# Cosmological Implications of Quantum Gravity

**Based on Publications:**

*Gravitational Fluctuations as an Alternative to Inflation* (arXiv:1807.10704)
*Gravitational Fluctuations as an Alternative to Inflation II. CMB Power Spectrum* (arXiv:1910.02990)
*Gravitational Fluctuations as an Alternative to Inflation III. Numerical Results* (arXiv:2005.09720)
*Dyson's Equations for Quantum Gravity in the Hartree-Fock Approximation* (arXiv:2010.10500)

University of Southampton
Lu Heng Sunny, YU
October 14, 2021

---

University of Southampton

Cosmological Implications of Quantum Gravity

**Based on Publications**

*Gravitational Fluctuations as an Alternative to Inflation* (arXiv:1807.10704)
*Gravitational Fluctuations as an Alternative to Inflation II. CMB Power Spectrum* (arXiv:1910.02990)
*Gravitational Fluctuations as an Alternative to Inflation III. Numerical Results* (arXiv:2005.09720)
*Dyson's Equations for Quantum Gravity in the Hartree-Fock Approximation* (arXiv:2010.10500)

Lu Heng Sunny, Yu
w/ Herbert W. Hamber
University of California, Irvine

---



## Cosmological Implications of Quantum Gravity

Q How can one connect cosmological phenomenology to microscopic aspects in QFT/ Particle Physics?

The Mystery in Cosmology
- Galaxy distributions
- Matter correlations
- CMB power spectra

The Solution from Quantum Physics
- Correlation Functions
- Running of couplings
- Vacuum Condensate

Our Results

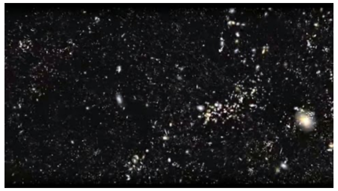


---



## The Mystery – Distributions of Galaxies







---



## Galaxies are not random



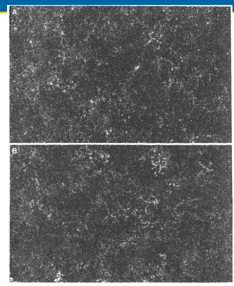


Figure 19.1. A model for the galaxy distribution. Panel A is a section from the Lick map in figure 3.9. Panel B is a map at the same depth and scale from a numerical model designed to reproduce the observed correlation functions through the four-point function (Soneira and Peebles 1978).

Forms *"clumps"* ①

of *different sizes* ②

*Q: How to measure/quantify them?*

A: Correlation Functions $\xi(r)$
Power Spectrums



---



## How to measure/quantify them?



"Density Contrast" $\delta(x)$

$$\delta(x) \coloneqq \frac{\delta\rho(x)}{\bar\rho} \equiv \frac{\rho(x)-\bar\rho}{\bar\rho}$$

("overdensity"(x) / "average density")

(i.e. *"fractional overdensity"*)

Correlation Functions $\xi(r)$

$$\xi(r) \stackrel{\text{def}}{=} \int_V d^3x\ \delta(x)\,\delta(x+r) \equiv \langle\, \delta(\boldsymbol{x})\delta(\boldsymbol{y})\, \rangle$$

(with $|x-y| = r$)



---



## Early measurements (1960-90's, Peebles, *et. al.*~)



TOPICS IN MODERN COSMOLOGY

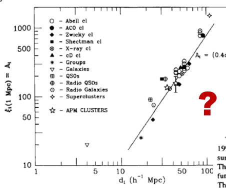



Figure 19.3. Scaling of the clustering amplitude with the density (Bahcall and West 1992). The vertical axis is the amplitude of t… correlation function scaled to $r = 1h^{-1}$ Mpc. The horizontal axi… distance $n_i^{-1/3}$ between clusters. The straight line is the scali… equation (19.3…

*Galaxy Correlation Functions*

The purpose of this subsection is to list some numerical results for the low-order galaxy correlation functions. Estimates of the galaxy two-point spatial correlation function $\xi(r)$ are discussed in section 7; they show that $\xi$ is usefully approximated as

$$\boxed{\xi(r) = \left(\frac{r_o}{r}\right)^{\gamma}},\quad r_0 = 5.4 \pm 1\, h^{-1}\,\mathrm{Mpc},\quad \gamma = 1.77 \pm 0.04, \qquad (19.18)$$

at the range of separations

$$10\,\mathrm{kpc} \lesssim hr \lesssim 10\,\mathrm{Mpc}. \qquad (19.19)$$

This power law model is a remarkably good approximation down to separations where the visible parts of the galaxies start to overlap (Gott and Turner 1979).

The surveys (Bahcall 1988; Bahcall and West 1992; and Luo and Schramm 1992) indicate that this *ansatz* is remarkably successful. Figure 19.3 shows measures of the correlation functions for clusters of different mean number densities. The vertical axis is the clustering amplitude, defined as the two-point correlation function fitted to $\xi \propto r^{-1.8}$, and scaled to the fiducial separation $r = 1h^{-1}$ Mpc. The horizontal axis is the mean distance between objects in the sample. The solid line is the scaling relation

$$\xi_i(r) = \left(\frac{0.4}{n_i^{1/3} r}\right)^{1.8} \qquad (19.39)$$

…ely has been introduced by the considerable differences in pro… …different classes of clusters have been identified, but the results …ell fit this universal dimensionless correlation function. At the …mber density in equation (20.33), $n_{cl} \sim 1 \times 10^{-5} h^3$ Mpc$^{-3}$, the length at which $\xi_i$ in equation (19.39) extrapolates to unity is the cluster-cluster clustering length $r_{cc} \sim 18h^{-1}$ Mpc.

…0 Mpc, (Groth …. More …ion are

Looks like a scaling law of QFT/ Statistical Mechanics!



---

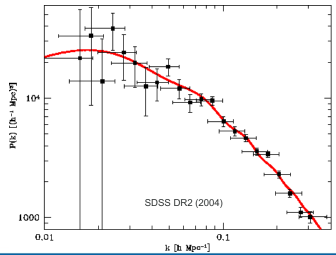


## Modern Data (2000-present, SDSS)



$$\xi(r) \equiv G_\rho(r) \equiv \langle\, \delta(x)\delta(y)\,\rangle = \left(\frac{r_0}{r}\right)^{\gamma} \quad \xrightarrow{\text{FT}} \quad P(k) \equiv \langle\, |\delta(k)|^2 \rangle = \frac{a_0}{k^s}$$

$$\Rightarrow\ s = 3 - \gamma$$





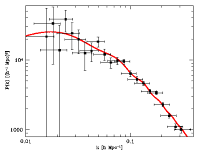

---



## The Mystery



$$G_\rho(r) = \left(\frac{r_0}{r}\right)^{\gamma}, \quad P(k) = \frac{a_0}{k^s}$$



Mystery (#1):

Why do galaxies follow this law?

Why is $\gamma \approx 2$ ?

(or equivalently, $s \approx 1$)

A long mystery since 1960's

Inflation?

Other possible explanation?

## A Possible Solution:



Theoretical observation 1:

- GR relates Gravity to Matter

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G\, T_{\mu\nu}$$
$$R = 8\pi G\, \rho$$

Theoretical observation 2:

- QFT is excellent at calculating correlation functions!!

Q:

- Can we apply *QFT* to *Gravity*?
- Are such predictions *reliable*?
- How well do they match with *data*?

before applying the functional derivative:

$$\frac{\delta}{\delta J(x)} \int d^4y\, \partial_\mu J(y) V^\mu(y) = -\partial_\mu V^\mu(x). \qquad (9.33)$$

...malism is the *generating functional* of corre-
...thors call it $W[J]$.) In a scalar field theory, $Z[J]$ is defined as

$$Z[J] \equiv \int \mathcal{D}\phi \exp\Big[ i \int d^4x \big[\mathcal{L} + J(x)\phi(x)\big]\Big]. \qquad (9.34)$$

This is a functional integral over $\phi$ in which we have added to $\mathcal{L}$ in the exponent a *source term*, $J(x)\phi(x)$.

