



# Constraining the microscopic properties of high-density QCD

Gravity Seminar, Southampton  
11 May 2023

*Based on 2303.11356, 2303.02175*

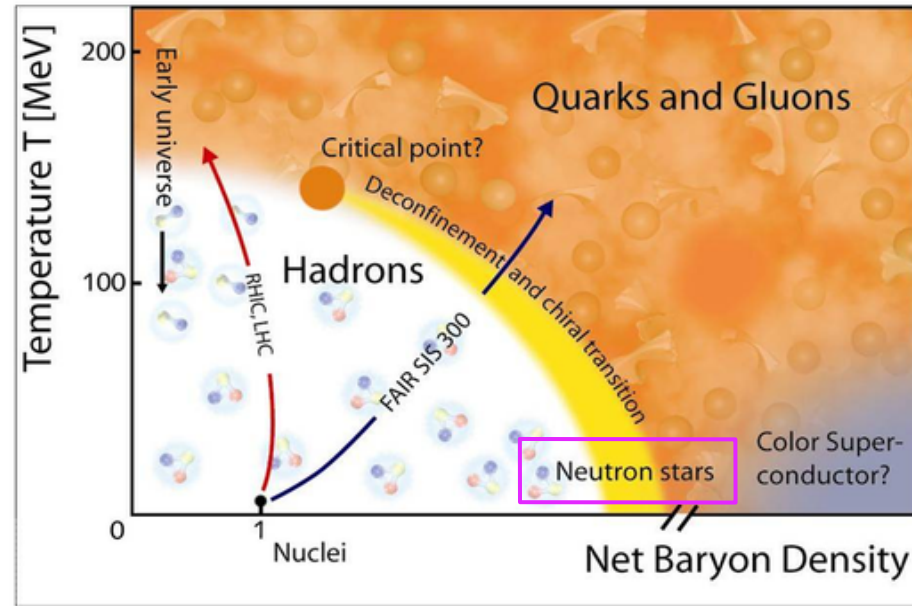
*In collaboration with*

E. Annala, J. Hirvonen, O. Komoltsev, A. Kurkela,  
A. Mazeliauskas, J. Näätälä and A. Vuorinen

Tyler Gorda  
TU Darmstadt

# How to study strongly interacting matter using NSs

- Strongly interacting matter at high density cannot be studied from first principles

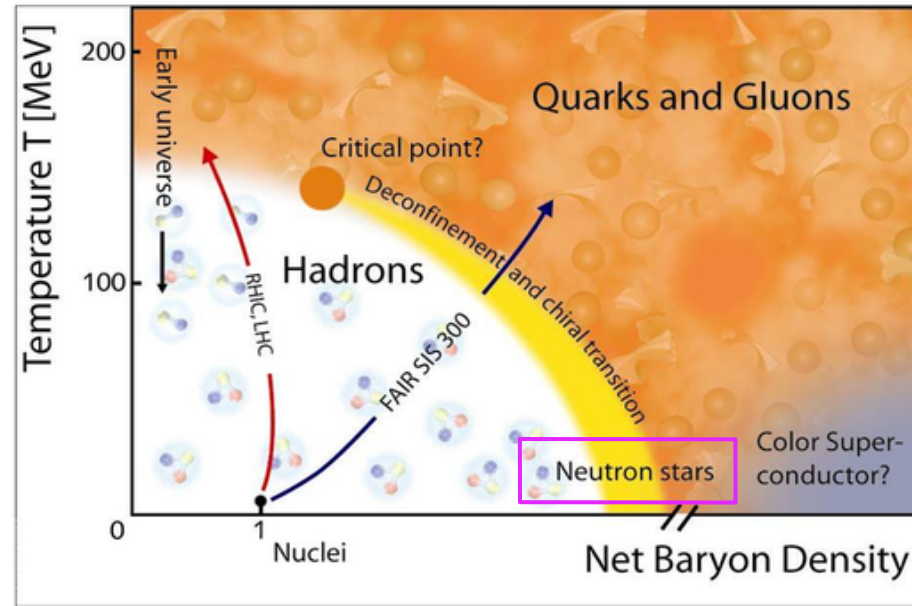


Compressed Baryonic Matter (CBM) experiment

# How to study strongly interacting matter using NSs

- Strongly interacting matter at high density cannot be studied from first principles

High-T lattice +  
experiment have  
identified a  
crossover  
deconfinement  
transition

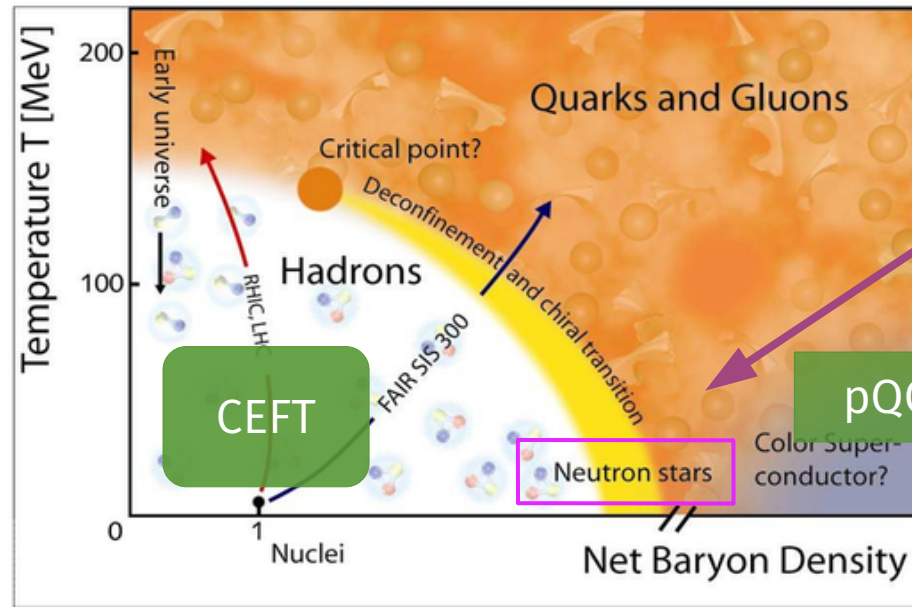


Compressed Baryonic Matter (CBM) experiment

# How to study strongly interacting matter using NSs

- Strongly interacting matter at high density cannot be studied from first principles

High-T lattice + experiment have identified a crossover deconfinement transition



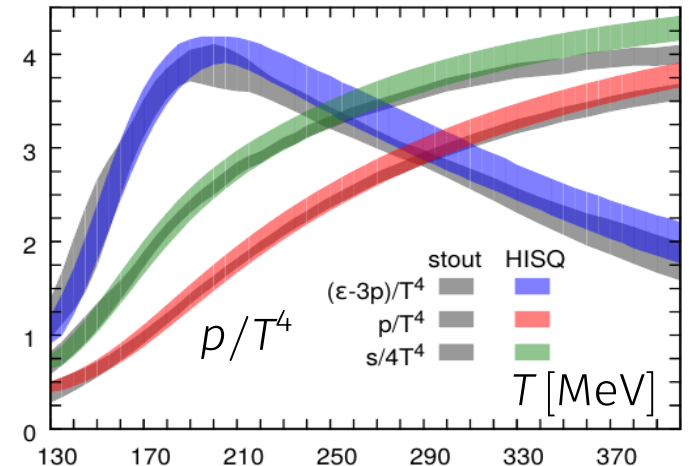
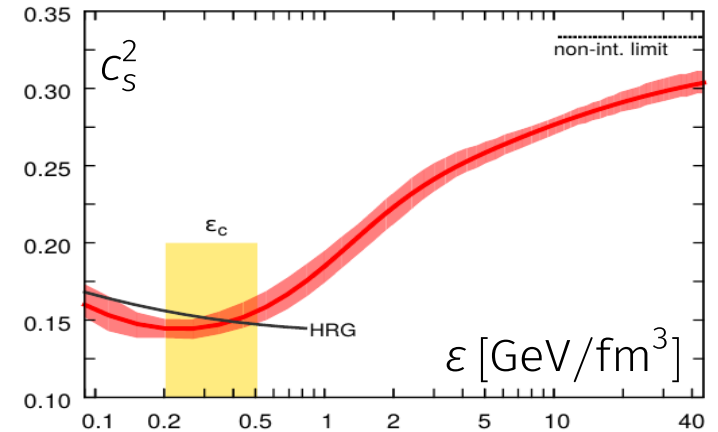
At high density and low T, have robust input only from CEFT, pQCD and NS observables

Combine all these to determine properties of dense matter

Compressed Baryonic Matter (CBM) experiment

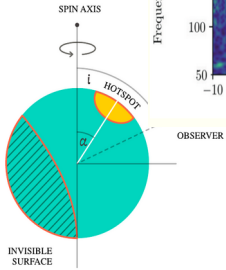
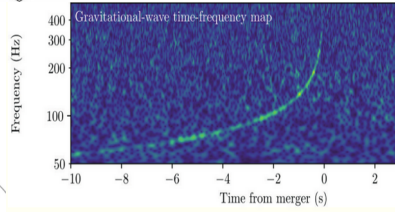
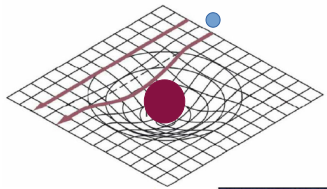
# Objective: Search for change in thermodynamic behavior

- **Strategy:**  
Identify where/if EoS changes physical properties from hadronic  $\rightarrow$  quark  
Want to be model agnostic
- Similar to isolating change in behavior of lattice results at high  $T$ .
- Identify change in phase from *change in physical properties* of matter

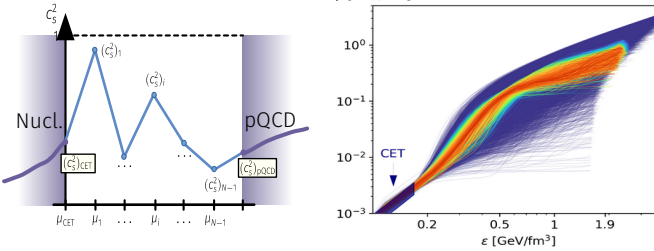


# Workflow

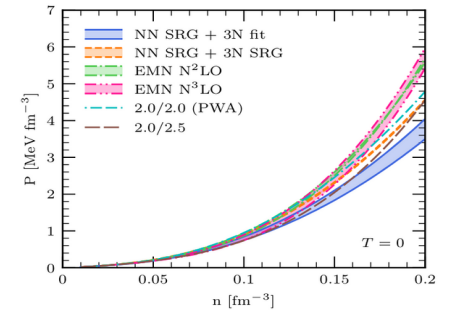
## NS observations



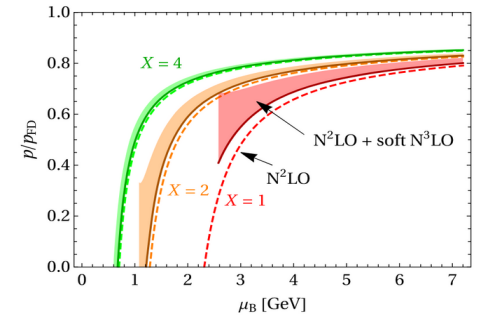
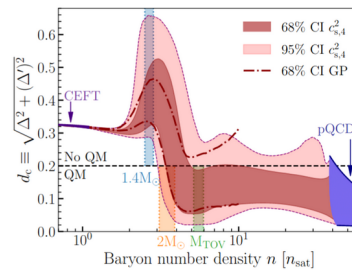
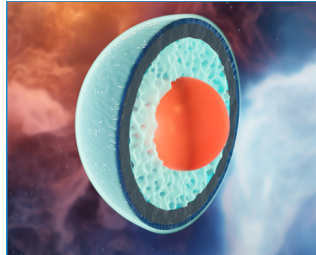
## EoS Inference



