

Junior Challenge '23

Year 8 or below

Illustrations by Theo Chaddock & Peter Ackerley

Rules

- 1) The challenge should be attempted in your own time. Your entry must be **your own work**, though you should ask for the meanings of unfamiliar words.
- 2) Marks are awarded for reasoning, not only for getting the right answer. It is also possible to win a prize or certificate even if you have not completed all the questions, so hand in your entry even if it is not quite finished.
- 3) Present your worked solutions **separately on A4 size papers**. Lined papers are recommended, but blank or graph paper are accepted – as long as they are neatly presented. Entries written on this question sheet or without any workings **will not be marked**.
- 4) Write **your name and school on every page neatly**.
- 5) Please **staple your pages** together at the top-left corner if submitting your entry by mail.

Deadline

Either you or your teacher needs to submit your entry by **17th March 2023**, by post or online. For submission instructions, see the end of this document or visit the Maths Challenge webpage at www.southampton.ac.uk/stag/mathschallenge.page

A Prize-Giving Evening will be held at the University of Southampton on 7th June 2023.

We hope that you enjoy the Challenge.

If you have any questions, please ask your teacher or parents to contact us at math4all@soton.ac.uk.

1. A Pyramid Paint Scheme

Imhotep the architect wants to paint the Great Pyramid. He wants one red side, one blue side, one green side and one yellow side.

How many different ways can he paint it?

His assistant is tasked with making models to show the possibilities. He realises he can make far fewer models as they can be turned around. How many different models does he need to make?

Show how he would need to paint each of the models.

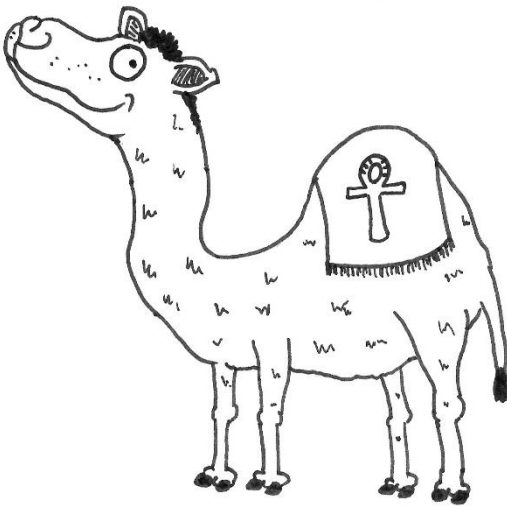


2. Don't Get the Hump

The old pharaoh dies and leaves his herd of 19 camels to his three daughters.

Hatshepsut, the eldest, is to receive $\frac{1}{2}$ of the camels; Isis, the middle daughter, is to receive $\frac{1}{4}$; and Nefertiti, the youngest, is to receive $\frac{1}{5}$. Their uncle offers to lend them a camel to make sure they can each receive a whole number of camels.

How many camels does each sister receive?



3. Four Flowers Fit

The pharaoh's gardener needs to plant 16 flowers into a four-by-four grid.

The Pharaoh wants there to be no repeated flowers in a given row or column.

The gardener has made a start.

How could the garden be finished, using only lotuses, roses, poppies, and jasmine flowers? Show all the possible ways you can find.

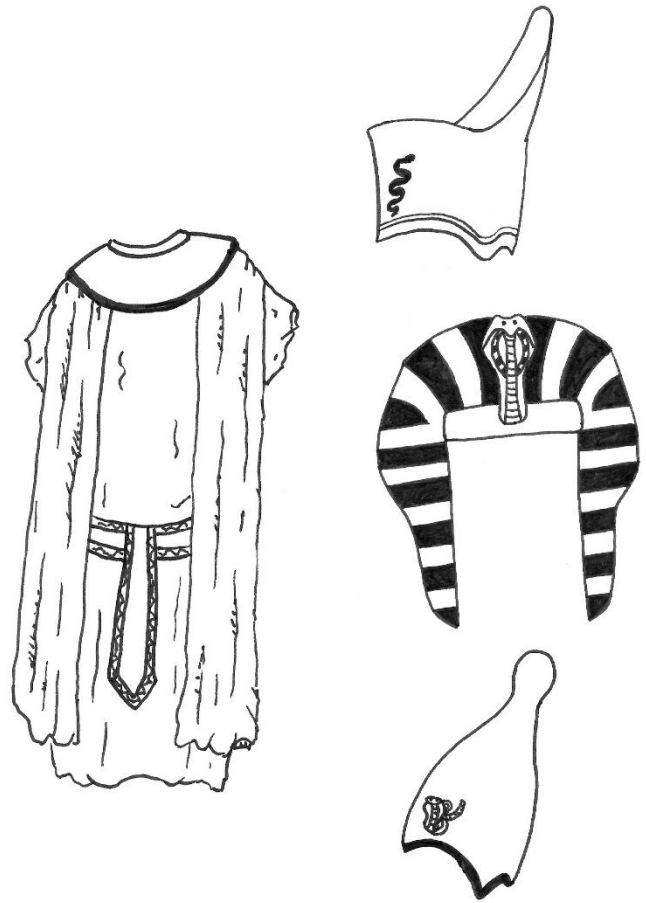
L	R	P	J
R			
P			
J			

L = Lotus
 R = Rose
 P = Poppy
 J = Jasmine

4. The Son has got his Crown On

The pharaoh's outfitters, 'Cloaks and Crowns', sell Cloaks and Crowns in 3 sizes – prince, king and emperor (in increasing size order).

