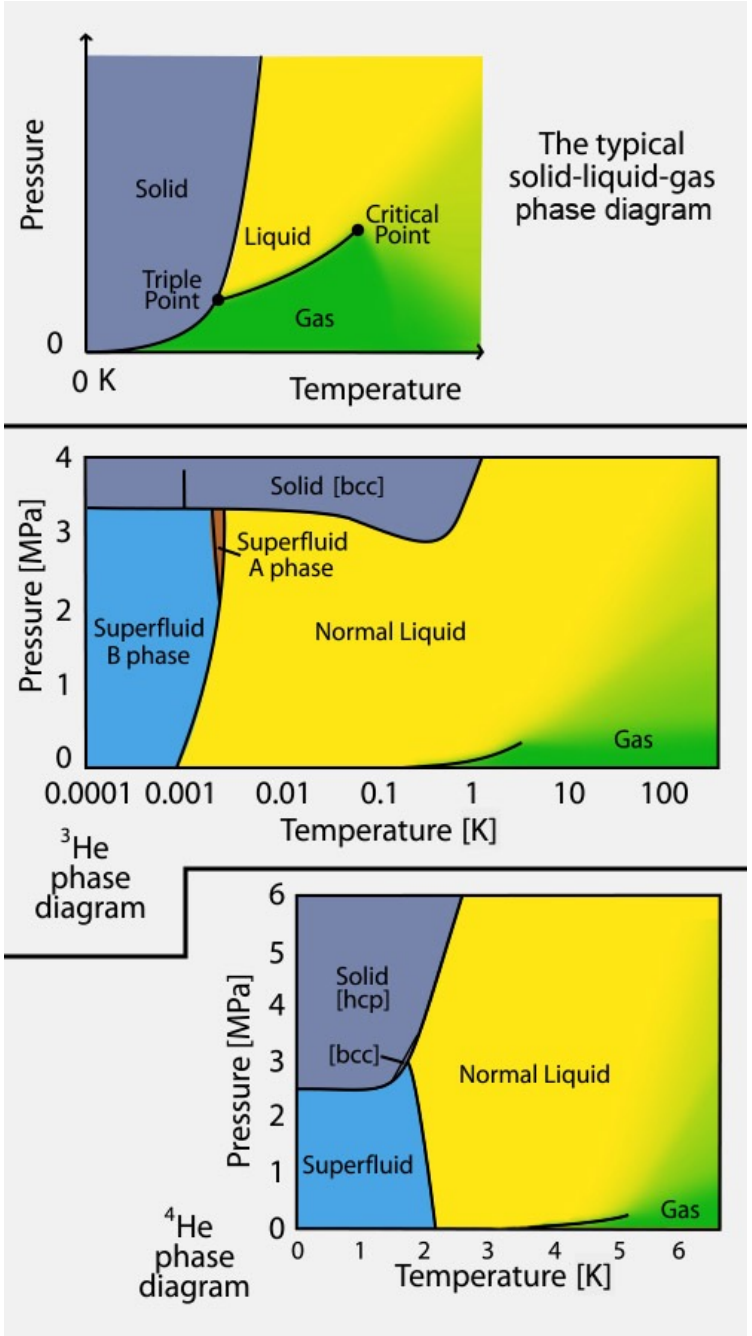
Experimental/Physical Systems

Superfluid Helium ($^4He/^3He3$)

- Helium is an intimate mix of inviscid superfluid component and a viscous normal fluid.

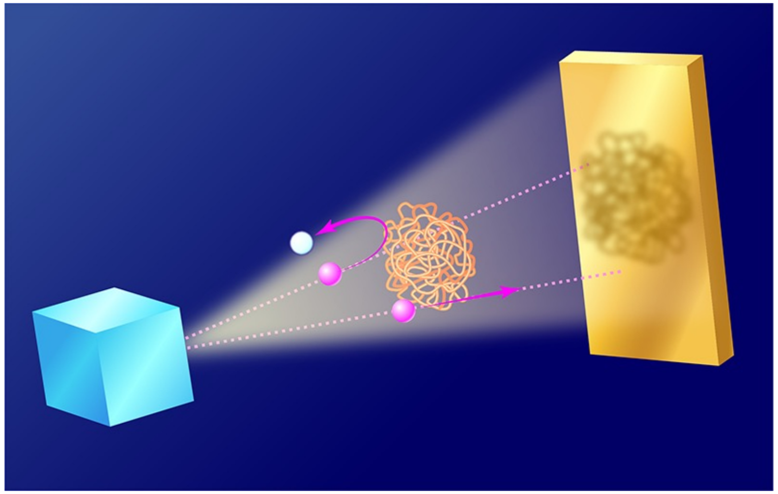
Andronikashvili [1946]

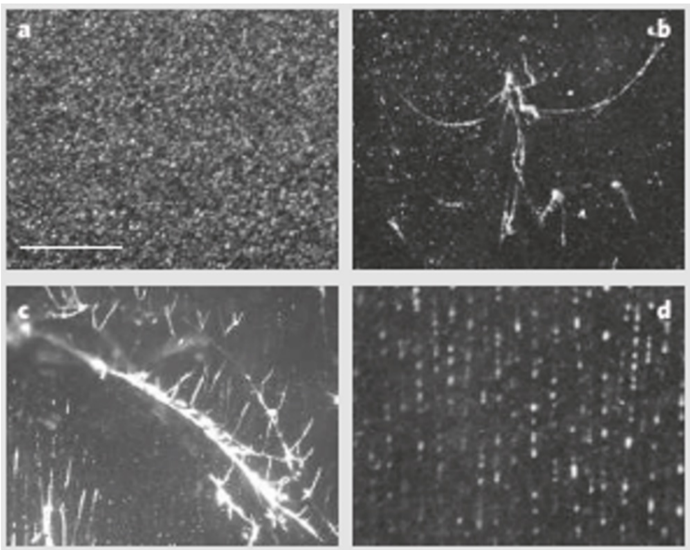


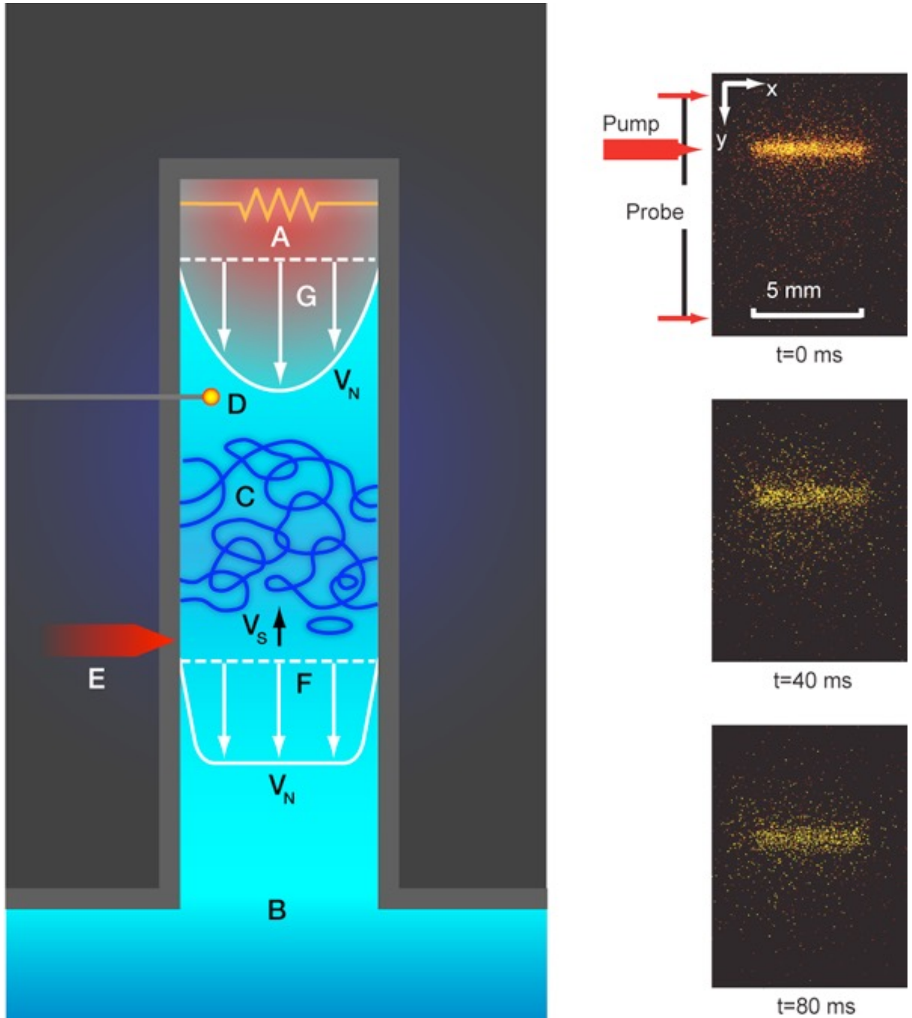


Experimental/Physical Systems

Superfluid Helium (^{4}He/^{3}He3)



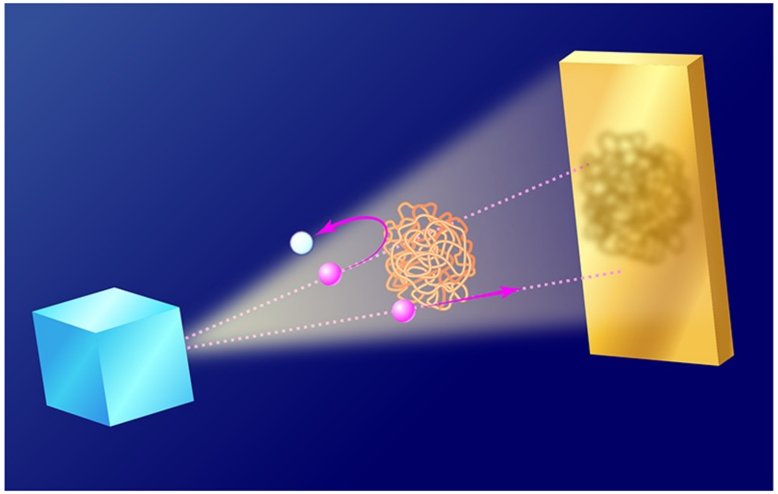


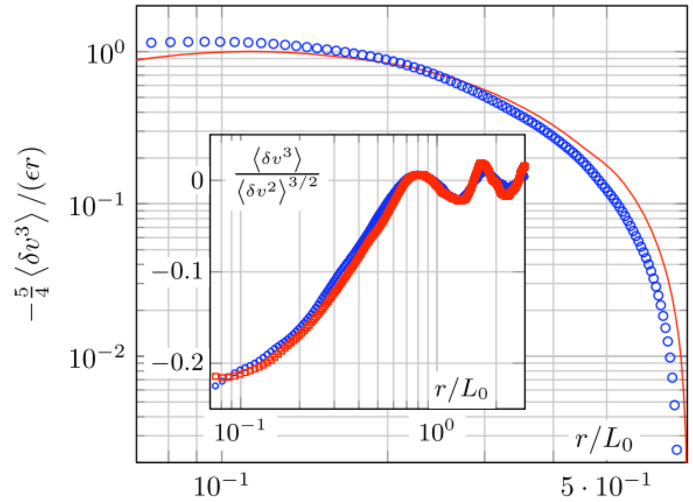


Guo et al., PRL, 2010

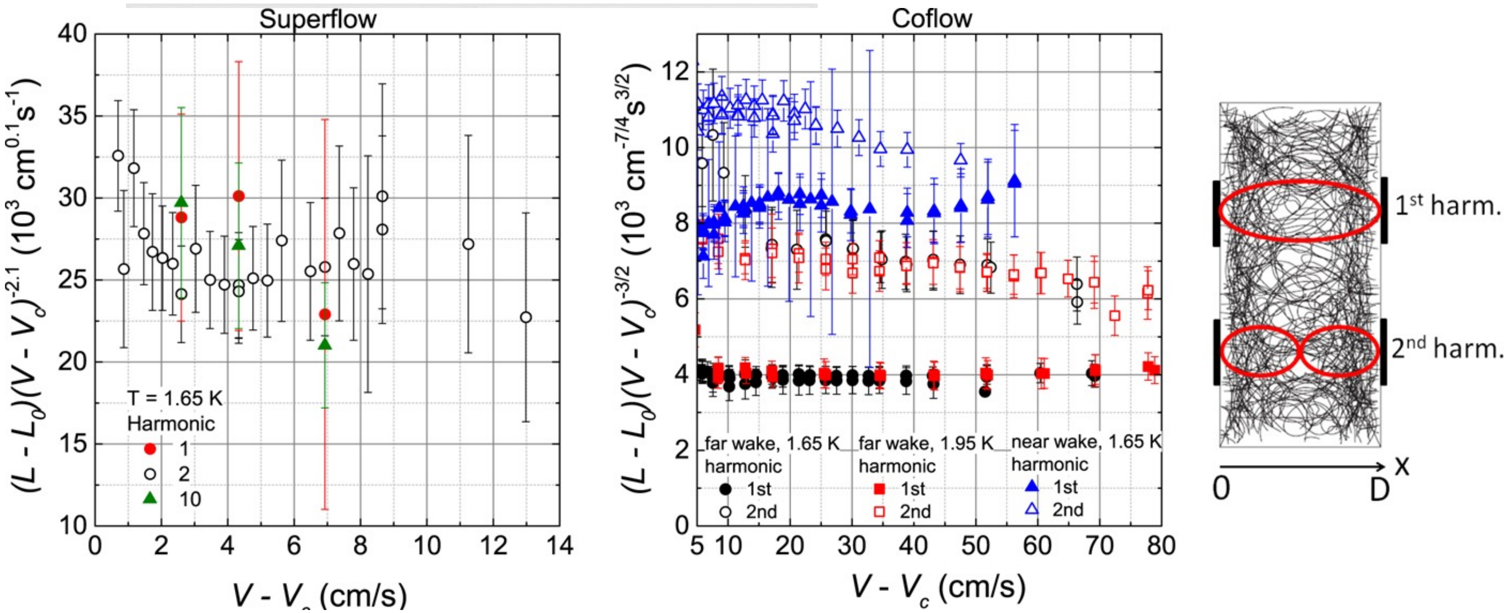
Experimental/Physical Systems

Superfluid Helium ($^4He/^3He3$)