## Theoretical Calculations:

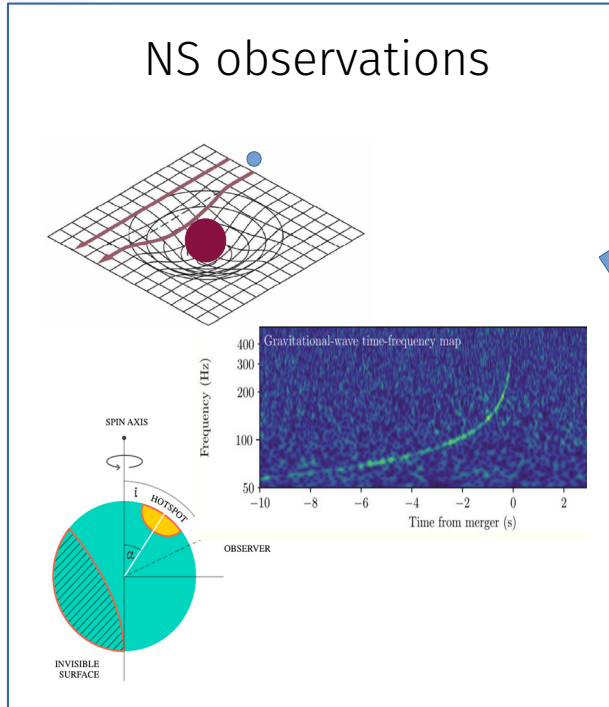
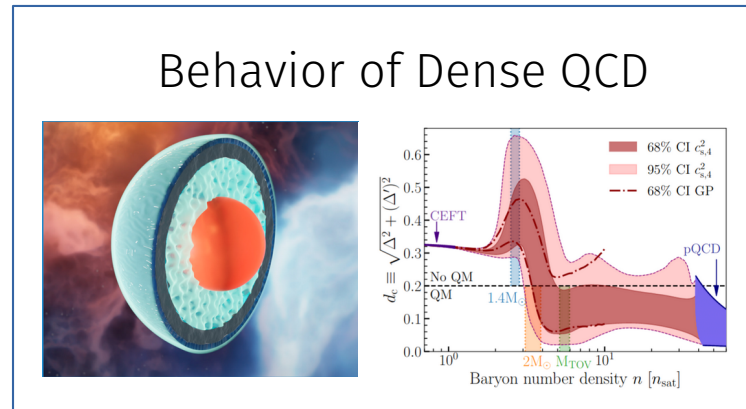
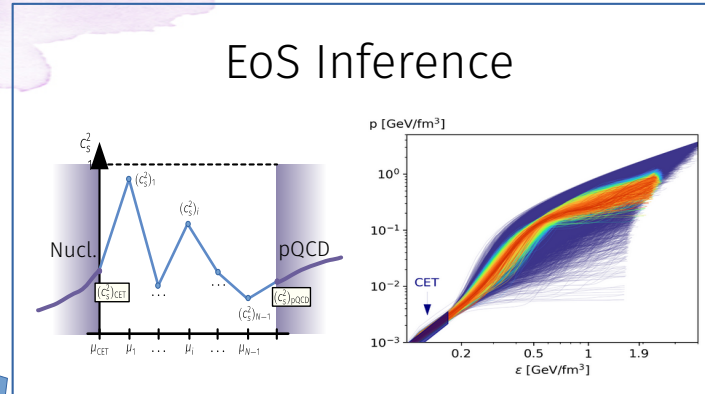
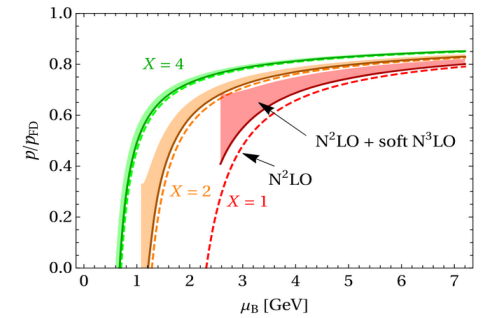
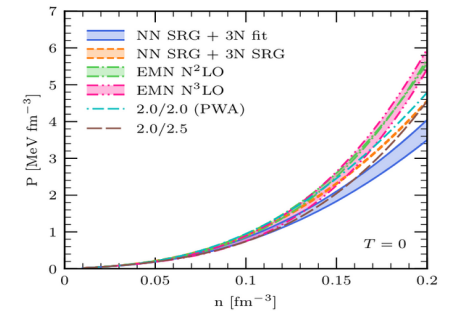


## Behavior of Dense QCD



What do we know from theory?

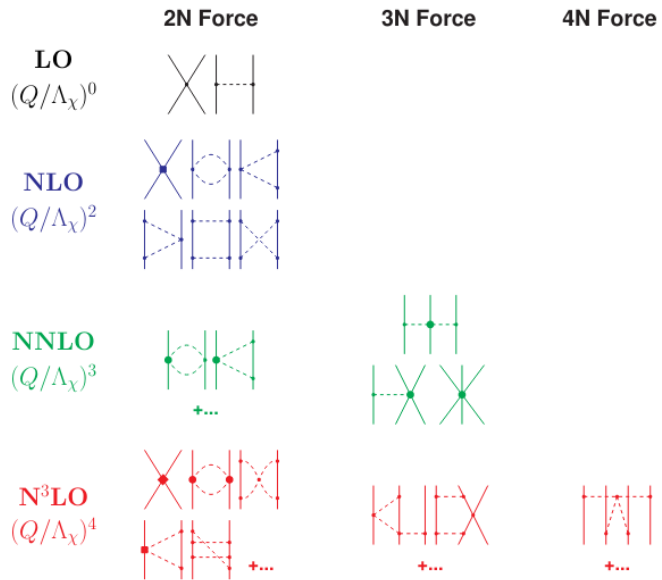
Theoretical Calculations:



# Theory predicts EoS at low and high Density

## Chiral EFT

Describes interactions between (*massive*) **nucleons**, via pion exchange and contact interactions, in a hierarchical manner



## Perturbative QCD

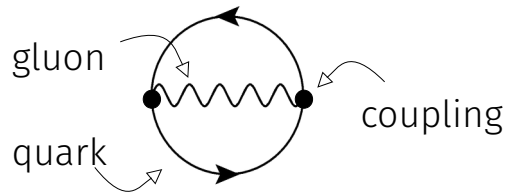
Describes interactions between *massless quarks and gluons*. Quarks are approximately free, up to [O(20%)] perturbative corrections

$$p = \underbrace{p_{\text{FD}}}_{\text{free quark gas}^*} + p_1 \alpha_s + p_2 \alpha_s^2 + \dots$$

*free quark gas*\*      \* ( $p_{\text{FD}} \propto \mu^4$ ,  $p_{\text{pairing}} \propto \mu^2 \Delta^2$ )

Alford+, Rev. Mod. Phys. 80, 1455 (2008)

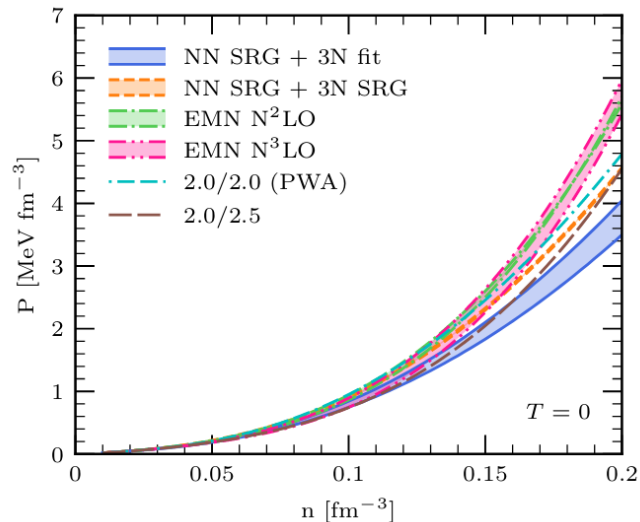
Language for this expansion is *Feynman diagrams*





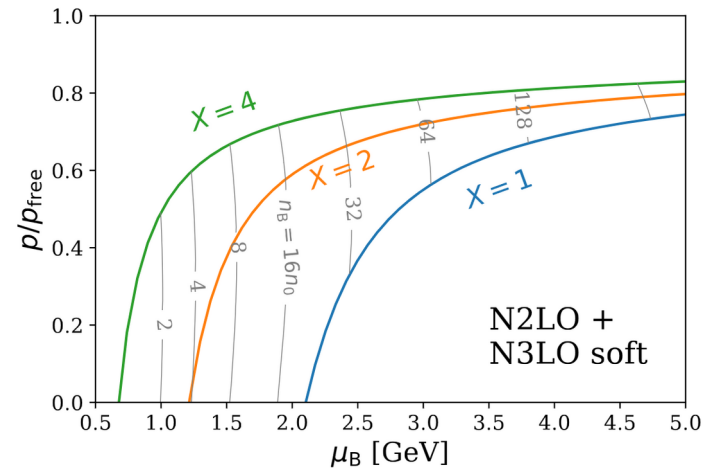
# Theory predicts EoS at low and high Density

## Chiral EFT



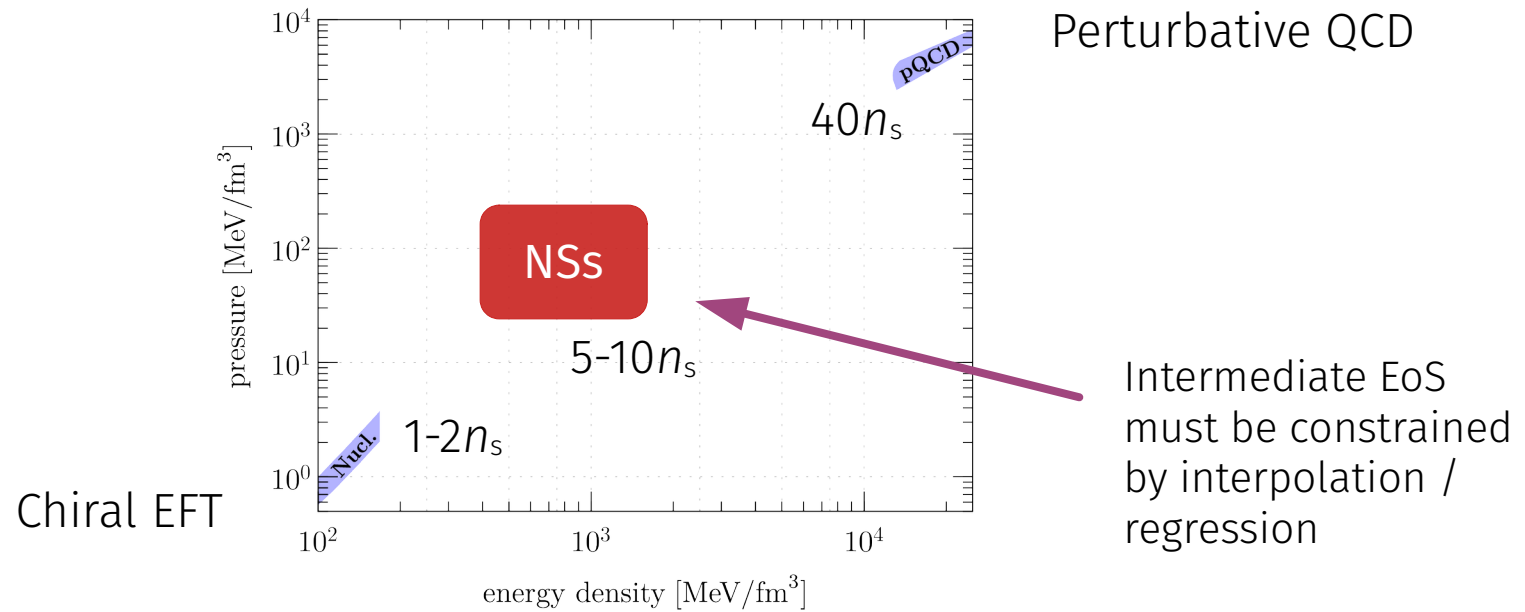
Hebeler, Lattimer, Pethick, Schwenk. *Astrophys. J.* 773 (2013);  
 Tews, Krüger, Hebeler, Schwenk, *PRL* 110, 032504 (2013)  
 Lynn, Tews, Carlson, et al., *PRL* 116, 062501 (2016),  
 Drischler, Hebeler, Schwenk, *PRL* 122, 042501 (2019),  
 Drischler, Furnstahl, Melendez, Phillips, *PRL* 125, 202702 (2020),  
 Keller, Hebeler, and Schwenk, *PRL* 130, 072701 (2023).