He wants to kit out his three sons for the forthcoming festival of Ra. Ahmose chooses a bigger cloak than Kamose, but a smaller crown than Thutmose. Both Thutmose's cloak and crown are larger than Kamose's, but the size of Thutmose's crown matches the size of Kamose's cloak. Which sizes did the pharaoh's sons each get?



5. The 13 Steps

The priestess at Horus' temple has to go up a flight of 5 steps each day. The high priest has decreed that she will ascend the steps one or two at a time. She wants to know if she can do this a different way each day for a week.

Is it possible?

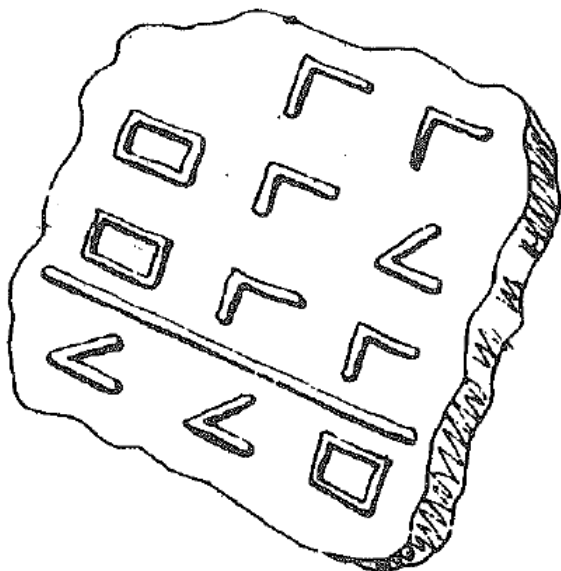
How many ways are there of ascending the steps?

The grand staircase has 13 steps. The priest wants to know if he can climb this a different way each day for a year using the same rules as the priestess.

Is it possible?

How many ways are there of ascending the grand staircase?

Explain why your answer is correct.



6. Rock Hard Sums

An ancient Egyptian stone tablet is found, as shown to the left. It is correctly deduced that it is a sum, with each of the symbols representing a different digit. Surprisingly, there are 2 ways it could be done.

What are they?

7. Plagues Palaver

It all began when the river's water turned to blood. Later, it rained hail & fire.

The flies were immediately after the lice.

3 days of darkness was the 3rd plague after the terrible boils, and the only insect plague between them was the locusts.

The frogs were 3 plagues before the livestock dying.

Hail & fire was before the locusts and the darkness, but after the boils and the livestock dying.

The death of the firstborn was the final plague.

Place the 10 plagues in the correct order. The marks for this question are all for the reasoning – no mark for just the list!

The list of plagues is given here in alphabetical order.



The Challenge is organised by the School of Mathematical Sciences outreach team, University of Southampton.

www.soton.ac.uk/maths/outreach/index.page.

We gratefully acknowledge MEM (Mathematical Education on Merseyside) for providing these questions and the concept of the Challenge.

www.mathsmerseyside.org.uk/

Instructions

Please note each school's submission should **include a marksheet**, which can be found at the Maths Challenge webpage: www.southampton.ac.uk/stag/mathschallenge.page

Instructions for students:

- 1) Follow the rules provided on the front of the question sheets.
- 2) Once you have finished the Challenge, return your entry to a designated person at your school. Entries should be submitted by the designated person whenever possible.
- 3) If for some reason you are unable to return your entry to a designated person, ask your parent or guardian to submit your entry and a completed Excel marksheet via email to math4all@soton.ac.uk, with your name in the email subject: "**Maths Challenge 23 submissions: *Student Name***". Please do not send high-resolution photos; instead follow the scanning instructions on the next page.

Instructions for schools/teachers:

- 1) Once you have received your students' entries, please **fill in the marksheets** (one each for the Junior and Senior Challenge separately).
- 2) Send the marksheets as Excel files to math4all@soton.ac.uk, with your school name in the email subject: "**Maths Challenge 23 submissions: *School Name***".
- 3) For the actual entries, you can submit them by post or online:
 - a. By post (preferred): Physical entries should be mailed to the address below.

Junior/Senior Challenge '23 Entries
Dr Sunny Yu
School of Mathematical Sciences
University of Southampton
Southampton, SO17 1BJ

- b. Online: In your email containing your marksheet, let us know that you would like to submit your students' entries electronically. We will provide a link for you to upload your files. Entries should be in the form of PDF scans.

Instructions for scanning your entry using a smart phone

If you are submitting an entry electronically, it should be in PDF format. Make sure you include all the pages and that all your writing (including your name and the name of your school) is clearly visible. If you do not have access to a flatbed scanner, then follow the instructions below.