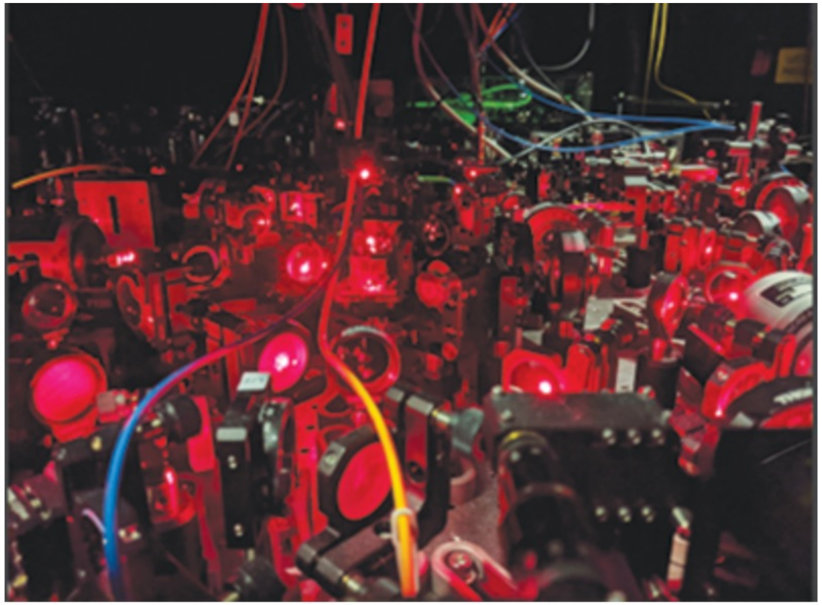
Salort at al., 2012

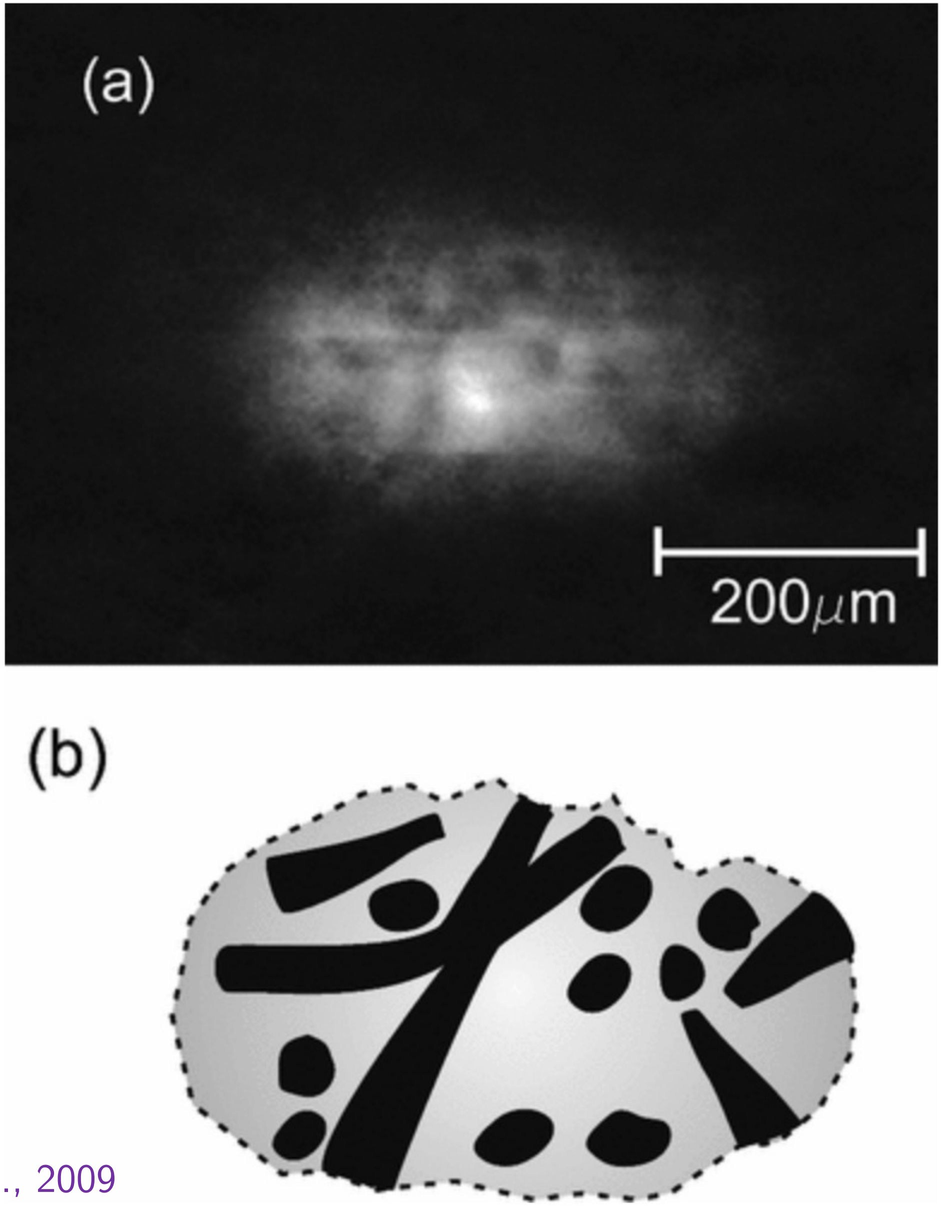


Varga et al., 2015

Experimental/Physical Systems

Atomic BECs

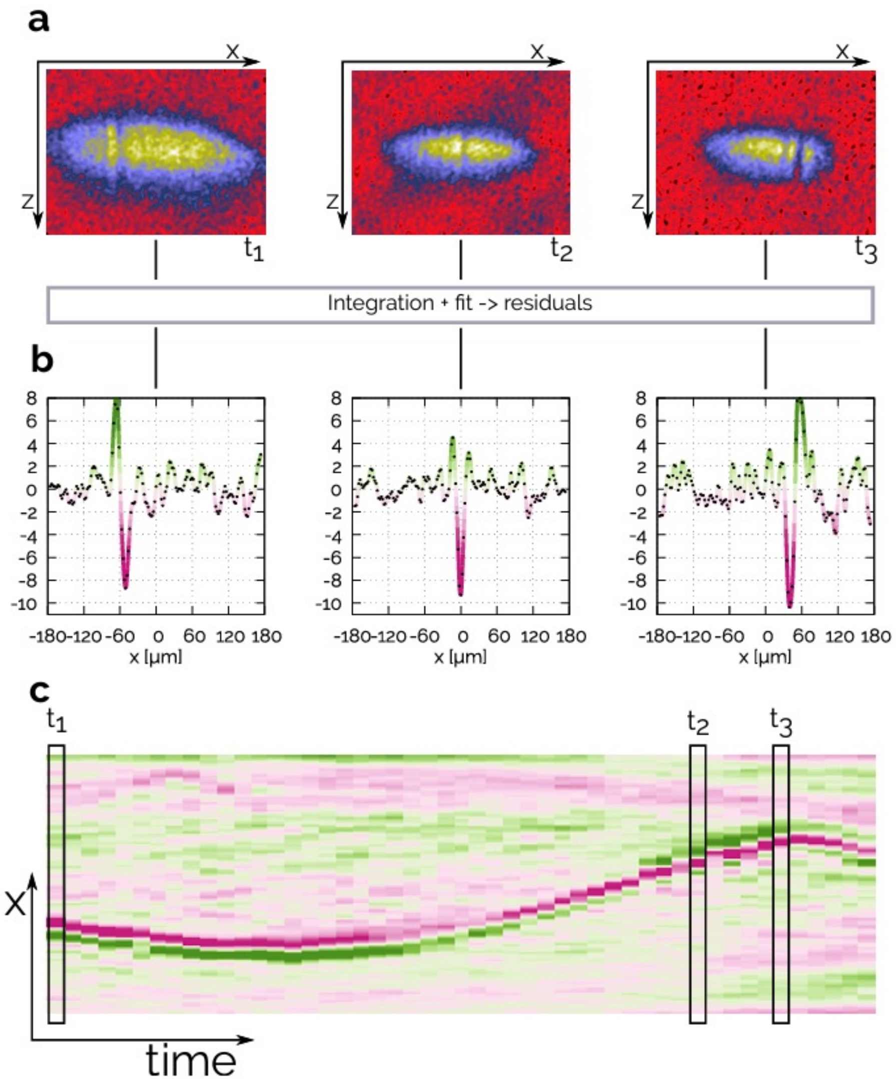


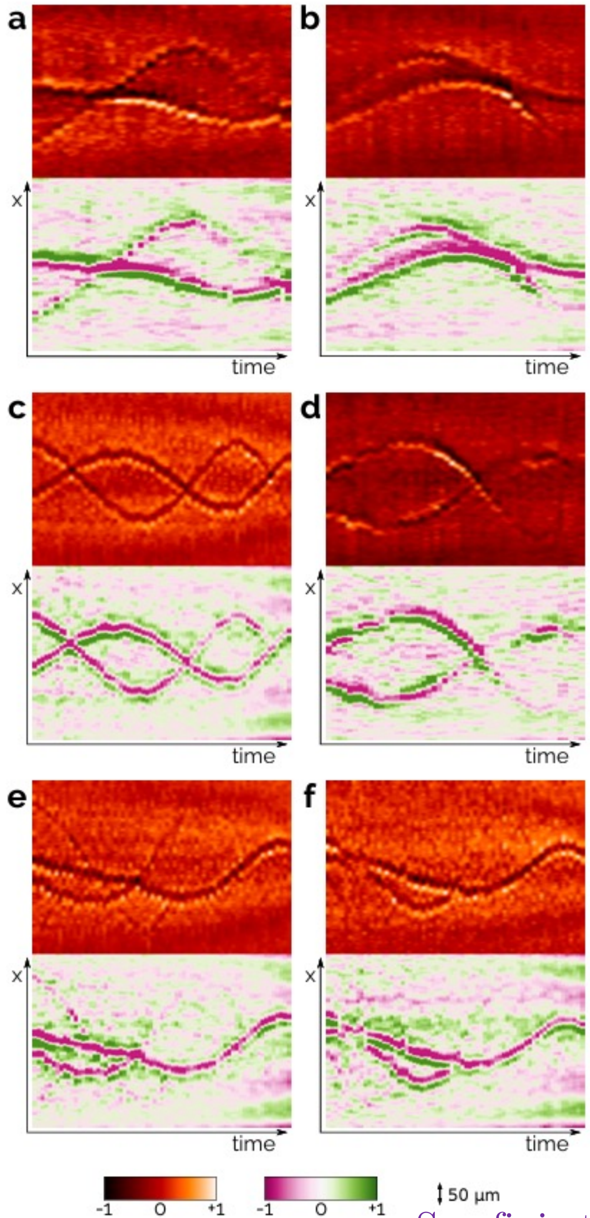


Henn, Bagnato et al., 2009

Experimental/Physical Systems

Atomic BECs

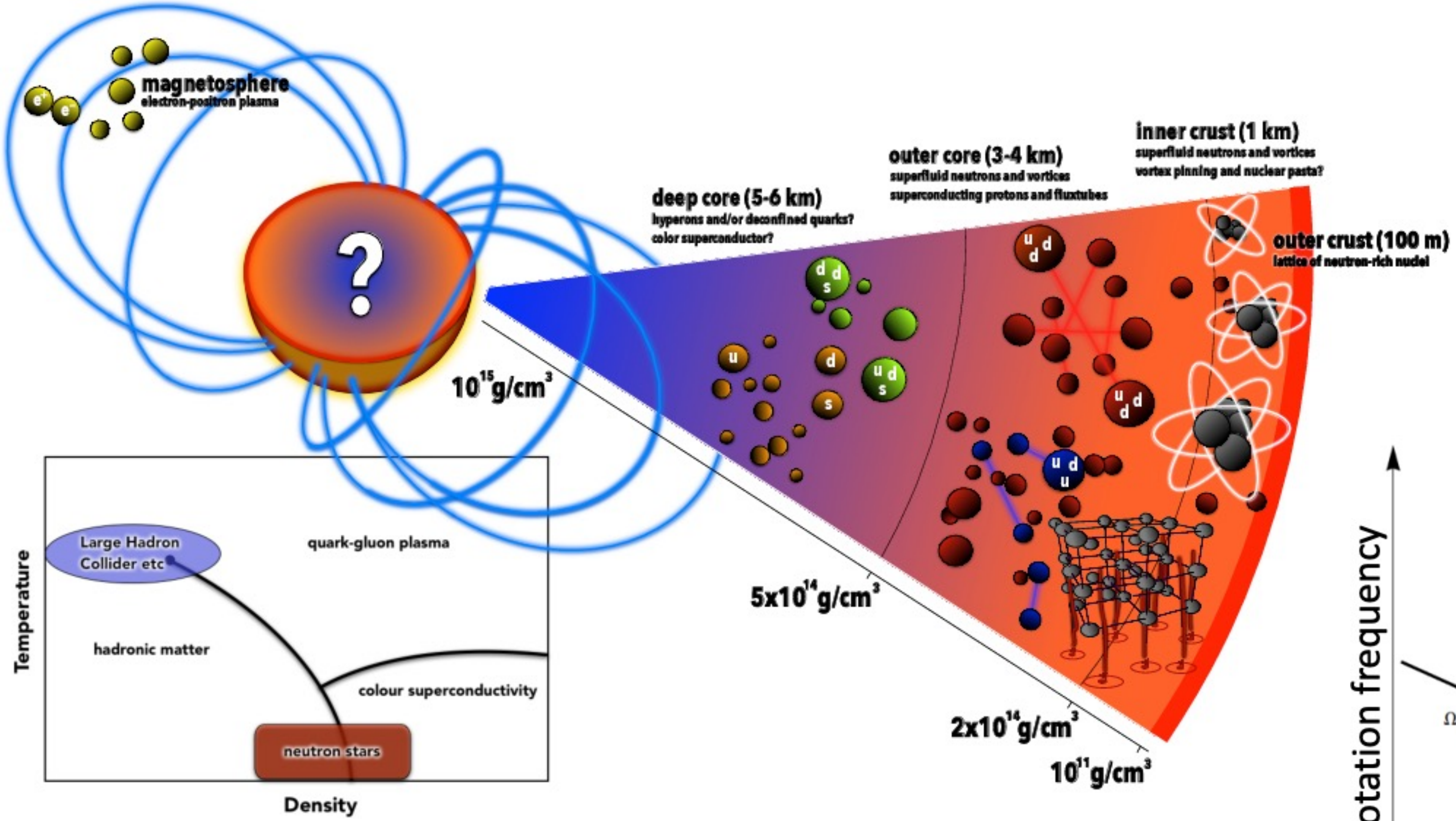


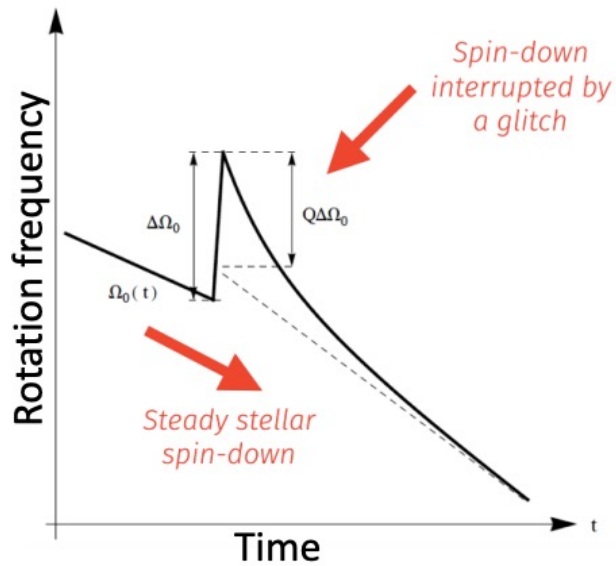


Serafini et al, PRX, 2017

Experimental/Physical Systems

Neutron Stars





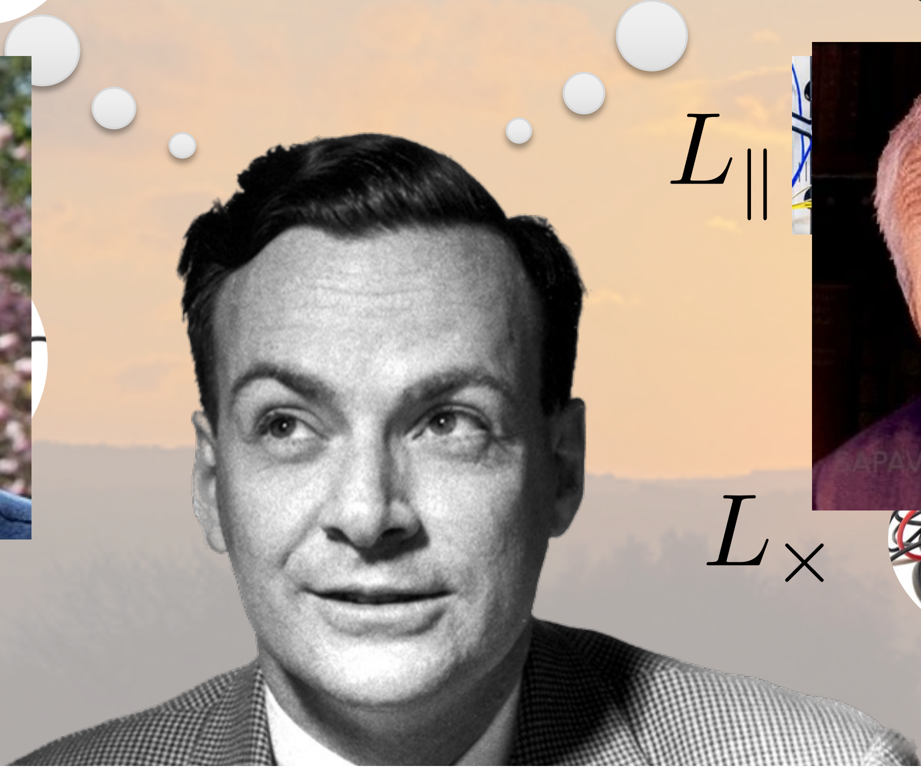
Graber et al., 2016

Stagg et al., 2017

Vinen

Kolmogorov

Muller, 2022



$L_{\parallel}$

$L_{\times}$

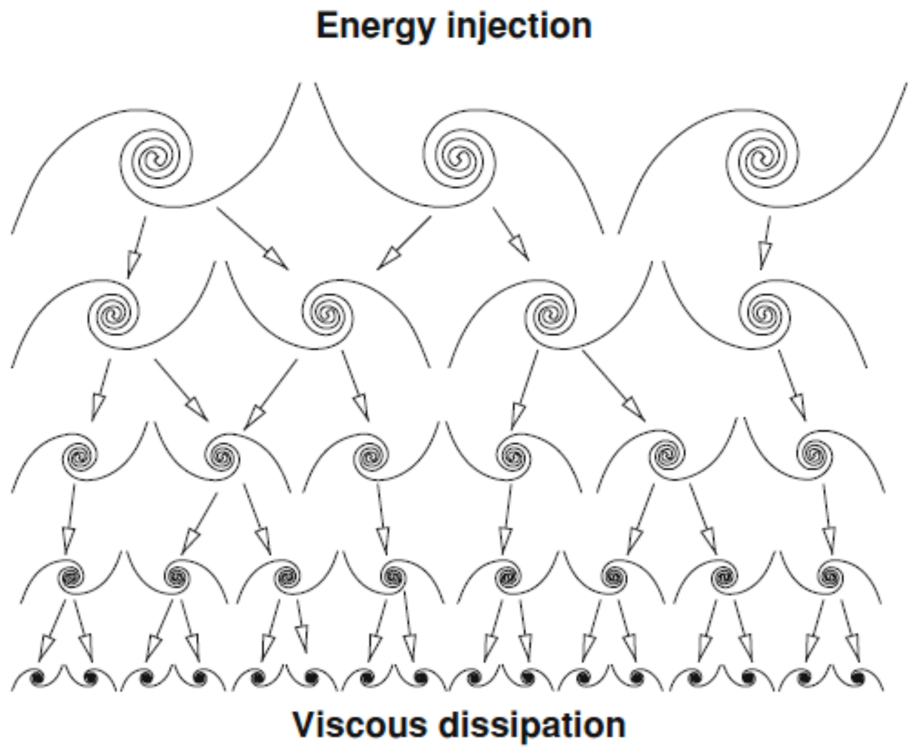
Roche & Barenghi, 2008

Classical (viscous) turbulence

- In a 3D classical turbulent flow, large scale eddies break up into smaller eddies, these into smaller ones and so on...(Richardson Cascade)
- If there is a large inertial range between the forcing and dissipation scale (i.e. high Re) then the flow of energy through scales is characterized by a constant energy flux .
- Dimensional analysis leads to a power-law scaling for the energy spectrum,

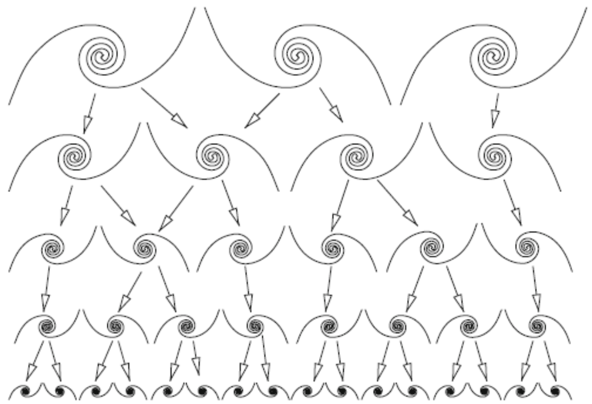
$$E(k) = C\epsilon^{2/3}k^{-5/3}$$

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\frac{1}{\rho}\nabla p + \nu \nabla^2 \mathbf{v}$$