## Perturbative QCD



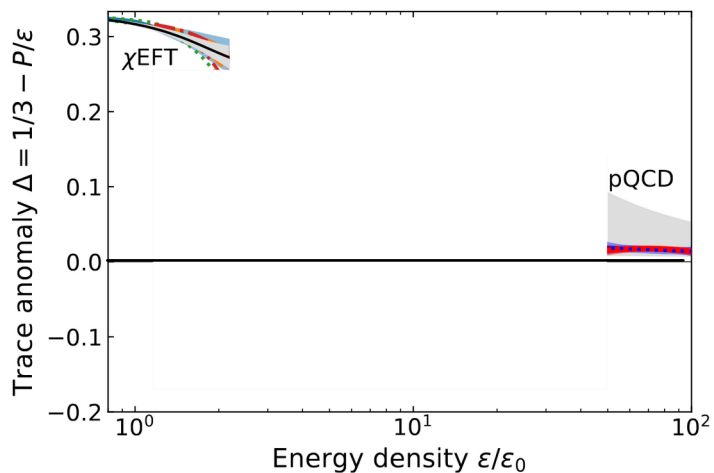
Kurkela, Romatschke, Vuorinen, *PRD* 81, 105021 (2010),  
 TG, Kurkela, Romatschke, Säppi, Vuorinen, *PRL* 121, 202701 (2018),  
 TG, Kurkela, Paatelainen, Säppi, Vuorinen, *PRD* 104, 074015 (2021),  
 TG, Kurkela, Paatelainen, Säppi, Vuorinen, *PRL* 127, 162003 (2021).

# Theory predicts EoS at low and high Density

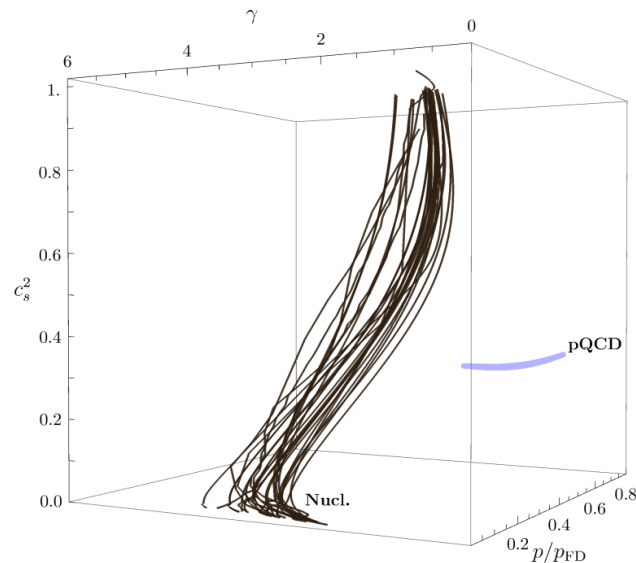


# Hadronic and Quark matter are different

QM has different physical properties than hadronic matter, in particular, it is approximately *conformal*, with many implications. One consequence is a vanishing trace of the energy-momentum tensor,  $\Delta \equiv T^\mu_\mu / (3\varepsilon) = (\varepsilon - 3p) / (3\varepsilon)$



ADAPTED from Fujimoto, Fukushima, McLerran+ PRL 129 (2022)  
25, 252702



Annala, TG, Kurkela, Nättilä, Vuorinen Nat. Phys. 16 (2020)

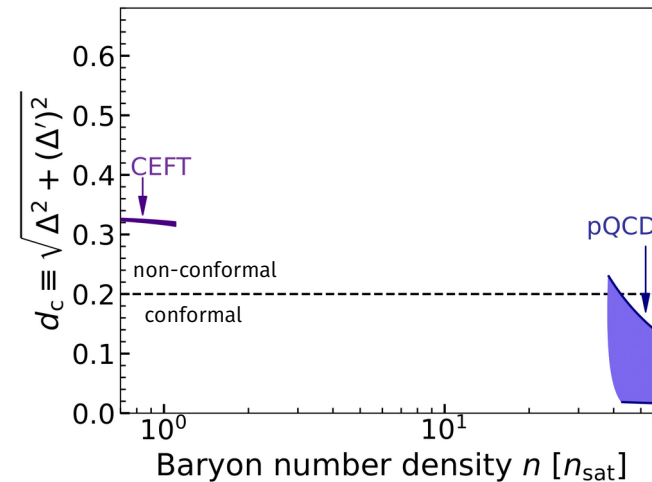
# Hadronic and Quark matter are different

QM has different physical properties than hadronic matter, in particular, it is approximately *conformal*, with many implications. One consequence is a vanishing trace of the energy-momentum tensor,  $\Delta \equiv T^\mu_\mu / (3\varepsilon) = (\varepsilon - 3p) / (3\varepsilon)$

For our recent work, we use a combination of the normalized trace anomaly and its derivative:  $\Delta' \equiv d \ln \Delta / d \ln \varepsilon$

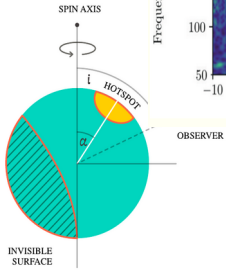
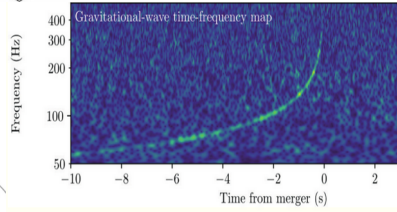
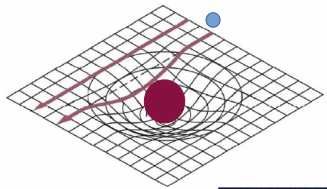
$$d_c \equiv \sqrt{\Delta^2 + (\Delta')^2}$$

This quantity is sensitive to the local trace anomaly and how quickly trace anomaly is changing

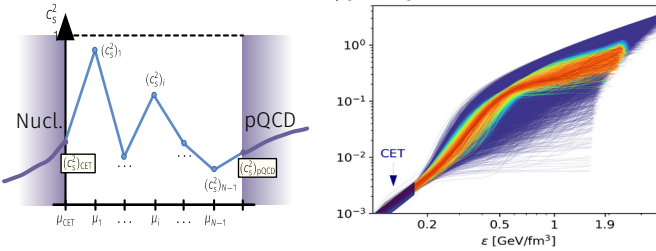


# What do we know from observations?

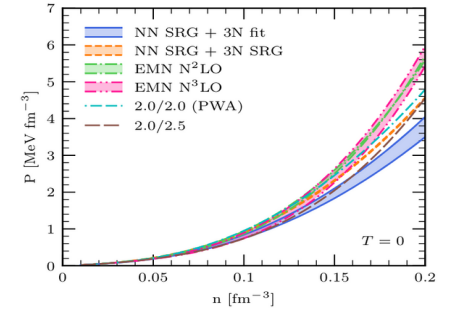
## NS observations



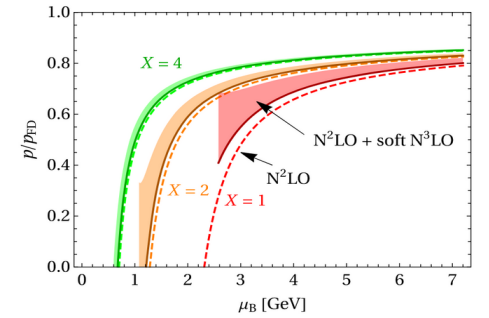
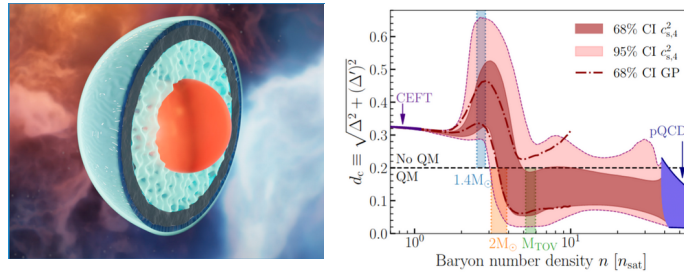
## EoS Inference



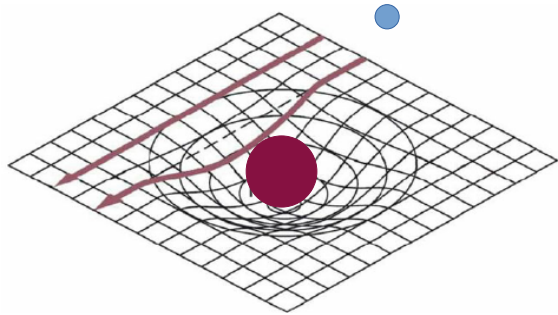
## Theoretical Calculations:



## Behavior of Dense QCD

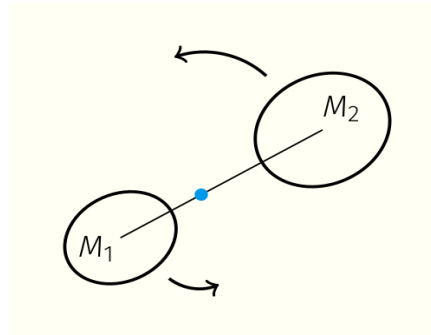


# Observations tell us about:



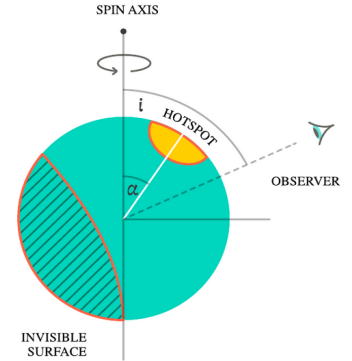
Masses

Demorest, Pennucci, Ransom, Roberts, Hessels. *Nature* 467 (2010) pp. 1081-1083;  
Antoniadis et al. *Science* 340 (2013) p. 6131;  
Cromartie et al. (NANOGrav). *Nature Astron.* 4.1 (2019).  
E. Fonseca et al. *Astrophys. J. Lett.* 915.1 (2021)



Deformabilities

Abbott et. al (LIGO Scientific, Virgo) *PRL* 119 (2017); *PRL* 121 (2018); *PRX* 9 (2019).