Scanning PDF files with an Android phone

We suggest you use one of three scanner apps: (a) Google Drive, (b) Adobe Scan, (c) Microsoft Lens. These are all free to use and can be downloaded from the Google Play store. We provide detailed instructions for using Google Drive.

Google Drive: You probably already have this on your phone. If not, it is downloadable from play.google.com/store/apps/details?id=com.google.android.apps.docs. Note that this is the full Google Drive app, not the standalone Sheets, Docs, or Slides apps.

- In the Google Drive app, click on the plus sign in the bottom-right corner and then click on “Scan”.
- Tap the large white button to scan a page. After taking the scan, click “OK” if you are happy with the scan or “Retry” if not. After clicking OK, click the plus sign on the bottom left **to add another page to the same PDF**. Please do this rather than submitting a separate PDF file for each page! When you have scanned all your pages, click “Save” to save the PDF file.

Scanning pdf files with an iPhone or iPad

For an iPhone, three options are (a) Notes, (b) Adobe Scan, and (c) Microsoft Lens. The first is built into your iPhone (as long as it is not too old) while the other two are free and can be downloaded from the App Store. We provide detailed instructions for using the Notes app.

Notes: This is the simplest in that Notes is already installed on your iPhone/iPad.

- Open Notes on your iPhone or iPad.
- Create a new note or tap on an existing one to add a document to it.
- Tap the camera button at the bottom of the screen or above the keyboard.
- Tap Scan Documents.
- Line up the document you want to scan.
- Tap the shutter button if the scanner doesn't automatically scan the document. Repeat this step for every page you want to scan.
- Tap Save **after** you've scanned all the pages of your entry. The button will have a count of how many pages you scanned.

Challenge '23 Solutions

1. A Pyramid Paint Scheme

There **are 24 ways** to paint the pyramid, $4 \times 3 \times 2 \times 1 = 24$.

6 models needed.

RGYB
 RYGB
 RBGY
 RBYG
 RBYG
 RYBG

2. Don't Get the Hump

The addition of the extra camel gives 20 camels

$\frac{1}{2}$ of 20 is **10 camels for Hatshepsut**

$\frac{1}{4}$ of 20 is **5 camels for Isis**

$\frac{1}{5}$ of 20 is **4 camels for Nefertiti**

Leaving one camel to return to their uncle.

3. Four Flowers Fit

L	R	P	J		L	R	P	J
R	L	J	P		R	L	J	P
P	J	L	R		P	J	R	L
J	P	R	L		J	P	L	R
L	R	P	J		L	R	P	J
R	P	J	L		R	J	L	P
P	J	L	R		P	L	J	R
J	L	R	P		J	P	R	L

Challenge '23 Solutions

4. The Son has got his Crown On

Cloaks: A and $T > K$
so K = prince with A and T king or emperor
OR K = king and A and T both emperor.

Crowns: $T > A$ and K ,
so T = king and A and K both prince
OR T = emperor and A and K are prince or king

T crown = K cloak \Rightarrow both are king
So

Son	Crown	Cloak
Ahmose	Prince	Emperor
Kamose	Prince	King
Tuthmose	King	Emperor

5. The 13 Steps

Yes, it is possible on the five steps.
There are 8 combinations.

Yes, it is possible on the 13 steps.
There are 377 possible combinations.

Fibonacci numbers
1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377

For $n+1$ steps, add up the number of ways for n and $n-1$ steps.

This is because you require a 1-step leap from the n th step or a 2-step leaps from the $(n-1)$ th step, so the number of ways is the sum of the number of ways of getting to the n th and $(n-1)$ th step.

6. Rock Hard Sums

Writing the sum as $aa+bac+baa=ccb$,
From the 100s column, b is less than 5 otherwise we would need 4 columns, so $b = 1, 2, 3, \text{ or } 4$
If $b = 1$, $c = 2, 3$ or 4 from the 100s column and only $c = 3$ works since it would need to be odd for the units column.
We can then see that $a = 4$ to make the sum work.

If $b=2$, $c=4, 5$ or 6 . C must now be even for the units column, so 4 or 6 . If $C=4$, then a needs to be 4 or 9 for the units column. Since they are distinct this eliminates 4 , and 9 doesn't work in the 10s column.

This leaves $C = 6$, so we now find A has to be 8 to make the sum work.

$B=3$ or 4 don't work.
 $44+143+144=331$ or $88+286+288=662$

Challenge '23 Solutions

7. Plagues Palaver

- Water to blood is first, death of 1st born is 10th.
- Frogs is 3 before livestock dying, Boils is 3 before darkness.
- Lice is immediately before flies.
- Only insects between boils and darkness are locusts, so lice and flies mustn't be between them.
- Hail and fire was before locusts and darkness, but after boils and livestock.
- This means the two between boils and darkness must be hail and fire and locusts in that order.
- This leaves lice and flies to slot between frogs and livestock which must then go before boils...darkness.

That gives the full list.

Water to blood
Frogs
Lice
Flies
Livestock dying
Boils
Hail and Fire
Locusts
3 days of darkness
Death of the 1st born