Correlation functions of the Klein-Gordon field theory can be simply computed by taking functional derivatives of the generating functional. For example, the two-point function is

$$\langle 0| T\phi(x_1)\phi(x_2) |0\rangle = \frac{1}{Z_0}\left(-i\frac{\delta}{\delta J(x_1)}\right)\left(-i\frac{\delta}{\delta J(x_2)}\right) Z[J]\Big|_{J=0}, \qquad (9.35)$$

where $Z_0 = Z[J=0]$. Each functional derivative brings down a factor of $\phi$ in the numerator of $Z[J]$; setting $J=0$, we recover expression (9.18). To compute higher correlation functions we simply take more functional derivatives.

Formula (9.35) is useful because, in a free field theory, $Z[J]$ can be rewritten in a very explicit form. Consider the exponent of (9.34) in the free Klein-Gordon theory. Integrating by parts, we obtain

$$\int d^4x \big[\mathcal{L}_0(\phi) + J\phi\big] = \int d^4x \big[\tfrac{1}{2}\phi(-\partial^2 - m^2 + i\epsilon)\phi + J\phi\big]. \qquad (9.36)$$

*"Introduction to Quantum Field Theory", Peskin and Schroeder*



## Quantum Gravity



### What is "Quantum Gravity"?

*Follow the path of QED/QCD!*

**QG** = direct combination of:

- Einstein's classical General Relativity (1916)

$$S\,[g_{\mu\nu}] = \frac{1}{16\pi G} \int d^4x \sqrt{g}\,(R - 2\lambda)$$

- Quantum Mechanics, in the form of Feynman's Path Integral:

$$Z = \int [d g_{\mu\nu}]\, e^{\frac{i}{\hbar} S[g_{\mu\nu}]}$$

*$S[g_{\mu\nu}]$ ≡ Einstein-Hilbert action w/ cos. constant*
*$[dg_{\mu\nu}]$ ≡ DeWitt functional measure*



## Quantum Gravity



$$S\,[g_{\mu\nu}] = \frac{1}{16\pi G} \int d^4x \sqrt{g}\,(R - 2\lambda) \qquad Z = \int [d g_{\mu\nu}]\, e^{\frac{i}{\hbar} S[g_{\mu\nu}]}$$

$$Z_{cont} = \int [d\, g_{\mu\nu}] \exp\Big\{ -\lambda_0 \int dx \sqrt{g} + \frac{1}{16\pi G} \int dx \sqrt{g} R \Big\}$$

### Main Challenges with Quantum Gravity:

(1) Short distance theory is ultimately uncertain.

(2) Theory not perturbatively renormalizable.



## Quantum Gravity



Main Challenges with Quantum Gravity:

(1) Short distance theory is ultimately uncertain.
(2) (Unlike QED) Theory not perturbatively renormalizable.

### Uniqueness

$$Z_{cont} = \int [d\, g_{\mu\nu}] \exp\Big\{ -\lambda_0 \int dx \sqrt{g} + \frac{1}{16\pi G} \int dx \sqrt{g} R \Big\}$$

*+ all higher order terms (consistent w/ general covariance)*

- *Higher order operators* ($R^2$) ⇒ Short distance physics ($< 10^{-33}$cm),
- Long distance theory is *unique**! (like *QED*!)
  - Feynman(1963) ⇒ *QG* = unique theory of m=0, s=2 field.
    …just like QED / QCD is for m=0 and s=1!



## Quantum Gravity



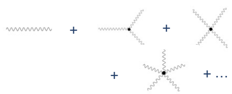
Main Challenges with Quantum Gravity:

(2) (Unlike QED) Theory not perturbatively renormalizable.

### Difficulty with Perturbative Gravity:

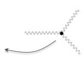
$$Z_{cont} = \int [d\, g_{\mu\nu}] \exp\Big\{ -\lambda_0 \int dx \sqrt{g} + \frac{1}{16\pi G} \int dx \sqrt{g} R \Big\}$$

- *Theory is Highly Non-Linear*
  - *"Gravity gravitates"*
  - *(∞ vertices!)*



- *Perturbation Theory in G is useless*
  - *"Not perturbatively renormalizable" !*
  - *$k$-integrals diverge!*

$$V(k) \sim h \cdot \partial h \cdot \partial h \sim k^2$$





## Non-perturbatively renormalizable theories



Q: Are there other QFTs where perturbation theory fails, yet makes useful/physical predictions?

- QCD
  Quarks and gluons are confined. *Main theoretical evidence for confinement and chiral symmetry breaking is possibly from the lattice.*
  $$V(r) \underset{r\to\infty}{\sim} \kappa r \qquad \kappa \simeq e^{-\frac{1}{2\beta_0 g^2}}$$
- Superconductor
  BCS Theory: *Fermions close to the Fermi surface condense into Cooper pairs.*
  $$\Delta = 2\hbar\omega_D\, e^{-\frac{1}{N(0)g}}$$
- Superfluid
  *Described by condensate density.*
  $$E_k = \hbar \sqrt{n_0(T)\, V_{\mathbf{k}=0}/m}\; k$$
- Degenerate Electron Gas
  Screening due to *Thomas-Fermi mechanism.*
  $$V(r) \sim \frac{e^2}{r} e^{-q_{TF} r} \qquad q_{TF} = e\sqrt{m\Lambda}$$
- Homogeneous Turbulence
  Observables $\sim (R - R_c)^\gamma$ $R_c$ *= critical Reynolds no. (Kolmogorov)*
- Ferromagnets
  Spontaneous Symmetry Breaking & dimensional transmutation
  $$\langle \frac{1}{V}\sum_i S_i \rangle \underset{T\to T_c}{\sim} (T_c - T)^\beta$$



## Non-perturbatively renormalizable theories



### Do these theories make accurate predictions?
### Yes!

*E.g.: *The Nonlinear Sigma-model* (Gell-Mann):

$$S(\phi) = \frac{1}{2} \int d^dx\, \partial_\mu \phi(x) \cdot \partial_\mu \phi(x)$$

$$Z[J] = \int [d\phi] \prod_x \delta[\phi(x)\cdot\phi(x) - 1] \exp\left( -\frac{\Lambda^{d-2}}{g^2} S(\phi) + \int d^dx\, J(x)\cdot\phi(x) \right)$$

- ⇒ a not perturbatively renormalizable theory in 3D,
- ⇒ *describes ferromagnets in 3D,*
- ⇒ *second most accurate* prediction (on critical exponents $\alpha, \beta, \ldots$) of QFT (after g-2 measurements of QED)!

