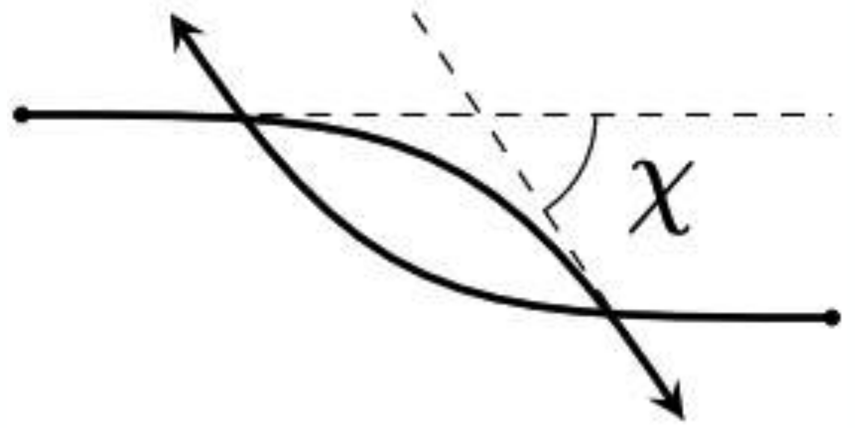
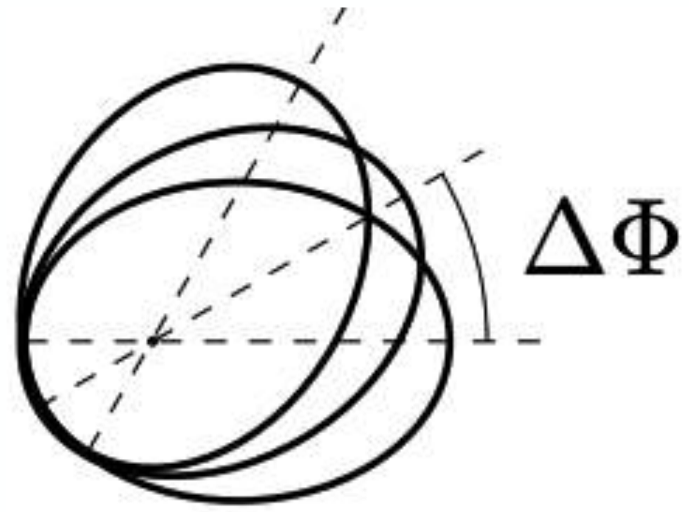
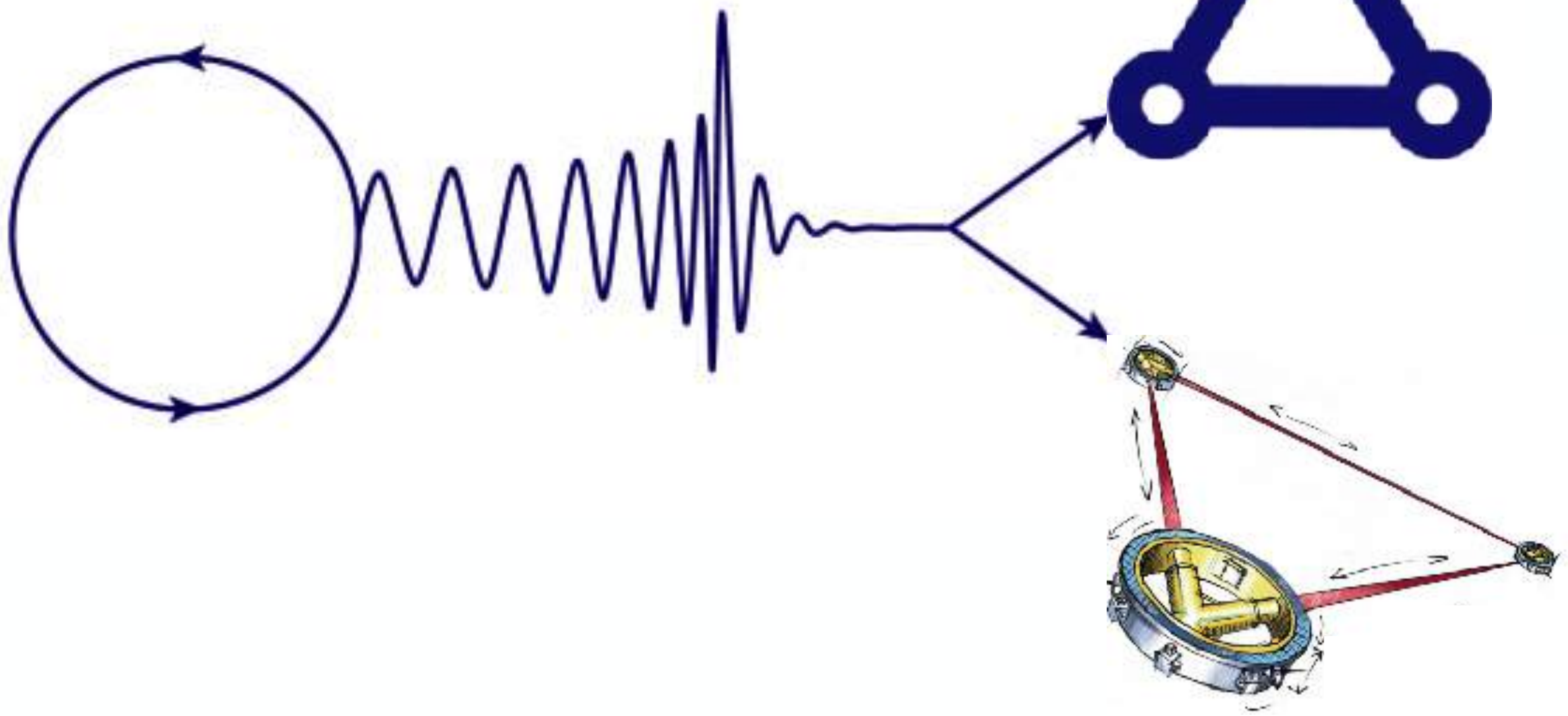
Precision Gravity: From the LHC to LISA and ET

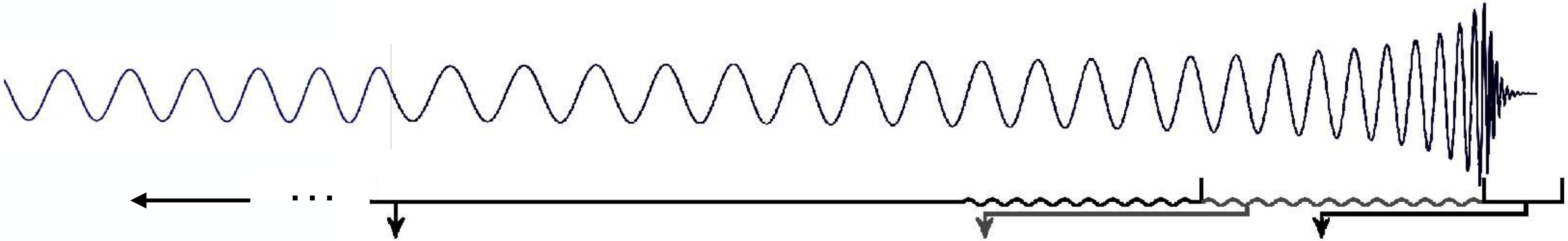


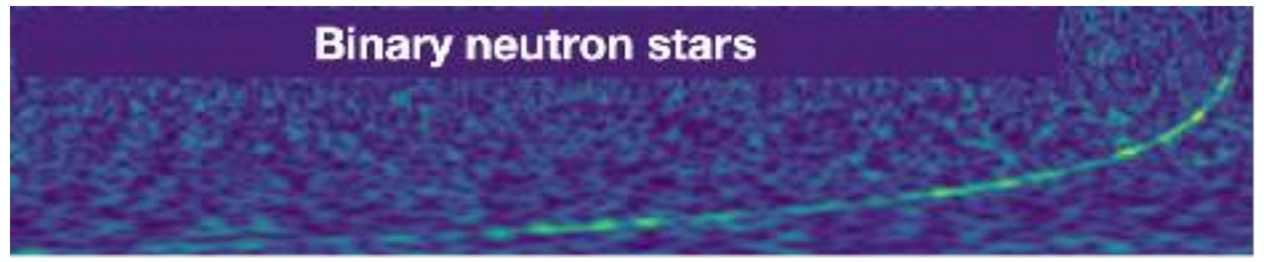


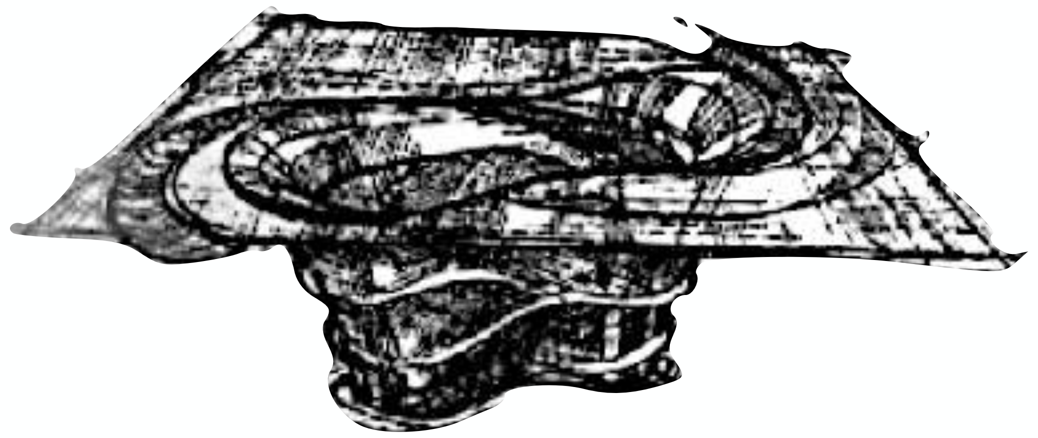


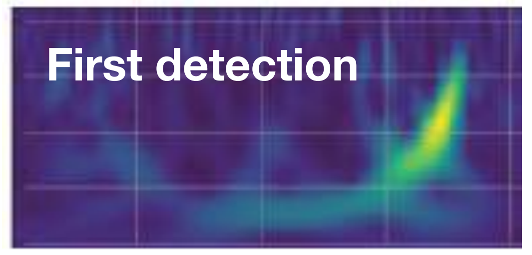
Rafael A. Porto

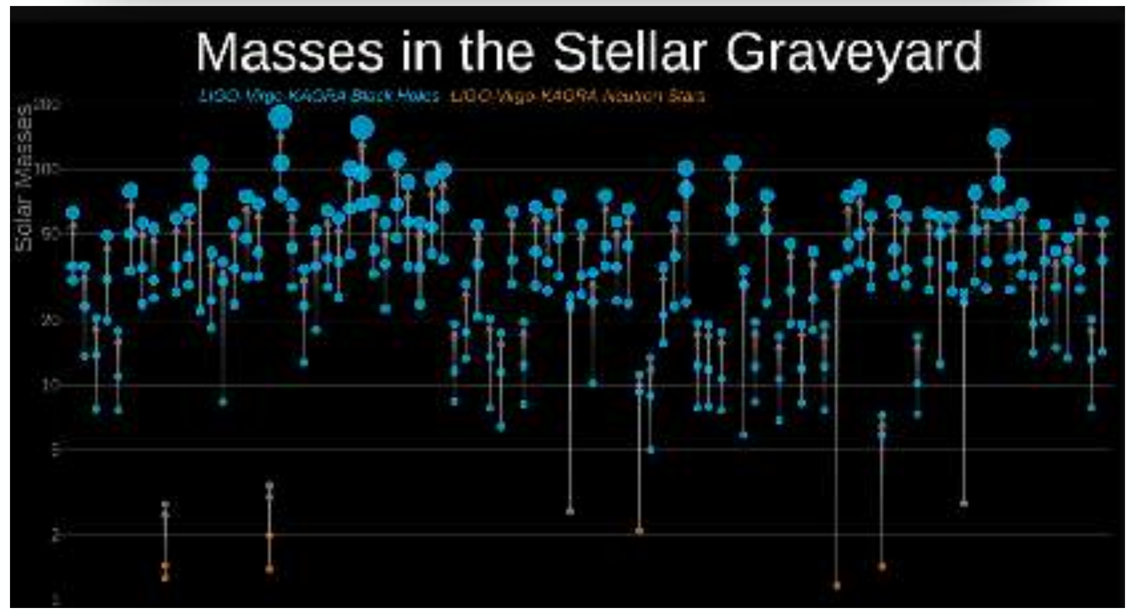
Precision Gravity: From the LHC to LISA and ET



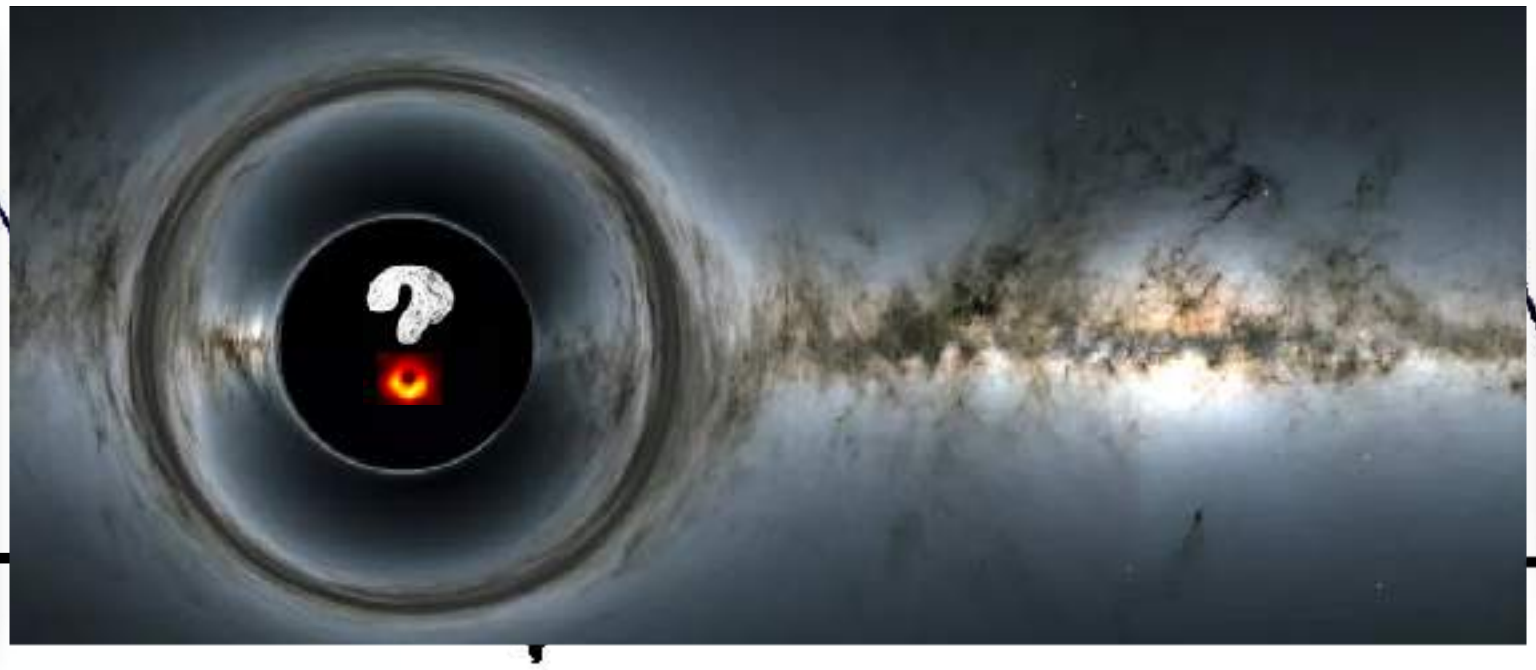






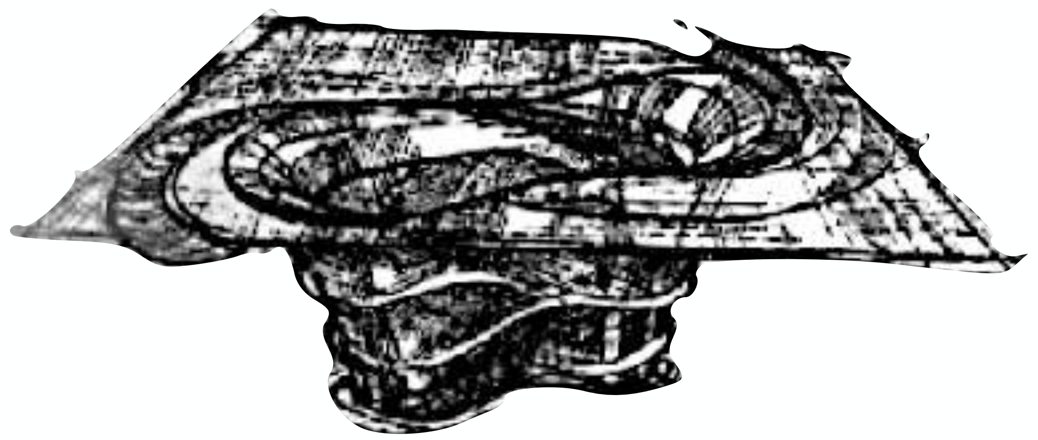


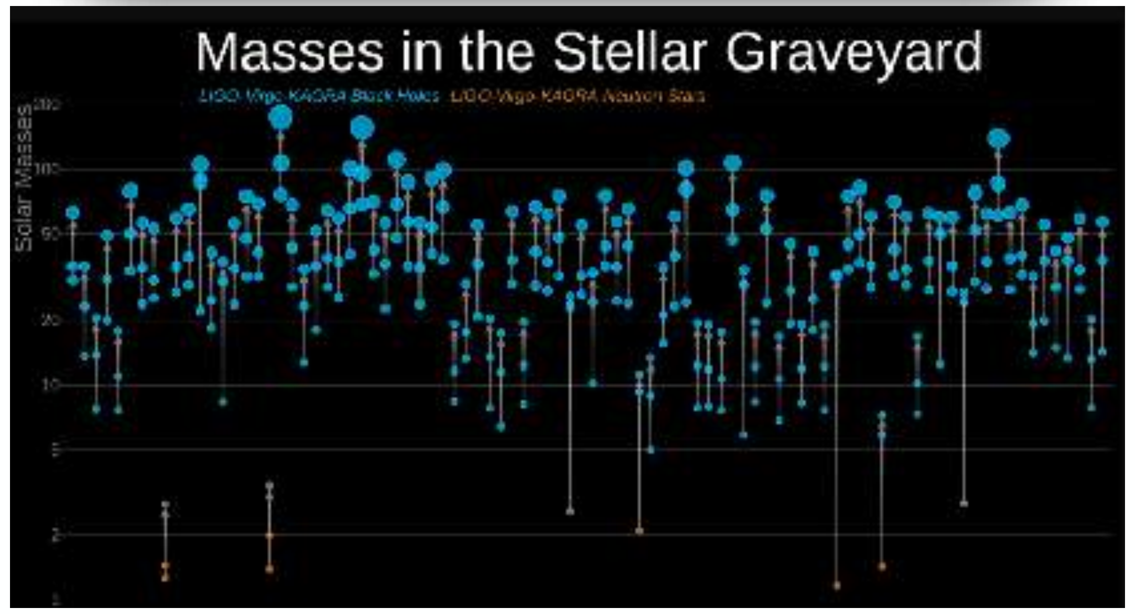
Precision Gravity:
From the LHC to LISA and ET