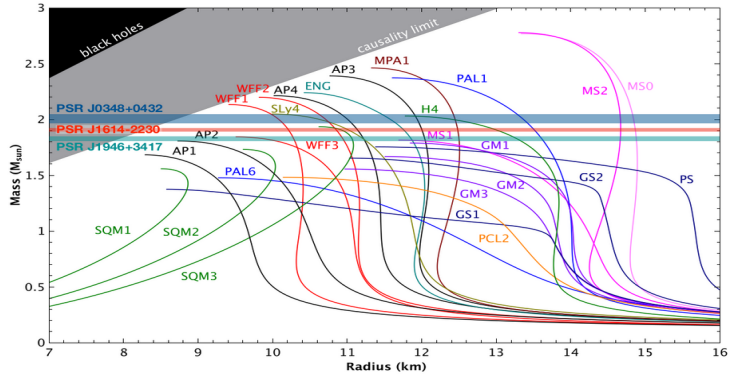


Radii, compactness

Miller et al. *Astrophys. J. Lett.* 918.2 (2021), p. L28. Riley et al. *Astrophys. J. Lett.* 918.2 (2021) p. L27.

# Observations tell us about:

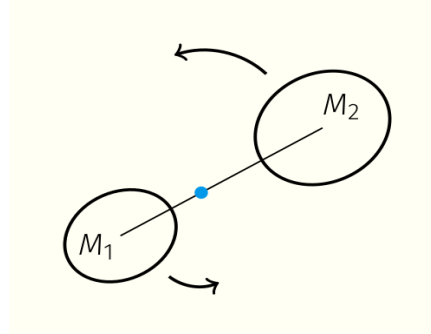
$$M_{\text{TOV}} \geq \begin{cases} 1.97 \pm 0.04 M_{\odot} & \text{PSR J1614-2230} \\ 2.01 \pm 0.04 M_{\odot} & \text{PSR J0348+0432} \\ 2.08 \pm 0.07 M_{\odot} & \text{PSR J0740+6620} \end{cases}$$



Norbert Wex

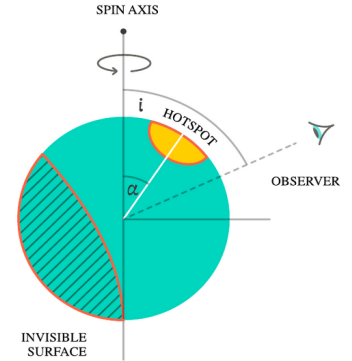
## Masses

Demorest, Pennucci, Ransom, Roberts, Hessels. *Nature* 467 (2010) pp. 1081-1083;  
 Antoniadis et al. *Science* 340 (2013) p. 6131;  
 Cromartie et al. (NANOGrav). *Nature Astron.* 4.1 (2019).  
 E. Fonseca et al. *Astrophys. J. Lett.* 915.1 (2021)



## Deformabilities

Abbott et. al (LIGO Scientific, Virgo) *PRL* 119 (2017); *PRL* 121 (2018); *PRX* 9 (2019).

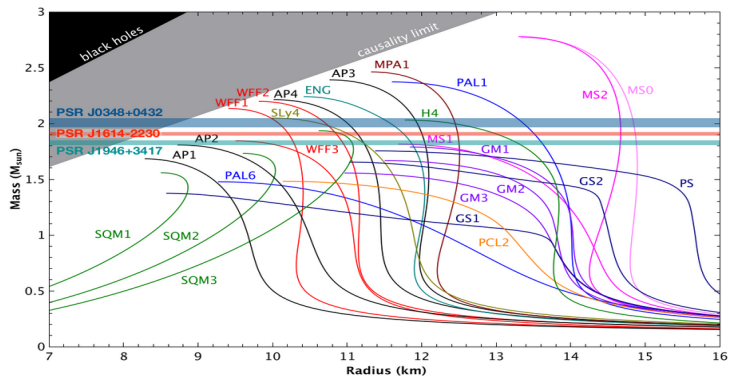


## Radii, compactness

Miller et al. *Astrophys. J. Lett.* 918.2 (2021), p. L28. Riley et al. *Astrophys. J. Lett.* 918.2 (2021) p. L27.

# Observations tell us about:

$$M_{\text{TOV}} \geq \begin{cases} 1.97 \pm 0.04 M_{\odot} & \text{PSR J1614-2230} \\ 2.01 \pm 0.04 M_{\odot} & \text{PSR J0348+0432} \\ 2.08 \pm 0.07 M_{\odot} & \text{PSR J0740+6620} \end{cases}$$

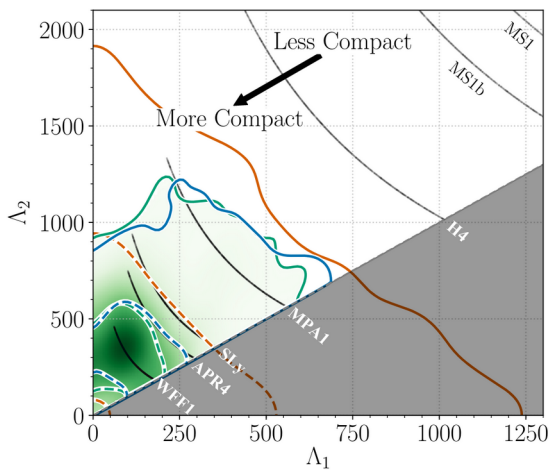


Norbert Wex

Masses

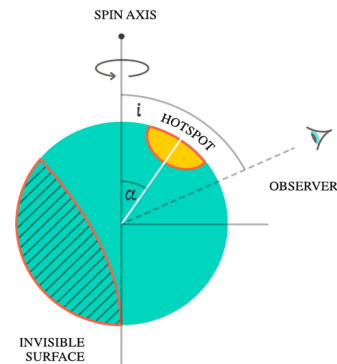
Demorest, Pennucci, Ransom, Roberts, Hessels. Nature 467 (2010) pp. 1081-1083;  
 Antoniadis et al. Science 340 (2013) p. 6131;  
 Cromartie et al. (NANOGrav). Nature Astron. 4.1 (2019).  
 E. Fonseca et al. Astrophys. J. Lett. 915.1 (2021)

$$\Lambda(M) \equiv |Q_{ij}/\mathcal{E}_{ij}|M^5$$



Deformabilities

Abbott et. al (LIGO Scientific, Virgo) PRL 119 (2017); PRL 121 (2018); PRX 9 (2019).



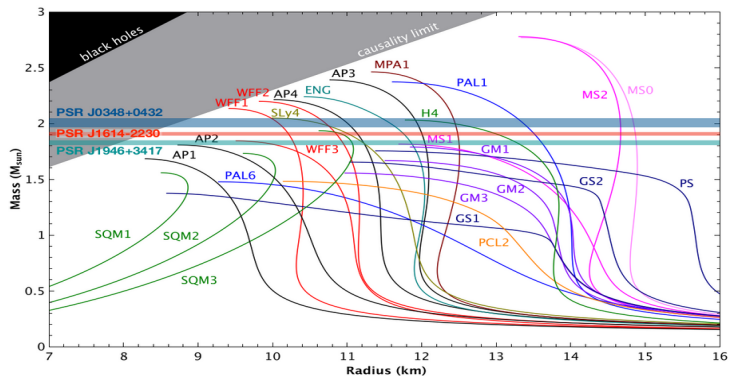
Radii, compactness

Miller et al. Astrophys. J. Lett. 918.2 (2021), p. L28. Riley et al. Astrophys. J. Lett. 918.2 (2021) p. L27.



# Observations tell us about:

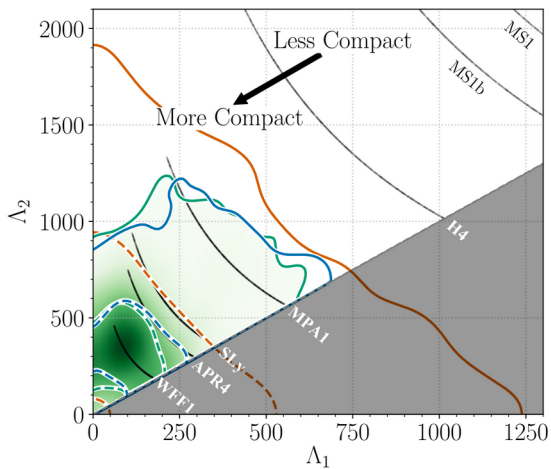
$$M_{\text{TOV}} \geq \begin{cases} 1.97 \pm 0.04 M_{\odot} & \text{PSR J1614-2230} \\ 2.01 \pm 0.04 M_{\odot} & \text{PSR J0348+0432} \\ 2.08 \pm 0.07 M_{\odot} & \text{PSR J0740+6620} \end{cases}$$



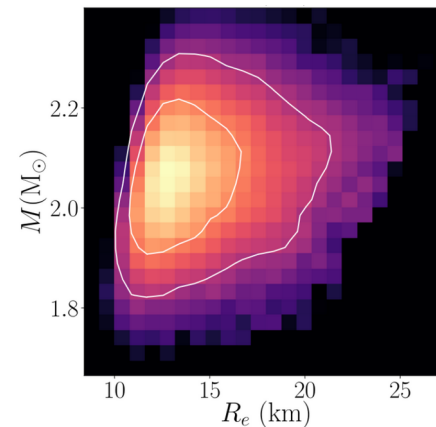
Norbert Wex

Masses

$$\Lambda(M) \equiv |Q_{ij}/\mathcal{E}_{ij}|M^5$$



Deformabilities



Radii, compactness

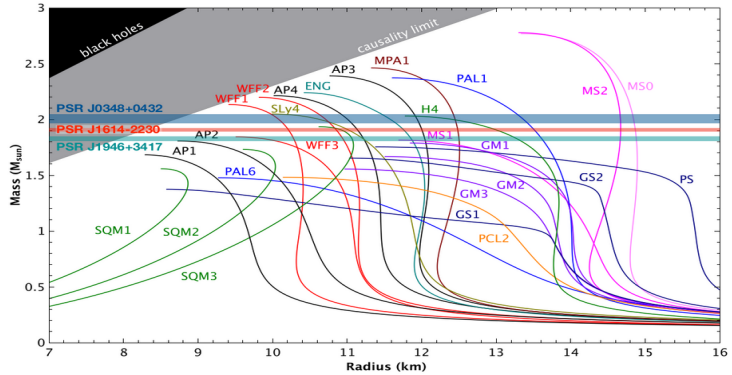
Demorest, Pennucci, Ransom, Roberts, Hessels. Nature 467 (2010) pp. 1081-1083;  
 Antoniadis et al. Science 340 (2013) p. 6131;  
 Cromartie et al. (NANOGrav). Nature Astron. 4.1 (2019).  
 E. Fonseca et al. Astrophys. J. Lett. 915.1 (2021)

Abbott et. al (LIGO Scientific, Virgo) PRL 119 (2017); PRL 121 (2018); PRX 9 (2019).

Miller et al. Astrophys. J. Lett. 918.2 (2021), p. L28. Riley et al. Astrophys. J. Lett. 918.2 (2021) p. L27.