*⇒ Nonperturbative methods*



## Non-perturbatively renormalizable theories



### Nonperturbative methods:

- Analytical methods:
  - d=2+ε dimensional expansion (Wilson/Weinberg),
  - Modern RG analysis (Wilson),
  - Large-N expansion ("Hartree-Fock" approximation),
  - Einstein-Hilbert RG-truncation method…
- Numerical methods:
  - Lattice calculations (M-C simulations)

*All these work very well with the NLSM!*



## Non-perturbatively renormalizable theories



**Nonperturbative methods:**

- Analytical methods: 2+ε expansion, Modern RG analysis, Large-N expansion ("Hartree-Fock" approximation), Einstein-Hilbert RG-truncation method…
- Numerical methods: Lattice calculations (M-C simulations)

### General Features from Non-pert. Renormalizable Theories:

- Non trivial RG fixed point, Phase Transitions!
- Nonperturbative scale $\xi$
- Nontrivial Critical Scaling exponents for correlation functions $\nu$
- RG Running of coupling constants $g \to g(k^2)$
- and so on…

*All play a role in gravity!*

## Non-perturbatively renormalizable theories



**Nonperturbative methods:**

- Analytical methods: 2+ε expansion, Modern RG analysis, Large-N expansion ("Hartree-Fock" approximation), Einstein-Hilbert RG-truncation method…
- Numerical methods: Lattice calculations (M-C simulations)

**General Features** from Non-pert. Renormalizable Theories:

- Non trivial RG fixed point, Phase Transitions!
- Nonperturbative scale $\xi$
- $\boxed{\text{Nontrivial Critical Scaling exponents for correlation functions } \nu}$
- RG Running of coupling constants $g \to g(k^2)$
- and so on…

*All play a role in gravity!*



## Non-perturbatively renormalizable theories


I. Introduction
II. Quantum Gravity
  I. Defining the Theory
  II. Main Challenges
  III. Nonperturbative Methods
  IV. **Nonperturbative effects**
III. Our Results
IV. Conclusion


General Features from Non-pert. Renormalizable Theories:

- Nonperturbative scale $\xi$
- $\boxed{\text{Nontrivial Critical Scaling exponents for correlation functions } \nu}$
- RG Running of coupling constants $g \to g(k^2)$

### Critical Exponents for Gravity

- *def:* Correlation function for scalar curvature ($R$):

$$\boxed{G_R(r) \equiv \langle\, \delta R(x)\delta R(y)\,\rangle \sim \frac{1}{r^{\,2(d-1/\nu\,)}}}$$

$|x-y| = r$ = geodesic distance
$\delta R$ = fluctuations in $R$

*In general one does not expect Gaussian exponents*



## Non-perturbatively renormalizable theories


I. Introduction
II. Quantum Gravity
  I. Defining the Theory
  II. Main Challenges
  III. Nonperturbative Methods
  IV. **Nonperturbative effects**
III. Our Results
IV. Conclusion


General Features from Non-pert. Renormalizable Theories:

- Nonperturbative scale $\xi$
- $\boxed{\text{Nontrivial Critical Scaling exponents for correlation functions } \nu}$
- RG Running of coupling constants $g \to g(k^2)$

### Critical Exponents for Gravity

$$\boxed{G_R(r) \equiv \langle\, \delta R(x)\delta R(y)\,\rangle \sim \frac{1}{r^{\,2(d-1/\nu\,)}}}$$

- Various methods:

| Method used to compute $\nu$ in d=4 | Universal Exponent $\nu$ |
|---|---|
| Euclidean Lattice Quantum Gravity | $\nu^{-1} = 2.997(9)$ |
| Perturbative $2+\epsilon$ expansion to one loop [22] | $\nu^{-1} = 2$ |
| Perturbative $2+\epsilon$ expansion to two loops [23] | $\nu^{-1} = 22/5 = 4.40$ |
| Einstein-Hilbert RG truncation [56] | $\nu^{-1} \approx 2.80$ |
| Recent improved Einstein-Hilbert RG truncation [57] | $\nu^{-1} \approx 3.0$ |
| Geometric argument [33] $\rho_{vac\ pol}(r) \sim r^{d-1}$ | $\nu^{-1} = d-1 = 3$ |
| Lowest order strong coupling (large G) expansion [29] | $\nu^{-1} = 2$ |
| Nonlocal field equations with $G(\Box)$ for the static metric [46] | $\nu^{-1} = d-1$ for $d \geq 4$ |



## Non-perturbatively renormalizable theories


I. Introduction
II. Quantum Gravity
  I. Defining the Theory
  II. Main Challenges
  III. Nonperturbative Methods
  IV. **Nonperturbative effects**
III. Our Results
IV. Conclusion


General Features from Non-pert. Renormalizable Theories:

- Nonperturbative scale $\xi$
- $\boxed{\text{Nontrivial Critical Scaling exponents for correlation functions } \nu}$
- RG Running of coupling constants $g \to g(k^2)$

### Critical Exponents for Gravity

$$\boxed{G_R(r) \equiv \langle\, \delta R(x)\delta R(y)\,\rangle \sim \frac{1}{r^{\,2(d-1/\nu\,)}}} \quad \boxed{\sim \frac{1}{r^2}}$$

- Various methods:

| Method used to compute $\nu$ in d=4 | Universal Exponent $\nu$ |
|---|---|
| Euclidean Lattice Quantum Gravity | $\nu^{-1} = 2.997(9)$ |
| Perturbative $2+\epsilon$ expansion to one loop [22] | $\nu^{-1} = 2$ |
| Perturbative $2+\epsilon$ expansion to two loops [23] | $\nu^{-1} = 22/5 = 4.40$ |
| Einstein-Hilbert RG truncation [56] | $\nu^{-1} \approx 2.80$ |
| Recent improved Einstein-Hilbert RG truncation [57] | $\nu^{-1} \approx 3.0$ |
| Geometric argument [33] $\rho_{vac\ pol}(r) \sim r^{d-1}$ | $\nu^{-1} = d-1 = 3$ |
| Lowest order strong coupling (large G) expansion [29] | $\nu^{-1} = 2$ |
| Nonlocal field equations with $G(\Box)$ for the static metric [46] | $\nu^{-1} = d-1$ for $d \geq 4$ |

$$\Rightarrow \boxed{\nu^{-1} \approx 3}$$



## Non-perturbatively renormalizable theories


I. Introduction
II. Quantum Gravity
  I. Defining the Theory
  II. Main Challenges
  III. Nonperturbative Methods
  IV. **Nonperturbative effects**
III. Our Results
IV. Conclusion


General Features from Non-pert. Renormalizable Theories:

- $\boxed{\text{Nonperturbative scale } \xi}$
- Nontrivial Critical Scaling exponents for correlation functions $\nu$
- RG Running of coupling constants $g \to g(k^2)$

$\boxed{\langle \delta R(x)\delta R(y)\rangle \sim \frac{1}{r^2}}$

### Nonperturbative scale

Nonperturbative scale $\xi$ ⟶ regulates expressions in *IR*-limit

| QCD case: | Gravity case: |
|---|---|
| $\langle F_{\mu\nu}^2\rangle \sim \frac{1}{\xi^4} \sim \Lambda_{QCD}^4$ | $\lambda_{\cos} \sim \langle R\rangle \sim \frac{1}{\xi^2}$ |
| $(\Lambda_{QCD} \simeq 200$ MeV$)$ | $(\xi \simeq \sqrt{\frac{1}{\lambda_{\cos}}} \simeq 5300$ Mpc $)$ |
| $\int dk \to \int_{\Lambda_{QCD}} dk$ | $\boxed{\int dk \to \int_{1/\xi} dk}$ |



## Non-perturbatively renormalizable theories


I. Introduction
II. Quantum Gravity
  I. Defining the Theory
  II. Main Challenges
  III. Nonperturbative Methods
  IV. **Nonperturbative effects**
III. Our Results
IV. Conclusion


General Features from Non-pert. Renormalizable Theories:

- Nonperturbative scale $\xi$
- Nontrivial Critical Scaling exponents for correlation functions $\nu$
- $\boxed{\text{RG Running of coupling constants } g \to g(k^2)}$

$\boxed{\int dk \to \int_{1/\xi} dk}$ $\boxed{\langle \delta R(x)\delta R(y)\rangle \sim \frac{1}{r^2}}$

### RG Running of coupling constants

Renormalization Group Eqns (RGE) ⇒ Running of coupling "constants"

**QCD case:**

$$\alpha_s \to \alpha_s(k)$$

$$\alpha_s(k) = \frac{1}{\beta_0 \ln\left(\frac{k^2}{\Lambda_{qcd}^2}\right)}$$

$\beta_0$ (Wilczek, Gross and Politzer)
$\Lambda_{qcd} \simeq 200$ MeV

No adjustable parameter (except $\Lambda_{qcd}$!)