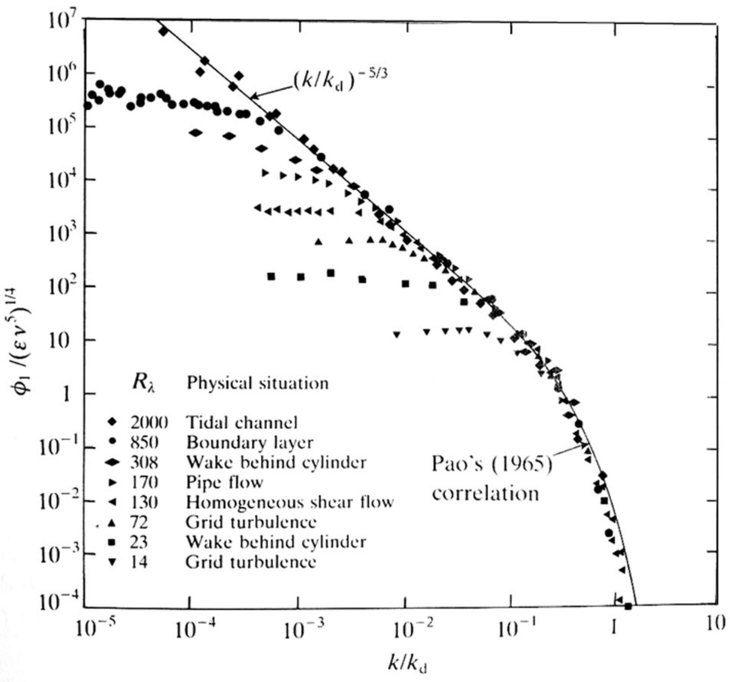
Classical (viscous) turbulence

$$S_p(r) = \langle (\delta u(r))^p \rangle, \quad \delta u(r) = u(x+r) - u(x)$$

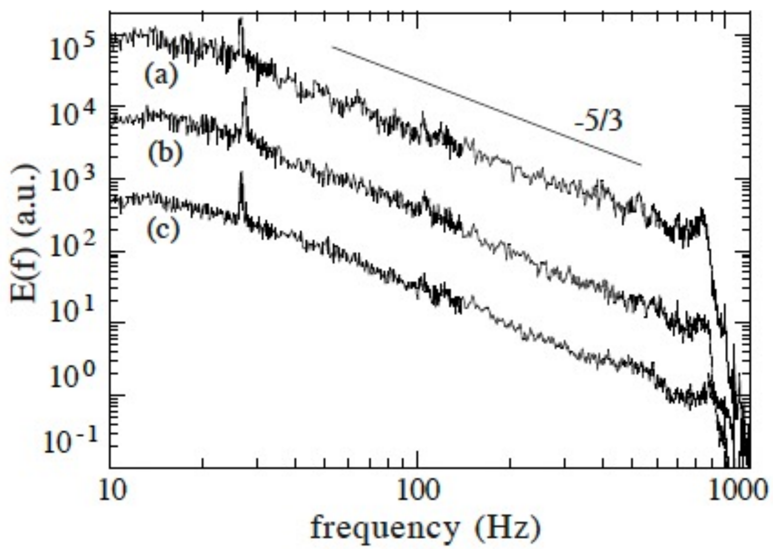


Self similarity K41:

$$S_p(r) \sim (\epsilon r)^{p/3}$$



$$E(k) = C\epsilon^{2/3} k^{-5/3}$$



Maurer & Tabeling, EPL, 1998

Batchelor & Townsend, Proc RS A, 1949

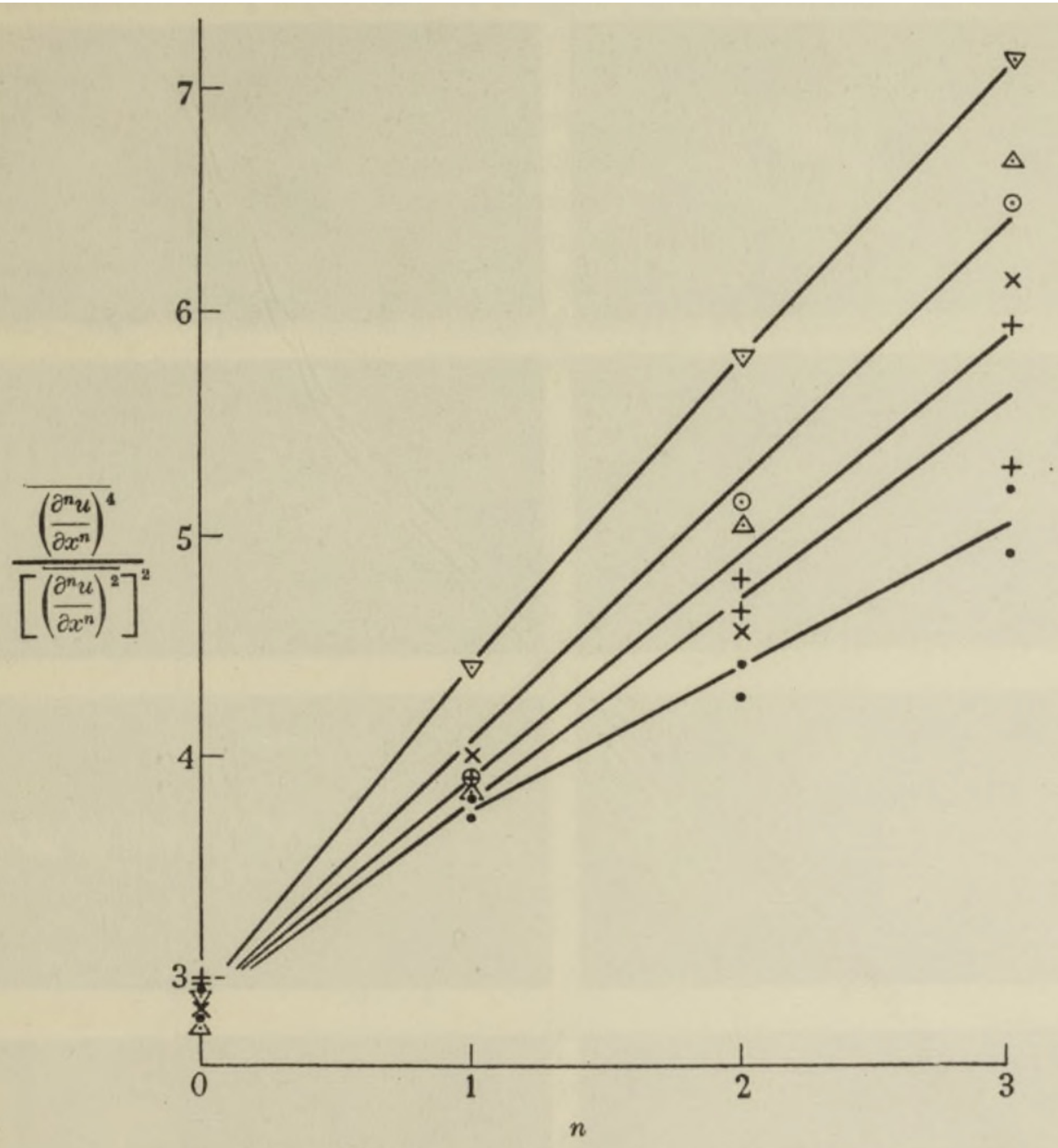


FIGURE 5. Flattening factors of velocity derivatives.

Isotropic turbulence, R_M: •, 2810; +, 5620; ×, 11,200; ⊙, 22,500.

Flatness factor:

$$\overline{(\delta_x^n u)^4} / \left[\overline{(\delta_x^n u)^2}\right]^2$$

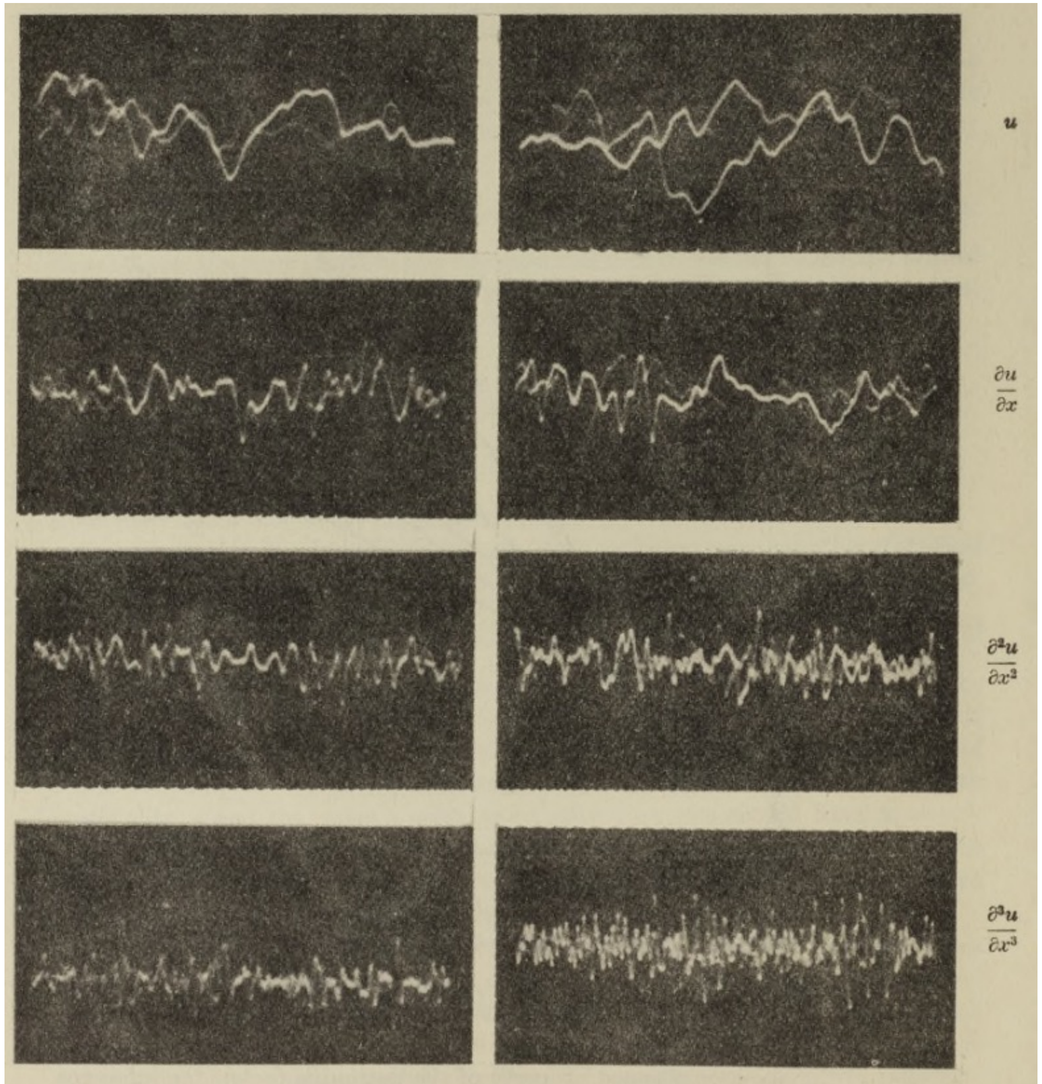
K41 implies a constant, independent of Reynolds number...

...problem

Interestingly it was probably Batchelor who introduced K41 to the west in his 1947 paper:

"Like a prospector going through a load of crushed rock, I suddenly came across rushed rock, I suddenly came across two articles about four pages in length, whose quality was immediately clear."

Prophetic words

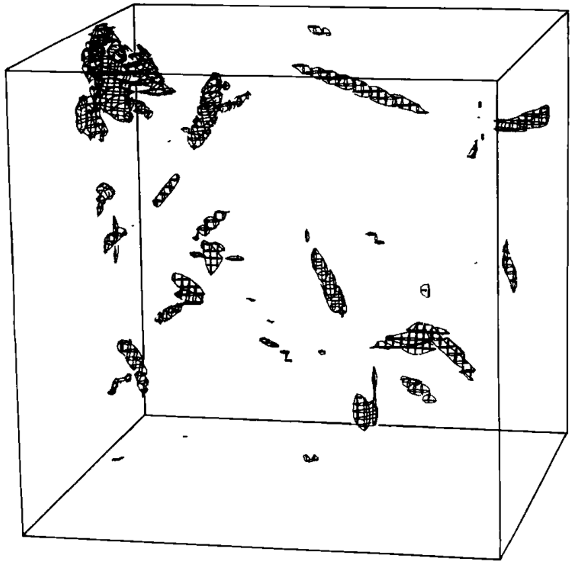


isotropic turbulence
$R_M = 22{,}500.$

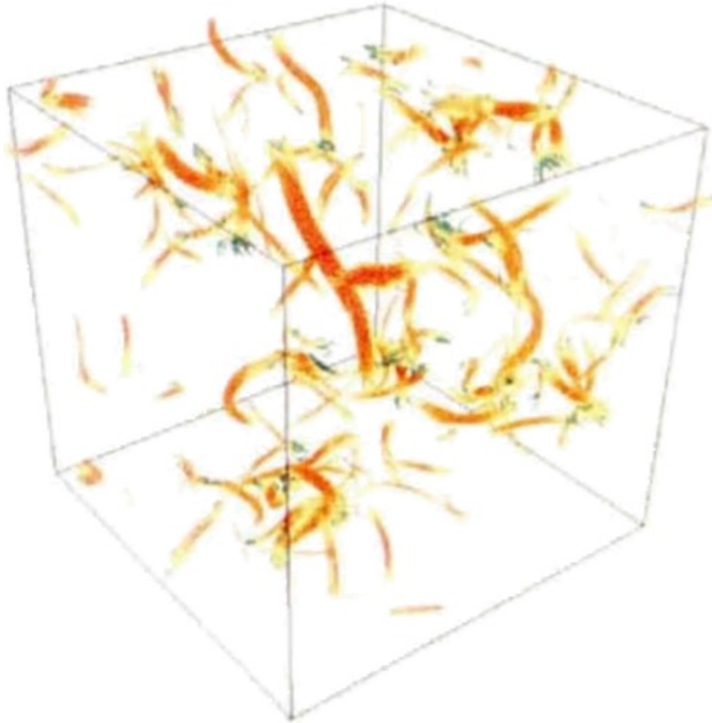
wake flow
$R_d = 4100.$

Measurements describing the probability distribution of $\partial u/\partial x$, $\partial^2 u/\partial x^2$ and $\partial^3 u/\partial x^3$ are also described. **These, and oscillograms of the velocity derivatives, show that the energy associated with large wave-numbers is very unevenly distributed in space**. There appear to be **isolated regions in which the large wave-numbers are 'activated', separated by regions of comparative quiescence**. This spatial inhomogeneity becomes more marked with increase in the order of the velocity derivative, i.e. with increase in the wave-number. It is suggested that the spatial inhomogeneity is produced early in the history of the turbulence by an intrinsic instability, in the way that a **vortex sheet quickly rolls up into a number of strong discrete vortices. Thereafter the inhomogeneity is maintained by the action of the energy transfer.**