# Observations tell us about:

$$M_{\text{TOV}} \geq \begin{cases} 1.97 \pm 0.04 M_{\odot} & \text{PSR J1614-2230} \\ 2.01 \pm 0.04 M_{\odot} & \text{PSR J0348+0432} \\ 2.08 \pm 0.07 M_{\odot} & \text{PSR J0740+6620} \end{cases}$$



Norbert Wex

## Masses

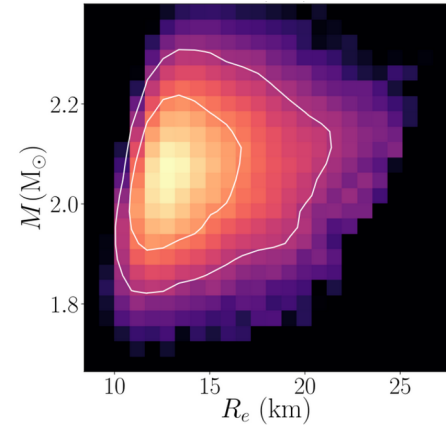
Demorest, Pennucci, Ransom, Roberts, Hessels. *Nature* 467 (2010) pp. 1081-1083;  
 Antoniadis et al. *Science* 340 (2013) p. 6131;  
 Cromartie et al. (NANOGrav). *Nature Astron.* 4.1 (2019).  
 E. Fonseca et al. *Astrophys. J. Lett.* 915.1 (2021)

\* EM counterpart evidence for collapse to BH (BH-hyp)

Margalit & Metzger. *ApJ. Lett.* 850.2 (2017);  
 Rezzolla, Most, Weih. *ApJ* 852.2 (2018);  
 Shibata+ *PRD* 96.12 (2017);

## Deformabilities

Abbott et. al (LIGO Scientific, Virgo) *PRL* 119 (2017); *PRL* 121 (2018); *PRX* 9 (2019).



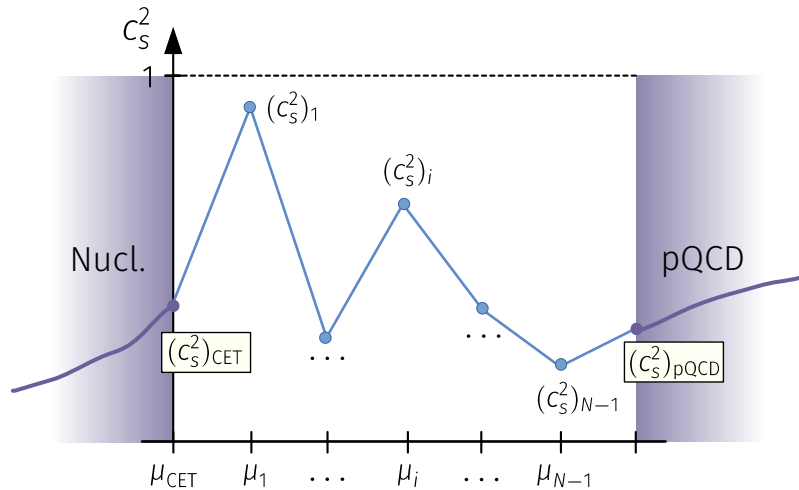
## Radii, compactness

Miller et al. *Astrophys. J. Lett.* 918.2 (2021), p. L28. Riley et al. *Astrophys. J. Lett.* 918.2 (2021) p. L27.

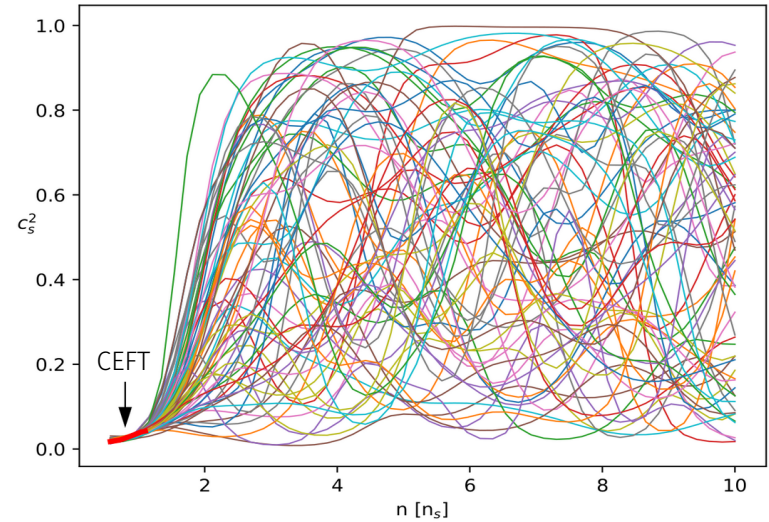


# Two approaches to EoS Inference

## Parametric interpolation



## Gaussian process regression



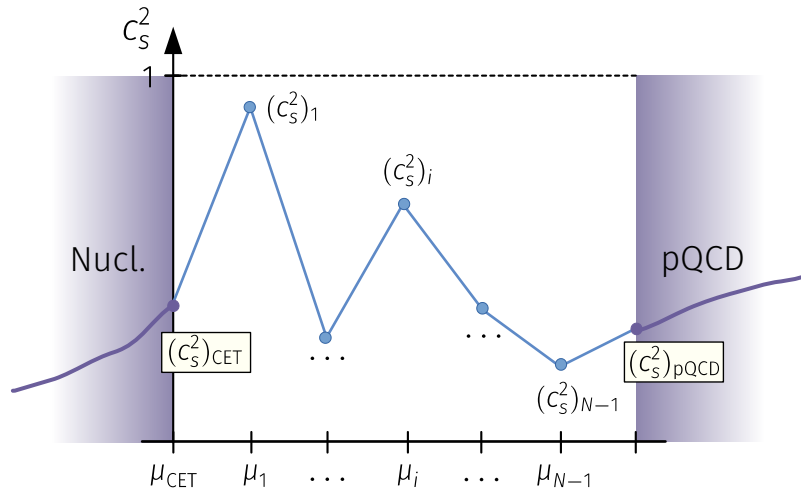
Annala, TG, Kurkela, Nättilä, Vuorinen. Nature Phys. 16.9 (2020)  
Annala, TG, Katerini, Kurkela, Nättilä, Paschalidis, Vuorinen.  
PRX 12.1 (2022)

Following Landry & Essick Phys. Rev. D 99 (2019)

TG, Komoltsev, Kurkela, 2204.11877, accepted to ApJ

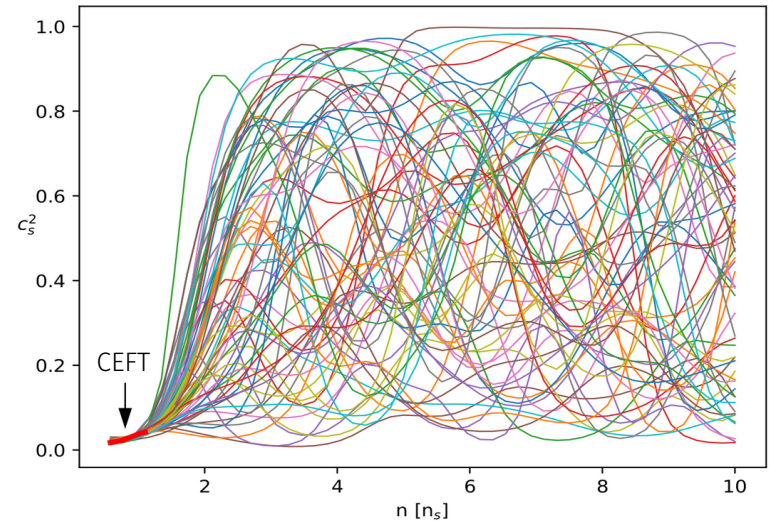
# Two approaches to EoS Inference

## Parametric interpolation



Cons: Lose many parameters interpolating from  $10-40n_s$ .

## Gaussian process regression

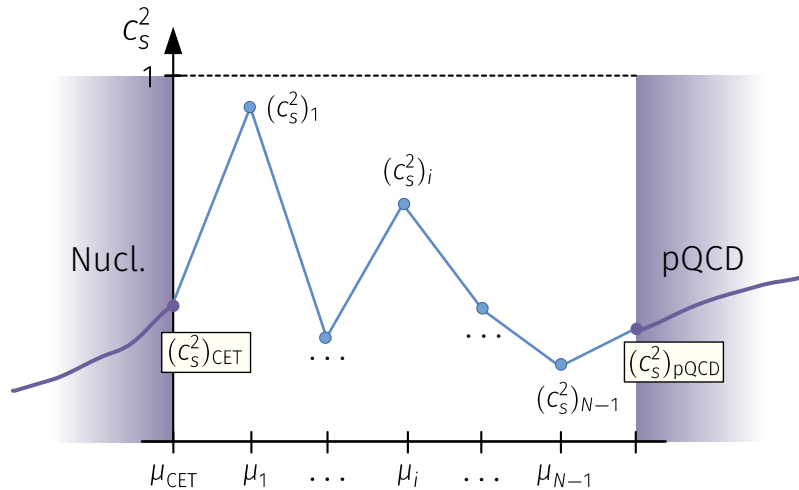


Following Landry & Essick Phys. Rev. D 99 (2019)

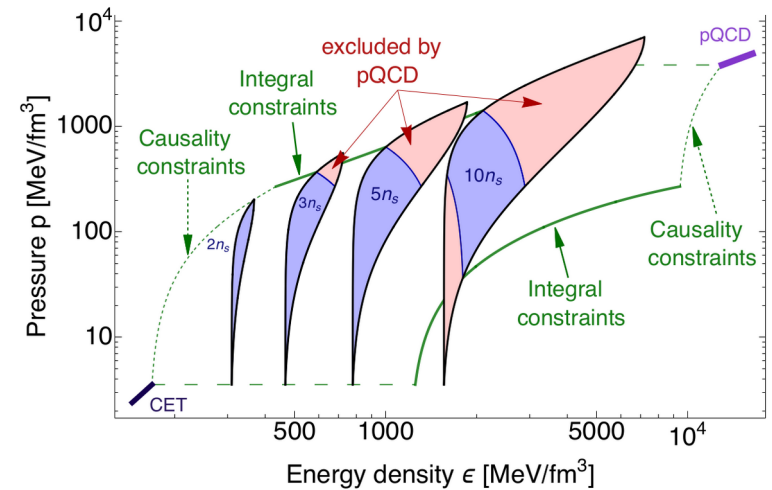
TG, Komoltsev, Kurkela, 2204.11877, accepted to ApJ

# Two approaches to EoS Inference

## Parametric interpolation



## Gaussian process regression

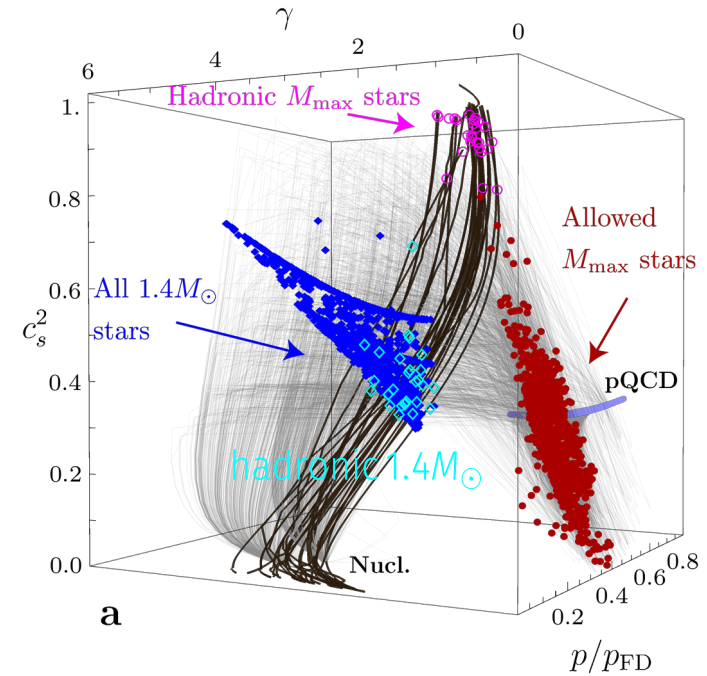
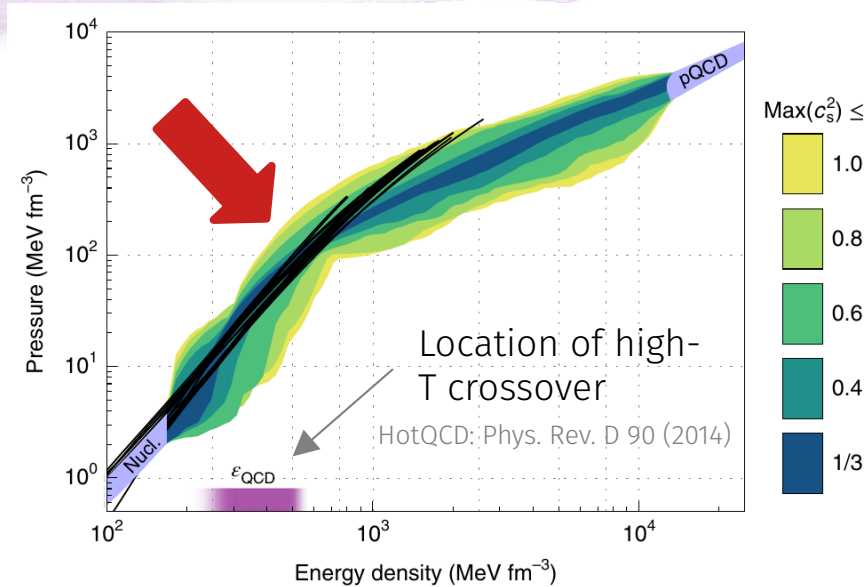


Cons: Lose many parameters interpolating from  $10$ - $40n_s$ .