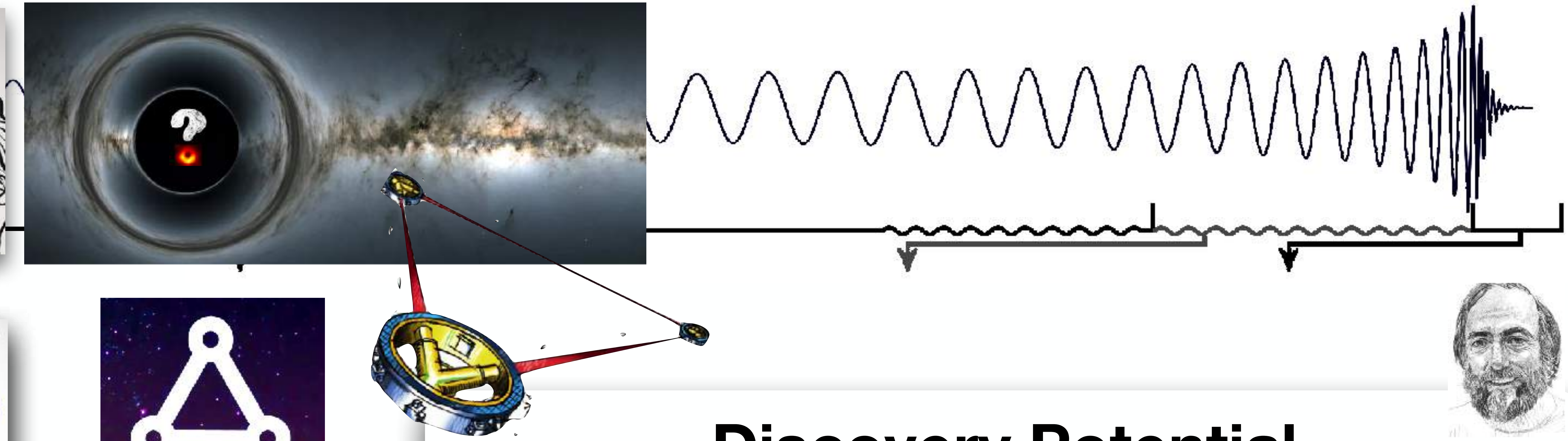
"for the discovery of a supermassive compact object at the centre of our galaxy"

"for the discovery that black hole formation is a robust prediction of the general theory of relativity"

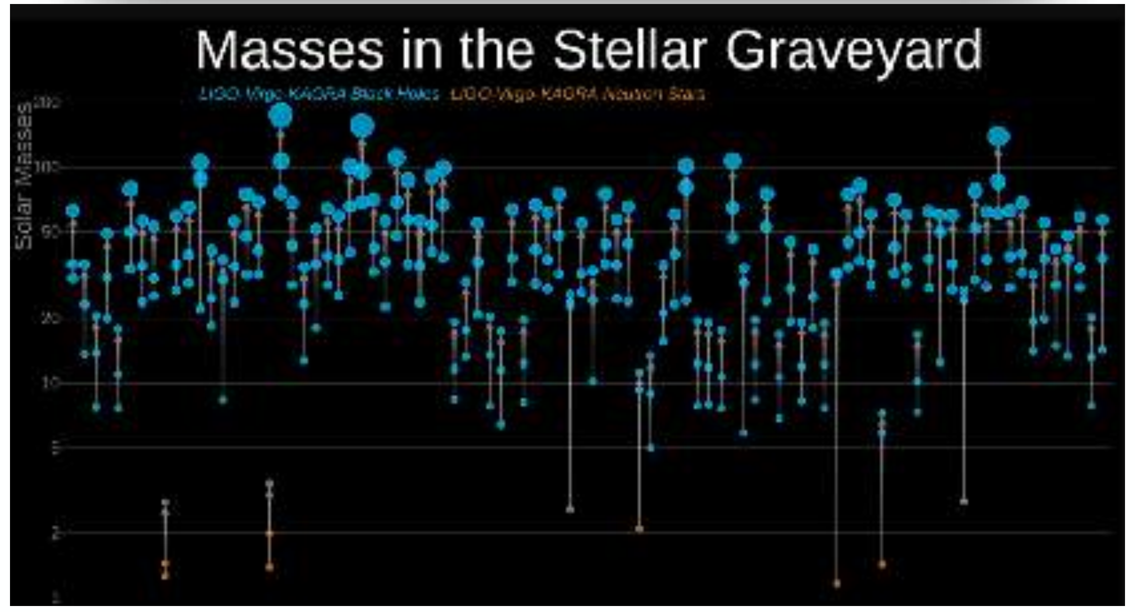




Precision Gravity: From the LHC to LISA and ET



1993



Einstein Telescope

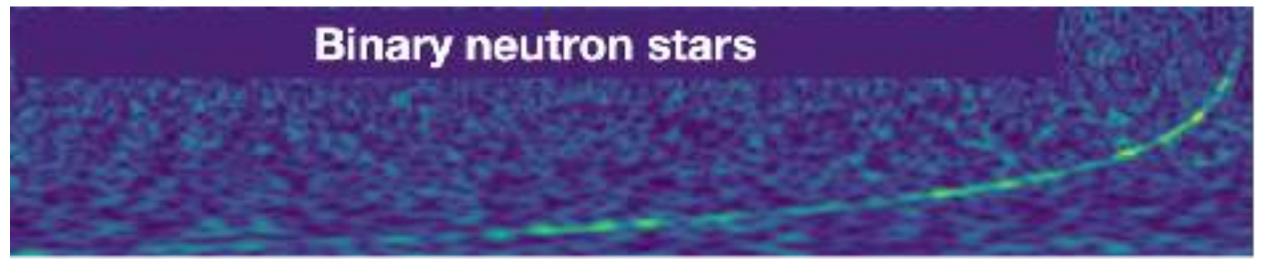
Discovery Potential =

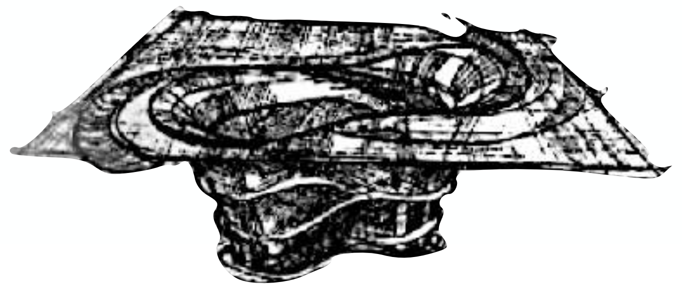
Precise Theoretical Predictions

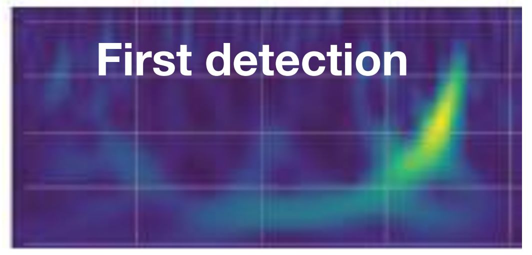
“Waveforms will be far more complex and carry more information than expected. Improved modeling will be needed for extracting the gravitational wave’s information”

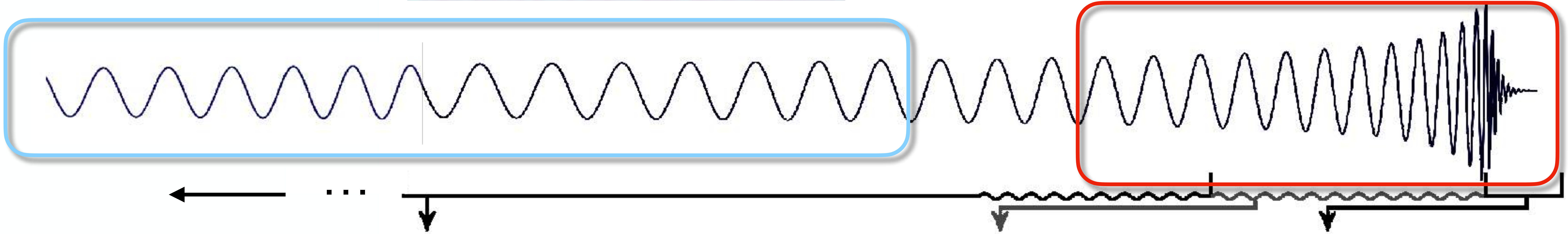
Challenge in GW Science

$$R_{im} = \sum_{l} \frac{\partial \Gamma^{l}_{im}}{\partial x_{l}} + \sum_{\rho l} \Gamma^{l}_{i\rho} \Gamma^{\rho}_{ml} = -\varkappa \left(T_{im} - \frac{1}{2} g_{im} T \right)$$

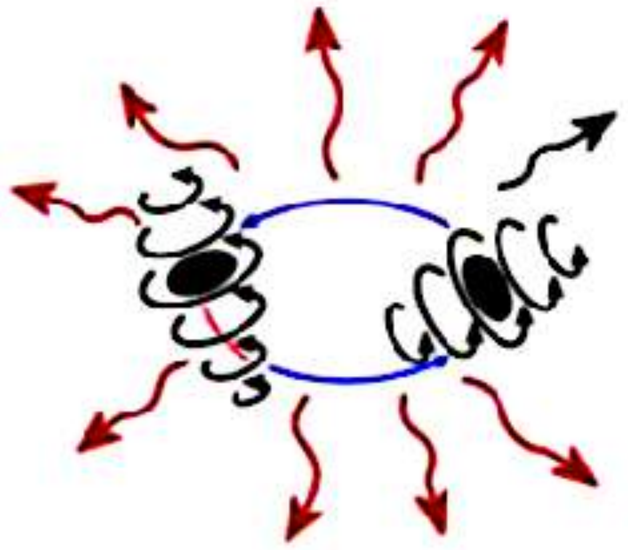








Inspiral

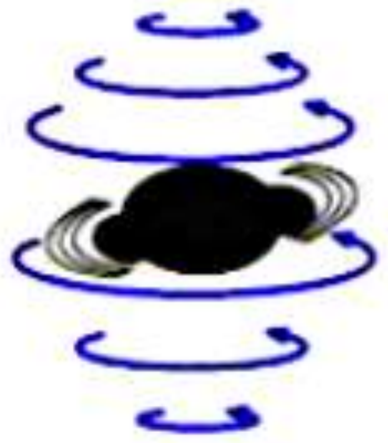


Analytic
(Approx. but fast)

Merger

Numerical
(exact but slow)

Ringing

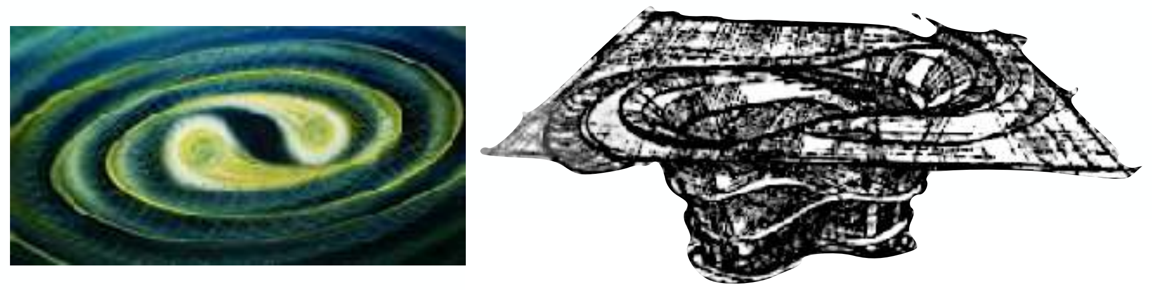


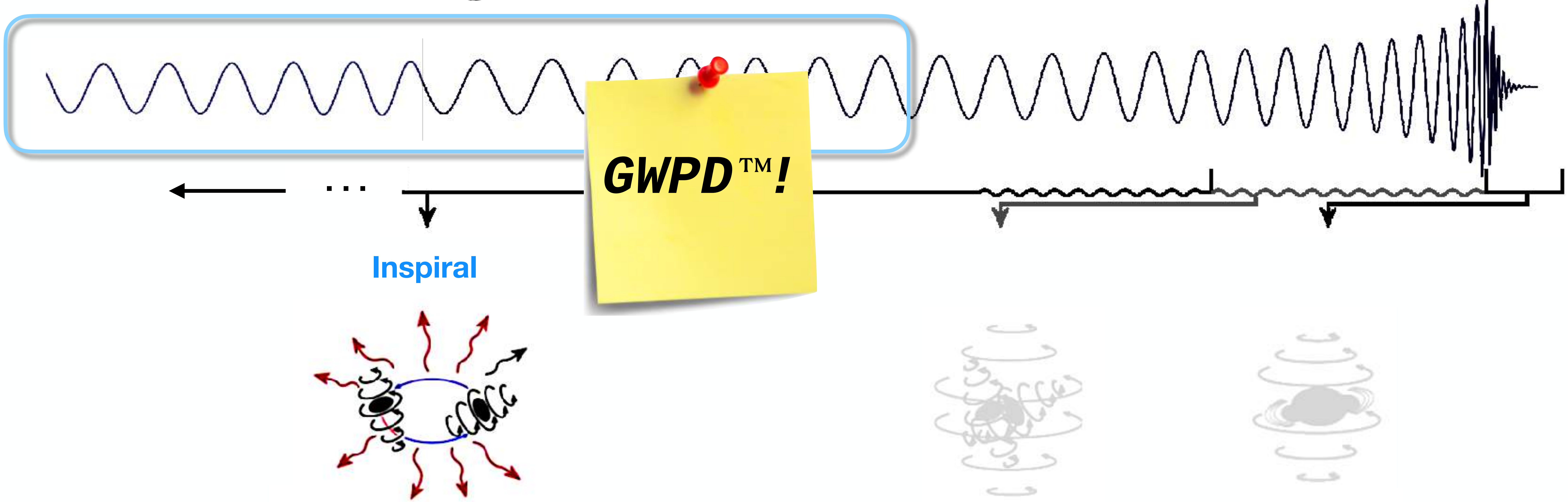
Analytic/
Perturbative

‘GW Precision Data’

1000+ cycles in band @ Design-Sensitivity

100+ events per year!





GWPD™!

Inspiral

Post-Newtonian

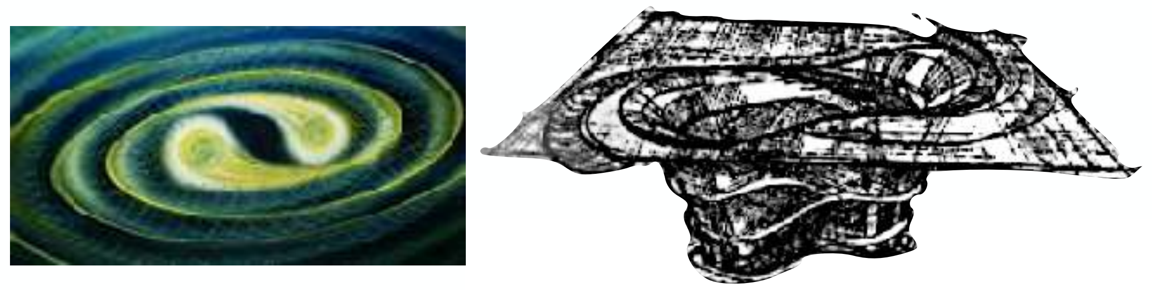
$$n\mathrm{PN} = \mathcal{O}(v^{2n})$$

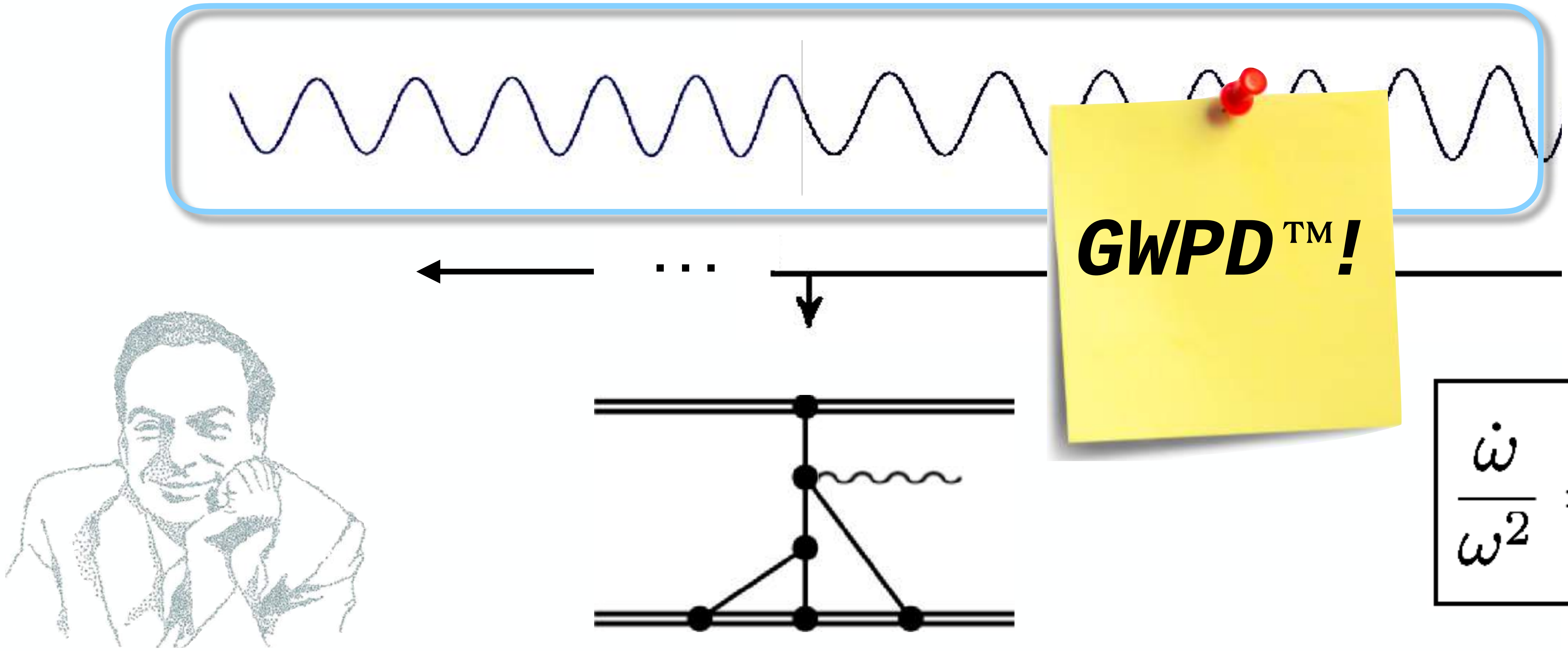
‘GW Precision Data’

1000+ cycles in band @ Design-Sensitivity

100+ events per year!

state of the art



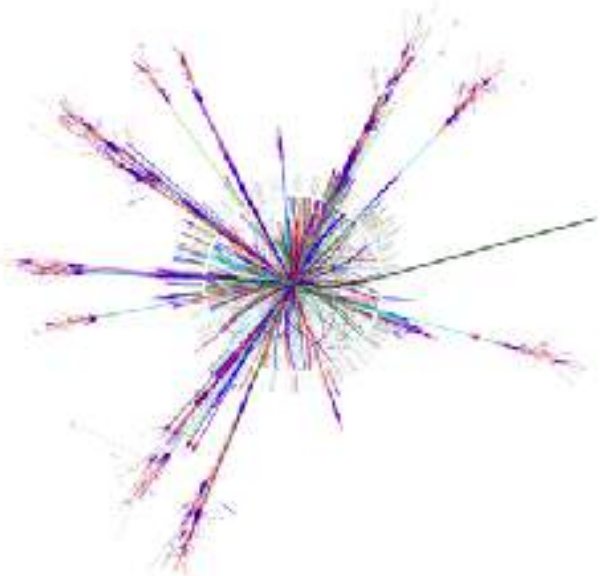


3.5PN order (almost 4PN)

$$\frac{\dot{\omega}}{\omega^2} = \underbrace{\frac{96}{5}\nu x^{5/2} \Big\{}1 + \overbrace{\cdots + [\cdots] x^{7/2}}\Big\}$$