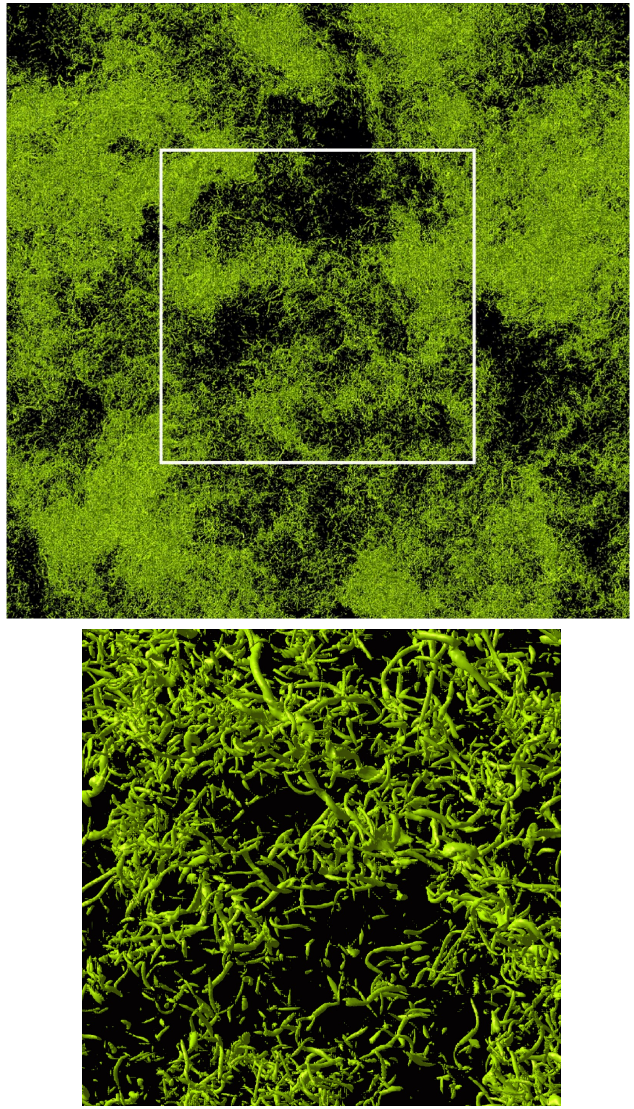
Numerical age



Siggia, JFM, 1981, 32^3

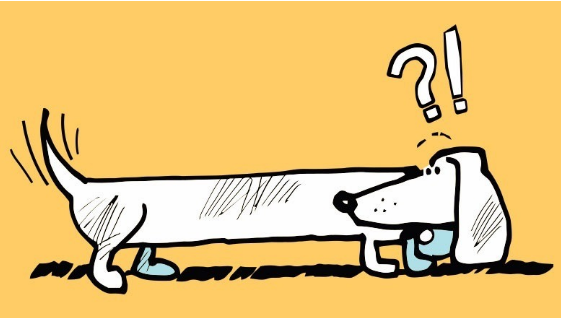


She et al., Nature, 1990, 96^3



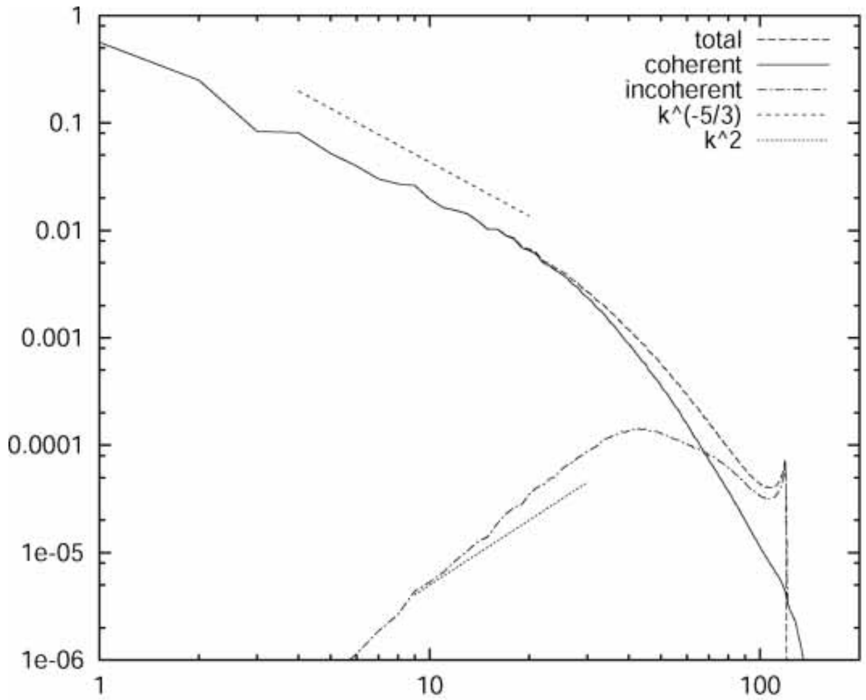
Ishihara et al. 4096^3

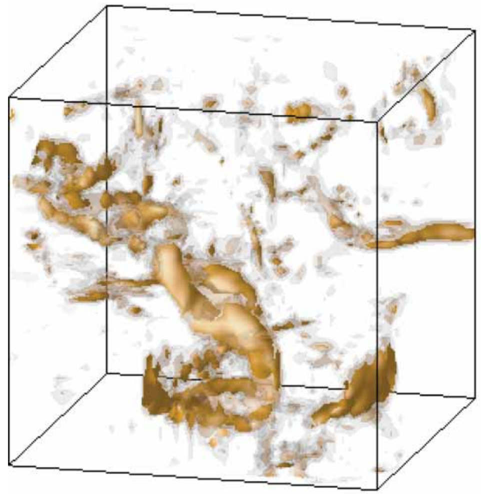
$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\frac{1}{\rho}\nabla p + \nu \nabla^2 \mathbf{v} \qquad \omega = \nabla \times \mathbf{v}$$

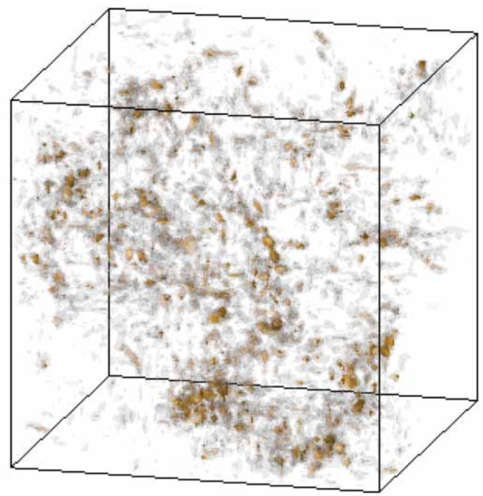


Frisch, 1995

Importance?







Roussel, Schneider & Farge, 2005

Vorticity

Local Energy Transfer

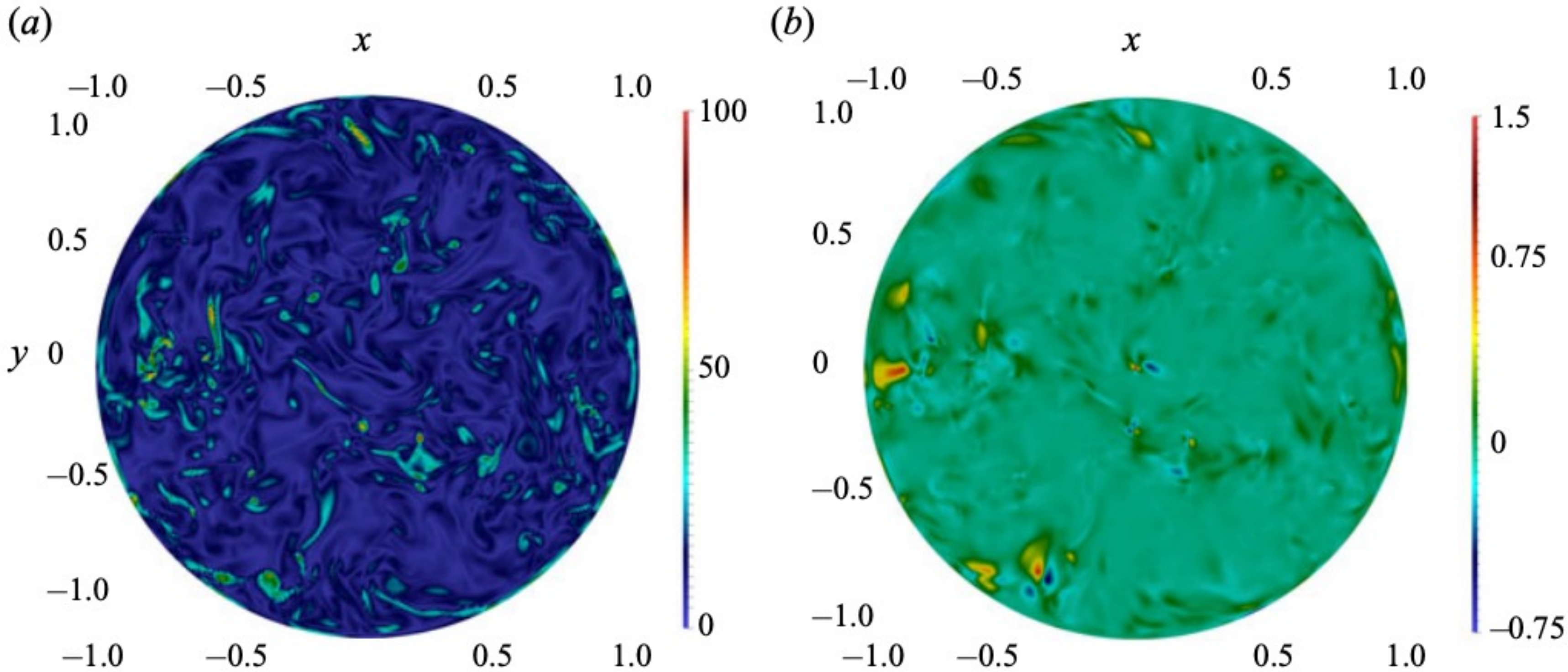
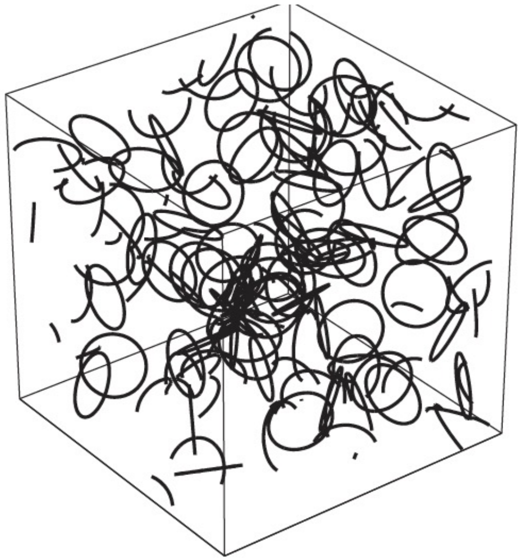


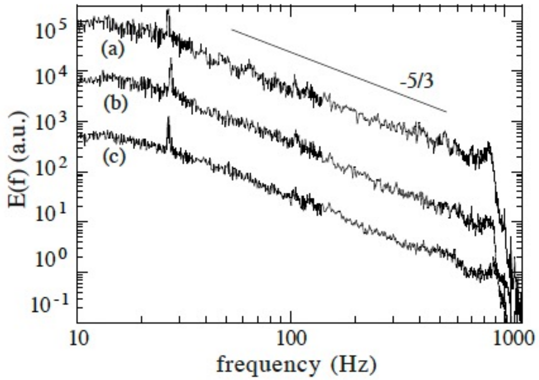
Figure 2. Visualization of the vorticity amplitude of ω (*a*) and $\mathscr{D}_\ell^I$ (*b*) for $\ell = 8\eta$ on a plane containing the centre perpendicular to the cylinder's axis from the DNS of table 1.

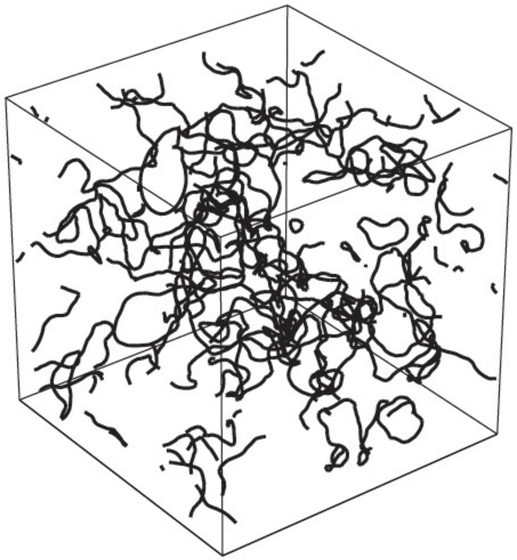
Faller et al., 2021

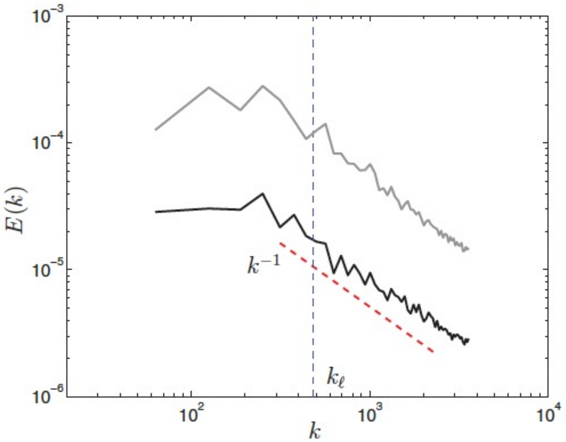
Vinen tangle – unstructured quantum turbulence

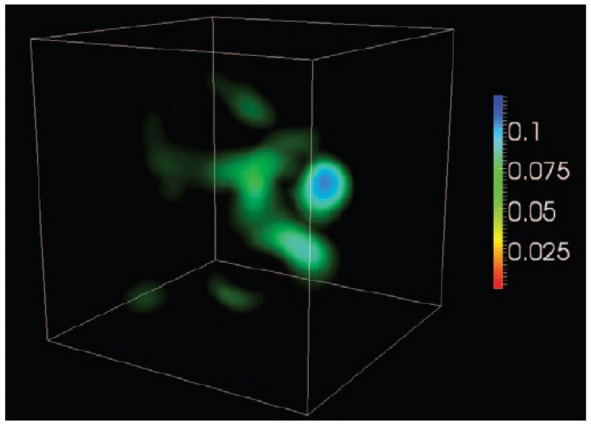


Course-grained (HVBK) picture: $\omega \approx 0$





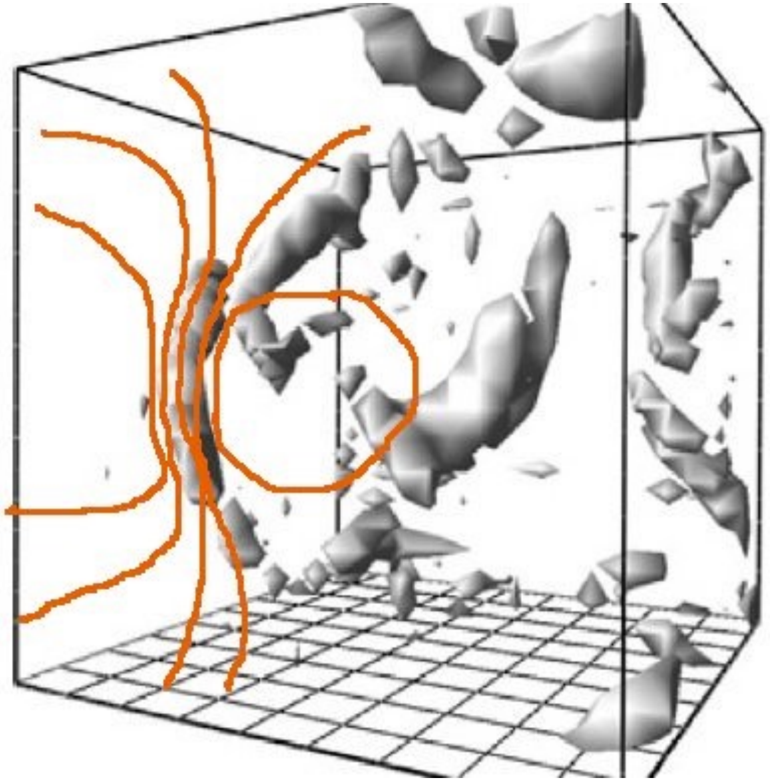




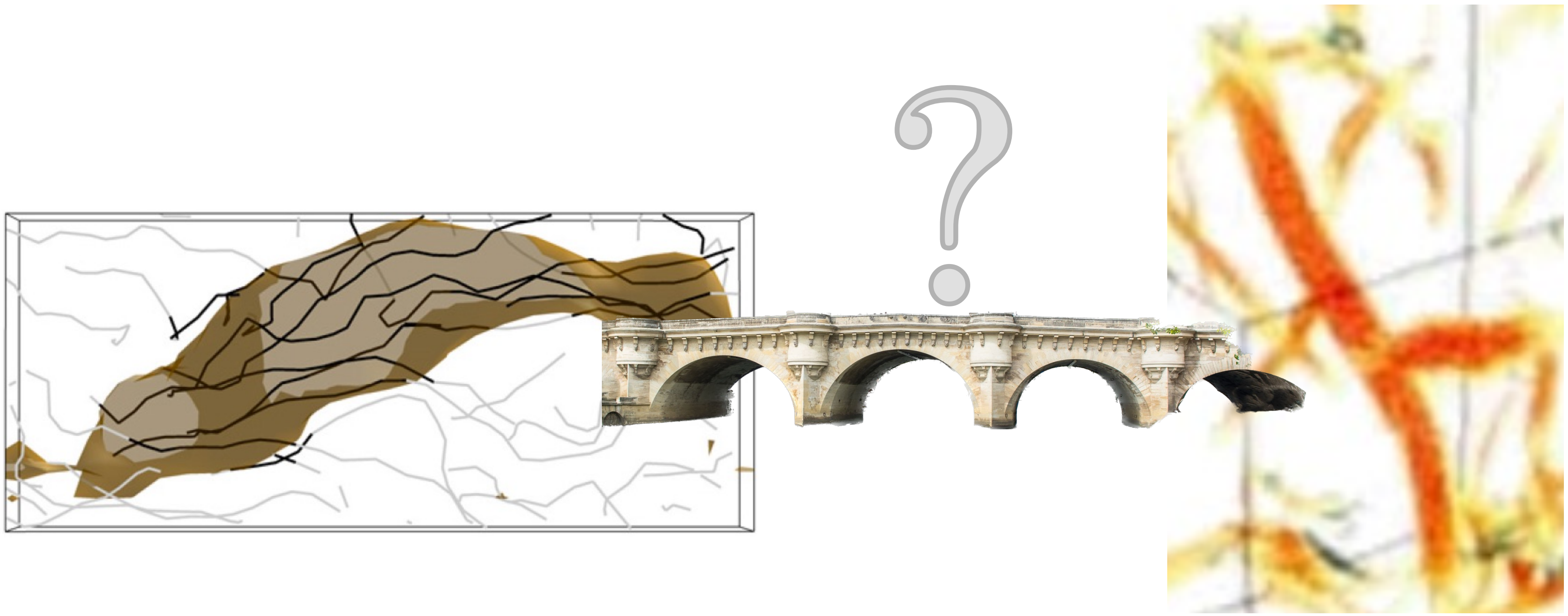
AWB, 2011

Coherent structures in QT?