Use Komoltsev and Kurkela, PRL 128 (2022) to impose most general  $\epsilon, p$  region consistent with pQCD at  $10n_s$ .

# Using hard cuts gave evidence for transition

Annala, TG, Kurkela, Näätä, and Vuorinen. Nature Phys. 16.9 (2020)



Previous evidence for **non-conformal to conformal transition**, based on  $\gamma \equiv d \ln p / d \ln \epsilon$

Also indicated by recent analysis of trace anomaly in Fujimoto, Fukushima, McLerran, Praszalowicz. PRL 129 (2022); Marczenko, McLerran, Redlich, Sasaki PRC 107 (2023)

- $1.4M_{\odot}$  stars *consistent* with cores of hadronic  $1.4M_{\odot}$
- (most)  $M_{\text{max}}$  stars *inconsistent* with centers of hadronic  $M_{\text{max}}$

# Full Bayesian analysis quantifies $P(\text{conformal})$

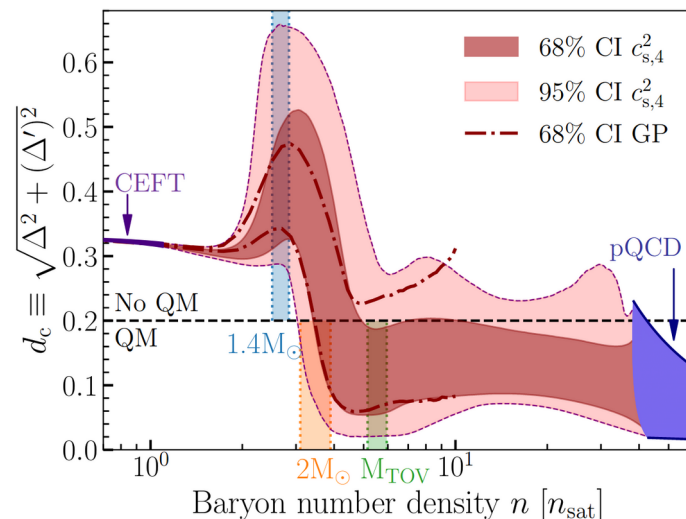
Annala, TG, Hirvonen, Komoltsev, Kurkela, Nättilä, Vuorinen 2303.11356

After folding in measurements of high-mass pulsars, GW observations, and X-ray observations, the new conformal measure  $d_c$  shows striking behavior inside neutron stars.

$P(\text{conformal}) = 88\%$  (75%) for the parametric (GP) approach, for TOV stars. (GP without pQCD is 50%).

Criterion is much stricter than previous work. (Would have found 99.8% previously.)

See also: Han, Huang, Tang, Fan+ arXiv:2207.13613





# Full Bayesian analysis quantifies $P(\text{conformal})$

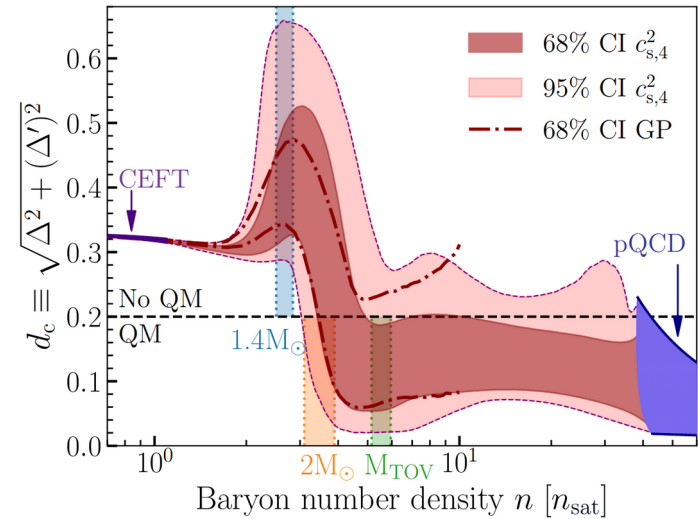
Annala, TG, Hirvonen, Komoltsev, Kurkela, Nättilä, Vuorinen 2303.11356

After folding in measurements of high-mass pulsars, GW observations, and X-ray observations, the new conformal measure  $d_c$  shows striking behavior inside neutron stars.

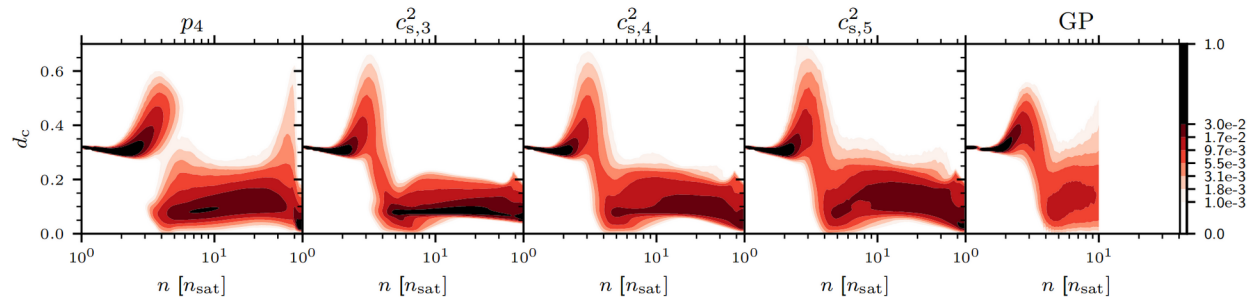
$P(\text{conformal}) = 88\%$  (75%) for the parametric (GP) approach, for TOV stars. (GP without pQCD is 50%).

Criterion is much stricter than previous work. (Would have found 99.8% previously.)

See also: Han, Huang, Tang, Fan+ arXiv:2207.13613



Robust to interpolants:

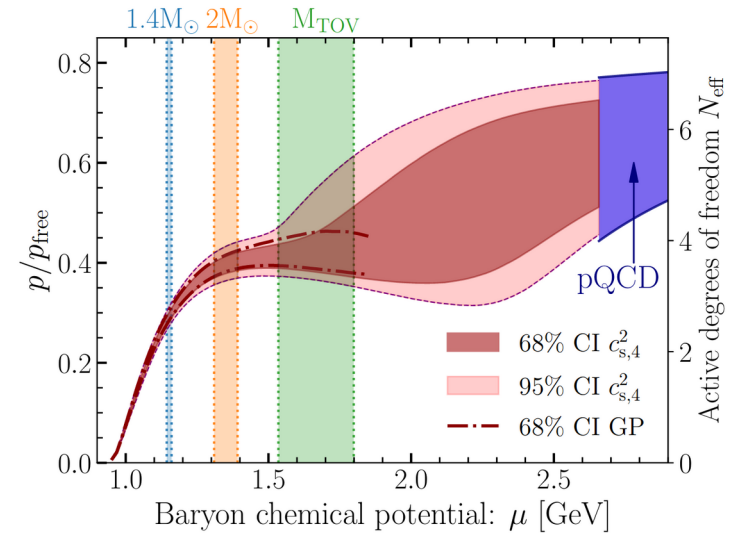


# Is it Quark matter?

Annala, TG, Hirvonen, Komoltsev, Kurkela, Nättilä, Vuorinen 2303.11356

Active degrees of freedom  $N_{\text{eff}}$  comparable to number in quark matter at high density (roughly 2/3) – **suggests: Yes, QM**

*“It is, however, interesting to ask whether the slightly reduced value of the quantity within maximally massive NSs may signal that matter in these stellar cores features strongly-coupled characteristics.”*

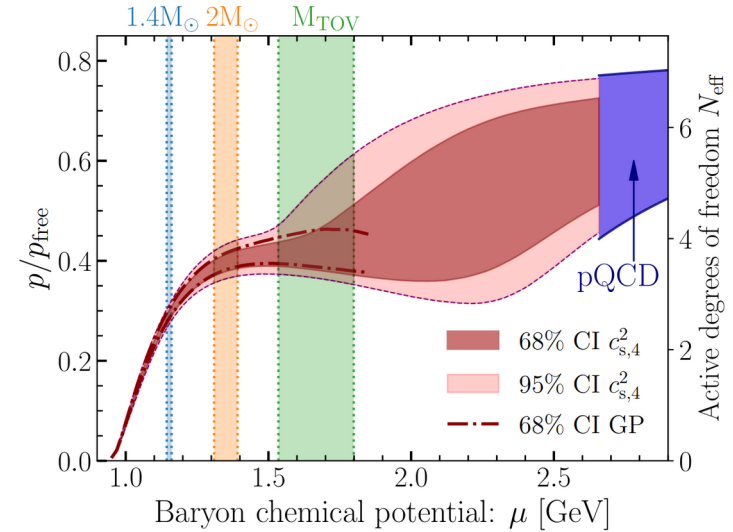


# Is it Quark matter?

Annala, TG, Hirvonen, Komoltsev, Kurkela, Nättilä, Vuorinen 2303.11356

Active degrees of freedom  $N_{\text{eff}}$  comparable to number in quark matter at high density (roughly 2/3) – **suggests: Yes, QM**

*“It is, however, interesting to ask whether the slightly reduced value of the quantity within maximally massive NSs may signal that matter in these stellar cores features strongly-coupled characteristics.”*



Also desirable to find other observables, arguments that support or contradict this EoS evidence for deconfined matter



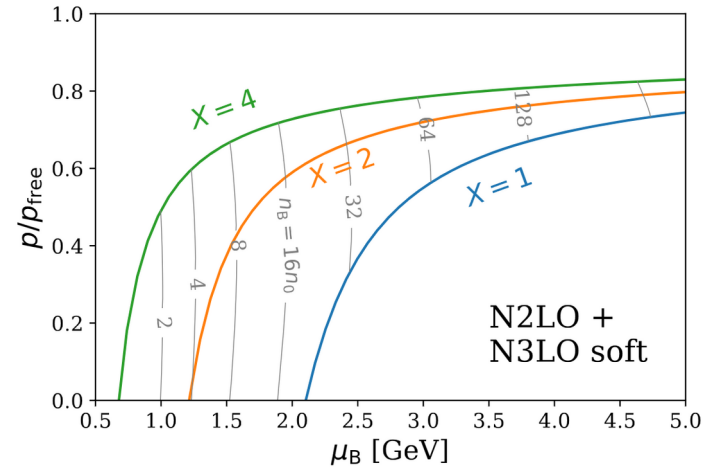
# Statistical treatment of pQCD errors

TG, Komoltsev, Kurkela, Mazeliauskas 2303.02175

# Estimates of truncation errors in pQCD

TG, Komoltsev, Kurkela, Mazeliauskas 2303.02175

Heretofore have estimated pQCD errors by varying renormalization scale  $X = \bar{\Lambda}/\mu_B$ , but this is just a proxy