$\nu \sim m_2/m_1$

$x \sim (v/c)^2$

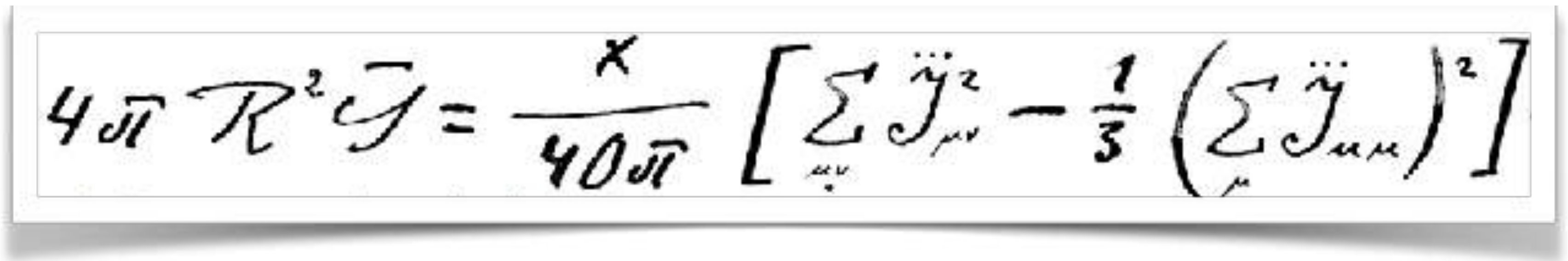


The effective field theorist's approach to gravitational dynamics

Physics Reports

Rafael A. Porto

Volume 633, 20 May 2016, Pages 1-104

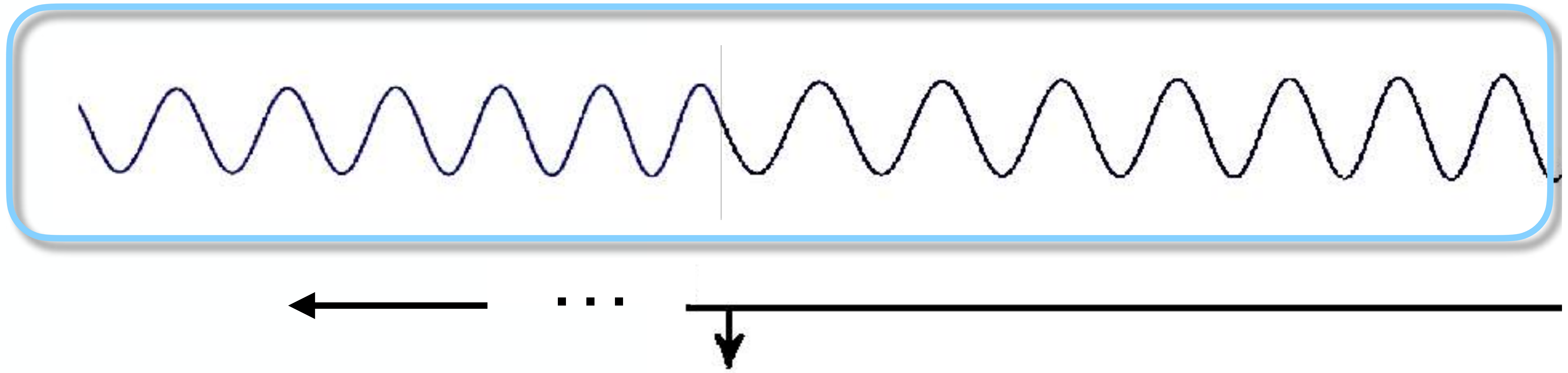


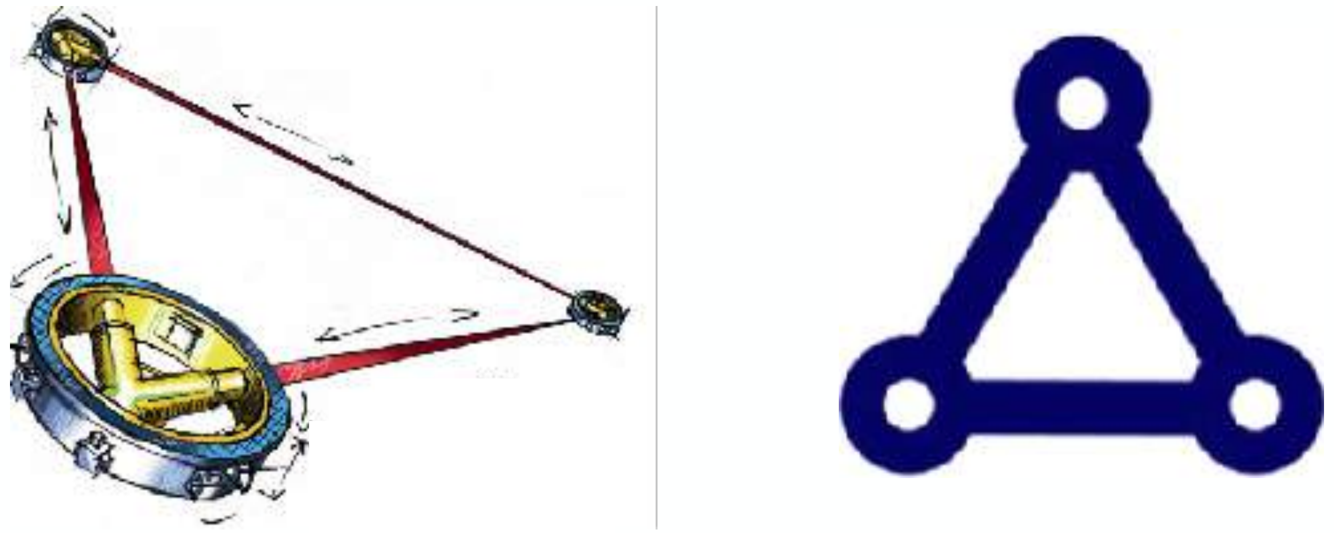
$$4\pi R^2 \bar{\mathcal{S}} = \frac{\varkappa}{40\pi}\left[\sum_{\mu\nu} \dddot{\mathcal{J}}_{\mu\nu}^2 - \frac{1}{3}\left(\sum_{\mu} \dddot{\mathcal{J}}_{\mu\mu}\right)^2\right]$$

‘GW Precision Data’

1000+ cycles in band @ Design-Sensitivity

100+ events per year!



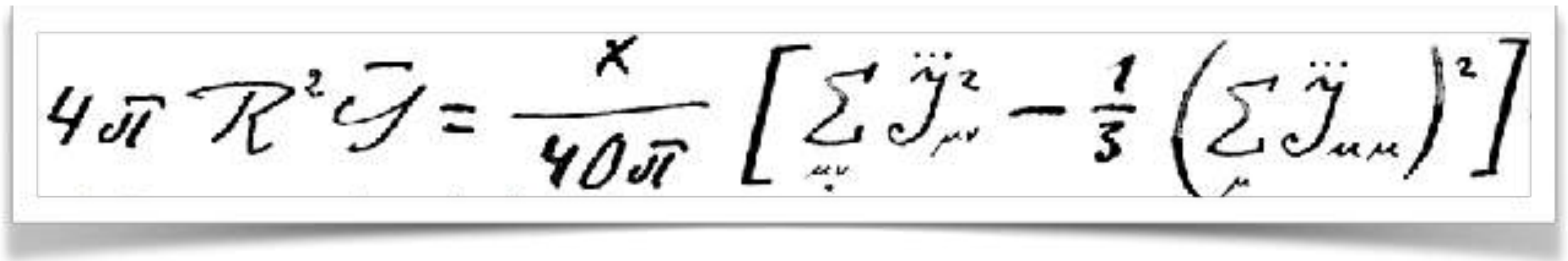


Are we ready for the future?

$$\boxed{\frac{\dot{\omega}}{\omega^2} = \frac{96}{5}\nu x^{5/2}\left\{1 + \cdots + [\cdots]x^{7/2}\right\}}$$

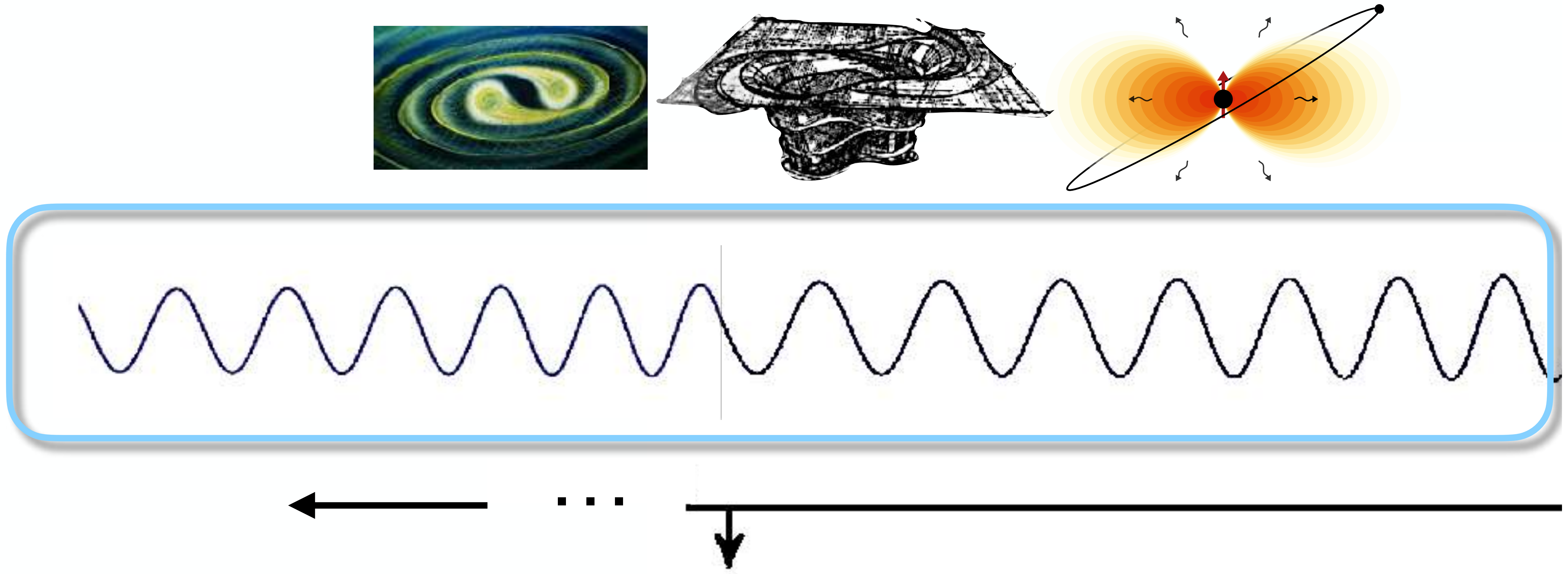
$\nu \sim m_2/m_1$

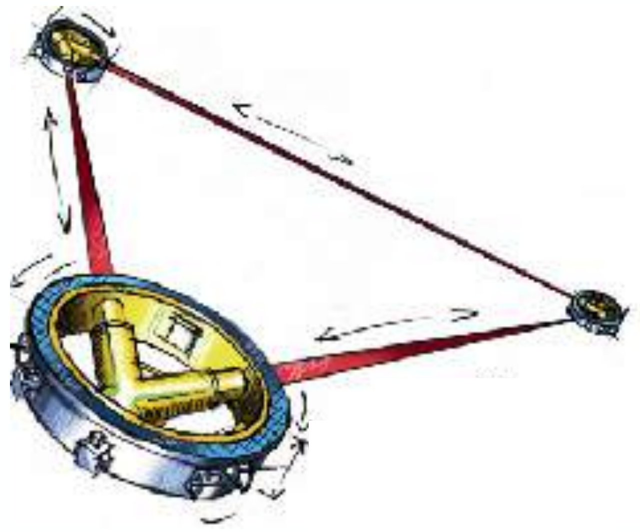
$x \sim (v/c)^2$



$$4\pi R^2 \bar{\mathfrak{J}} = \frac{\varkappa}{40\pi}\left[\sum_{\mu\nu} \dddot{\mathfrak{J}}_{\mu\nu}^2 - \frac{1}{3}\left(\sum_{\mu} \dddot{\mathfrak{J}}_{\mu\mu}\right)^2\right]$$

Theoretical uncertainties dominate over planned empirical reach





Not Good Enough

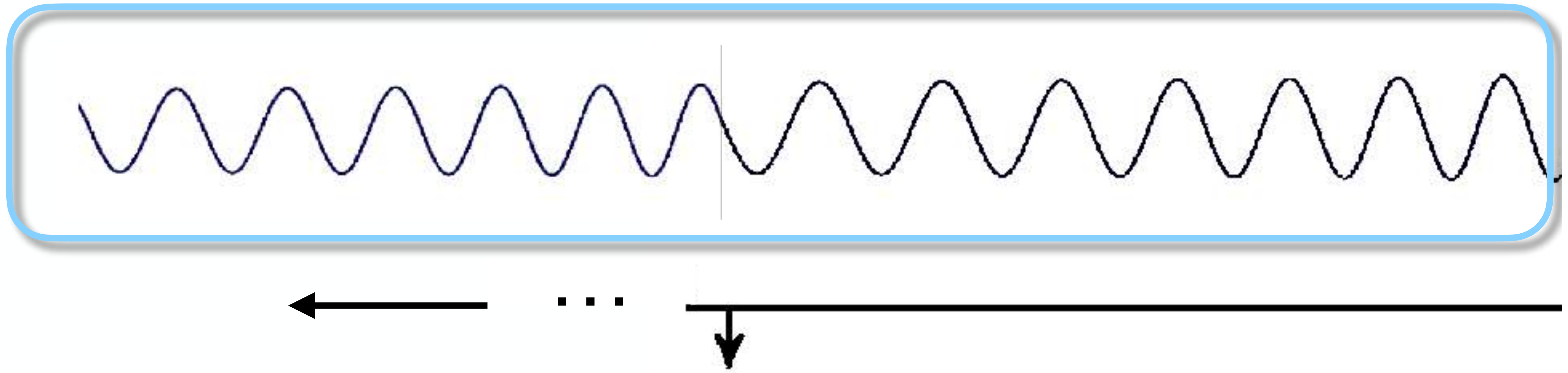
$$\frac{\dot{\omega}}{\omega^2} = \frac{96}{5}\nu x^{5/2}\left\{1 + \cdots + [\cdots]x^{7/2}\right\}$$

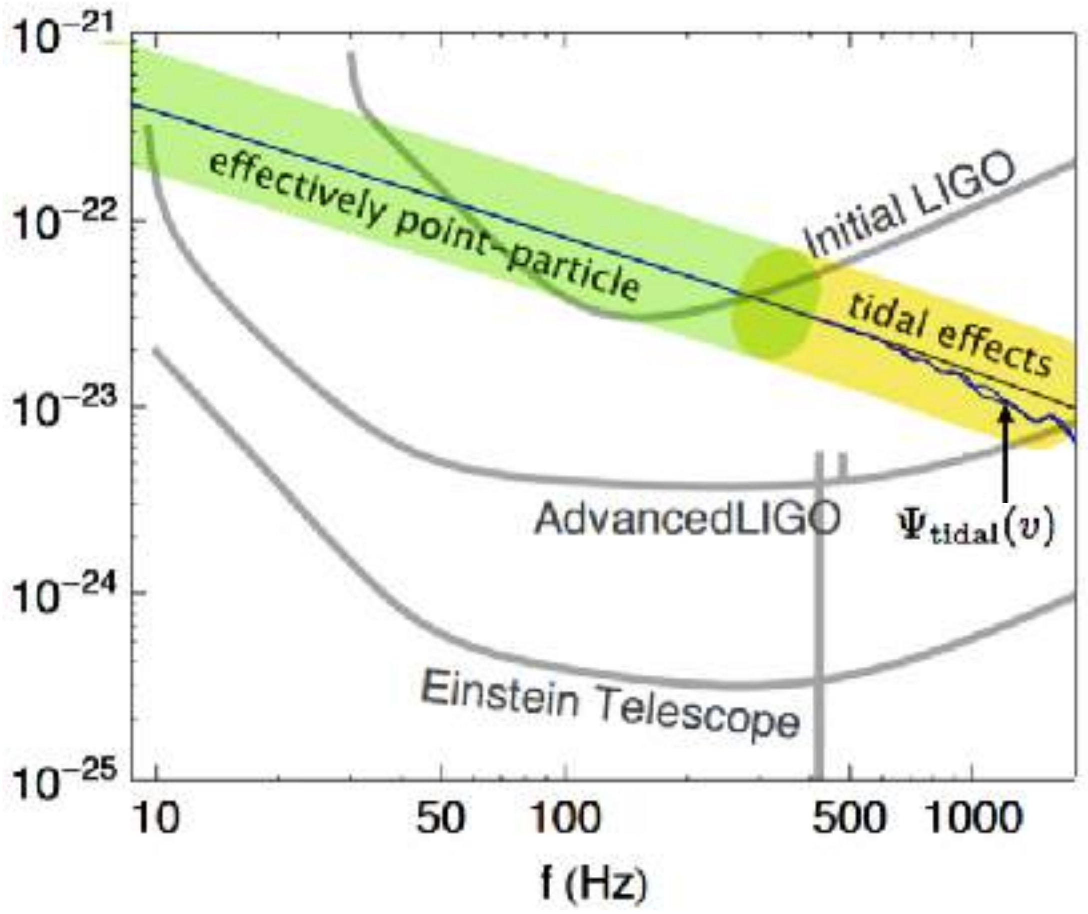
$\nu \sim m_2/m_1$

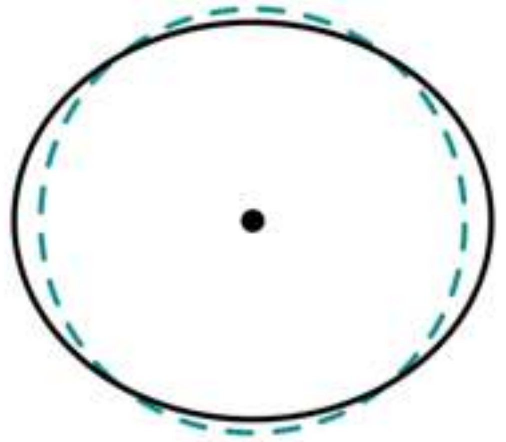
$x \sim (v/c)^2$

- **Gravitational-wave experiments on ground and in space** require **more accurate waveform** models: new **theoretical challenges** and **opportunities**.