**Gravity case:**

$$G \to G(k)$$

$$\boxed{G(k) = G\left[1 + c_0\left(\frac{m^2}{k^2}\right)^{\frac{1}{2\nu}} + \mathcal{O}\left(\frac{m^2}{k^2}\right)^{1/\nu}\right]}$$

$c_0 \approx 16.04$ (HWH)
$m = 1/\xi \sim 1/\ 5300\text{Mpc} \sim 2{\times}10^{-4}\ \text{Mpc}^{-1}$
$1/\nu \simeq 3$

**Q: Relate these to Cosmology?**



## Cosmological Implication 1: Galaxy Power Spectrum



Relate these to Cosmology:

- Nonperturbative scale $\xi$
- $\boxed{\text{Nontrivial Critical Scaling exponents for correlation functions } \nu}$
- RG Running of coupling constants $G \to G(k^2)$

$\int dk \to \int_{1/\xi} dk$ $\boxed{\langle \delta R(x)\delta R(y)\rangle \sim \frac{1}{r^2}}$

$G(k) = G\left[1 + c_0\left(\frac{m^2}{k^2}\right)^{\frac{1}{2\nu}} + \mathcal{O}\left(\frac{m^2}{k^2}\right)^{1/\nu}\right]$

### Gravitational Fluctuations ⇒ Matter Fluctuation

$$\langle\, \delta R(x)\delta R(y)\,\rangle \sim \frac{1}{r^2}$$

$$R_{\mu\nu} - \tfrac{1}{2}\, g_{\mu\nu} R + \lambda g_{\mu\nu} = 8\pi G\, T_{\mu\nu}$$

$$R - 4\lambda = -8\pi G\, T$$

trace gives $T = 3p - \rho$, and thus $T \simeq -\rho$ for a non-rel.

$$R - 4\lambda = 8\pi G\ \rho$$

$$\langle\, \delta R\, \delta R\,\rangle = (8\pi G)^2\, \langle\, \delta\rho\, \delta\rho\,\rangle$$

$$G_R(r) \equiv \langle\, \delta R(x)\delta R(y)\,\rangle \sim \frac{1}{r^2} \quad \Rightarrow \quad G_\rho(r) \equiv \langle\, \delta\rho(x)\delta\rho(y)\,\rangle = \left(\frac{r_0}{r}\right)^2$$

$$\Leftrightarrow \quad \boxed{P(k) = \frac{a_0}{k}} \qquad \gamma = 2, \text{ or } s = 1$$

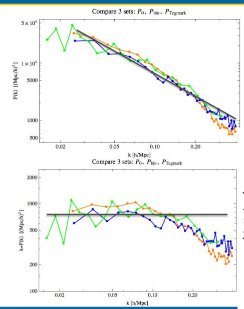


## Cosmological Implication 1: Galaxy Power Spectrum


I. Introduction
II. Quantum Gravity
III. Our Results

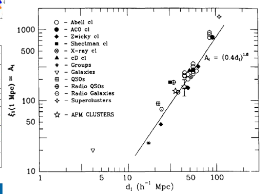


IV. Conclusion


SDSS DR14 (2016)
Tegmark SDSS DR2 (2004)

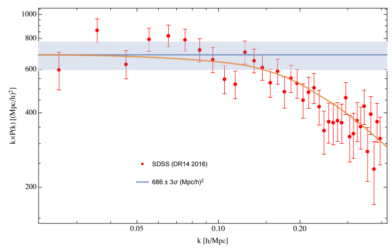
$$G_\rho = \left(\frac{r_0}{r}\right)^2 \qquad P(k) = \frac{a_0}{k}$$




## Cosmological Implication 1: Galaxy Power Spectrum


I. Introduction
II. Quantum Gravity
III. Our Results
  The Matter Power Spectrum
  I. **The Galactic Regime**
  II. The CMB Regime
  III. The Extreme Small-*k* Regime
  IV. Constraints on parameters
  The Angular Temperature Spectra
  Numerical Results
IV. Conclusion

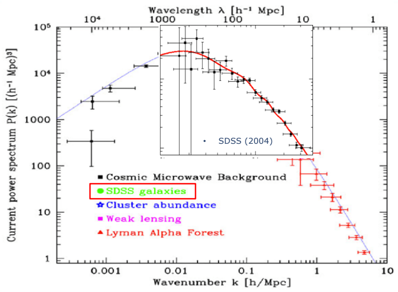

Expect: $k \times P(k) \to a_0$ (for $r \to \infty$, $k \to 0$)




H. W. Hamber and L.H.S. Yu, "Gravitational Fluctuations as an alternative to Inflation II. CMB Angular Power Spectrum", Oct 2019

## Extending to CMB Scales



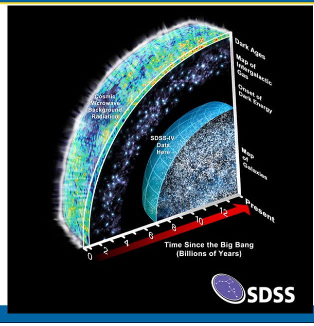



*Ref:* M. Tegmark, et. al. (2004) astro-ph/0107418, *ApJ*, **571**, 191



## Extending to CMB Scales

I. Introduction
II. Quantum Gravity
III. Our Results
The Matter Power Spectrum
I. The Galactic Regime
**II. The CMB Regime**
III. The Extreme Small-k Regime
IV. Constraints on parameters
The Angular Temperature Spectra
Numerical Results
IV. Conclusion






## Extending to CMB Scales

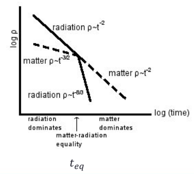
I. Introduction
II. Quantum Gravity
III. Our Results
The Matter Power Spectrum
I. The Galactic Regime
**II. The CMB Regime**
III. The Extreme Small-k Regime
IV. Constraints on parameters
The Angular Temperature Spectra
Numerical Results
IV. Conclusion

Larger distance → earlier in time
→ switches from **matter dominated** → **radiation dominated**

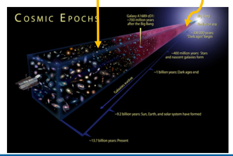
But radiation **behaves** quantitatively **different** than matter in realtivity.

∴ expects $P(k)$ to behave differencely **beyond critical scale** $k_{eq}$.

Matter Dom.
Radiation Dom.




$t_{eq}$



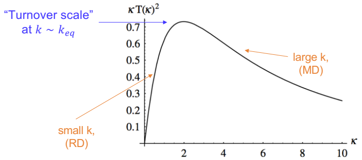



## Extending to CMB Scales

I. Introduction
II. Quantum Gravity
III. Our Results
The Matter Power Spectrum
I. The Galactic Regime
**II. The CMB Regime**
III. The Extreme Small-k Regime
IV. Constraints on parameters
The Angular Temperature Spectra
Numerical Results
IV. Conclusion

- Classical GR ⇒ *"Transfer function"*
- Provides an unambiguous relation between matter-dominated regime to radiation-dominated regime **via Boltzmann & Friedmann Eqns**.