# Estimates of truncation errors in pQCD

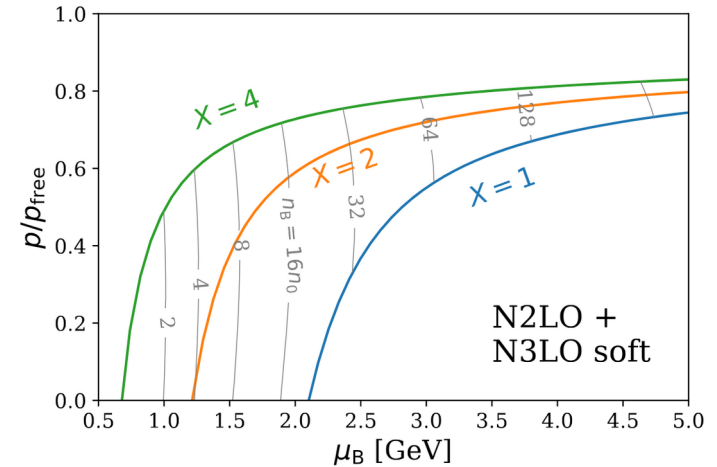
TG, Komoltsev, Kurkela, Mazeliauskas 2303.02175

Heretofore have estimated pQCD errors by varying renormalization scale  $X = \bar{\Lambda}/\mu_B$ , but this is just a proxy

Decided to investigate using a Bayesian framework, assuming the perturbative coefficients are independent draws from distributions of a statistical model of convergent series.

M. Bonvini, Eur. Phys. J. C 80, 989 (2020); Duhr, Huss, Mazeliauskas, Szafron, JHEP 09, 122 (2021)

Performing Bayesian inference on the known orders constrains the model parameters and returns a probability distribution for the next term in the series.



# The *abc* model of convergent series

Duhr, Huss, Mazeliauskas, Szafron, JHEP 09, 122 (2021)

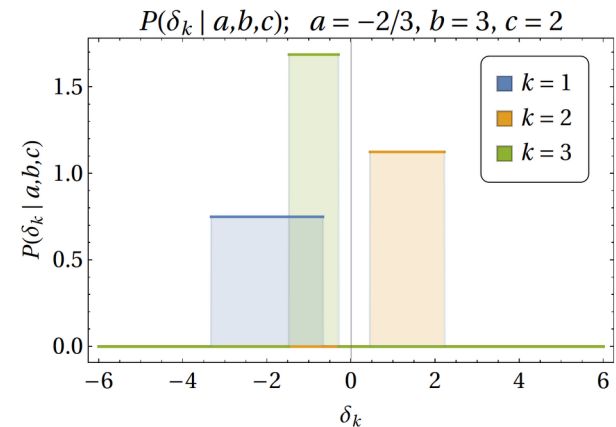
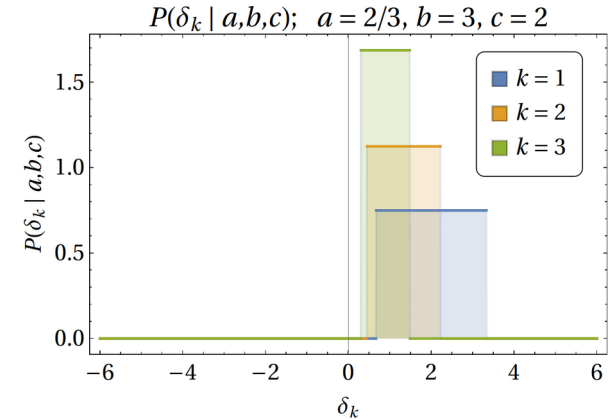
Consider a perturbative sequence, normalized by the LO term

$$\delta_k \equiv \frac{\alpha^k O_k}{O_0} \quad (\delta_0 = 1)$$

The model assumes that these  $\delta_k$  are bounded by some geometric series defined by  $(a, b, c)$

$$(-c + b)a^k < |\delta_k| < (c + b)a^k,$$

Take flat likelihoods on all  $\delta_k$  satisfying this equation. (\*also specify a prior for  $a, b, c$  which favor smaller values of  $|a|, b, c$ )



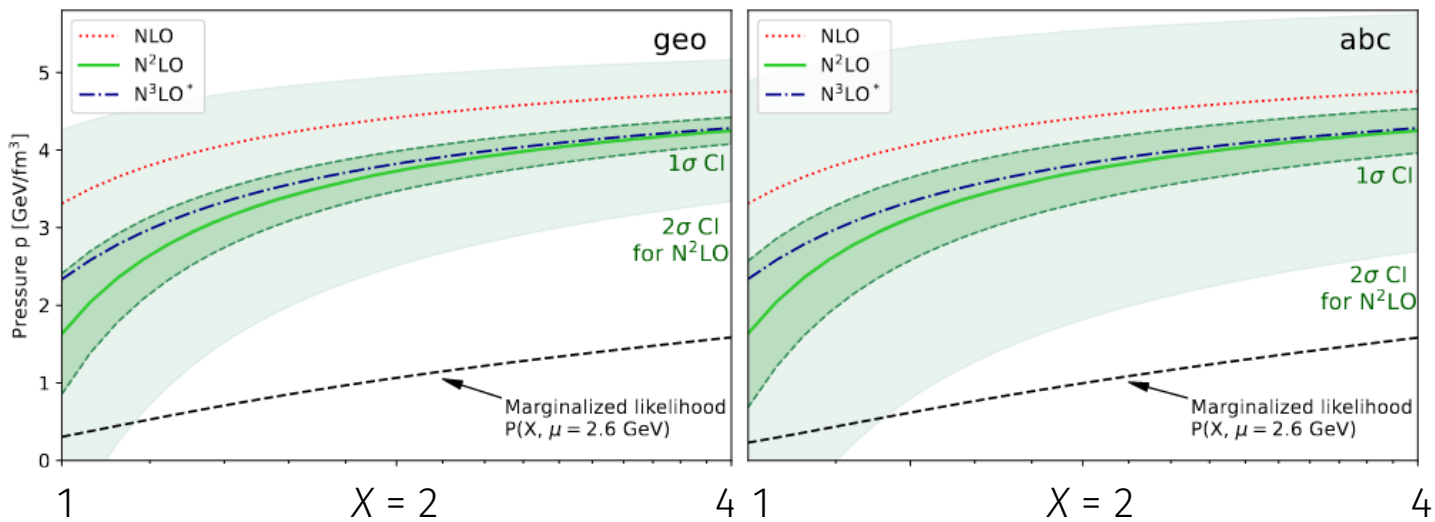
\*geo model defined by 2 parameters, with  $c = 0$

# Bayesian inference of the next term in the pressure

1.  $P(\vec{\delta}_k|abc) = \prod_{n=0}^k P(\delta_n|abc)$  (What is the likelihood of the current terms?)
2.  $P(\vec{\delta}_k) \equiv \int dadbdc P(\vec{\delta}_k|abc) P_0(abc)$  (marginal likelihood / evidence)
3.  $P(abc|\vec{\delta}_k) = \frac{P(\vec{\delta}_k|abc) P_0(abc)}{P(\vec{\delta}_k)}$  (Bayes theorem; posterior of model parameters)
4.  $P(\delta_{k+1}|\vec{\delta}_k) \equiv \int dadbdc P(\delta_{k+1}|abc) P(abc|\vec{\delta}_k) = \frac{P(\vec{\delta}_{k+1})}{P(\vec{\delta}_k)}$  (Prediction for next term)



# Bayesian inference of the next term in the pressure



Posteriors for  $p$  for different perturbative orders, as function of  $X$ , with  $\mu_B = 2.6 \text{ GeV}$

# Incorporating the scale dependence

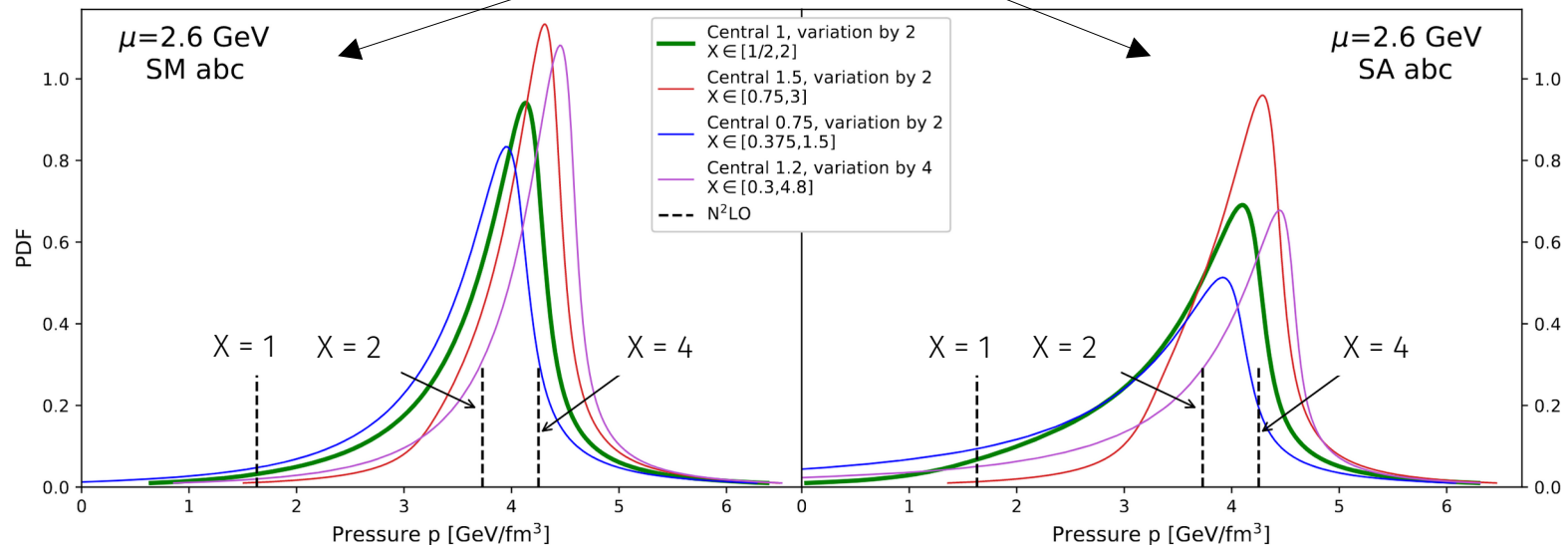
## 1. Scale Averaging :