- Do coherent structures exist in quantum turbulence?
- What are these structures, bundles? How do they form and evolve?
- Would allow a mechanism for vortex stretching, i.e. stretch the bundle.



$$\frac{D\boldsymbol{\omega}}{Dt} = (\boldsymbol{\omega} \cdot \nabla)\mathbf{v} + \nu \nabla^2 \boldsymbol{\omega}$$

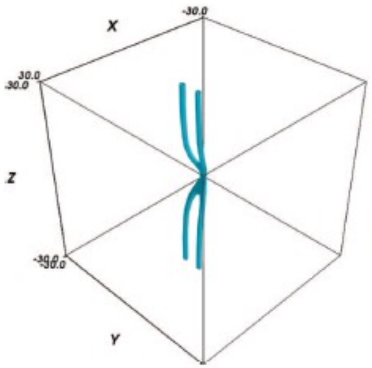


Modelling approach

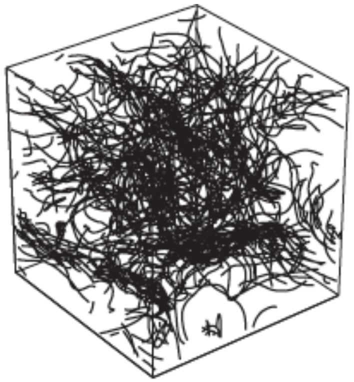
3 distinct scales/numerical approaches

Barenghi *et al.* (2014)

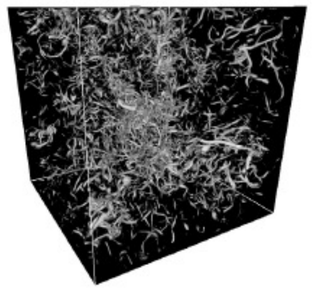
Gross-Pitaevskii



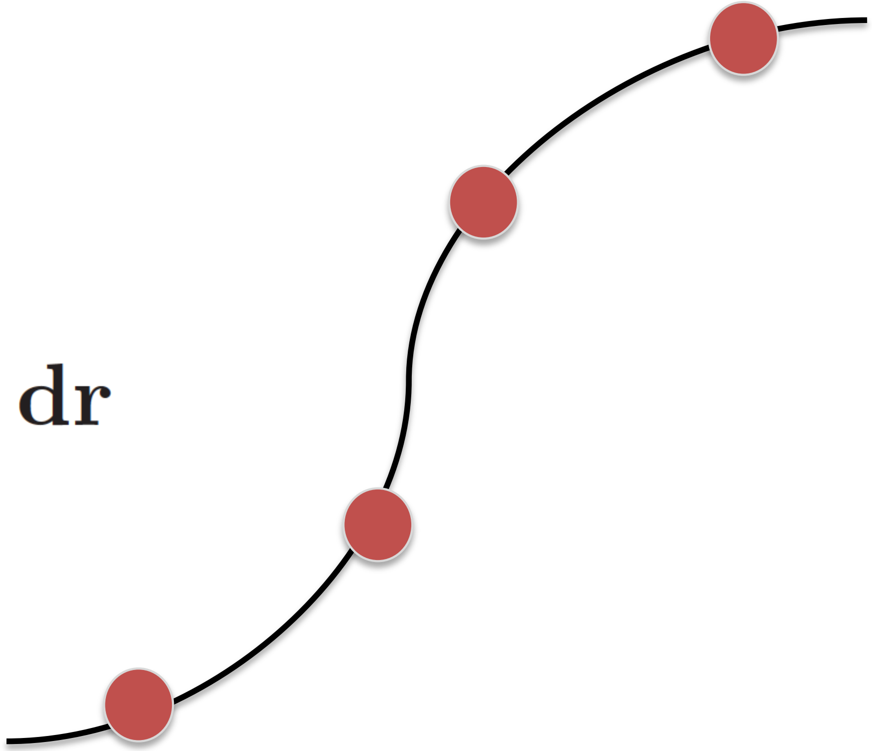
Point Vortex/VFM



Course-Grained
HVBK

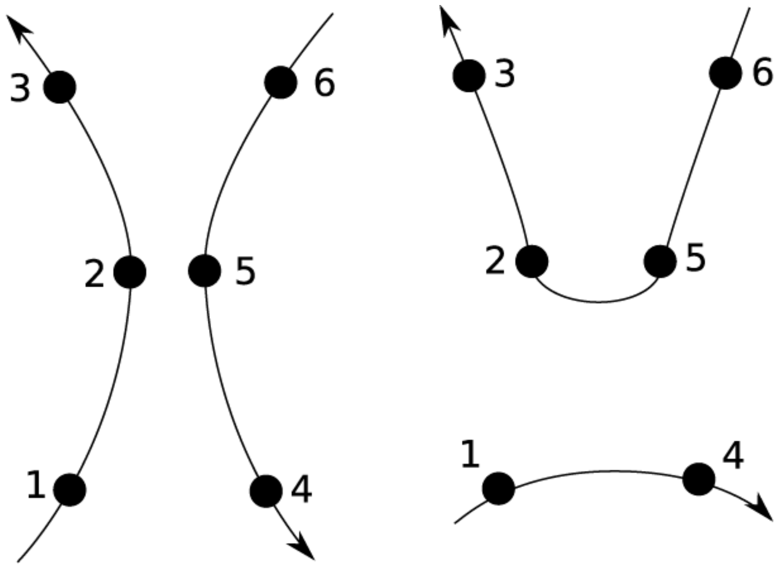


Vortex filament method



Biot-Savart Integral

$$\frac{d\mathbf{s}}{dt} = -\frac{\Gamma}{4\pi}\oint_{\mathcal{L}} \frac{(\mathbf{s}-\mathbf{r})}{|\mathbf{s}-\mathbf{r}|^3} \times \mathbf{dr}$$



Model reconnections algorithmically ‘cut and paste’

Mutual friction

$$\frac{d\mathbf{s}}{dt} = \mathbf{v}_s^{\mathrm{tot}} + \alpha\mathbf{s}' \times (\mathbf{v}_n^{\mathrm{ext}} - \mathbf{v}_s^{\mathrm{tot}}) - \alpha'\mathbf{s}' \times [\mathbf{s}' \times (\mathbf{v}_n^{\mathrm{ext}} - \mathbf{v}_s^{\mathrm{tot}})]$$

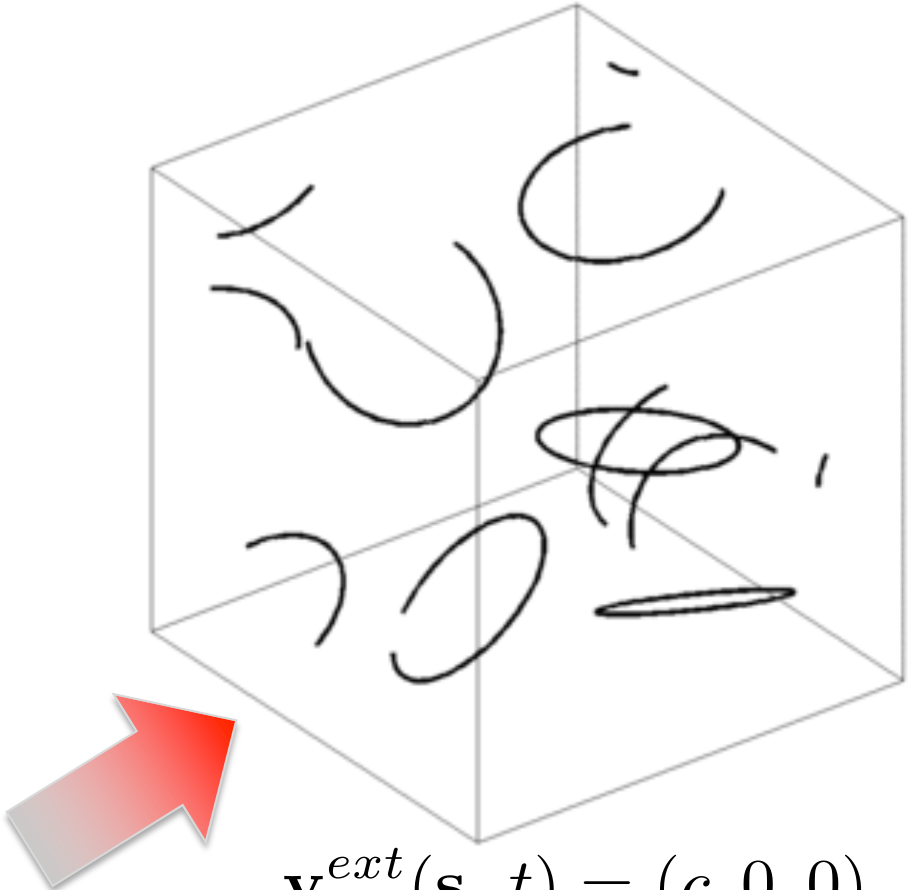
Normal viscous fluid coupled to inviscid superfluid via mutual friction.

Superfluid component extracts energy from normal fluid component via Donelly-Glaberson instability, amplification of Kelvin waves.

Kelvin wave grows with amplitude $A(t) = A(0)e^{\sigma t}$

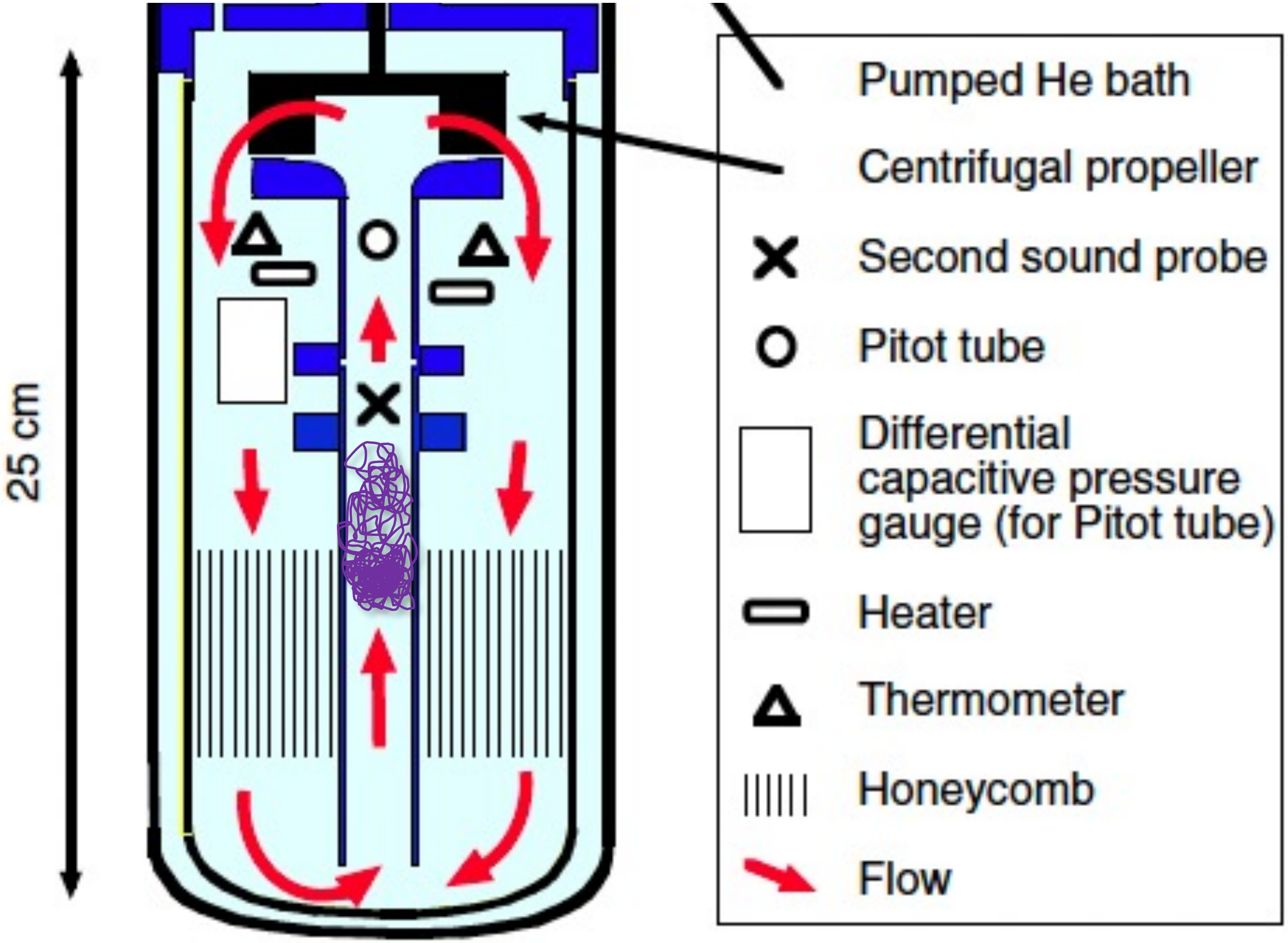
$$\sigma(k) = \alpha(kV - \nu' k^2)$$

Counterflow Turbulence



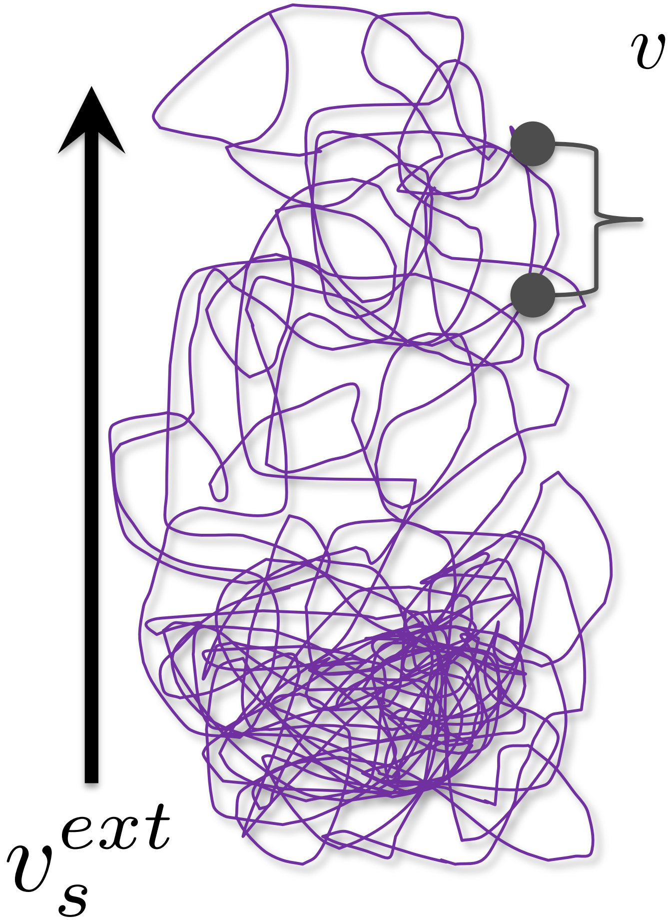
$$\mathbf{v}_n^{ext}(\mathbf{s},\, t) = (c, 0, 0)$$

A surprising result



Roche *et al.*, EPL, 2007

A surprising result



$v_s(t) \approx v_s(x/v_s^{ext})$ G.I. Taylor, 1938

$$\kappa L \sim \omega_s$$

Expectation, in K41 picture

$$E(k) \sim v_s^2(k) \sim k^{-5/3}$$

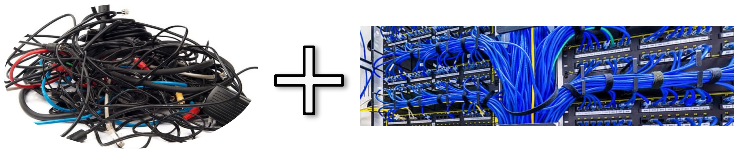
Thus (naively),

$$\omega_s^2(k) \sim k^2 v_s^2(k) \sim k^{1/3}$$

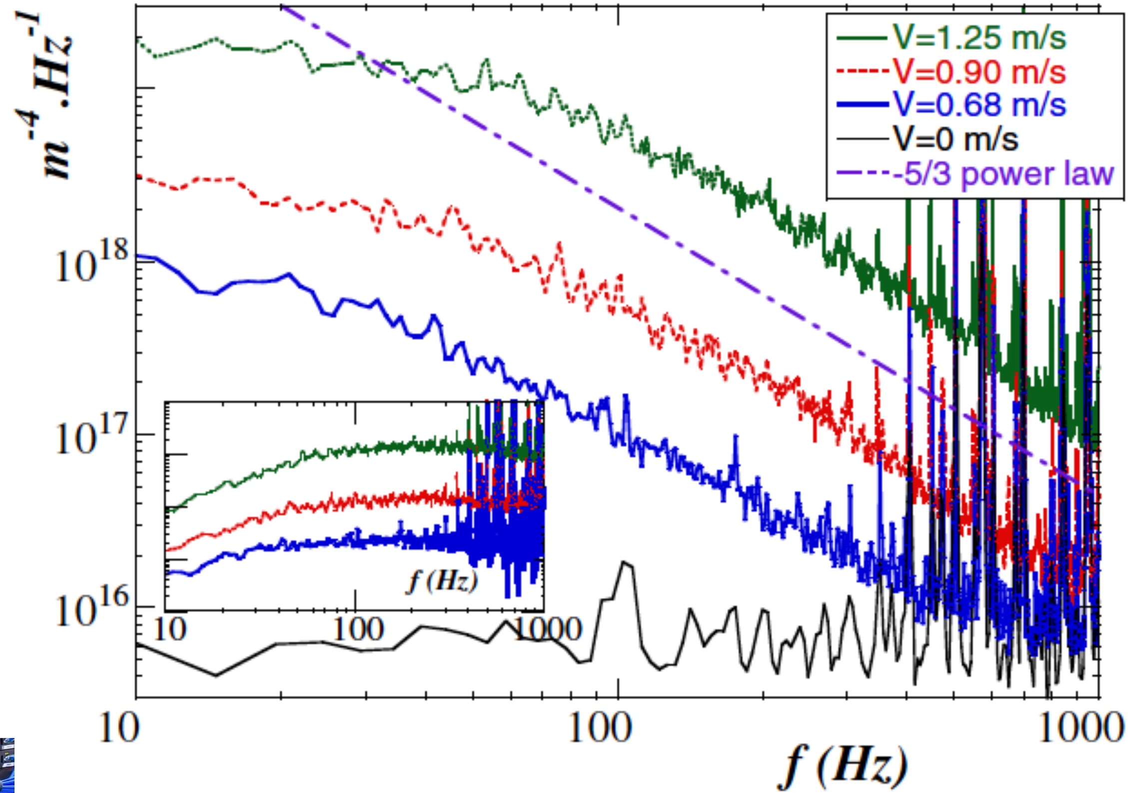
But how can we measure spatial spectrum with one probe?

A surprising result

- Fluctuations of vortex line density scale as $f^{-5/3}$.
- Contradiction of the classical scaling of vorticity expected from K41.
- Roche & Barenghi (EPL, 2008) - vortex line density field is decomposed into a polarised component, and a random component.



- Random component advected as a passive scalar explaining -5/3 scaling.