We haven’t reached the analytic precision to distinguish between compact bodies!







$$Q_{ij} = C_E E_{ij}$$

$$C_E \sim R^5$$

(“susceptibility”)

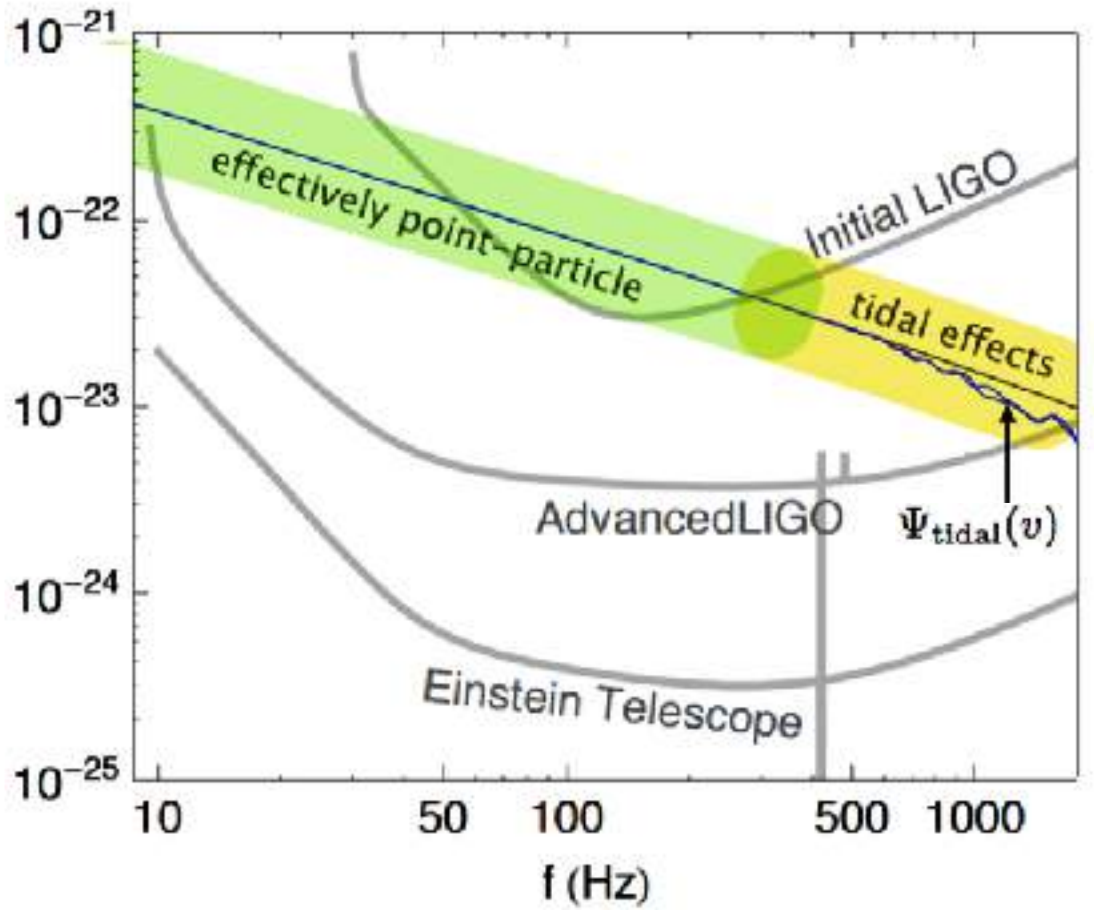
$$\frac{\dot{\omega}}{\omega^2} = \frac{96}{5}\nu x^{5/2}\left\{1 + \cdots + [\cdots]x^{7/2} + \mathcal{O}(x^4) + \mathcal{O}(x^5)\right\}$$

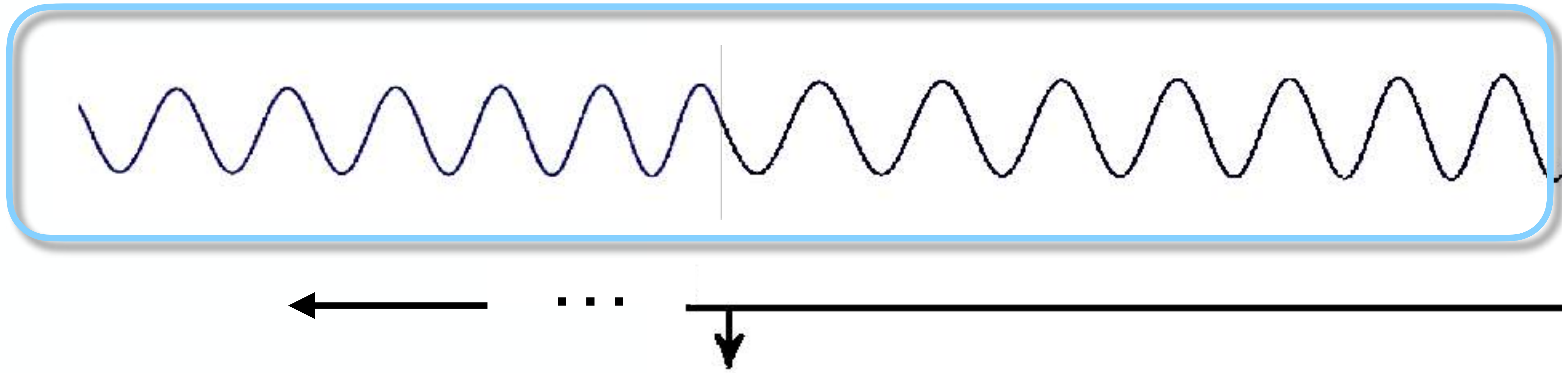
N⁵LO
5PN

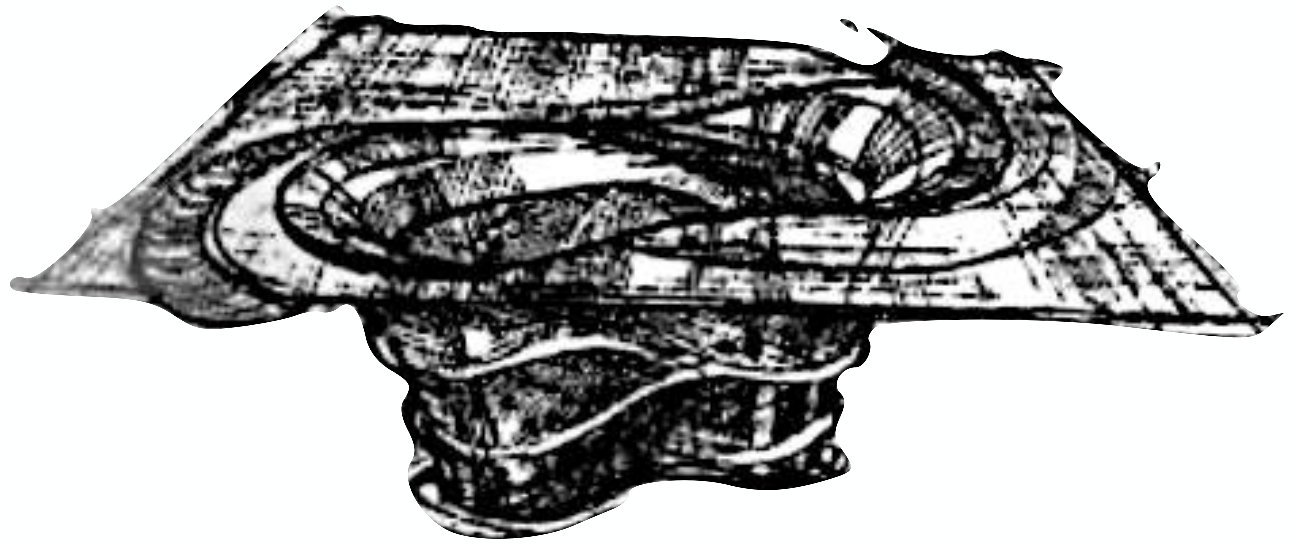
$$\Psi(v) = \Psi_{\rm PP}(v) + \Psi_{\rm tidal}(v)$$

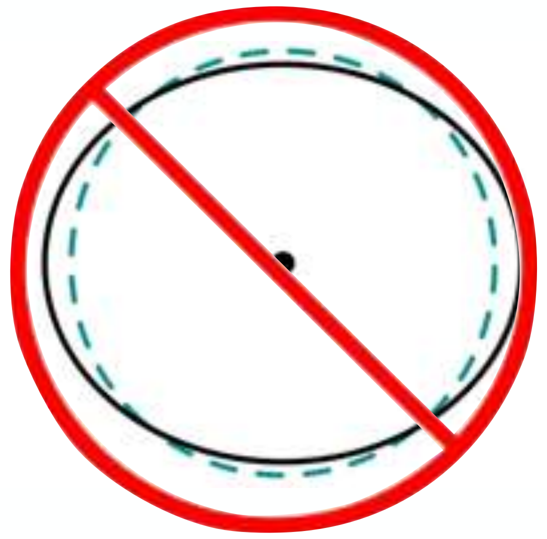
e.g. Equation of State of Neutron Stars

We haven’t reached the analytic precision to distinguish between compact bodies!









$$\frac{\dot{\omega}}{\omega^2} = \frac{96}{5}\nu x^{5/2}\left\{1+\cdots+[\cdots]x^{7/2} + \mathcal{O}(x^4) + \mathcal{O}(x^5)\right\}$$

N⁵LO

5PN

$$\Psi(v) = \Psi_{\rm PP}(v) + \cancel{\Psi_{\rm tidal}(v)}$$

vanishes for black-holes in Einstein’s gravity (4d)

Fortschr. Phys. 64, No. 10, 723–729 (2016) / DOI 10.1002/prop.201600064

The tune of love and the *nature(ness)* of spacetime

*Rafael A. Porto**

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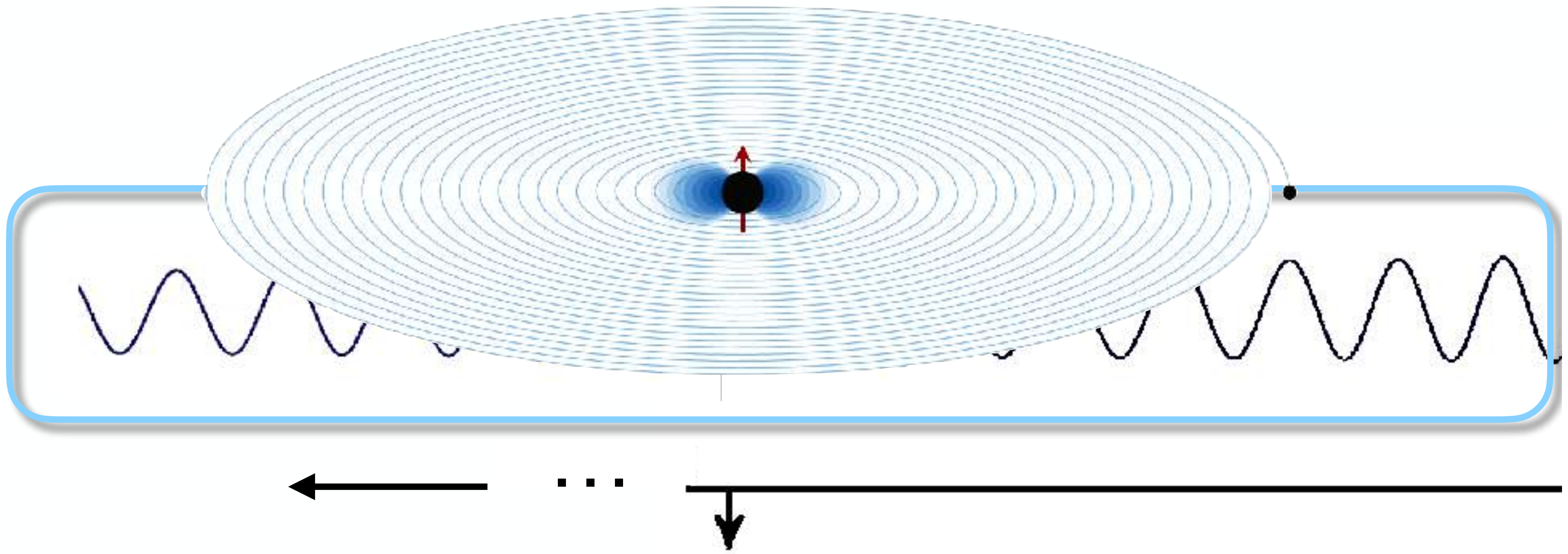
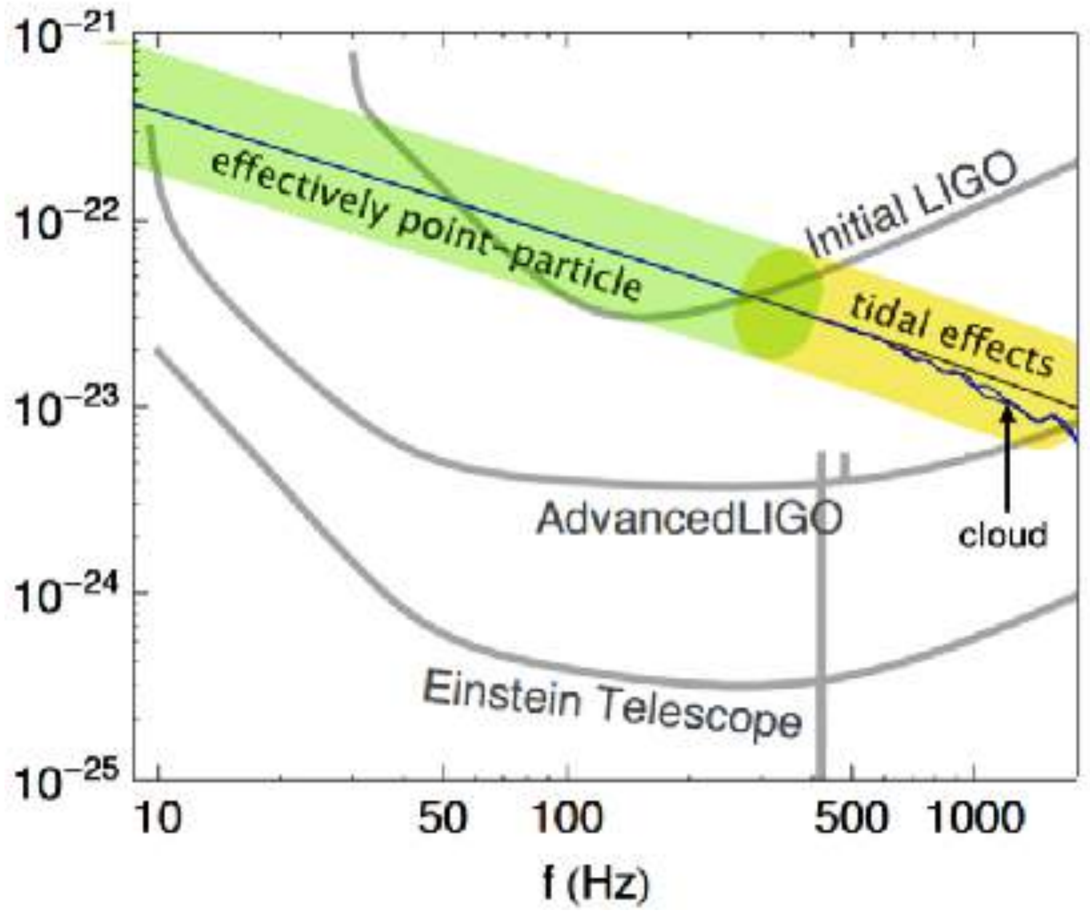
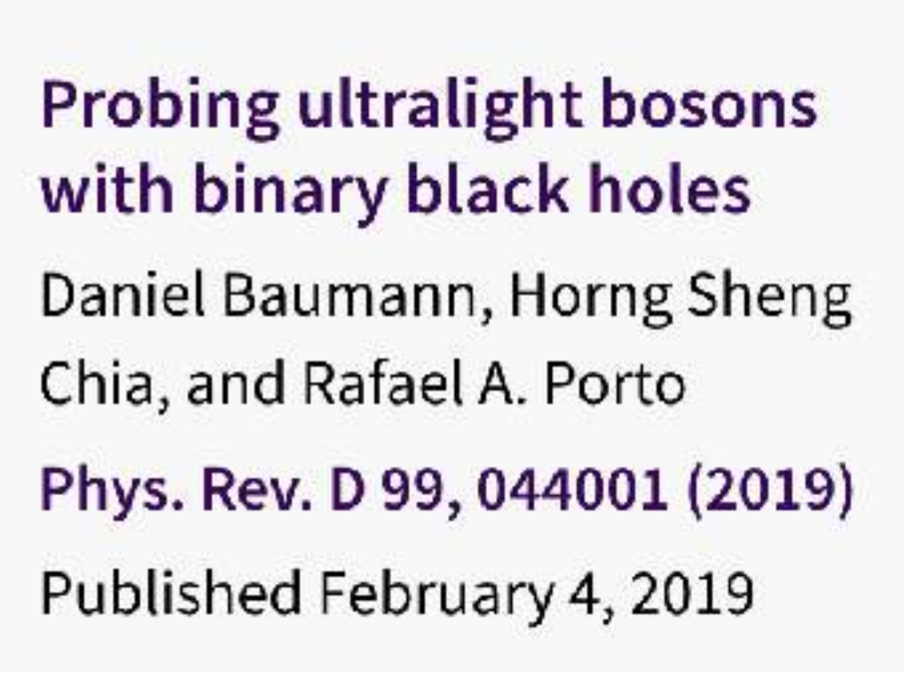






Probing ultralight bosons with binary black holes

Daniel Baumann, Horng Sheng Chia, and Rafael A. Porto

Phys. Rev. D 99, 044001 (2019)

Published February 4, 2019

Physics See Synopsis:

Black Holes Could Reveal New Ultralight Particles

$$\frac{\dot{\omega}}{\omega^2} = \frac{96}{5}\nu x^{5/2}\left\{1 + \cdots + [\cdots]x^{7/2} + \mathcal{O}(x^4) + \mathcal{O}(x^5)\right\} \quad N^5LO \quad 5PN$$

$$\Psi(v) = \Psi_{\rm PP}(v) + \Psi_{\rm tidal}(v)$$

‘New Physics’
Threshold

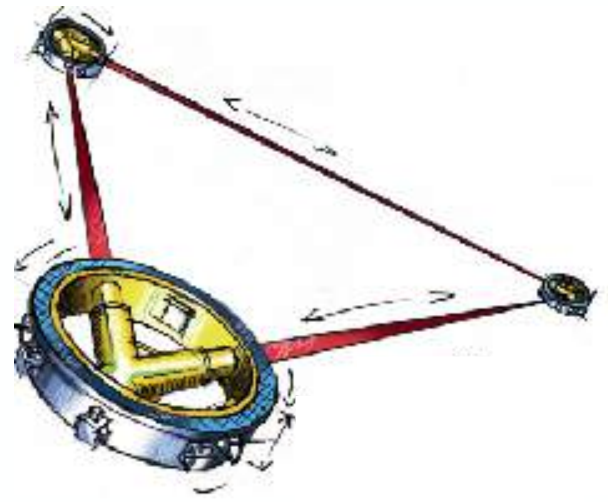
~~‘Standard Model’ Background!~~

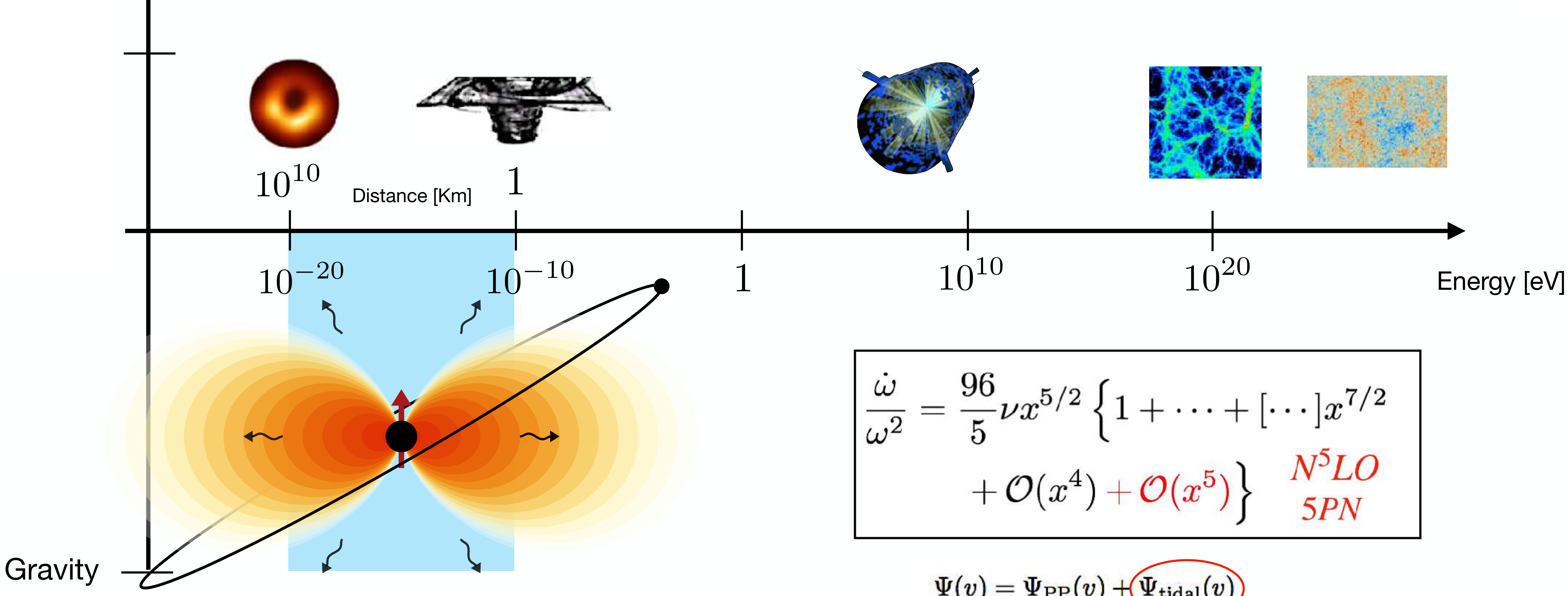
NEW frontier in particle physics

Gravitational Collider Physics

Baumann Chia **RAP** 19’

Baumann Chia **RAP** Stout 20’





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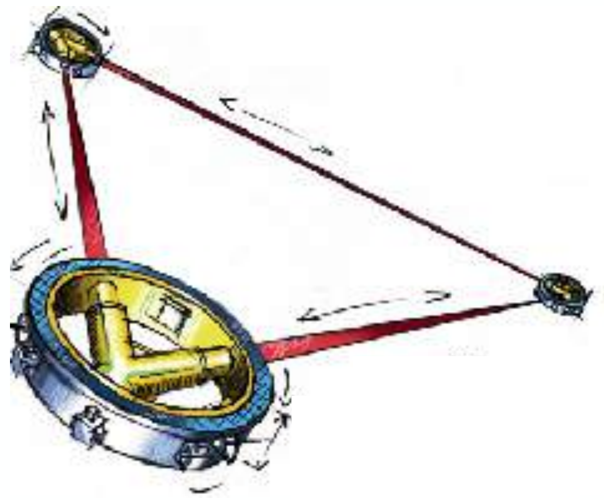
‘New Physics’
Threshold

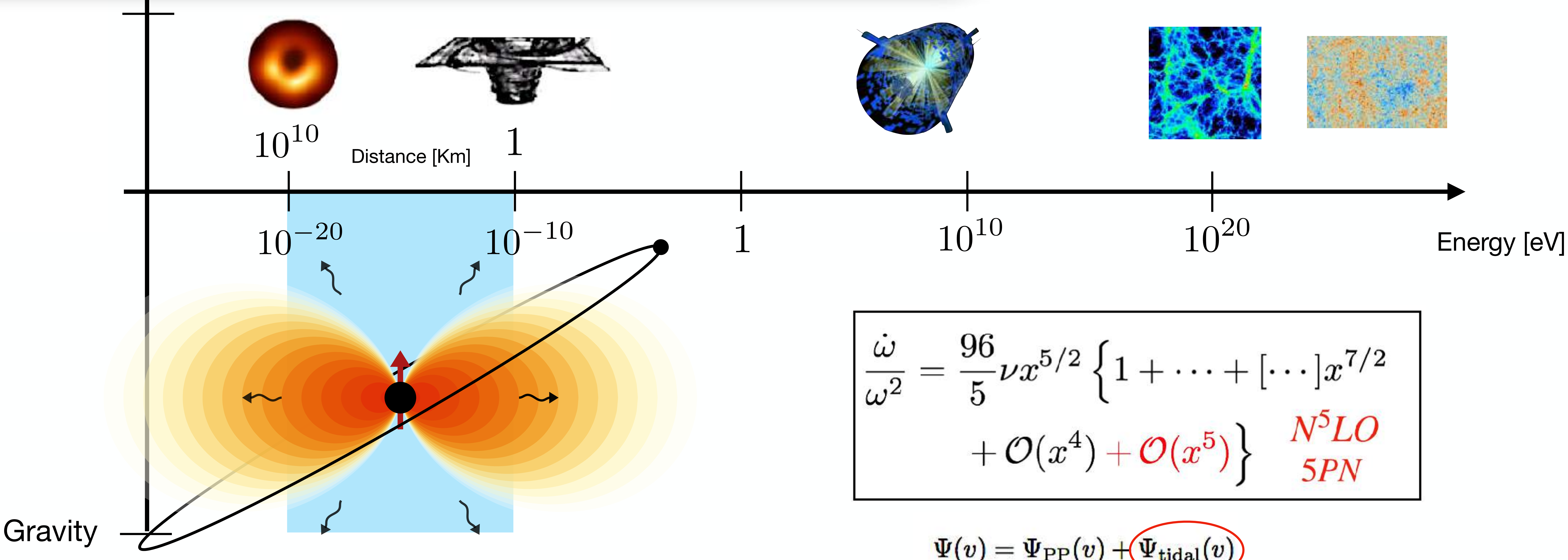
‘Standard Model’
Background!

NEW frontier in particle physics



Discovery Potential =

Precise Theoretical Predictions



$$\frac{\dot{\omega}}{\omega^2} = \frac{96}{5}\nu x^{5/2}\left\{1 + \cdots + [\cdots]x^{7/2} + \mathcal{O}(x^4) + \mathcal{O}(x^5)\right\} \quad N^5LO \quad 5PN$$

$$\Psi(v) = \Psi_{\mathrm{PP}}(v) + \Psi_{\mathrm{tidal}}(v)$$

'New Physics'
Threshold

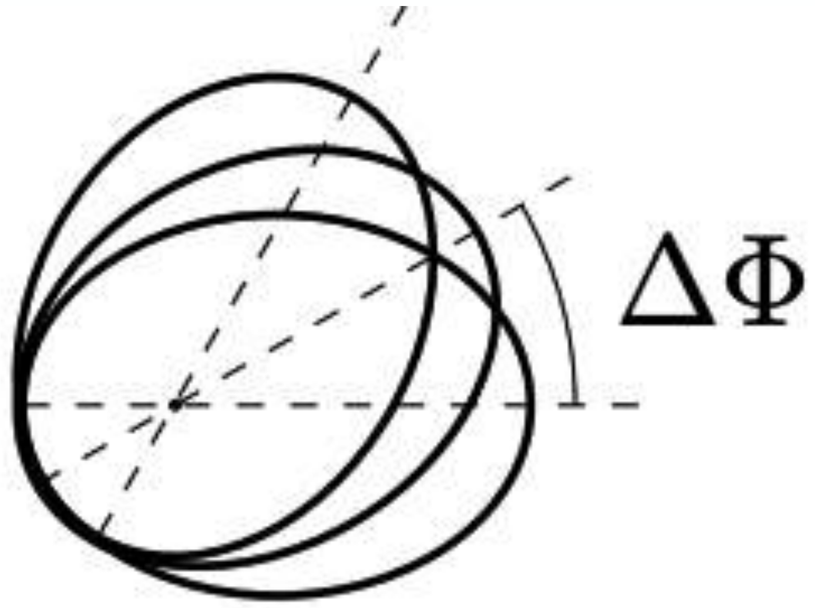
~~**'Standard Model'**~~
~~**Background!**~~

Outline remaining of the talk…

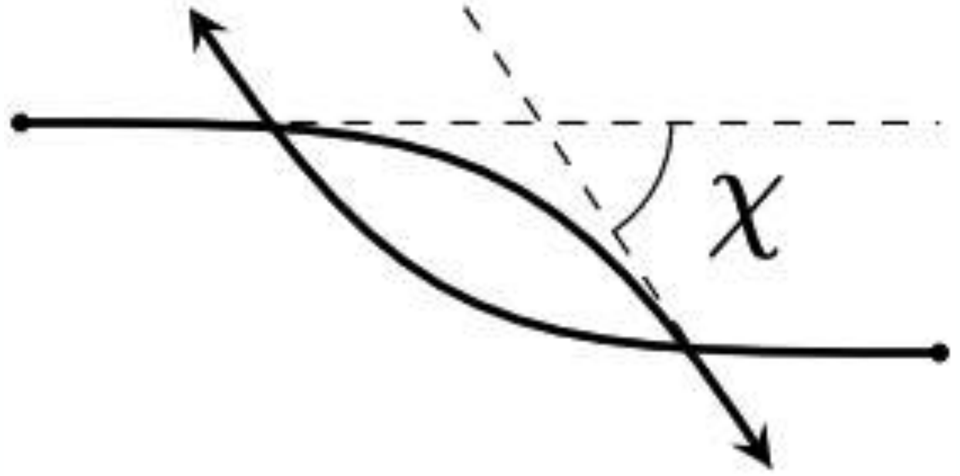
Discovery Potential =

Precise Theoretical Predictions

- **Part I: Bound**

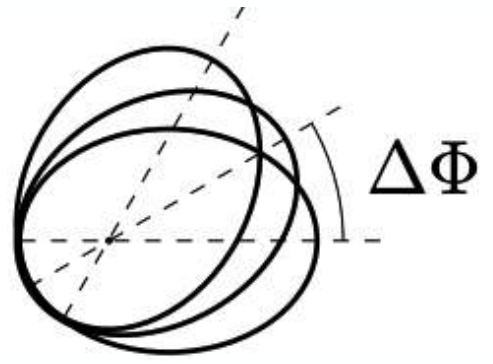


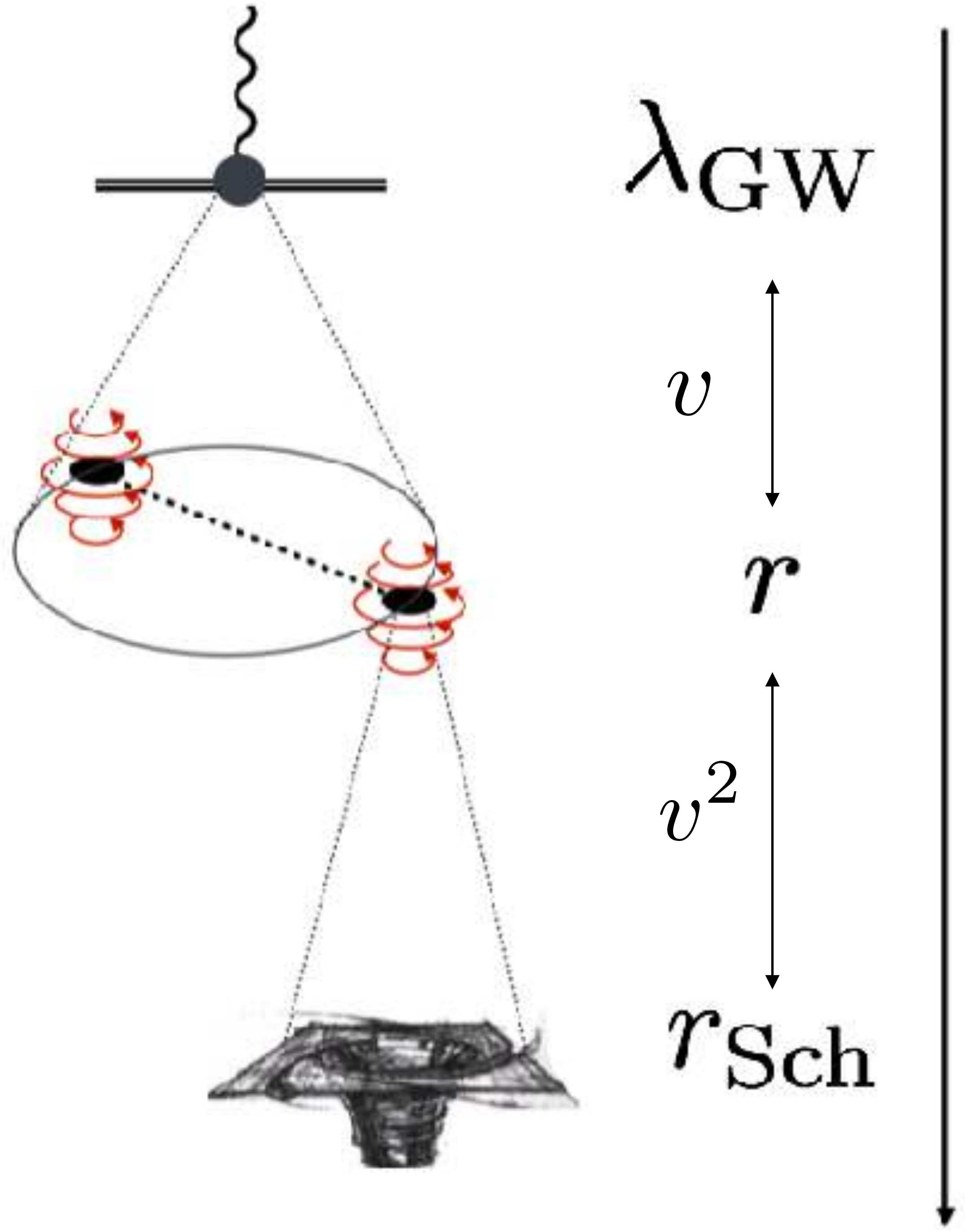
- **Part II: Boundary2Bound**



EFT approach to GW physics *PN*

Goldberger Rothstein (2006)
Porto (2006)
Goldberger Ross (2009)





- **Separation of Scales (2-body in GR):**

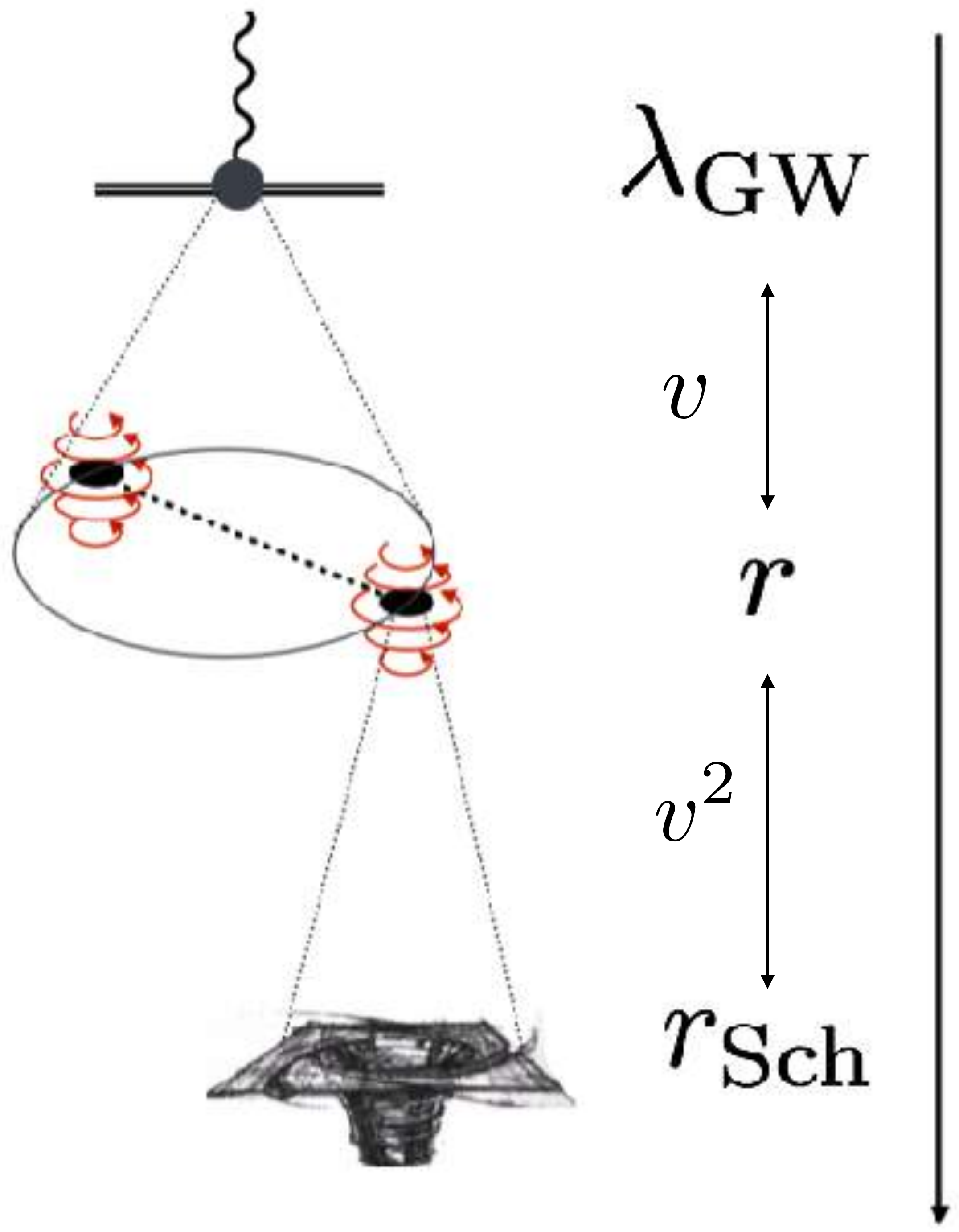
$$r_{\rm Sch} \ll r \ll \lambda_{\rm GW}$$

- **Effective Field Theory:**
  One scale at a time

- **Tools from HEP:** Feynman diagrams, regularization/renormalization/RG-flow

The effective field theorist's approach to gravitational dynamics
Physics Reports
Rafael A. Porto
Volume 633, 20 May 2016, Pages 1-104

EFT approach to GW physics PN



$$e^{iW} = \int D[\lambda_{\rm rad}^{-1}]\, D[r^{-1}]\, D[r_s^{-1}]\, e^{iS_{\rm full}}$$