"Turnover scale" at $k \sim k_{eq}$




$$\mathcal{T}(\kappa) \simeq \frac{\ln[1+(0.124\kappa)^2]}{(0.124\kappa)^2}\left[\frac{1+(1.257\kappa)^2+(0.4452\kappa)^4+(0.2197\kappa)^6}{1+(1.606\kappa)^2+(0.8568\kappa)^4+(0.3927\kappa)^6}\right]^{1/2}$$

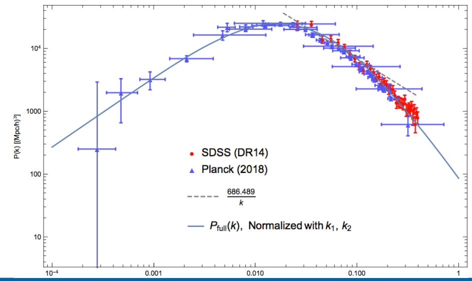
Weinberg p.412



## Extending to CMB Scales

I. Introduction
II. Quantum Gravity
III. Our Results
The Matter Power Spectrum
I. The Galactic Regime
**II. The CMB Regime**
III. The Extreme Small-k Regime
IV. Constraints on parameters
The Angular Temperature Spectra
Numerical Results
IV. Conclusion

- Normalized result for full $P(k)$, extrapolated to small $k$: (blue curve)




H. W. Hamber and **L.H.S. Yu**, "Gravitational Fluctuations as an alternative to Inflation II. CMB Angular Power Spectrum", Oct 2019



## Additional Quantum Effects (when $r \to \sim\xi$, $k \to \sim m$)

I. Introduction
II. Quantum Gravity
III. Our Results
The Matter Power Spectrum
I. The Galactic Regime
II. The CMB Regime
**III. The Extreme Small-k Regime**
IV. Constraints on parameters
The Angular Temperature Spectra
Numerical Results
IV. Conclusion

In extreme small k, additional effects important in QG:
(1) Nonperturbative scale $\xi$ regulates expressions.

Just like QCD! → $\Lambda_{QCD}$ QG → "$m$" $\sim \frac{1}{\xi}$ ($\xi = \sqrt{\frac{3}{\lambda_{cos}}} \simeq 5300$ Mpc)

"Gluon condensate" "*Curvature Condensate*"!

Acts as "*infrared regulator*"

(modifies small-k) ⇒ $\int dk \to \int_m dk$ , $\boxed{\frac{1}{k^2} \to \frac{1}{k^2+m^2}}$

(2) RG Running of G

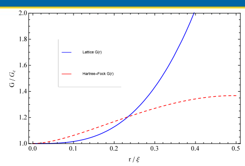
Recall field eqns ⇒ $G_\rho = \frac{1}{(8\pi G)^2} G_R$

So: → $G_\rho = \frac{1}{(8\pi G(r))^2} G_R$ , or $\boxed{P(k) = \frac{1}{(8\pi G(k))^2} P_R(k)}$



## Antiscreening of Gravity!

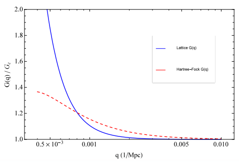
I. Introduction
II. Quantum Gravity
III. Our Results
The Matter Power Spectrum
I. The Galactic Regime
II. The CMB Regime
**III. The Extreme Small-k Regime**
IV. Constraints on parameters
The Angular Temperature Spectra
Numerical Results
IV. Conclusion




virtual graviton cloud


$$G(k) = G_0\left[1 + 2c_0\left(\frac{m^2}{k^2}\right)^{3/2} + \mathcal{O}\left(\left(\frac{m^2}{k^2}\right)^3\right)\right]$$

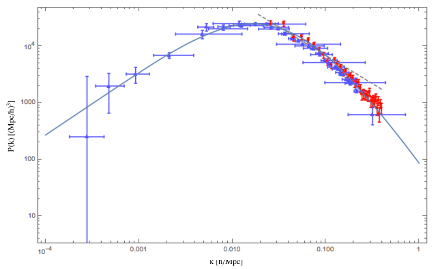
$$P(k) = \frac{1}{(8\pi G)^2}(\bar{\rho})^{-2} P_R(k)$$






## Additional Quantum Effects when $k \to \sim m$

I. Introduction
II. Quantum Gravity
III. Our Results
The Matter Power Spectrum
I. The Galactic Regime
II. The CMB Regime
**III. The Extreme Small-k Regime**
IV. Constraints on parameters
The Angular Temperature Spectra
Numerical Results
IV. Conclusion

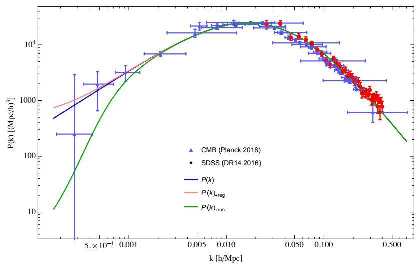



H. W. Hamber and **L.H.S. Yu**, "Gravitational Fluctuations as an alternative to Inflation II. CMB Angular Power Spectrum", Oct 2019



## Additional Quantum Effects when $k \to \sim m$

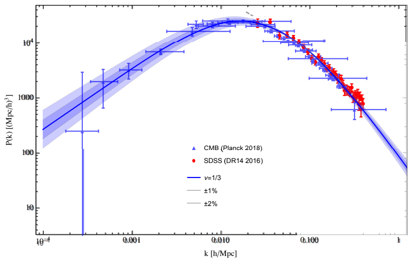
I. Introduction
II. Quantum Gravity
III. Our Results
The Matter Power Spectrum
I. The Galactic Regime
II. The CMB Regime
**III. The Extreme Small-k Regime**
IV. Constraints on parameters
The Angular Temperature Spectra
Numerical Results
IV. Conclusion




H. W. Hamber and **L.H.S. Yu**, "Gravitational Fluctuations as an alternative to Inflation II. CMB Angular Power Spectrum", Oct 2019

## Constraints on $\nu$





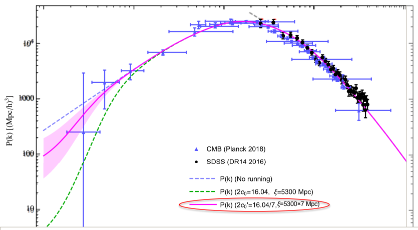

⇒ QG's prediction of $\nu$ agrees with observations!



## Constraints on $c_0$ / $\xi$



$$G(k) = G_0\left[1 + 2c_0\left(\frac{m^2}{k^2+m^2}\right)^{3/2} + \mathcal{O}\left(\left(\frac{m^2}{k^2+m^2}\right)^3\right)\right]$$



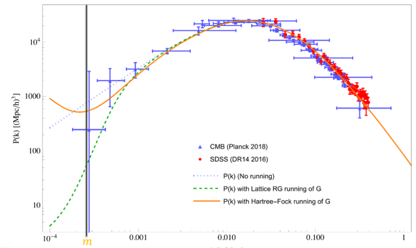

* Data seems to favor a *smaller* value of $c_0$ (or $1/\xi$) from the lattice.