Roche *et al.*, EPL, 2007

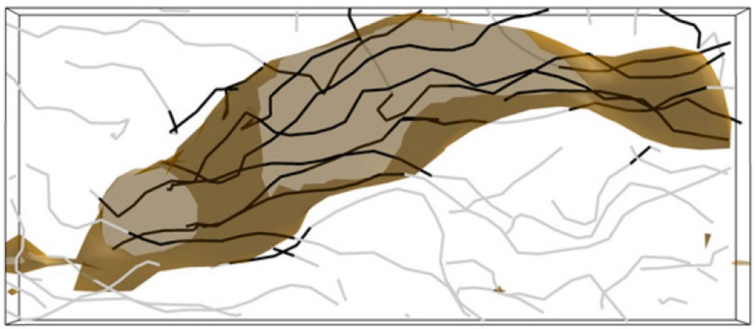
Quantum turbulence at finite temp.

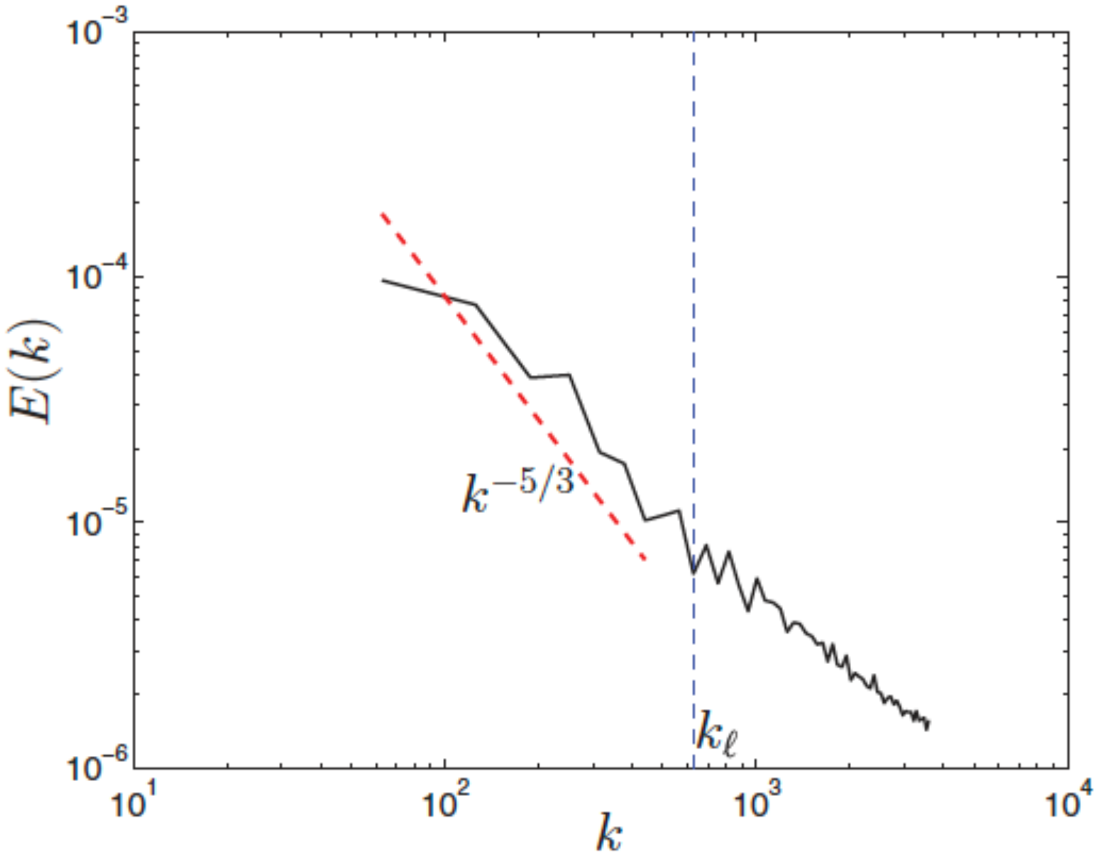
Drive turbulence in superfluid component to a steady state with imposed normal 'fluid turbulence'.

$$\mathbf{v}_n^{ext}(\mathbf{s},\, t) = \sum_{m=1}^{m=M} (\mathbf{A}_m \times \mathbf{k}_m \cos\phi_m + \mathbf{B}_m \times \mathbf{k}_m \sin\phi_m)$$

Identify regions of high course-grained vorticity

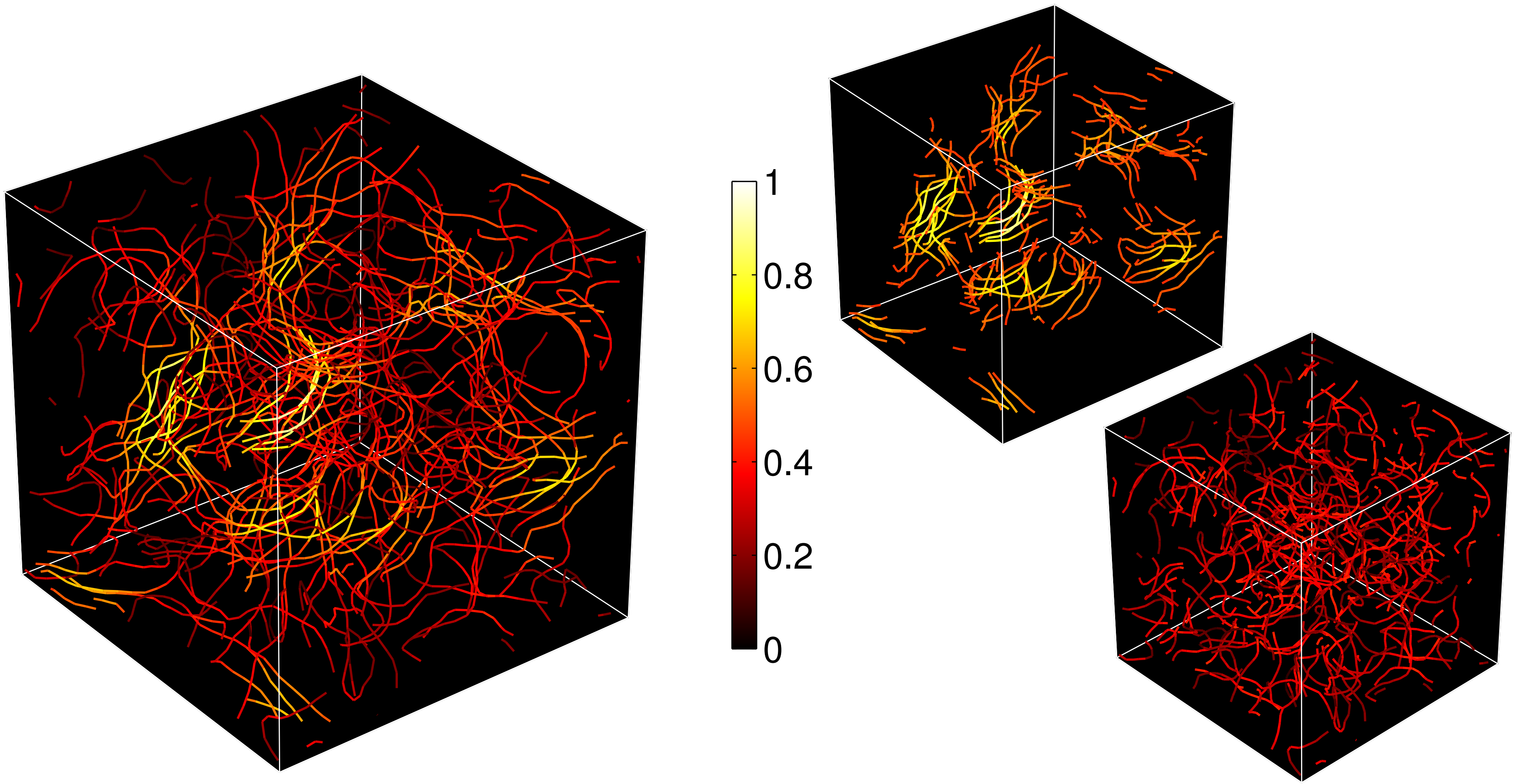
$$\boldsymbol{\omega}(\mathbf{r},\, t) = \kappa \sum_{i=1}^{N} \frac{\mathbf{s}_i'}{(2\pi\sigma^2)^{3/2}} \exp(-|\mathbf{s}_i - \mathbf{r}|^2/2\sigma^2)$$





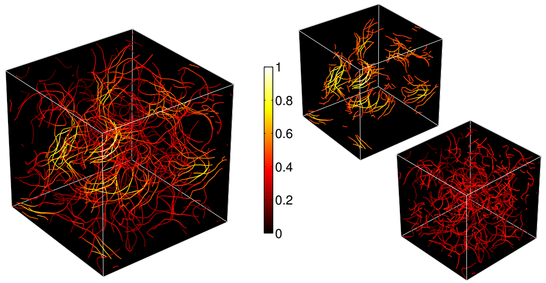
AWB, Laurie & Barenghi, 2012

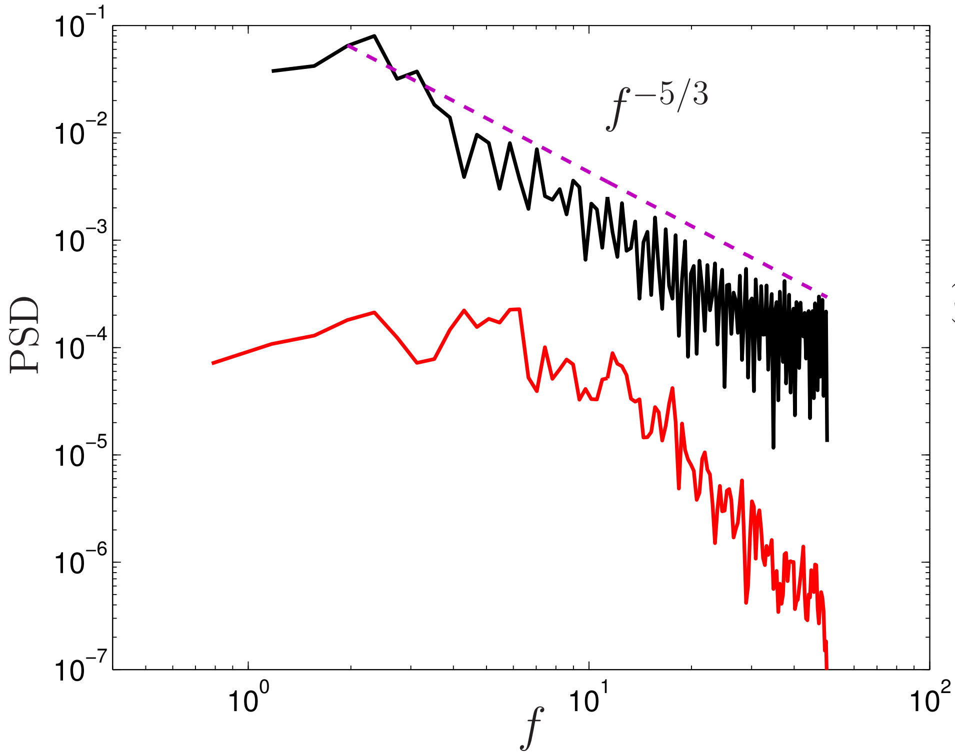
Decomposition of a tangle

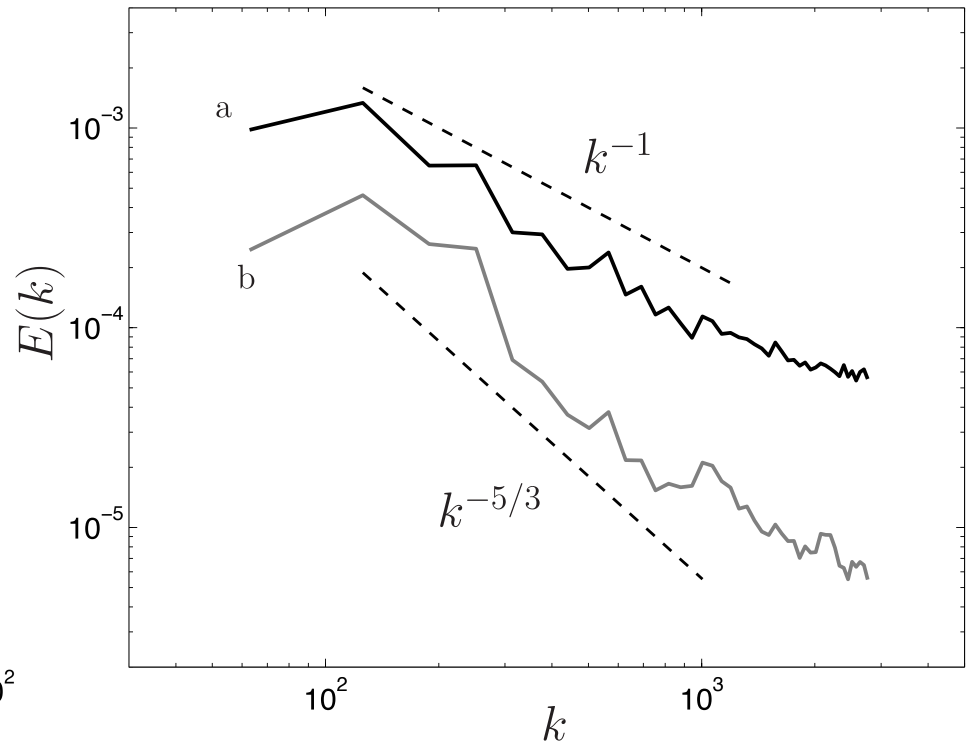


AWB, Laurie & Barenghi, 2012

Numerical results



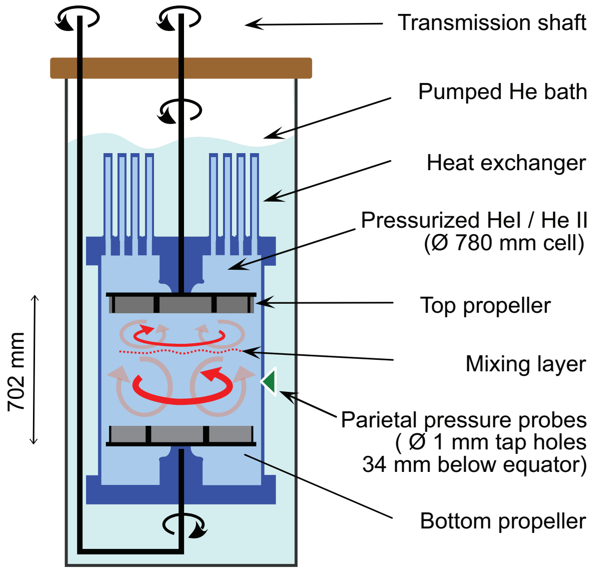


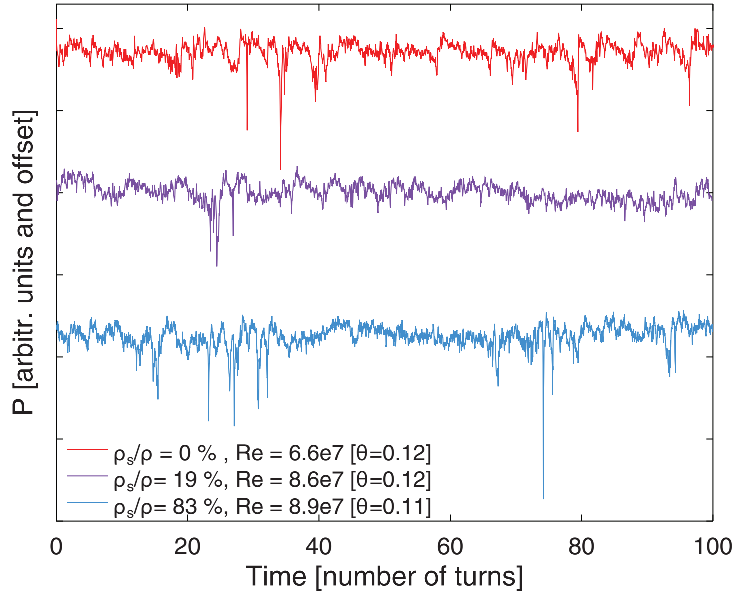


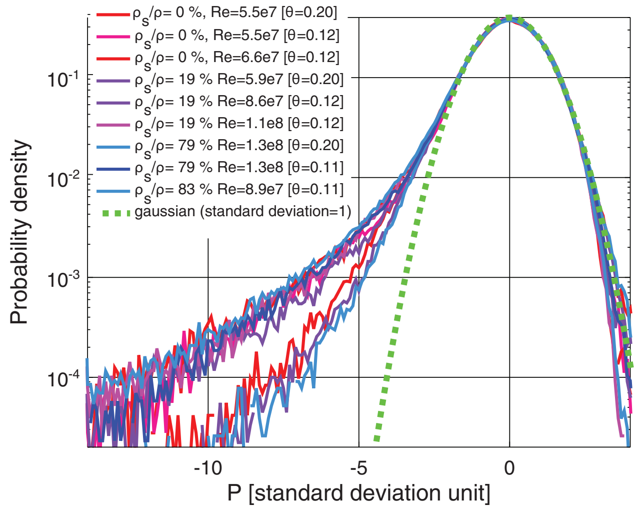
Left, frequency spectra (red polarised ; black total), right energy spectrum, upper random component, lower polarised component.