Radiation Modes

$(k_0 \sim |\boldsymbol{k}|)$

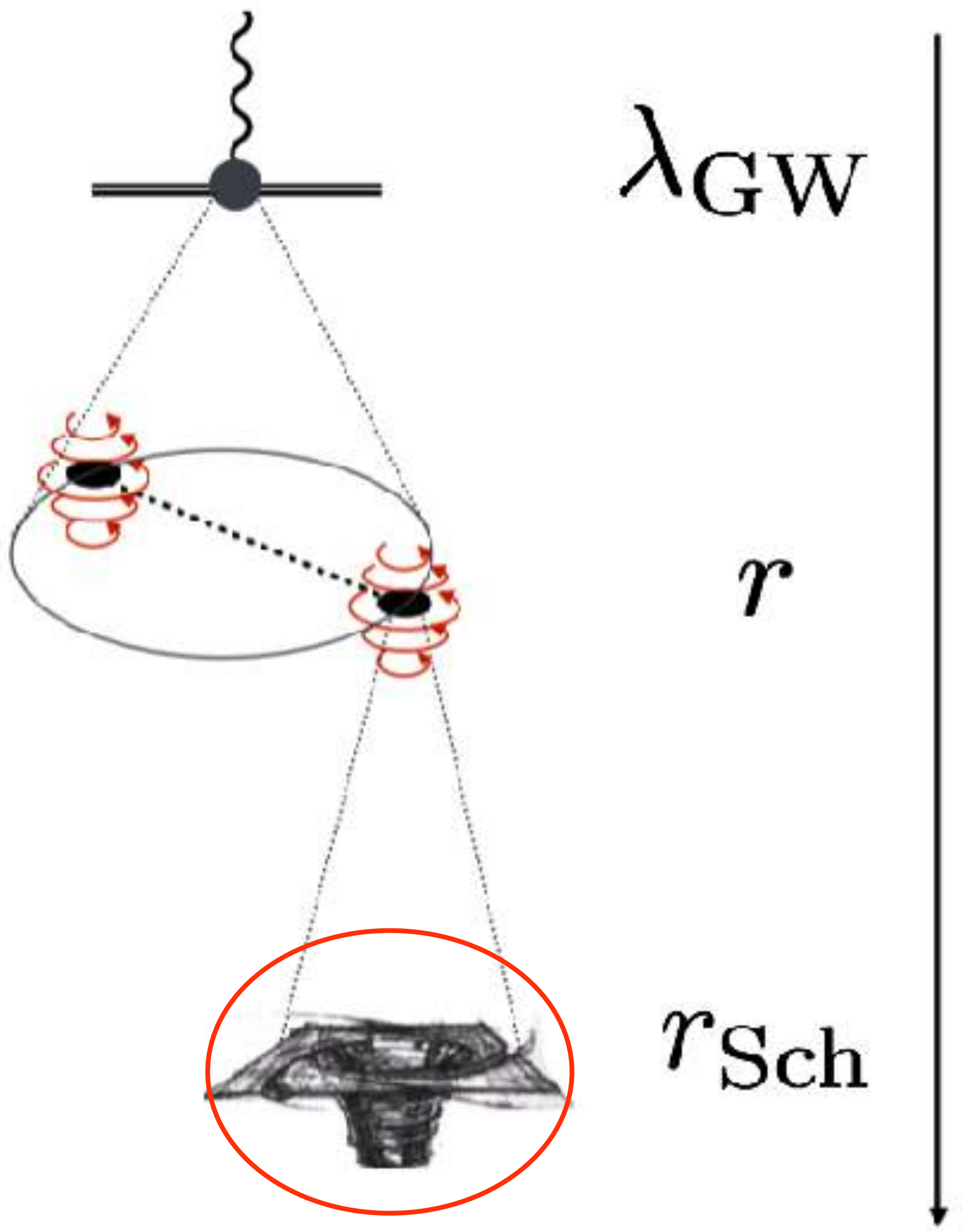
Potential Modes

$(k_0 \ll |\boldsymbol{k}|)$

Finite size

$$\int D[\mu]\, e^{iS} \to e^{iS_{\rm eff}}$$

EFT approach to GW physics PN



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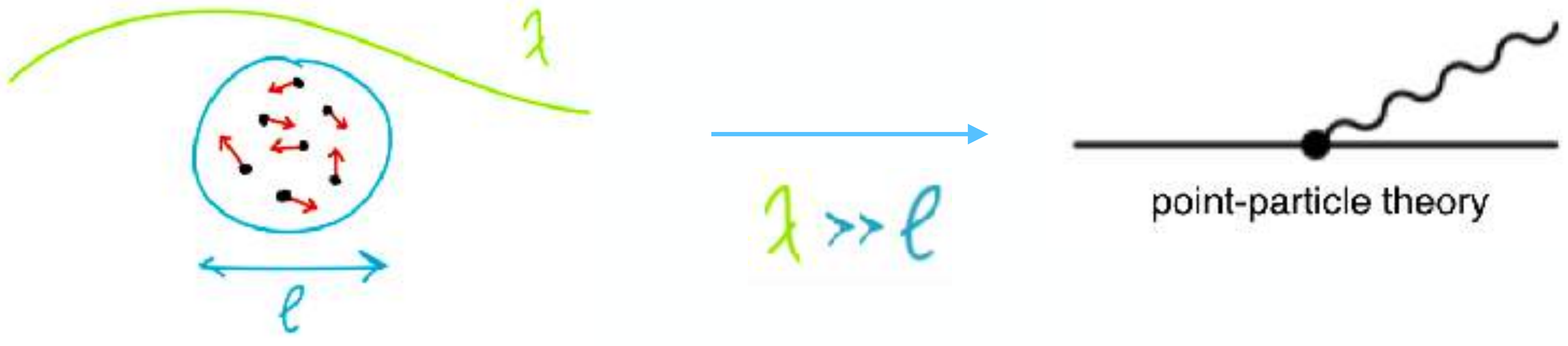
Radiation Modes

Potential Modes

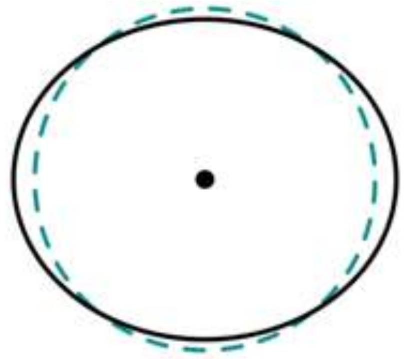
Finite size



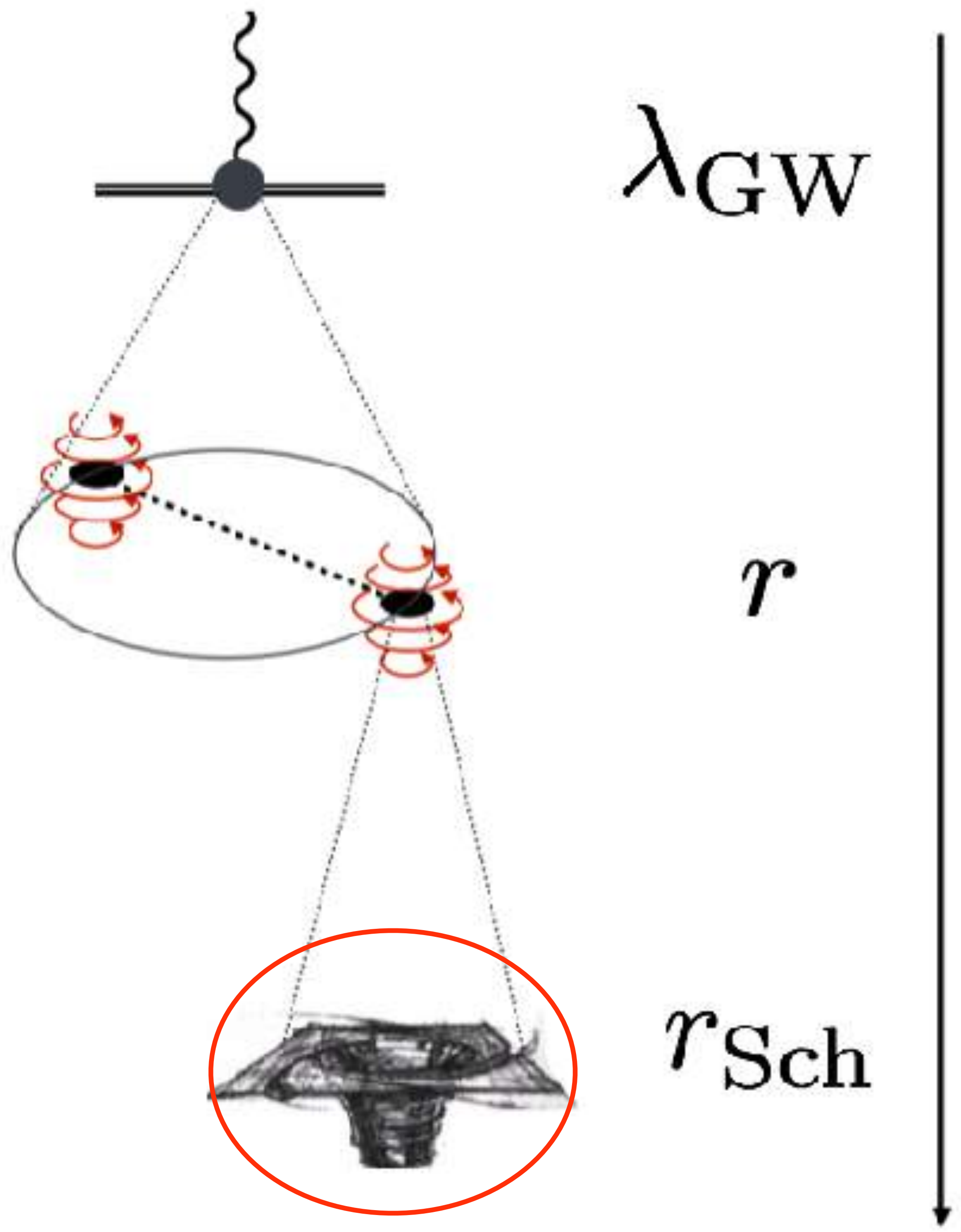
$$S_{\rm pp} = -\sum_A \frac{m_A}{2} \int \mathrm{d}\tau_A g_{\mu\nu}\left(x_A\left(\tau_A\right)\right) v_A^\mu\left(\tau_A\right) v_A^\nu\left(\tau_A\right) - \frac{1}{2} \int \mathrm{d}\tau_A S_{ab}(\tau_A) \omega_\mu^{ab}(\tau_A) v_A^\mu(\tau_A).$$

finite-size effects

$$\frac{C_{ES^2}}{2m_A} \int \underbrace{d\tau_A E_{ab} S_A^{ac} S_{cA}^b}_{\textbf{spin-induced moments}} + c_{E^2} \int \underbrace{d\tau_A E_{\mu\nu} E^{\mu\nu}}_{\textbf{tidal effects}} + \ldots$$



EFT approach to GW physics PN



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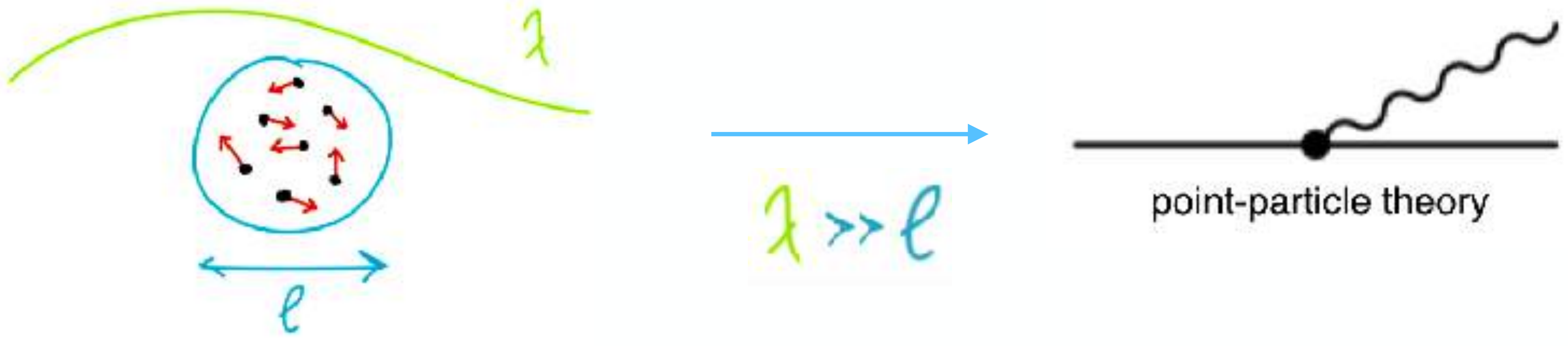
Potential Modes

Finite size



$$\frac{\dot\omega}{\omega^2} = \frac{96}{5}\nu x^{5/2}\left\{1+\cdots+[\cdots]x^{7/2} + \mathcal{O}(x^4) + \mathcal{O}(x^5)\right\}$$

N^5LO

$5PN$

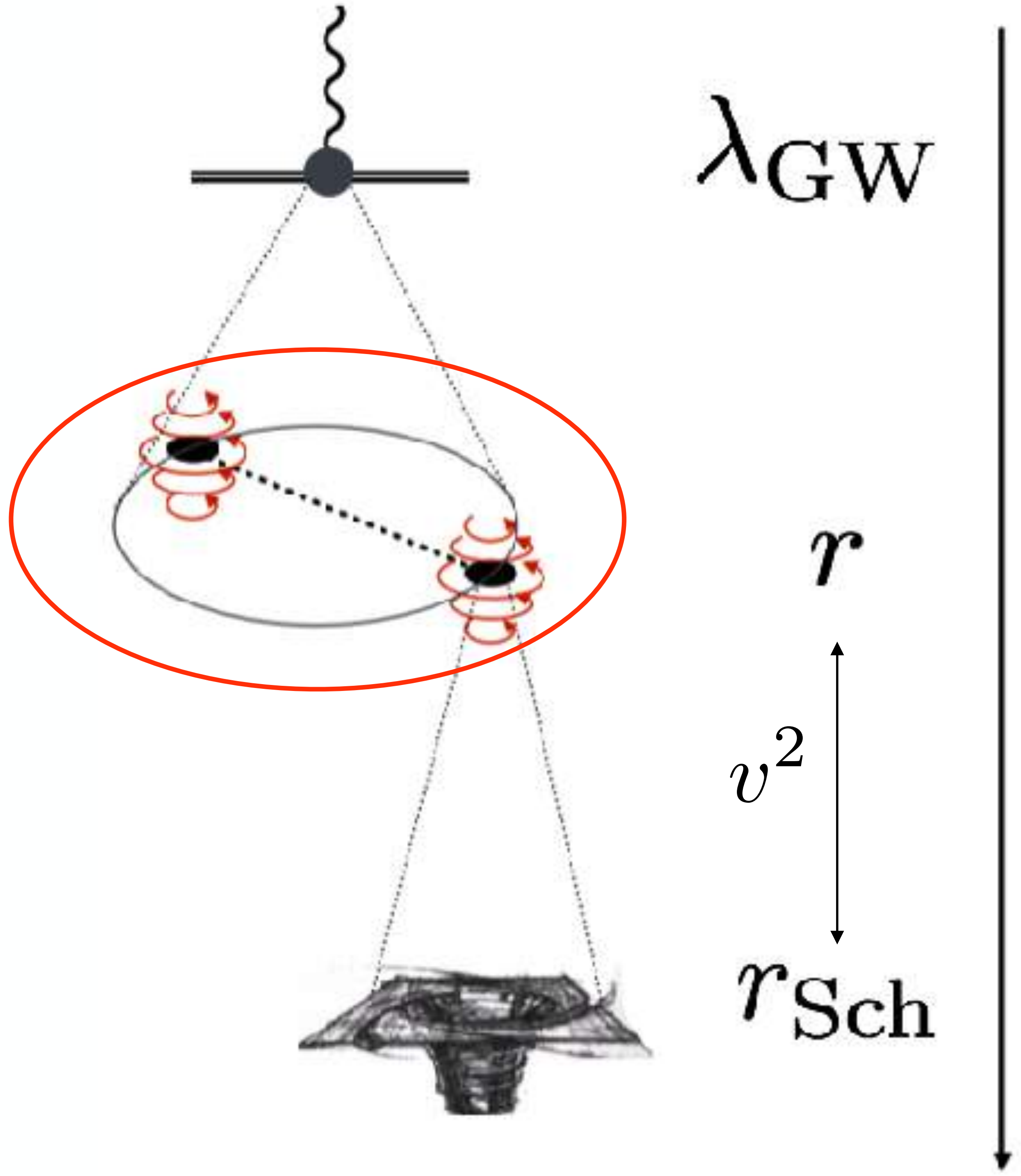
'New Physics'
Threshold

~~'Standard Model'
Background!~~

vanishes for BHs in d=4!

$$\cancel{c_{E^2}} \int \underbrace{d\tau_A E_{\mu\nu} E^{\mu\nu}}_{\text{tidal effects}}$$

EFT approach to GW physics *PN*



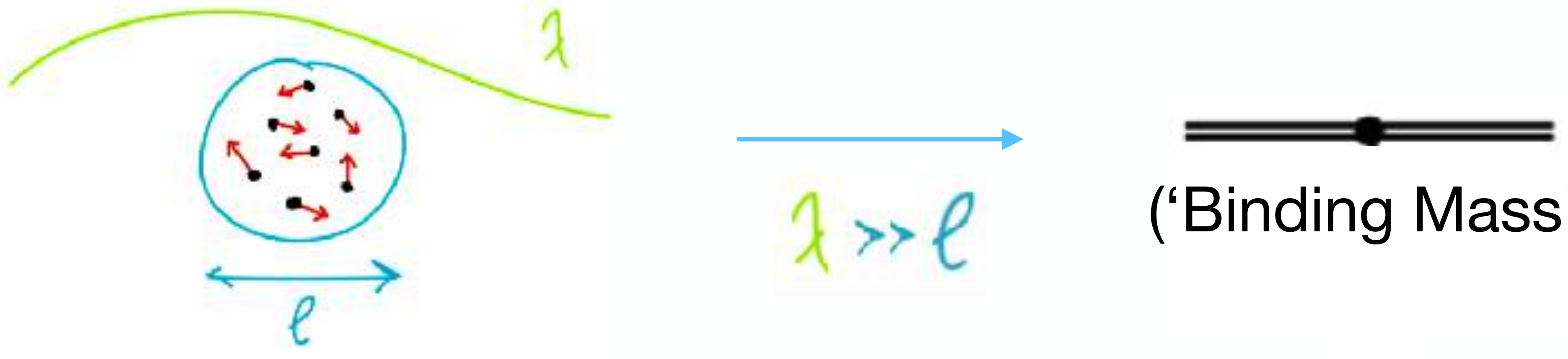
$$e^{iW} = \int D[\lambda_{\rm rad}^{-1}]\, D[r^{-1}]\, D[r_s^{-1}]\, e^{iS_{\rm full}}$$