## Hartree-Fock Running of G



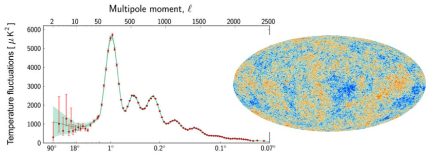
$$G \;\rightarrow\; G_{\mathrm{HF}}(k) = G_0\left[1 - \frac{3m^2}{2k^2}\log\left(\frac{3m^2}{2k^2}\right)\right]$$






## Angular CMB Power Spectrum $C_l$






$$C_l \;\equiv\; \frac{1}{4\pi}\int d^2\hat{n}\int d^2\hat{n}'\; L_l(\hat{n}\cdot\hat{n}')\,\langle\, \Delta T(\hat{n})\,\Delta T(\hat{n}')\,\rangle$$



## Angular CMB Power Spectrum $C_l$



$$\underbrace{C_l \;\equiv\; \frac{1}{4\pi}\int d^2\hat{n}\int d^2\hat{n}'\; L_l(\hat{n}\cdot\hat{n}')\,\langle\, \Delta T(\hat{n})\,\Delta T(\hat{n}')\,\rangle}$$

$$C_l = 16\pi^2\, T_0^2 \int_0^\infty q^2 dq\, P(q)\left[C_0\, k^4\, \mathcal{T}(\kappa)^2\right]^{-1}\left[j_l(qr_L)\widetilde{F_1}(q) + j_l'(qr_L)\widetilde{F_2}(q)\right]^2$$

Transfer functions:
(*completely determined by classical GR*)

$\mathcal{T}(\kappa)$, $\mathcal{S}(\kappa)$ and $\Delta(\kappa)$

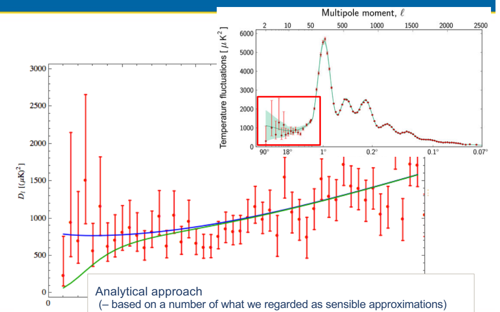
$$F_1(q) = \frac{\mathcal{R}_q^0}{5}\left[3\,\mathcal{T}\left(\frac{q\,d_T}{a_L}\right)R_L \;-\; (1+R_L)^{-\frac{1}{4}}\, e^{-\left(\frac{q\,d_D}{a_L}\right)^2}\mathcal{S}\left(\frac{q\,d_T}{a_L}\right)\cos\left[\frac{q\,d_H}{a_L} + \Delta\left(\frac{q\,d_T}{a_L}\right)\right]\right]$$

$$F_2(q) = \sqrt{2}\,\frac{\mathcal{R}_q^0}{5}\,(1+R_L)^{-\frac{3}{4}}\, e^{-\left(\frac{q\,d_D}{a_L}\right)^2}\mathcal{S}\left(\frac{q\,d_T}{a_L}\right)\sin\left[\frac{q\,d_H}{a_L} + \Delta\left(\frac{q\,d_T}{a_L}\right)\right]$$



## Angular CMB Power Spectrum $C_l$






Analytical approach
(– based on a number of what we regarded as sensible approximations)



## Numerical Programs

with H. E. Pituwala, H.W. Hamber



Modified Gravity Programs:

$G \rightarrow G(k)$

MGCAMB

MGCLASS

ISiTGR

- $P(k)$
- $C_l^{TT}$

CAMB

CLASS

Original CMB Programs



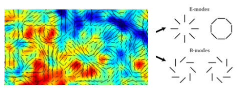
## Other Cosmological Power Spectra



### What else can we measure?

- $\delta\rho(x)$ = density fluctuations (at $x$) $\rightarrow P(k)$
- $\Delta T(\hat{n})$ = temperature fluctuations of CMB $\gamma$'s (in the direction $\hat{n}$) $\rightarrow C_l^{TT}$

Other information:
- E-mode polarization of CMB $\gamma$'s
- B-mode polarization of CMB $\gamma$'s




- Lensing potential $\phi$ of CMB $\gamma$'s
- $\Delta\theta_a = \sum_b M_{ab}(\hat{n})\,\theta_b$ where $\theta_a$ = deflection angle ($\perp$ to $\hat{n}$), $M$ = "shear matrix"
- $\mathrm{Tr}M(\hat{n}) \sim \nabla^2\phi(\hat{n})$ where $\phi$ = *lensing potential.*

Can measure and plot a spectrum for these.
Does QG has any effect on them?



## Other Cosmological Power Spectra



- $P(k)$
- $C_l^{TT}$

$$\Delta T(\hat{n}) \equiv T(\hat{n}) - T_0 = \sum_{lm} a_{lm}^T\, Y_l^m(\hat{n})$$

$$\left\langle \Delta T(\hat{n})\,\Delta T(\hat{n}')\right\rangle = \sum_{lm} C_l^{TT}\, Y_l^m(\hat{n})Y_l^{-m}(\hat{n}') = \sum_l C_l^{TT}\left(\frac{2l+1}{4\pi}\right)L_l(\hat{n}\cdot\hat{n}')$$

$$\left\langle a_{lm}^T\, a_{l'm'}^T\right\rangle \equiv \delta_{ll'}\delta_{m-m'}\, C_l^{TT}$$

- $C_l^{EE}$
- $C_l^{BB}$
- $C_l^{TE}$

$$\left\langle a_{E,lm}^*\, a_{E,l'm'}\right\rangle = C_l^{EE}\,\delta_{ll'}\delta_{mm'}$$

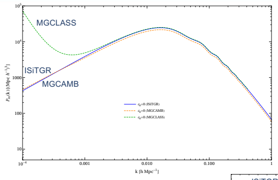
$$\left\langle a_{T,lm}^*\, a_{E,l'm'}\right\rangle = C_l^{TE}\,\delta_{ll'}\delta_{mm'}$$

$$\left\langle a_{B,lm}^*\, a_{B,l'm'}\right\rangle = C_l^{BB}\,\delta_{ll'}\delta_{mm'}$$

- $C_l^{\phi\phi}$
- $C_l^{T\phi}$
- $C_l^{E\phi}$

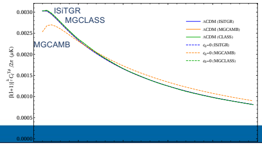
E- & B-modes also measured by BICEP
$\phi$-modes also measured by ACT and SPT

- $C_l^{EB} = C_l^{TB} = C_l^{B\phi} = 0$




$$G(k) = G_0\left[1 + 2c_0\left(\frac{m^2}{k^2+m^2}\right)^{3/2} + \mathcal{O}\left(\left(\frac{m^2}{k^2+m^2}\right)^3\right)\right]$$

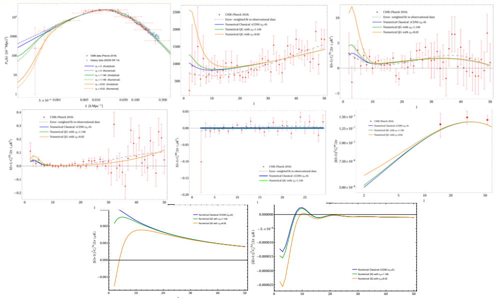
Calibrating: by running $c_0 = 0$ against base default "ΛCDM" program