Experimental detection

Rusaouen *et al.*, 2017

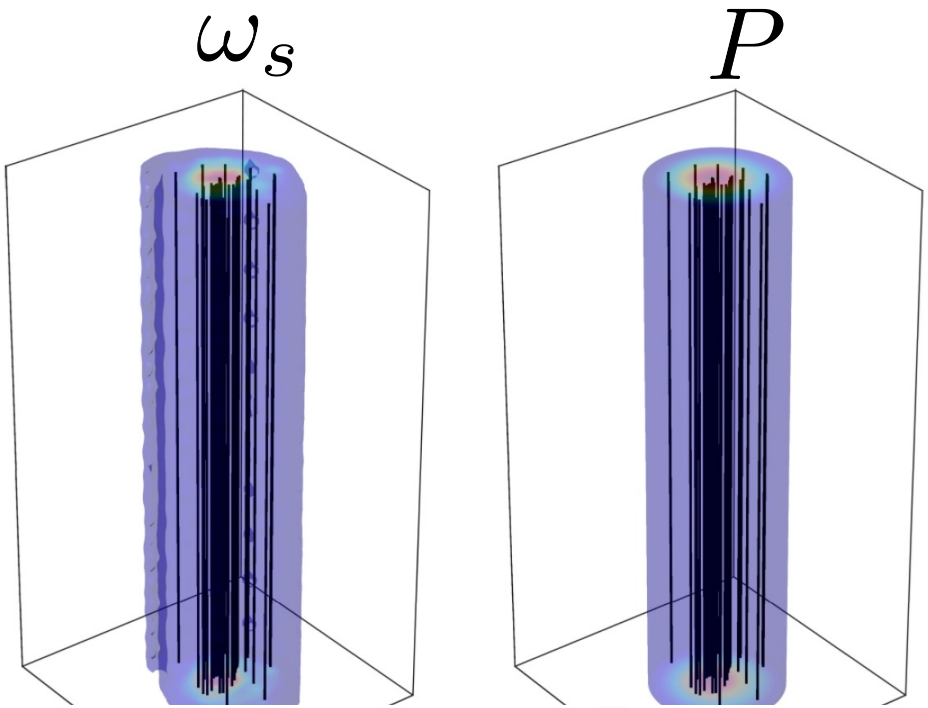


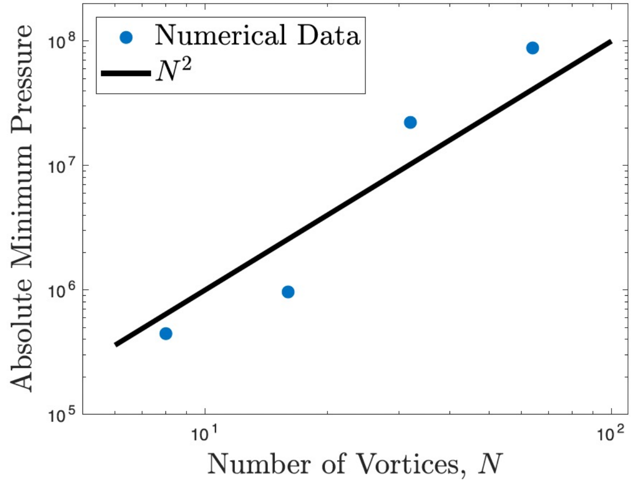




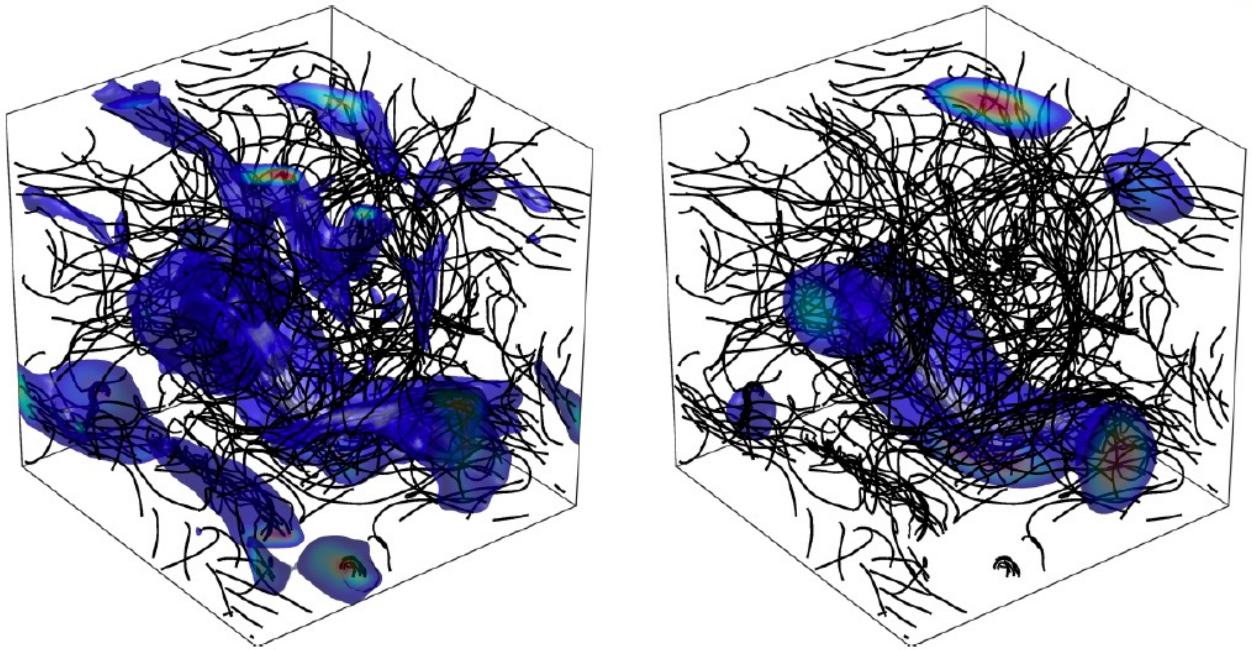
Presence of coherent structures inferred from intermittent pressure drops, HVBK:

$$\nabla^2 P = \frac{\rho_s}{2}(\omega_s^2 - \sigma_s^2) + \frac{\rho_n}{2}(\omega_n^2 - \sigma_n^2)$$





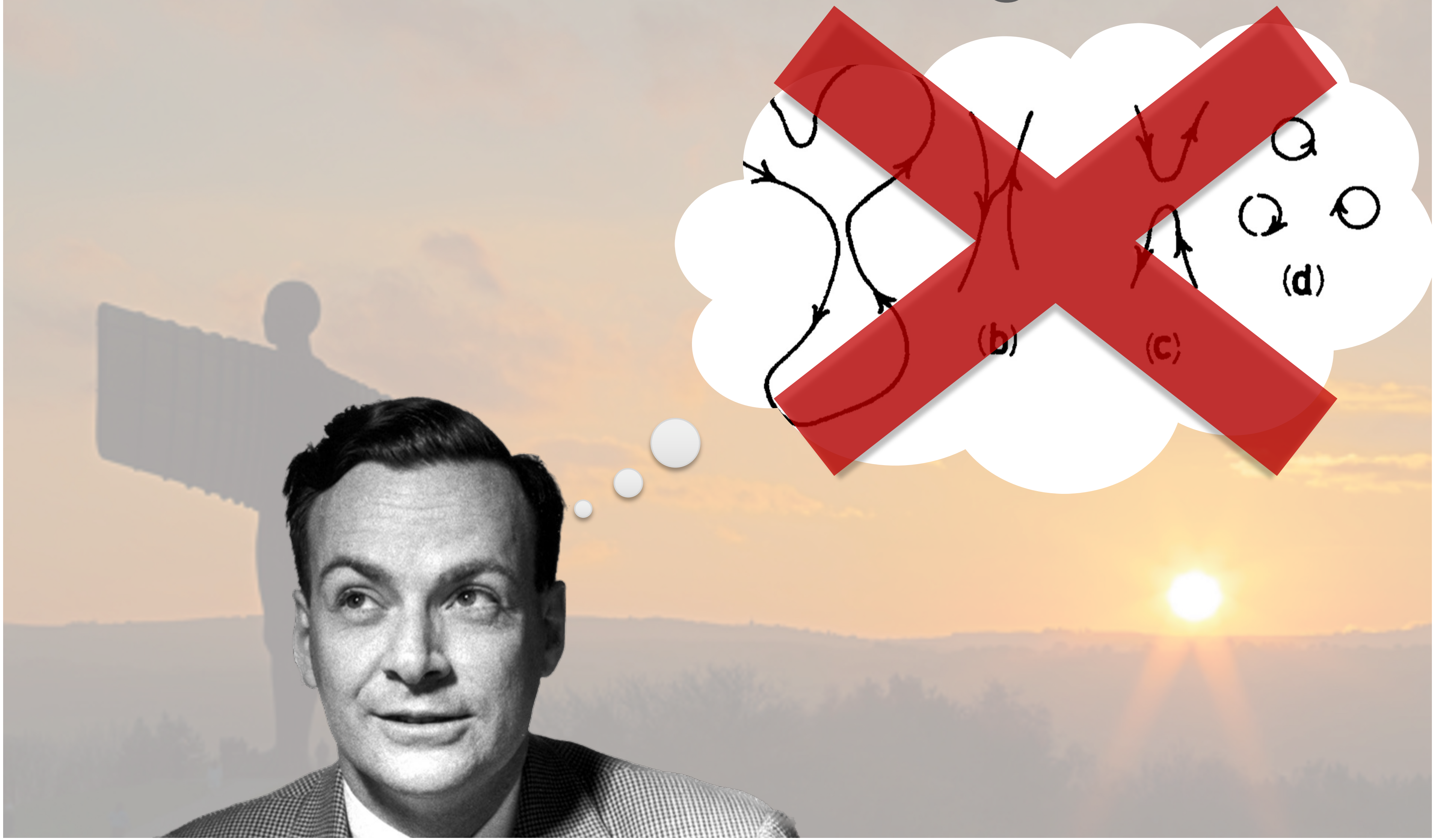
$$P \sim P_0 - \frac{\rho_s N^2 \Gamma^2}{8\pi^2 r^2},$$



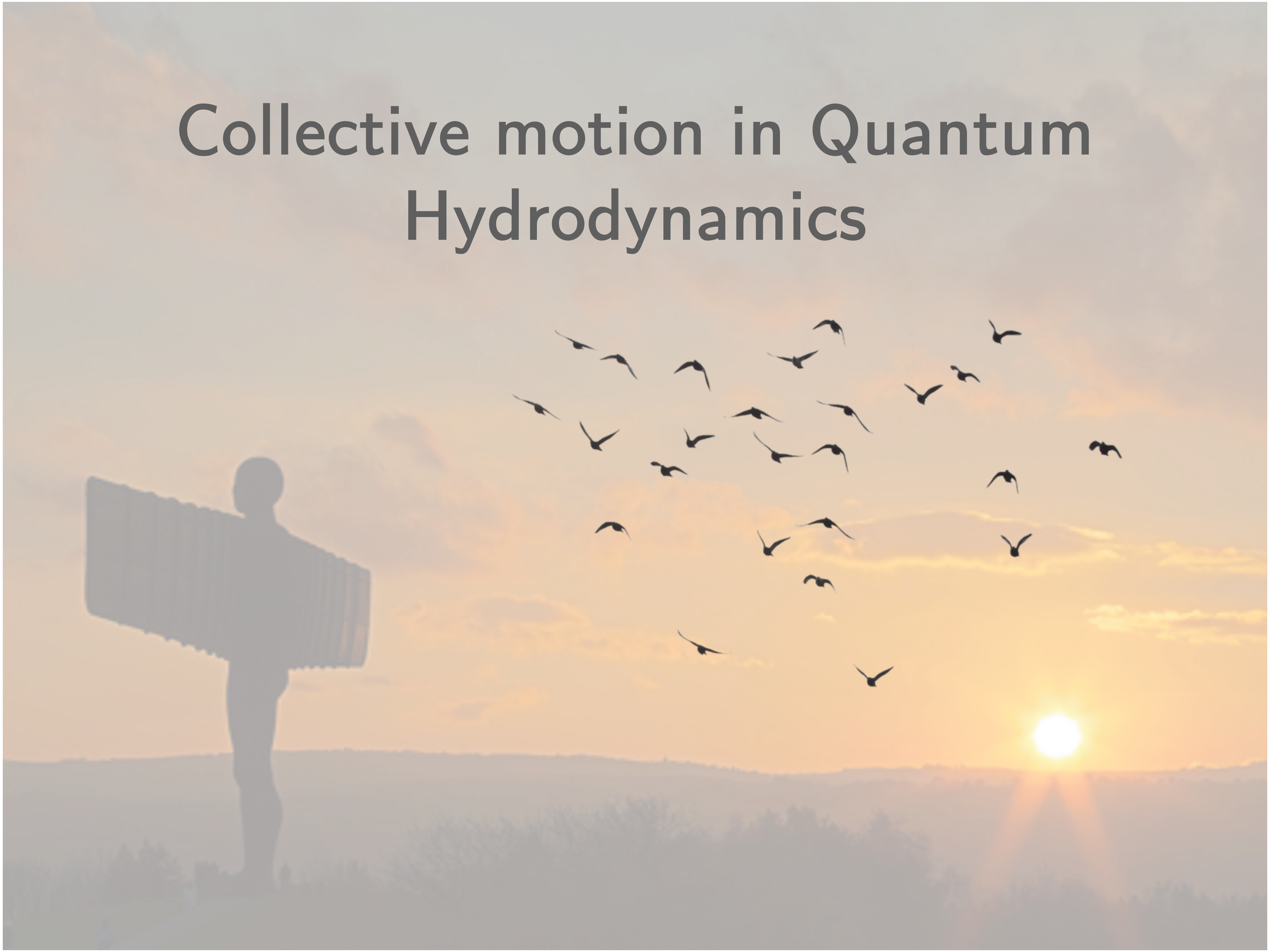
Baggaley. & Laurie, 2020

FIG. 5. Isosurfaces of the coarse-grained standardized vorticity magnitude (left) and negative pressure (right) fields of the static quasiclassical tangle depicted in Fig. 3 using $l_f = 2\ell$. The isosurfaces are taken at $\omega/a_{\text{vort}} > 2.5$ and $P/\sigma_{\text{press}} < -1.5$, respectively.

Take-home message

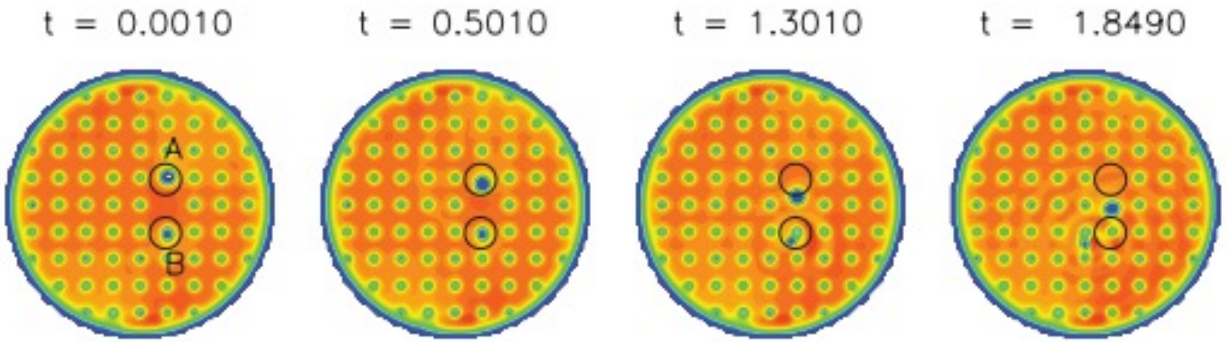


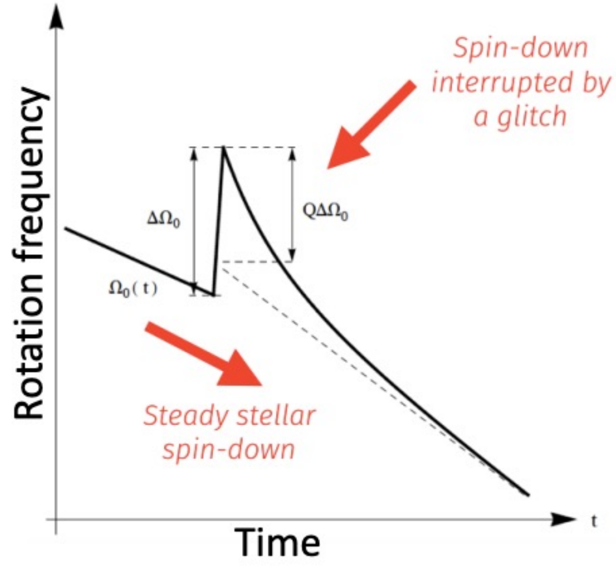
Collective motion in Quantum Hydrodynamics