Radiation Modes

Potential Modes

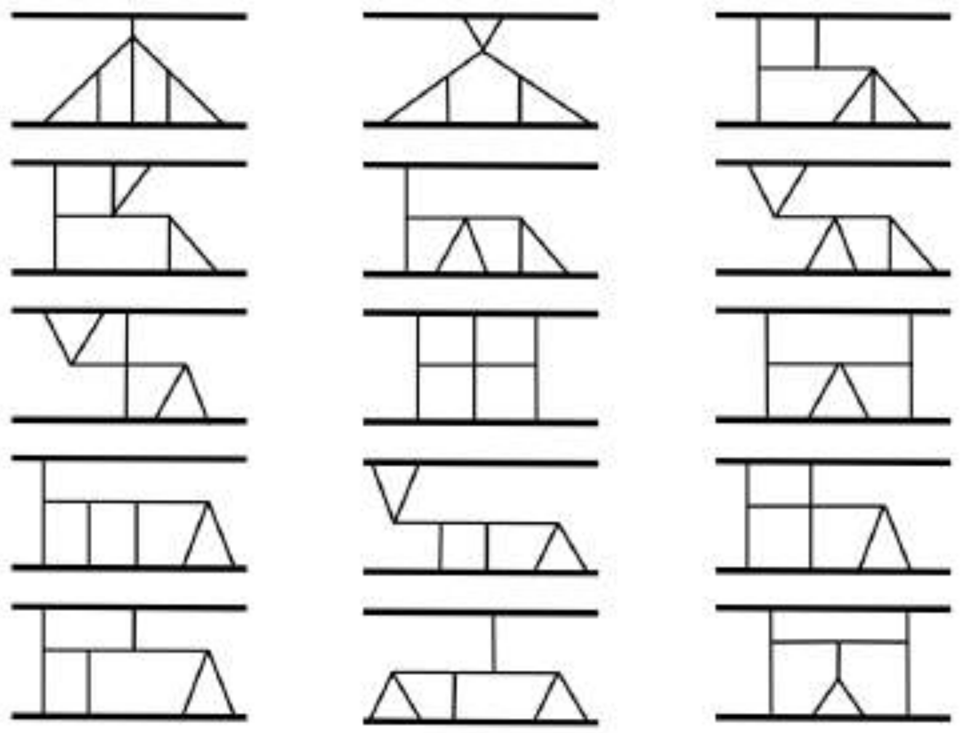
$(k_0 \ll |\boldsymbol{k}|)$

Finite size



('Binding Mass')

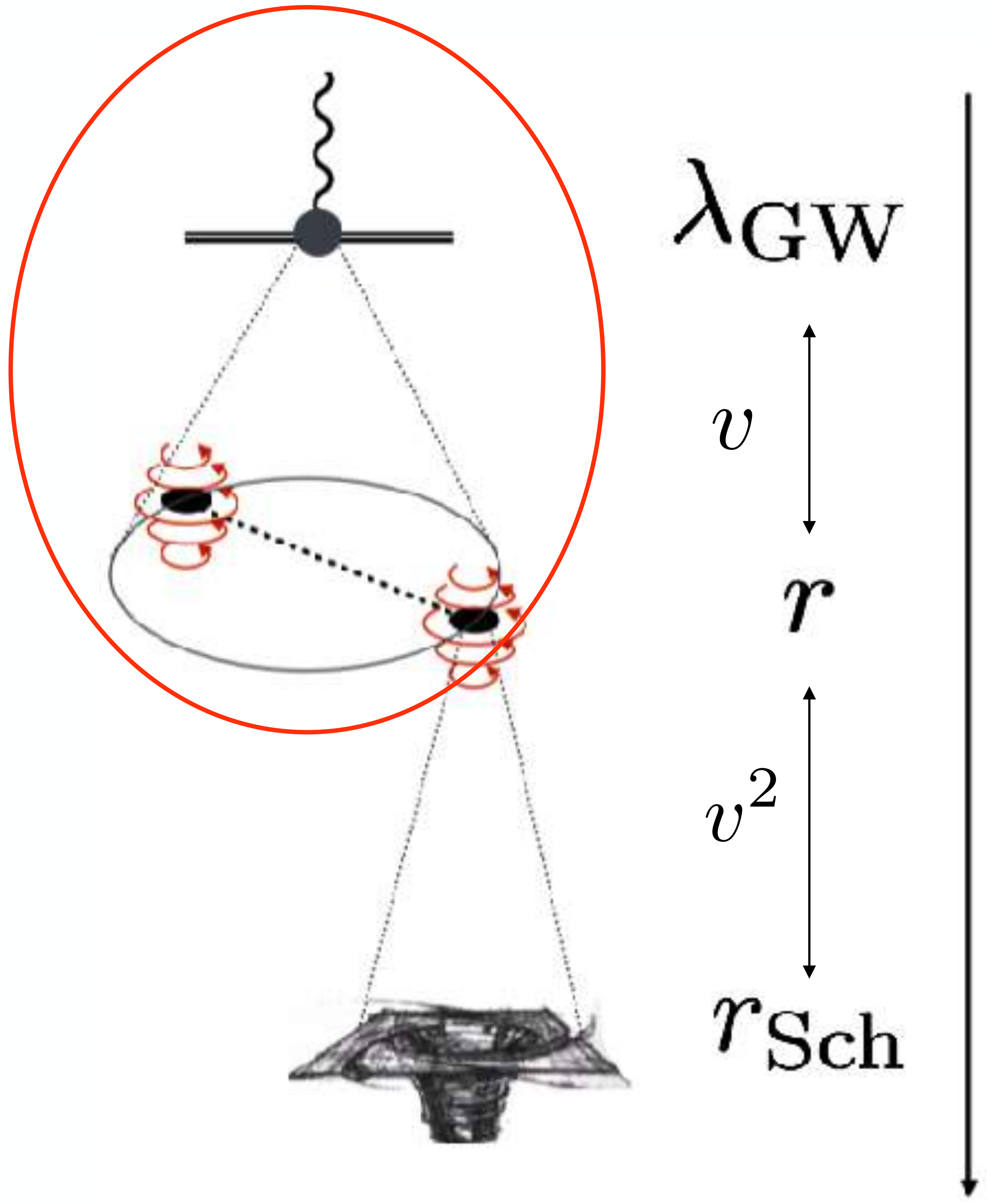
$$\underbrace{\mathrm{Re}\, W[\boldsymbol{x}_a]}_{\text{binding}}$$



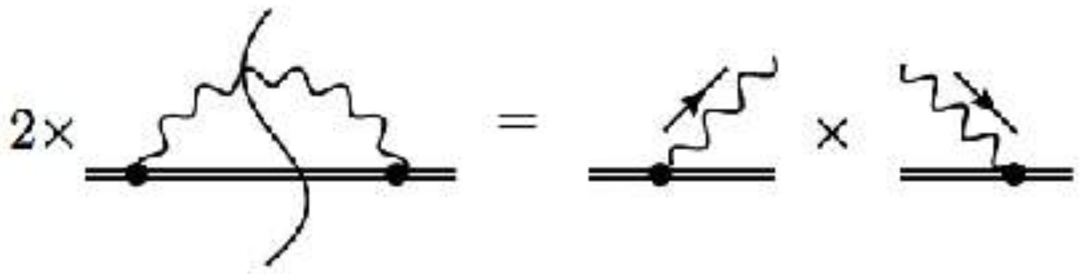
$$\frac{1}{p_0^2 - \boldsymbol{p}^2} \simeq -\frac{1}{\boldsymbol{p}^2}\left(1 + \frac{p_0^2}{\boldsymbol{p}^2} + \cdots\right) .$$

$i\epsilon$ prescription
irrelevant

EFT approach to GW physics *PN*



$$\underbrace{\mathrm{Re}\, W[x_a]}_{\text{binding}} + i\, \underbrace{\mathrm{Im}\, W[x_a]}_{\text{radiation}}$$



classical optical theorem!

$$e^{iW} = \int \overbrace{D[\lambda_{\rm rad}^{-1}]\, D[r^{-1}]}^{\text{decoupling only in space!}}\, D[r_s^{-1}]\, e^{iS_{\rm full}}$$

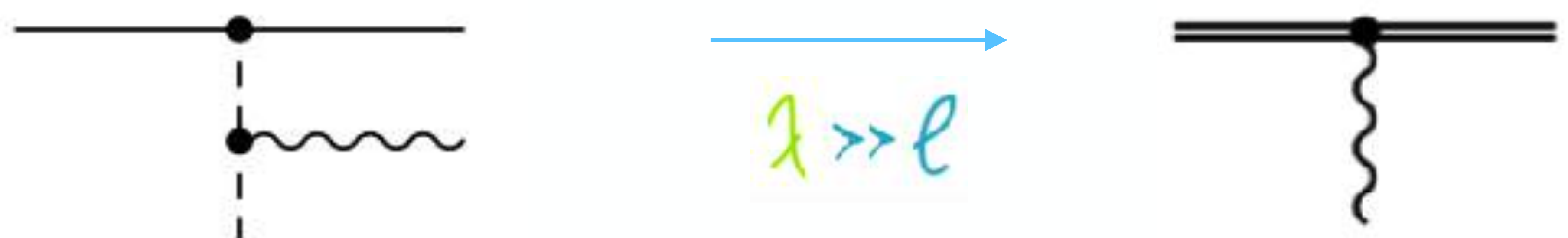
Radiation Modes

$(k_0 \sim |\boldsymbol{k}|)$

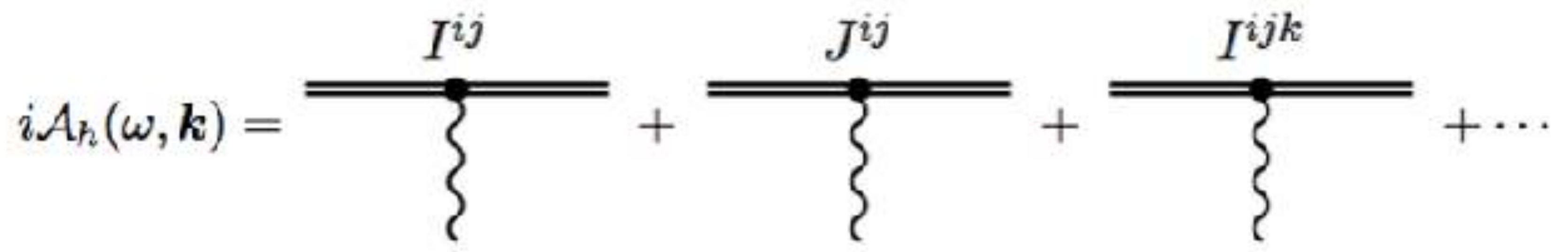
Potential Modes

$(k_0 \ll |\boldsymbol{k}|)$

Finite size



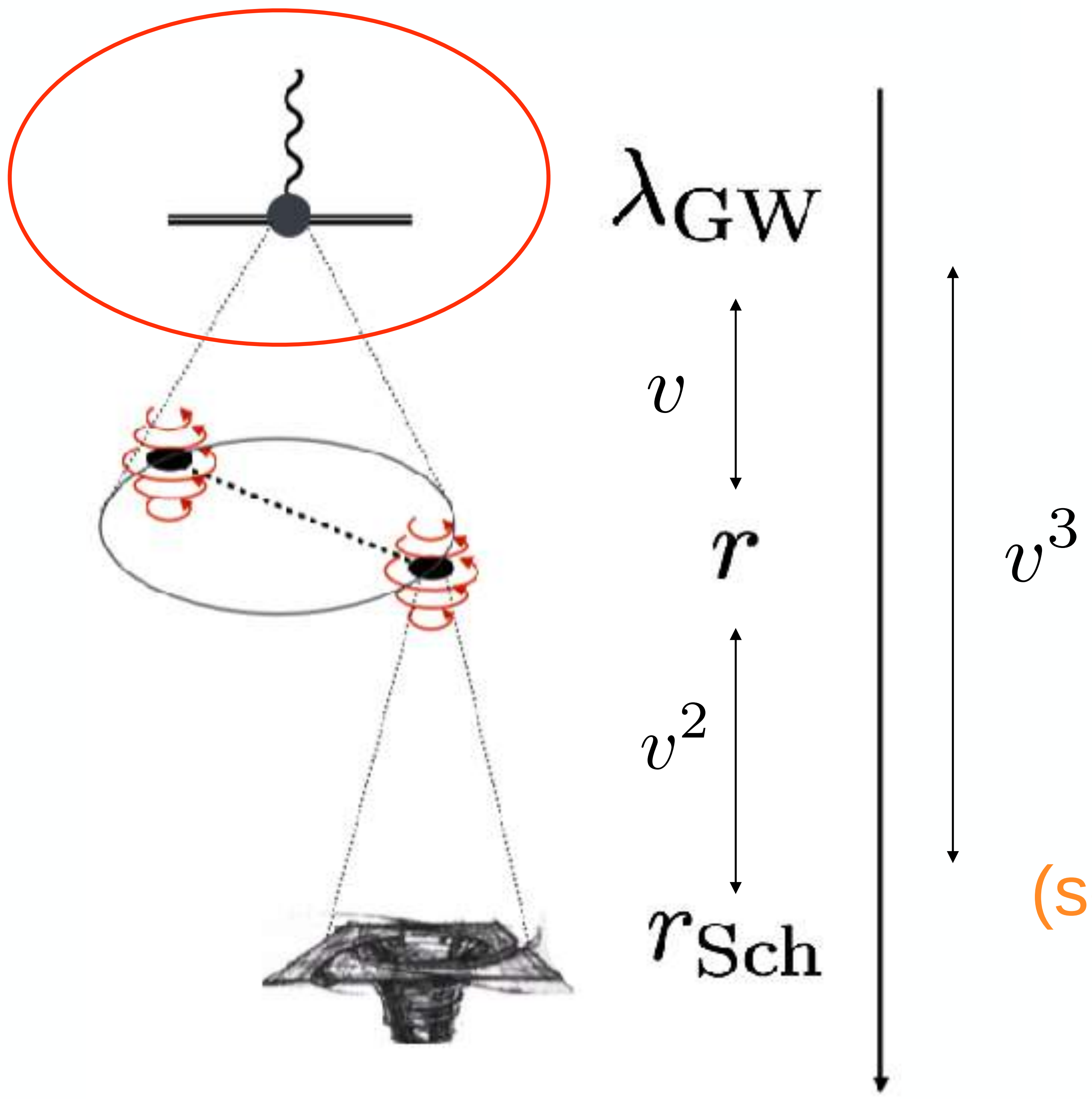
$$e^{i\mathbf{k}\cdot\mathbf{x}} = 1 + i\mathbf{k}\cdot\mathbf{x} + \cdots$$



Time-dependent multipole moments

$$\sum_{\ell=2}\left(\frac{1}{\ell!}I^L(\tau)\nabla_{L-2}E_{i_{\ell-1}i_\ell} - \frac{2\ell}{(2\ell+1)!}J^L(\tau)\nabla_{L-2}B_{i_{\ell-1}i_\ell}\right)$$

EFT approach to GW physics



$$e^{iW} = \int D[\lambda_{\rm rad}^{-1}]\, D[r^{-1}]\, D[r_s^{-1}]\, e^{iS_{\rm full}}$$

Radiation Modes ($k_0 \sim |\boldsymbol{k}|$)

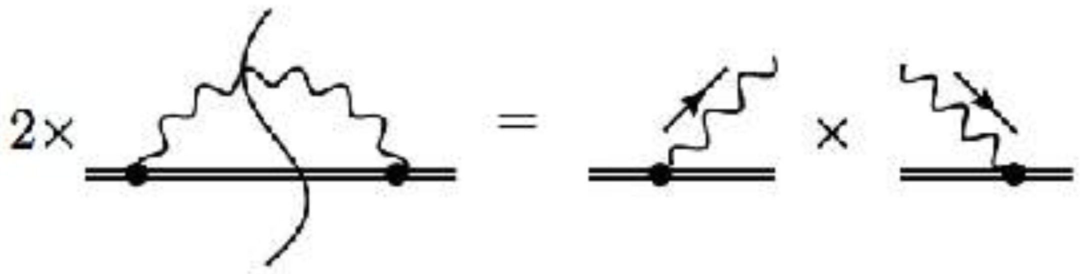
Potential Modes ($k_0 \ll |\boldsymbol{k}|$)

Finite size

“Tail effect”
(scattering off the geometry)

$$i\mathcal{A}^{(1)}_{\rm tail}(\boldsymbol{k}) =$$

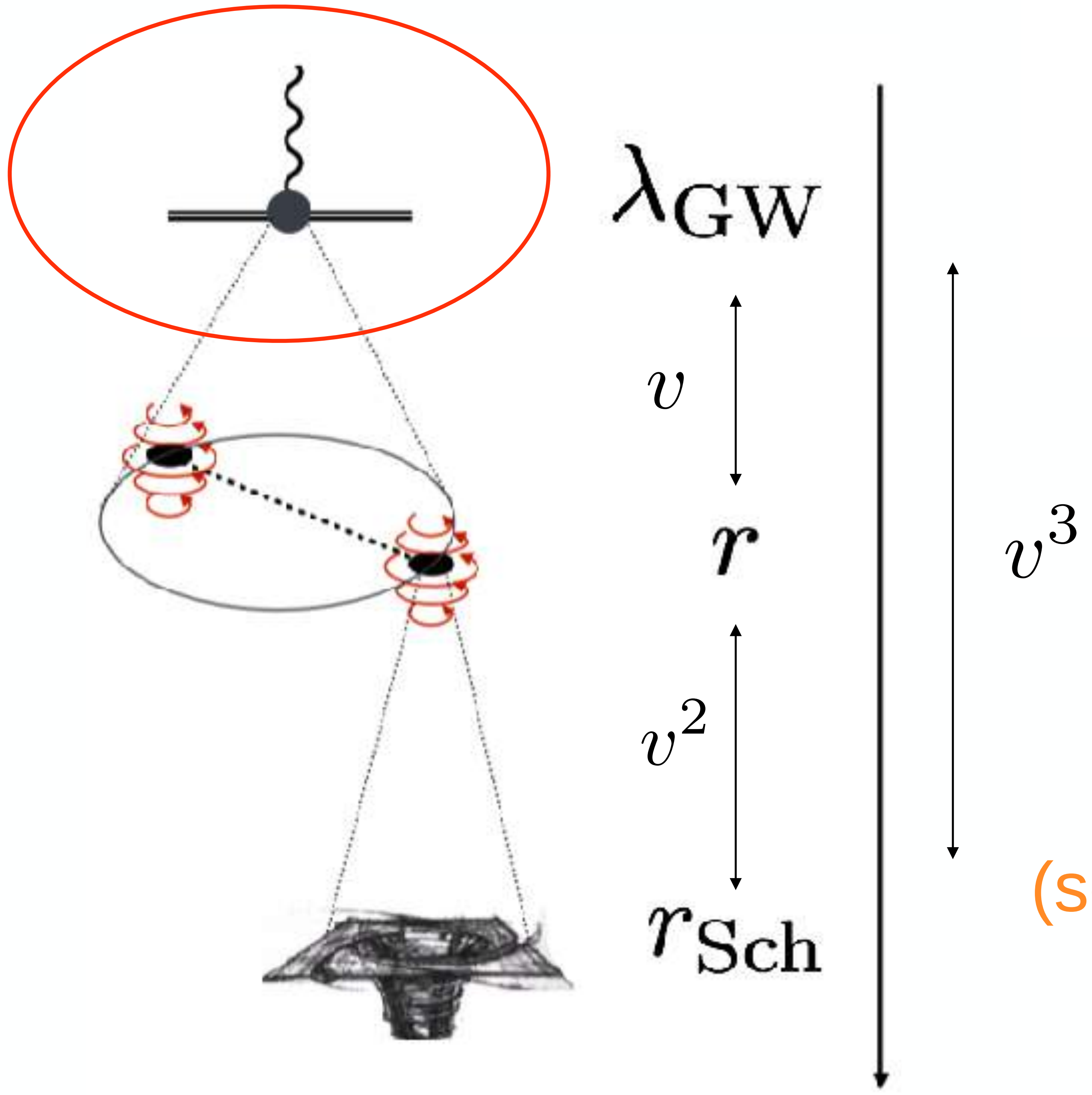
$$\underbrace{\operatorname{Re} W[\boldsymbol{x}_a]}_{\text{binding}} + i\, \underbrace{\operatorname{Im} W[\boldsymbol{x}_a]}_{\text{radiation}}$$



classical optical theorem!