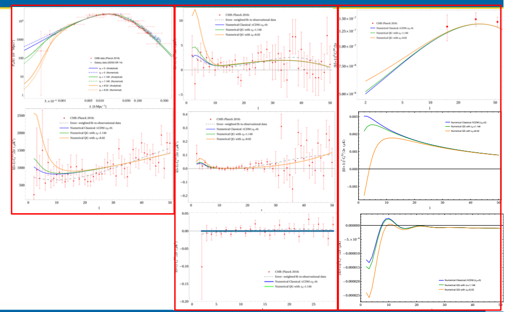






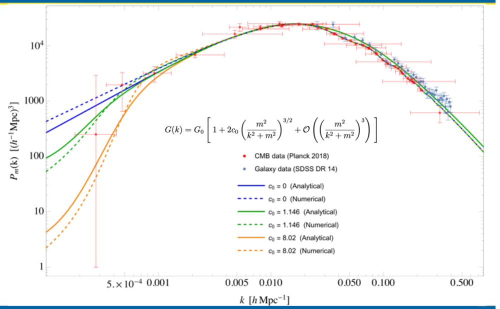








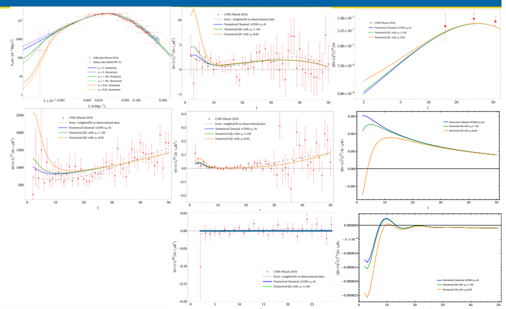








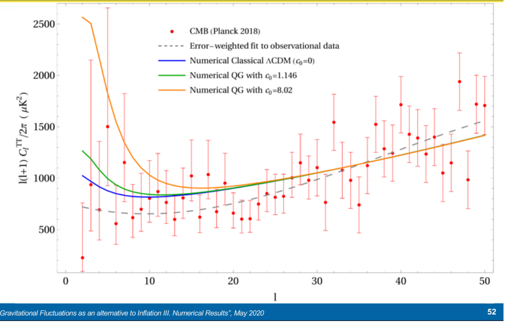






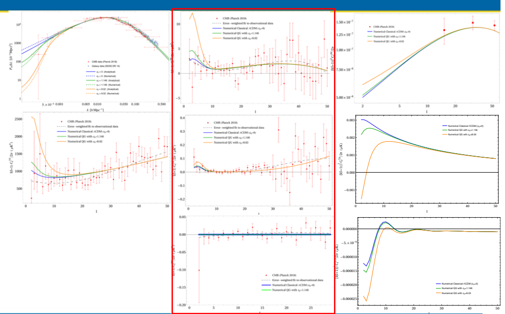








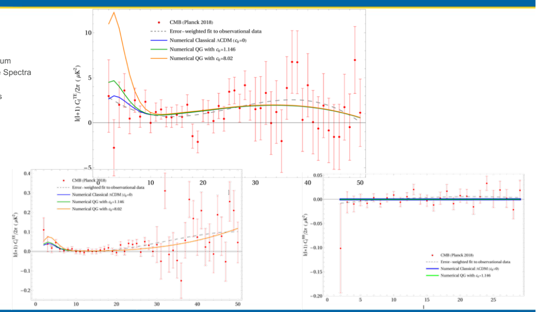






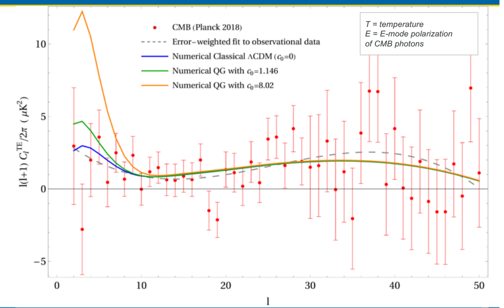




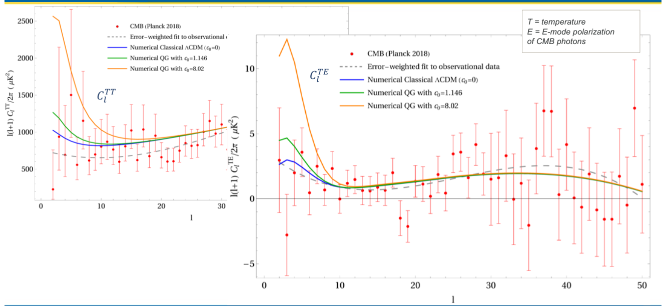






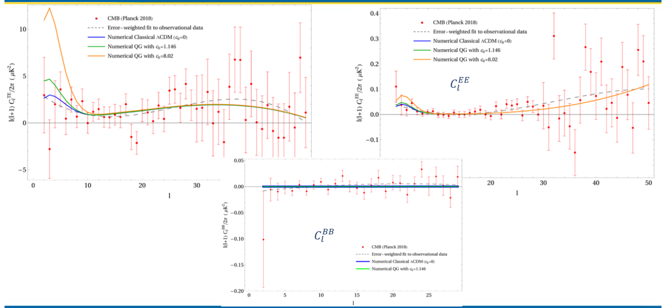



## Cross Temperature-$E$-mode Spectrum $C_l^{TE}$





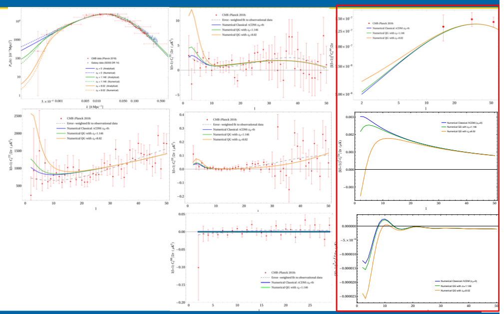

## Numerical Results





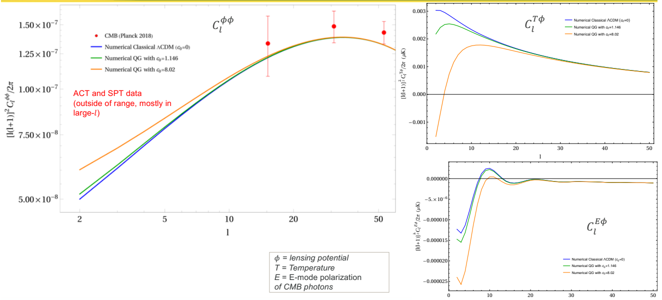
## Numerical Results







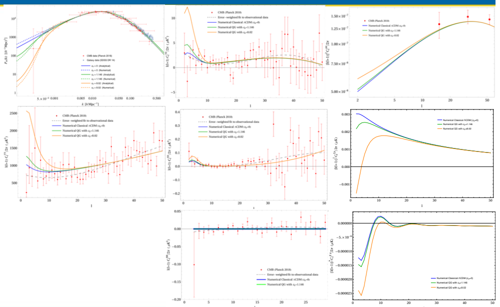
## Numerical Results






## Numerical Results







## Comparison with Inflation

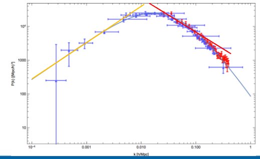


Inflation – the original picture

- Proposes → scalar field $\phi$
- Fluctuations from $\langle\phi\phi\rangle$
- Calculated with *pert-QFT* methods
- Relates $\langle\phi\phi\rangle \xrightarrow{M-S} \langle\delta\rho\delta\rho\rangle$
- Normalizes on CMB scales (small k)
- Extrapolate to SDSS scale (large k)

QG – an *alternative* picture

- Curvature/Gravitational field $R$
- Fluctuations from $\langle RR\rangle$
- Calculated with *nonpert-* methods
- Relates $\langle RR\rangle \xrightarrow{EFE} \langle\delta\rho\delta\rho\rangle$
- Normalizes on SDSS scales (large k)
- Extrapolate to CMB scale (small k)





## Comparison with Inflation



*Radically different perspective!*

*Arguable more grounded?*

- 1. No new fields required
- 2. Standard techniques based on Feynman P.I. & Wilson's RG analysis
- 3. Very limited free/adjustable parameters in the theory

- Or, Possibly coexists??