## 2. Scale Marginalization :

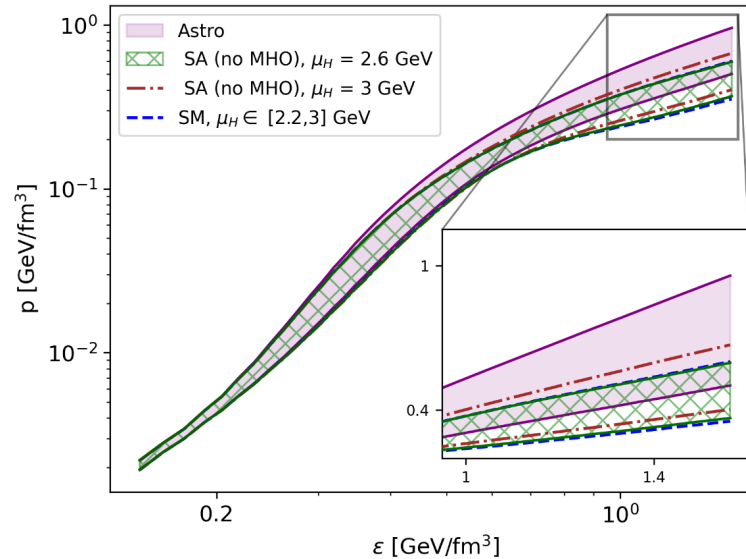
Treat all  $\log(X)$  in some interval as equally likely

Weight different  $X$  by likelihood of  $X$

We used this in Gorda, Komoltsev, Kurkela 2204.11877  
(accepted to Apj)



In the end, the effect on the NS EoS inference is small



**Takeaway:** Impact of pQCD input insensitive to the exact treatment of the  $X$  dependence (and even the matching  $\mu_{\text{pQCD}}$ )



# Outlook and future directions

# Ongoing work to improve pQCD to full N3LO

TG, Paatelainen, Säppi, Seppänen (Ongoing work)

Organized the N3LO into kinematic sectors:

**Soft:** 2 interacting gluons screened at LO

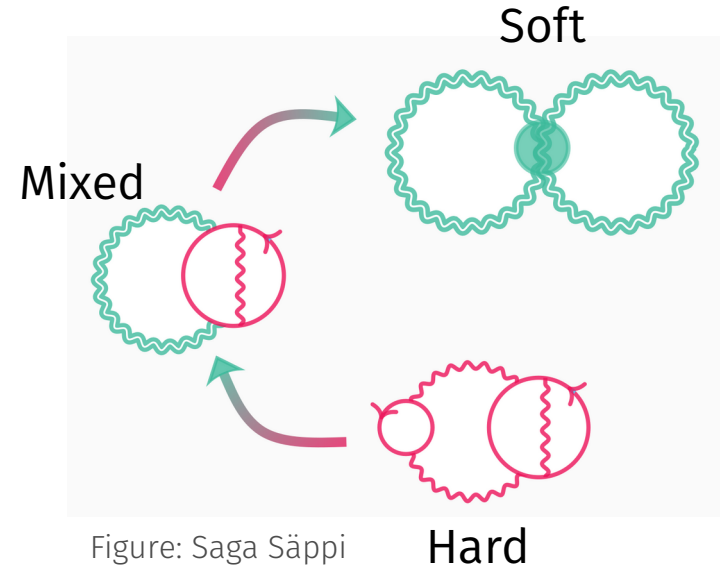
**Mixed:** 1 gluon screened at NLO

**Hard:** gluons are unscreened

Soft computed in 2021, and now the gluon screening at NLO was computed last month

TG, Kurkela, Paatelainen, Säppi, Vuorinen, PRL 127, 162003 (2021)

TG, Paatelainen, Säppi, Seppänen, 2304.09187.



Mixed contribution is on overleaf :)

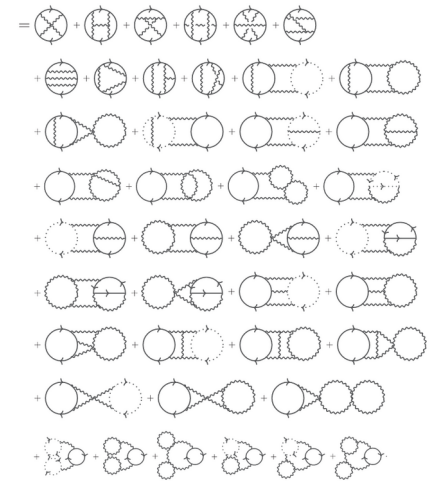
# Ongoing work to improve pQCD to full N3LO

TG, Paatelainen, Säppi, Seppänen (Ongoing work)

$$\frac{p}{p_{\text{free}}} = 1 - 2 \left( \frac{\alpha_s}{\pi} \right) - \left( \frac{\alpha_s}{\pi} \right)^2 \left[ 3 \ln \left( 6 \frac{\alpha_s}{\pi} \right) + 9 \ln \frac{\bar{\Lambda}}{2\mu} + 12.9268 \right] + \left( \frac{\alpha_s}{\pi} \right)^3 \left[ c_{3,2} \ln^2 \left( 6 \frac{\alpha_s}{\pi} \right) + c_{3,1}(\bar{\Lambda}) \ln \left( 6 \frac{\alpha_s}{\pi} \right) + c_{3,0}(\bar{\Lambda}) \right] + O(\alpha_s^4),$$

TG, Kurkela, Romatschke, Säppi, Vuorinen Phys. Rev. Lett. 121 (2018)

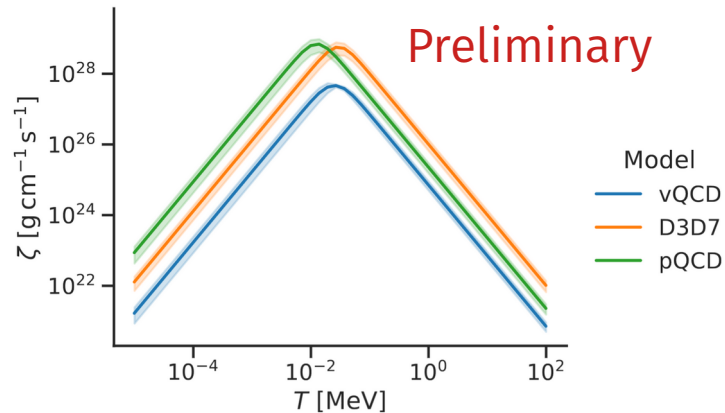
Overleaf :) Looks surprisingly big



Clearly requires sophisticated techniques

# Ongoing/Future directions

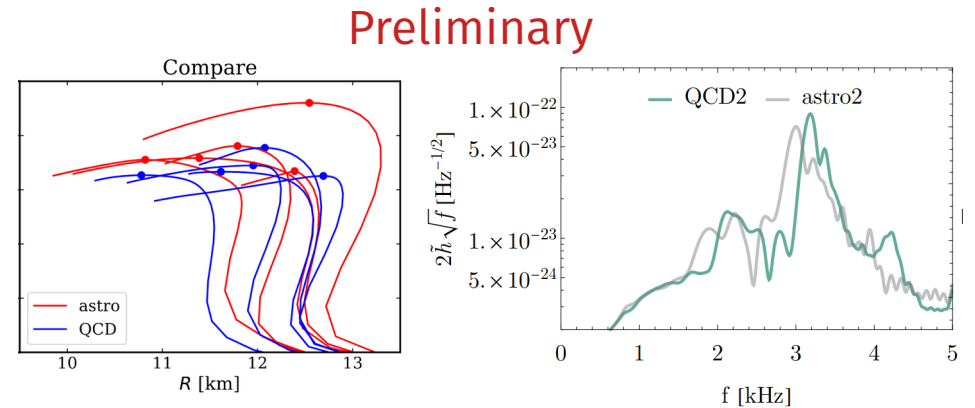
## Transport Properties



Bulk viscosity in unpaired quark matter

Ongoing work with Cruz Rojas, TG Hoyos, Jokela, Järvinen, Kurkela, Paatelainen, Säppi, Vuorinen

## Impact of the pQCD input on postmerger dynamics

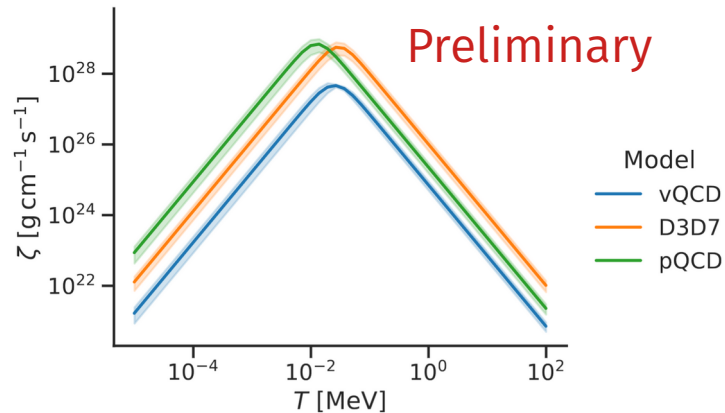


Representative “astro” and “astro+QCD” EOSs, effects on waveforms etc.

Ongoing work with Ecker, Kurkela, Rezzolla

# Ongoing/Future directions

## Transport Properties



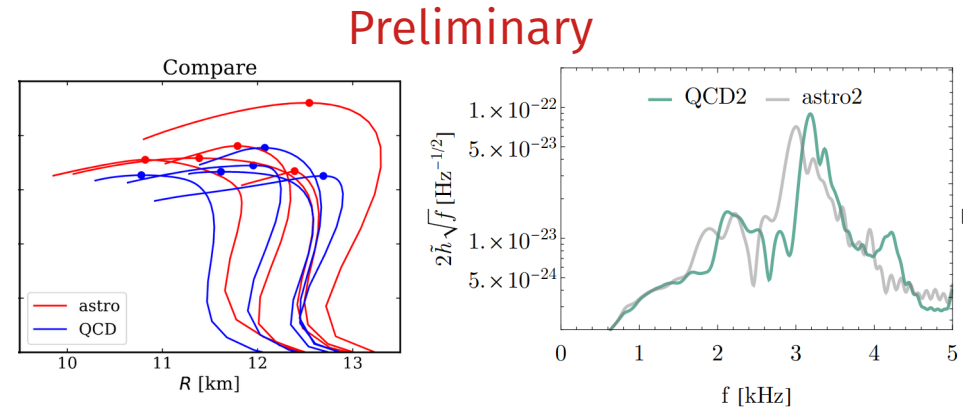
Bulk viscosity in unpaired quark matter

Ongoing work with Cruz Rojas, TG Hoyos, Jokela, Järvinen, Kurkela, Paatelainen, Säppi, Vuorinen

## Also looking into pairing effects in pQCD

Ongoing work with Braun, Geissel

Impact of the pQCD input on postmerger dynamics



Representative “astro” and “astro+QCD” EOSs, effects on waveforms etc.

Ongoing work with Ecker, Kurkela, Rezzolla



# Summary...

- Bayesian EOS analysis suggests conformal matter in the cores of massive NSs is likely, number of DOF points to quark matter.  
Other observables / arguments to support (or contradict!) this evidence? (Transport properties...?)
- Have completed a **statistical analysis** of the truncation errors of the pQCD input—**find that they do not strongly affect the NS-EOS analysis**

# And outlook...

- Ongoing work to improve pQCD, pushing to full N3LO pressure
- Working on finite- $T$ , dynamic regime, for applications to merger simulations, and the post-merger phase, and thinking about pairing effects in pQCD

# Summary...

- Bayesian EOS analysis suggests conformal matter in the cores of massive NSs is likely, number of DOF points to quark matter.  
Other observables / arguments to support (or contradict!) this evidence? (Transport properties...?)
- Have completed a **statistical analysis** of the truncation errors of the pQCD input—find that they do not strongly affect the NS-EOS analysis

# And outlook...

- Ongoing work to improve pQCD, pushing to full N3LO pressure
- Working on finite- $T$ , dynamic regime, for applications to merger simulations, and the post-merger phase, and thinking about pairing effects in pQCD

*Thanks for your attention!*