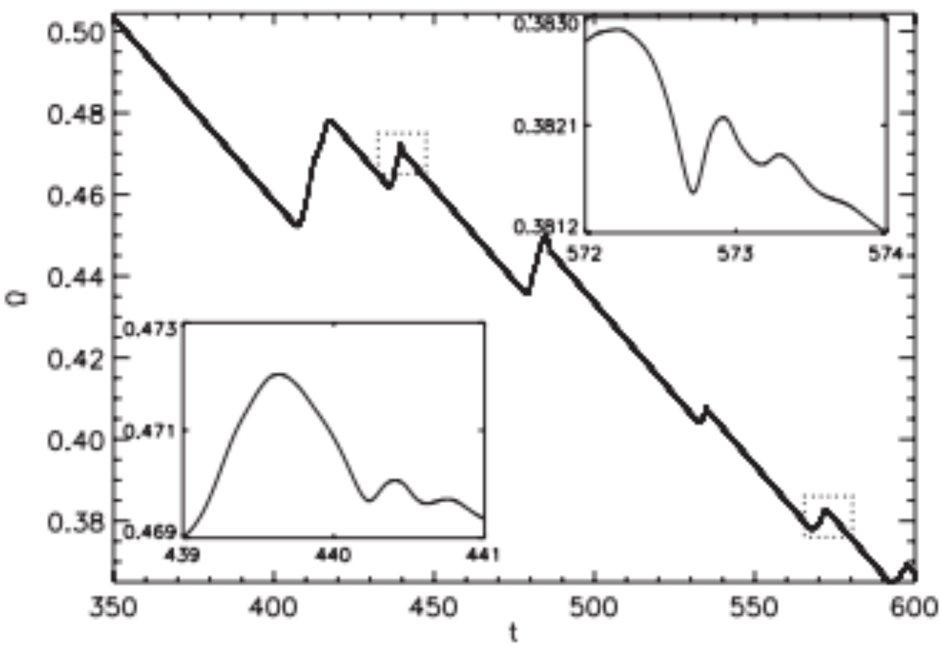


Motivation

Warszawski *et al.*, 2012

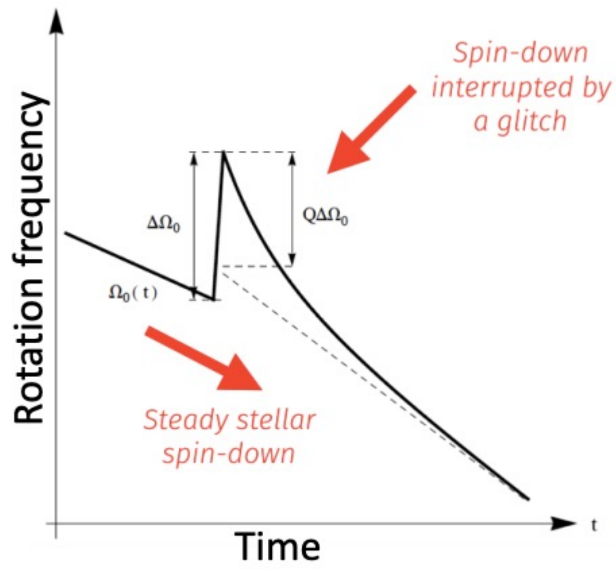


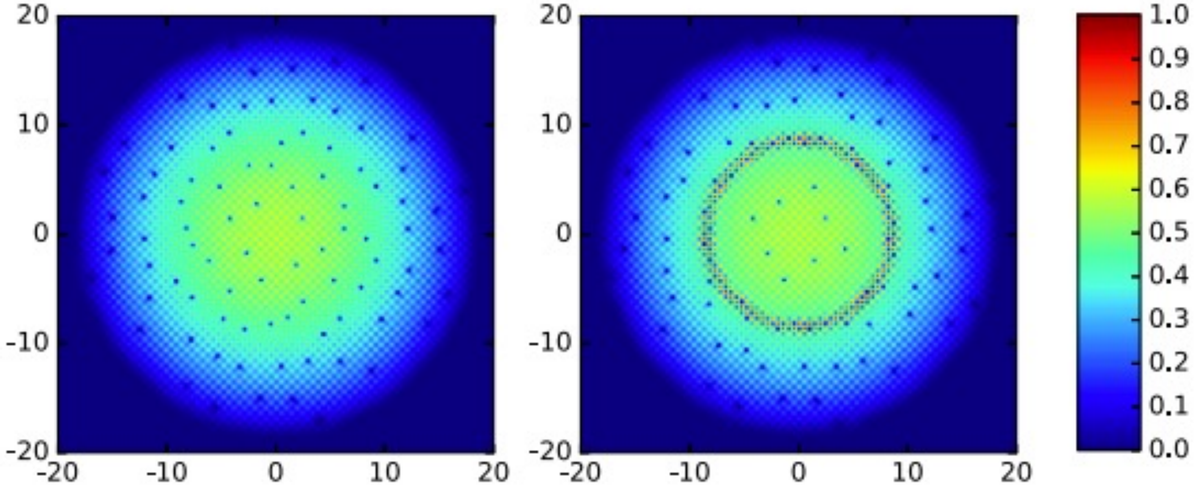


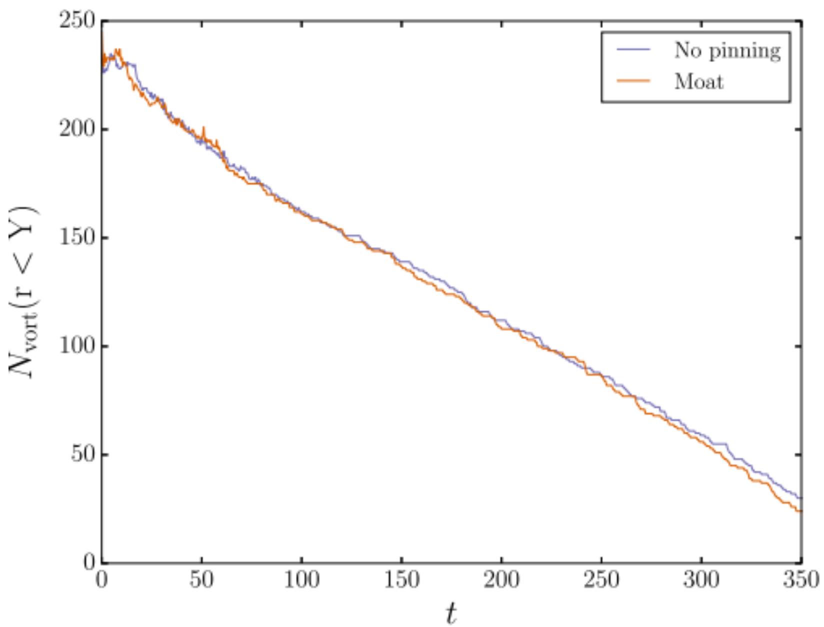


Motivation

Lonnborn *et al.*, 2019







Numerical setup

- Different approach/motivation from part I
- 3D $\rightarrow$ 2D
- Vortex filament $\rightarrow$ GPE

$$i\hbar\frac{\partial\psi}{\partial t} = (1 - i\gamma)\left[-\frac{\hbar^2\nabla^2}{2m_{\mathrm{eff}}} + V + g\left|\psi\right|^2 - \mu - \Omega(t)\hat{L}_z\right]\psi$$

- Dissipative GPE with time dependent rotation (spin-down)

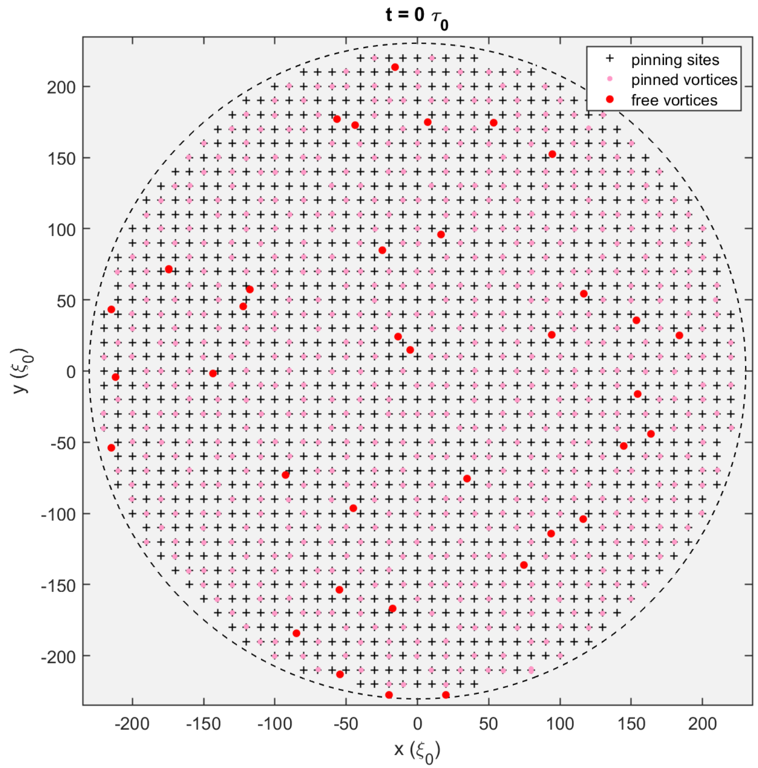
$$i\hbar\frac{\partial\psi}{\partial t} = (1 - i\gamma)\left[-\frac{\hbar^2\nabla^2}{2m_{\text{eff}}} + V + g\left|\psi\right|^2 - \mu - \Omega(t)\hat{L}_z\right]\psi$$

$$V(\mathbf{r}, t) = V_{\text{con}}(\mathbf{r}) + V_{\text{pin}}(\mathbf{r}) + V_{\text{eff}}(\mathbf{r}, t)$$

cylindrically hard-wall potential

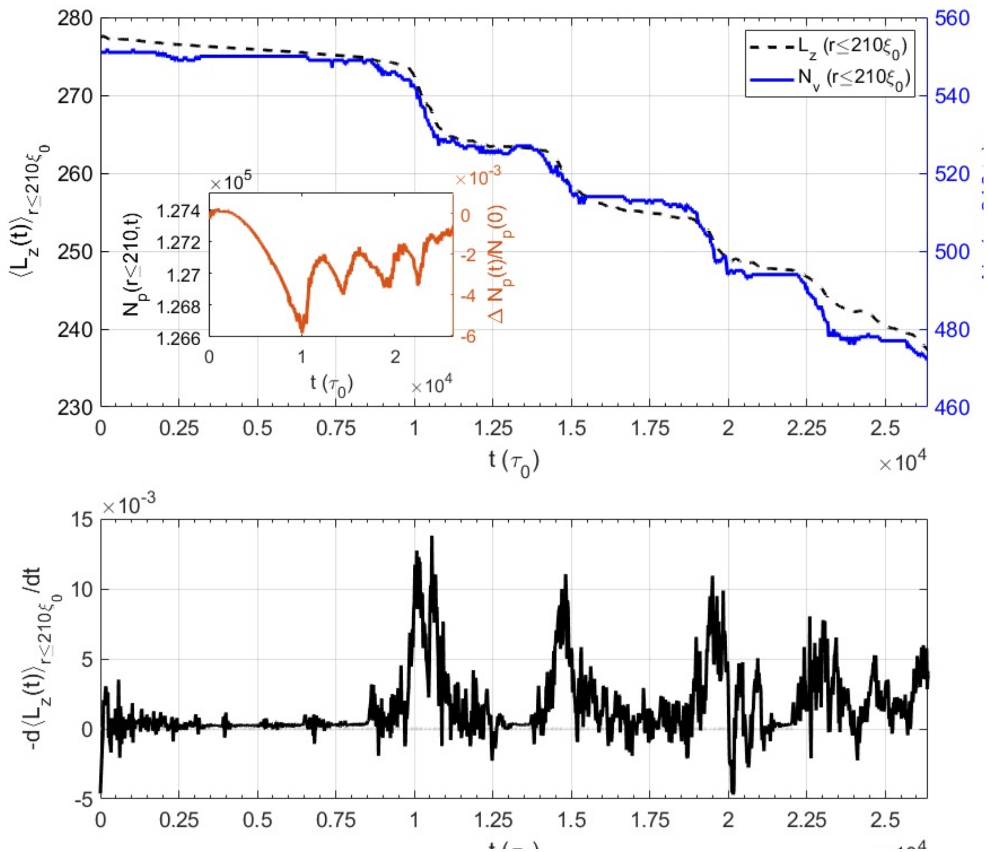
Lattice of Gaussian pinning potentials

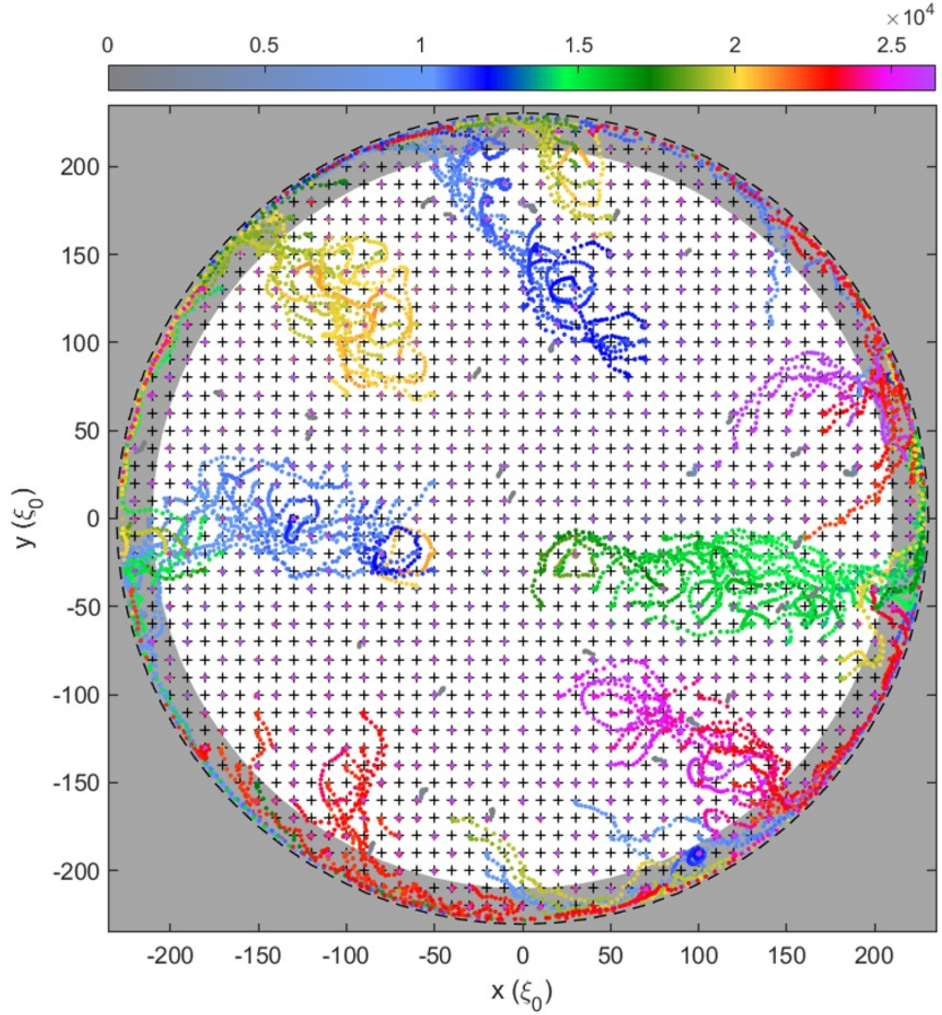
Anti-centrifugal potential to ensure constant density



$$\Omega(t) = \Omega_0 - \dot{\Omega}t.$$

Preliminary Results

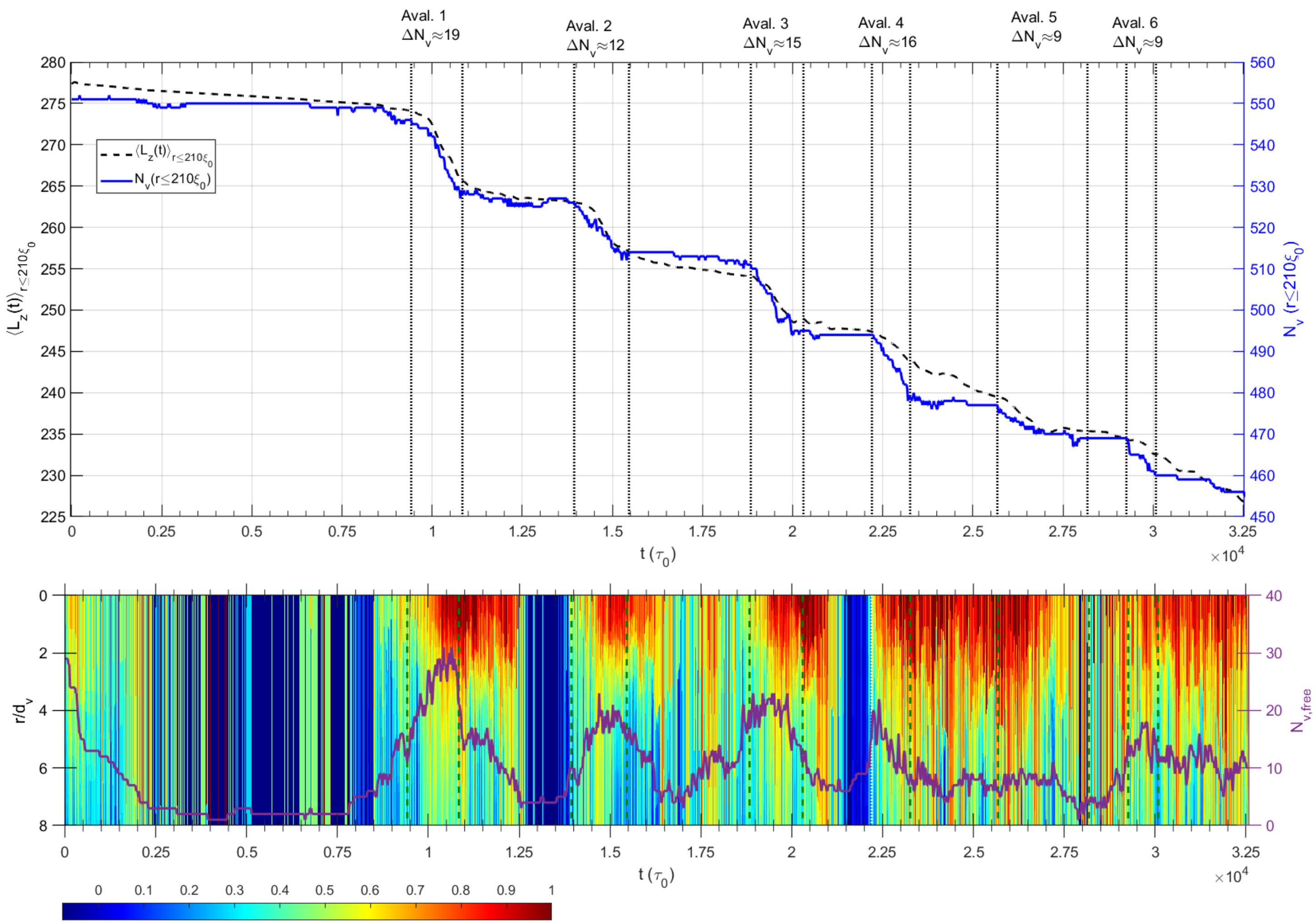




Collective motion?

Vicsek & Zafeiris, 2012

$$\varphi(r,t) = \frac{1}{N_{\mathrm{v,free}}(t)} \sum_{j=1}^{N_{\mathrm{v,free}}(t)} \frac{1}{\mathcal{N}_{j,r}(t)} \left| \sum_{|\vec{r}_j(t)-\vec{r}_k(t)|\le r} \vec{v}_k(t) \right|$$



Thankyou for listening!

Questions?