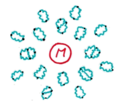
- Hopefully testable in near future,
- 1. more precision data on large scales
- 2. more theoretical studies,
  - (a). better understanding of effects on spectra (programs, parameters)
  - (b). other cosmological implications of QG



## Conclusion



1) Nonperturbative methods provide a useful theory of quantum gravity
2) Provides radically alternative picture to Inflation
3) Provides testable predictions for future



- *Possible Future work:*
  - Further investigation into numerical programs of modified gravity
  - Explore current measurements with upcoming observations
    - (SPT-3G, CLASS, BICEP3, ACTPol…)
  - Other cosmological effects:
    - Bispectrums, Tri-spectrums, Tensor Perturbations, tensor-to-scalar ratio…

*Publications*


- *H. W. Hamber and **L.H.S. Yu**, "Gravitational Fluctuations as an alternative to Inflation", Jul 2018*
- *H. W. Hamber and **L.H.S. Yu**, "Gravitational Fluctuations as an alternative to Inflation II. CMB Angular Power Spectrum", Oct 2019*
- *H. W. Hamber H. E. Pituwala K. and **L.H.S. Yu**, "Gravitational Fluctuations as an alternative to Inflation III. Numerical Results", May 2020*
- *H. W. Hamber and **L.H.S. Yu**, "Dyson's Equation for Quantum Gravity in the Hartree-Fock Approximation", Oct 2020*

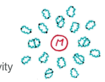

*Special thanks to Prof. Stan Brodsky (SLAC) for discussions over Fall 2019.*



## Conclusion



1) Nonperturbative methods provide a useful theory of quantum gravity
2) Provides radically alternative picture to Inflation
3) Provides testable predictions for future



*Possible Future work:*
- Further investigation into numerical programs of modified gravity
- Explore current measurements with upcoming observations
  - (SPT-3G, CLASS, BICEP3, ACTPol…)
- Other cosmological effects:
  - Bispectrums, Tri-spectrums, Tensor Perturbations, tensor-to-scalar ratio…

*Publications*


*H. W. Hamber and **L.H.S. Yu**, "Gravitational Fluctuations as an alternative to Inflation", Jul 2018*
*H. W. Hamber and **L.H.S. Yu**, "Gravitational Fluctuations as an alternative to Inflation II. CMB Angular Power Spectrum", Oct 2019*
*H. W. Hamber H. E. Pituwala K. and **L.H.S. Yu**, "Gravitational Fluctuations as an alternative to Inflation III. Numerical Results", May 2020*
*H. W. Hamber and **L.H.S. Yu**, "Dyson's Equation for Quantum Gravity in the Hartree-Fock Approximation", Oct 2020*


***Thank you!***

# Cosmological Implications of Quantum Gravity

*Thank you!*

Southampton

**Extra slides**

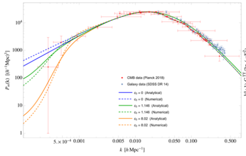
Southampton

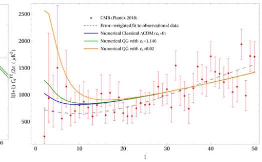
## Compare $P(k)$ to $C_l$



$$C_l \equiv \frac{1}{4\pi}\int d^2\hat{n}\int d^2\hat{n}'\, L_l(\hat{n}\cdot\hat{n}')\,\langle\, \Delta T(\hat{n})\,\Delta T(\hat{n}')\,\rangle$$

$$C_l = 16\pi^2\, T_0^2 \int_0^\infty q^2 dq\, P(q)\, \left[C_0\, k^4\, \mathcal{T}(\kappa)^2\right]^{-1} \left[j_l(qr_L)\widetilde{F_1}(q) + j_l'(qr_L)\widetilde{F_2}(q)\right]^2$$



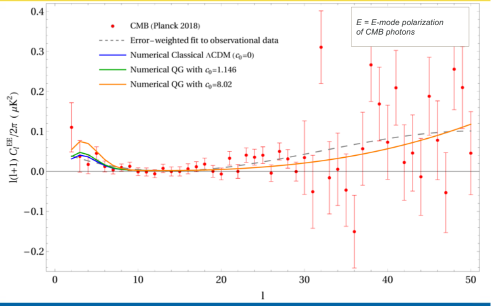





Southampton

## *E*-mode Polarization Spectrum $C_l^{EE}$



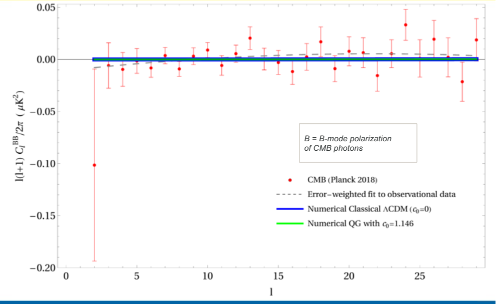





Southampton

## *B*-mode Polarization Spectrum $C_l^{BB}$



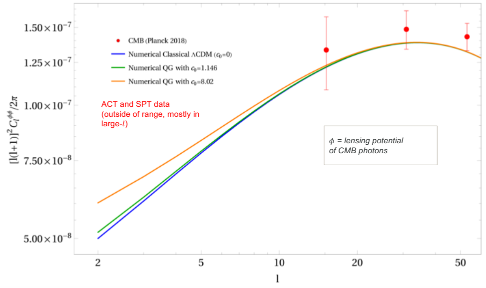





Southampton

## Photon Lensing Potential Spectrum $C_l^{\phi\phi}$



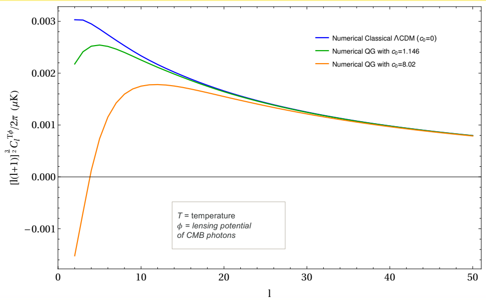





Southampton

## Cross Temperature-Lensing-Potential Spectrum $C_l^{T\phi}$



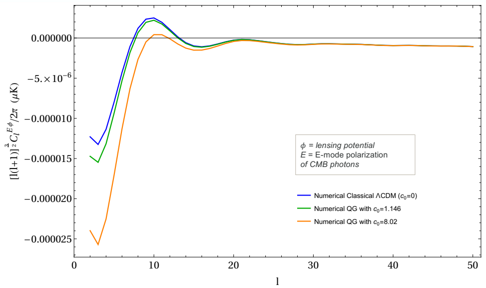





Southampton

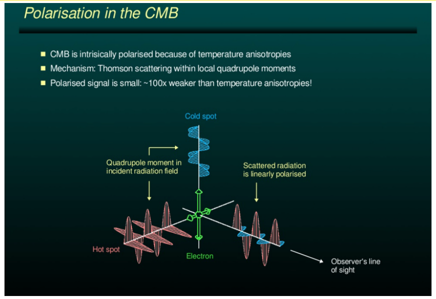
## Cross E-mode-Lensing-Potential Spectrum $C_l^{E\phi}$





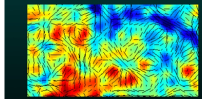



Southampton



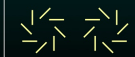
## Two flavors of polarisation

We can decompose a polarisation map...



“E” or “gradient” mode polarisation has no handedness

“B” or “curl” mode polarisation has handedness, i.e. rotation direction







## The buzz about B modes

E modes are the CMB's “intrinsic polarisation”

- We expect them to be there because of scattering processes in the CMB
- Temperature anisotropies predict E-mode spectra with almost no extra information
- Not only that, but “standard” CMB scattering physics generates ONLY E modes.

So then where do B modes come from?

- **Inflation**: exponential expansion of universe (x $10^{25}$) at $10^{-35}$ sec after big bang. “Smoking gun” signature = gravitational wave background that leaves a B-mode imprint on CMB polarization!
- **Gravitational lensing** by large scale structure converts some of the E-mode polarisation to B-mode. Use this to study structure formation, “weigh” neutrinos.
- How can we tell the difference between the above two? **Degree** vs. **arcminute** angular scales.

The moral of the story: B modes tell us things about the universe that temperature and E modes *can't*.