Galley Leibovich
RAP Ross
1511.07379

EFT approach to GW physics



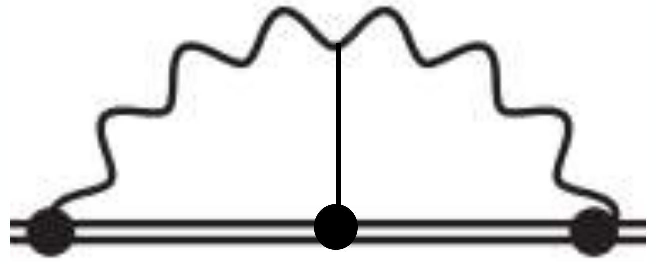
$$e^{iW} = \int D[\lambda_{\rm rad}^{-1}] D[r^{-1}] D[r_s^{-1}] e^{iS_{\rm full}}$$

Radiation Modes
$(k_0 \sim |\boldsymbol{k}|)$

Potential Modes
$(k_0 \ll |\boldsymbol{k}|)$

Finite size

“Tail effect”
(scattering off the geometry)



IR log from ‘radiation/soft’ modes!

$$\mu \frac{d}{d\mu} \langle M_{\rm ren}(t,\mu)\rangle = -2G_N^2 M \left\langle I_{ij}^{(3)}(t) I_{ij}^{(3)}(t)\right\rangle$$

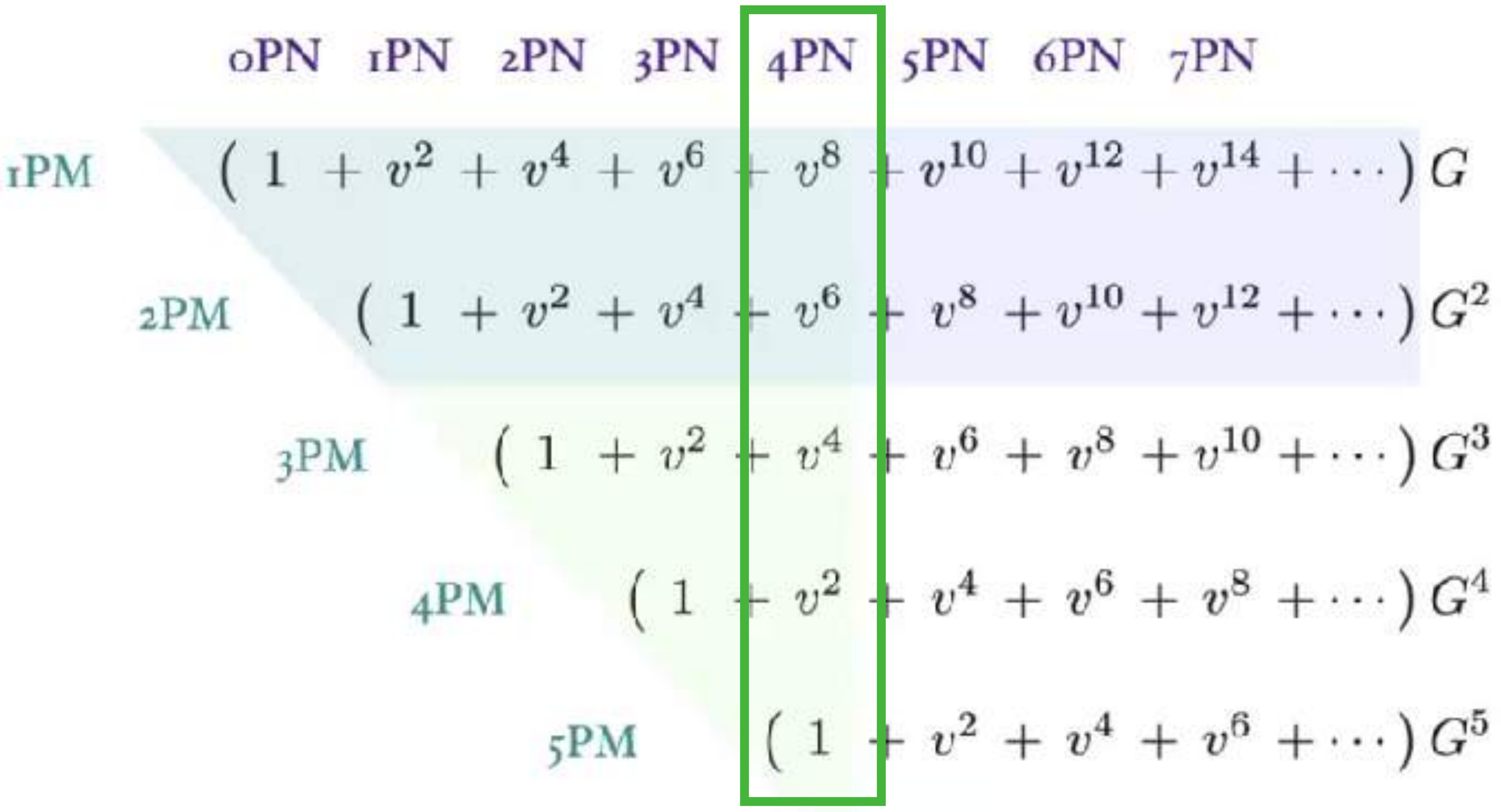
$$\underbrace{{\rm Re}\, W[\boldsymbol{x}_a]}_{\text{binding}}$$

$\uparrow$

radiation-reaction

Galley Leibovich
RAP Ross
1511.07379

EFT approach to GW physics PM



	0PN	1PN	2PN	3PN	4PN	5PN	6PN	7PN	
1PM	$(1$	$+v^2$	$+v^4$	$+v^6$	$+v^8$	$+v^{10}$	$+v^{12}$	$+v^{14}$	$+\cdots)\,G$
2PM		$(1$	$+v^2$	$+v^4$	$+v^6$	$+v^8$	$+v^{10}$	$+v^{12}$	$+\cdots)\,G^2$
3PM			$(1$	$+v^2$	$+v^4$	$+v^6$	$+v^8$	$+v^{10}$	$+\cdots)\,G^3$
4PM				$(1$	$+v^2$	$+v^4$	$+v^6$	$+v^8$	$+\cdots)\,G^4$
5PM					$(1$	$+v^2$	$+v^4$	$+v^6$	$+\cdots)\,G^5$

$$e^{iW} = \int D[\lambda_{\rm rad}^{-1}]\,D[r^{-1}]\,D[r_s^{-1}]\,e^{iS_{\rm full}}$$

Radiation
Modes
$(k_0 \sim |\boldsymbol{k}|)$

Potential
Modes
$(k_0 \ll |\boldsymbol{k}|)$

Finite
size

UV/IR
cancelation!

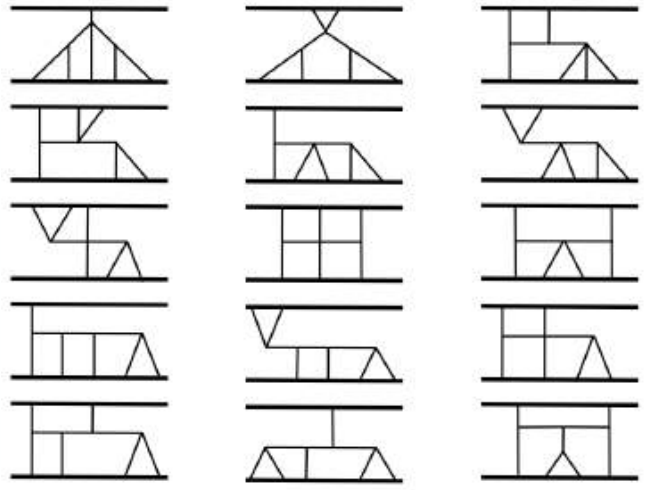
spoils
IR!

spoils
UV!

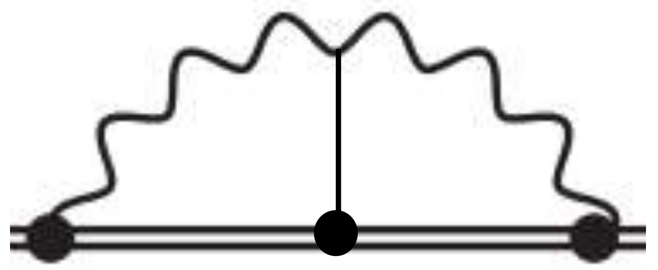
RAP Rothstein
1703.06433

Foffa **RAP**
Rothstein Sturani
1903.05118

Cho **RAP** Yang
2201.05138
(spin)



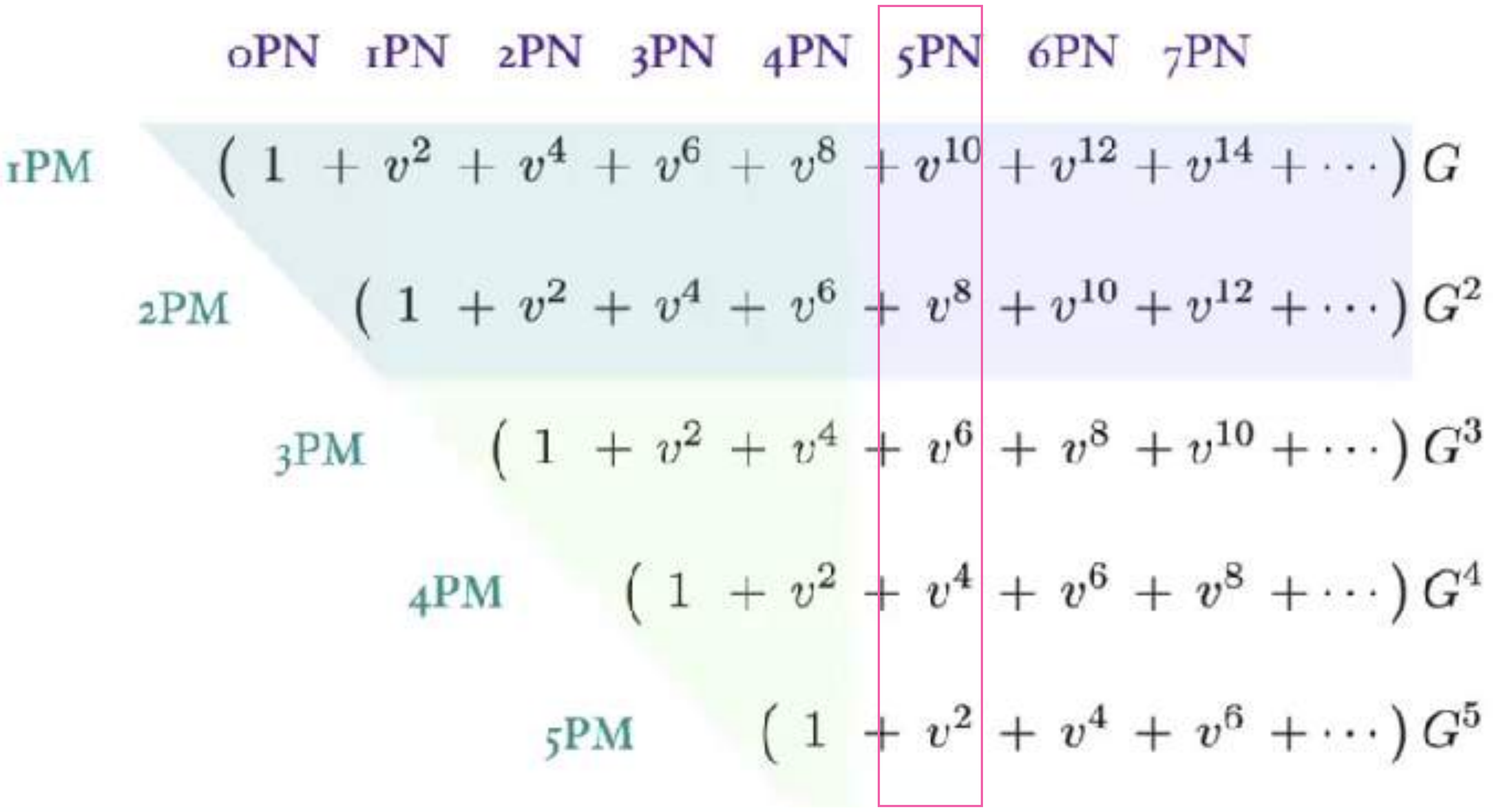
$$\frac{1}{p_0^2 - \boldsymbol{p}^2} \simeq -\frac{1}{\boldsymbol{p}^2}\left(1 + \frac{p_0^2}{\boldsymbol{p}^2} + \cdots\right) .$$



$$e^{i\mathbf{k}\cdot\mathbf{x}} = 1 + i\mathbf{k}\cdot\mathbf{x} + \cdots$$

EFT approach to GW physics PM

Galley Leibovich
RAP Ross
1511.07379



	0PN	1PN	2PN	3PN	4PN	5PN	6PN	7PN	
1PM	$(1$	$+v^2$	$+v^4$	$+v^6$	$+v^8$	$+v^{10}$	$+v^{12}$	$+v^{14}$	$+\cdots)G$
2PM		$(1$	$+v^2$	$+v^4$	$+v^6$	$+v^8$	$+v^{10}$	$+v^{12}$	$+\cdots)G^2$
3PM			$(1$	$+v^2$	$+v^4$	$+v^6$	$+v^8$	$+v^{10}$	$+\cdots)G^3$
4PM				$(1$	$+v^2$	$+v^4$	$+v^6$	$+v^8$	$+\cdots)G^4$
5PM					$(1$	$+v^2$	$+v^4$	$+v^6$	$+\cdots)G^5$

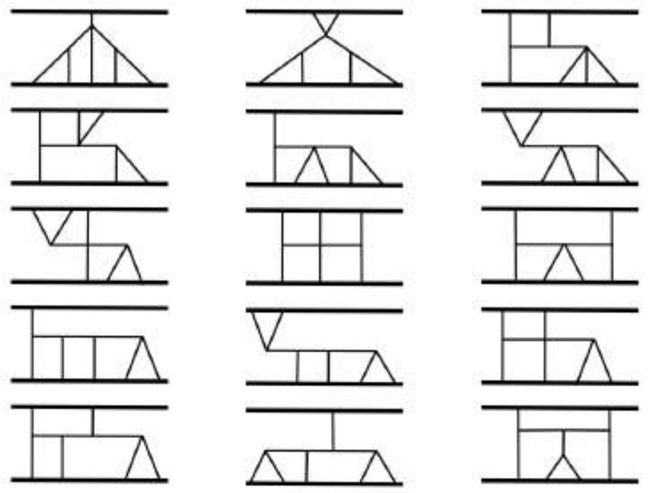
RAP Rothstein
1703.06433

Foffa **RAP**
Rothstein Sturani
1903.05118

Cho **RAP** Yang
2201.05138
(spin)

Bluemlein
Marquard Meier
2110.13822

Foffa Sturani
2110.14146



$$e^{iW} = \int D[\lambda_{\rm rad}^{-1}]\, D[r^{-1}]\, D[r_s^{-1}]\, e^{iS_{\rm full}}$$

Radiation Modes
$(k_0 \sim |\boldsymbol{k}|)$

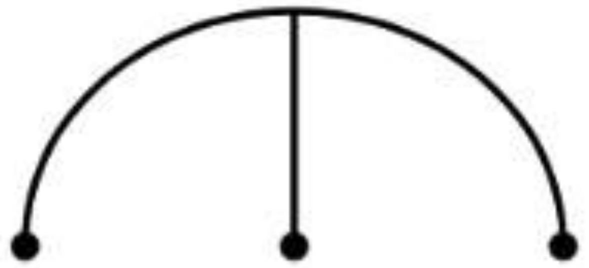
Potential Modes
$(k_0 \ll |\boldsymbol{k}|)$

Finite size

UV/IR cancelation!

spoils IR!

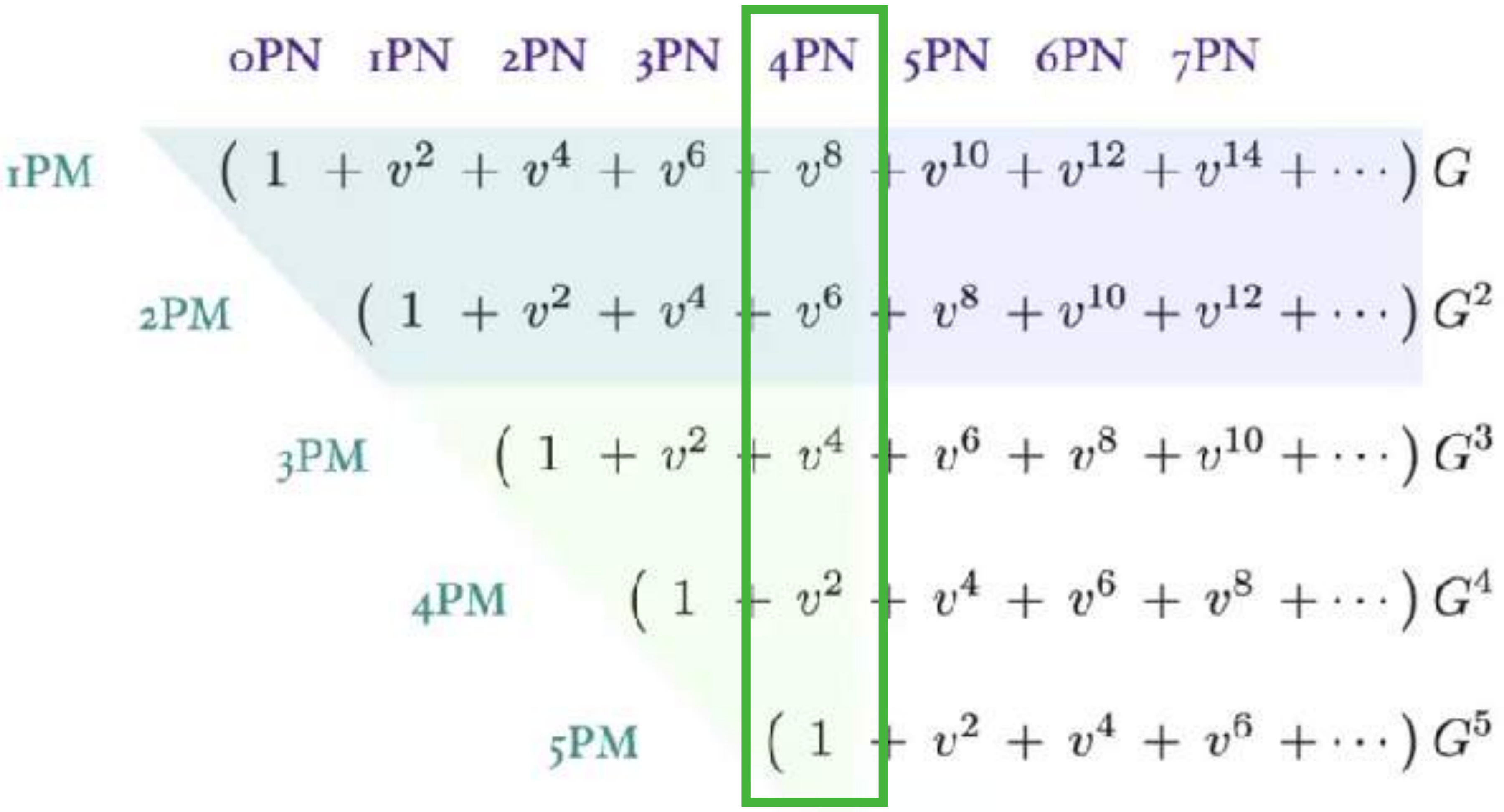
spoils UV!



in the entire soft region!

Galley Leibovich
RAP Ross
1511.07379

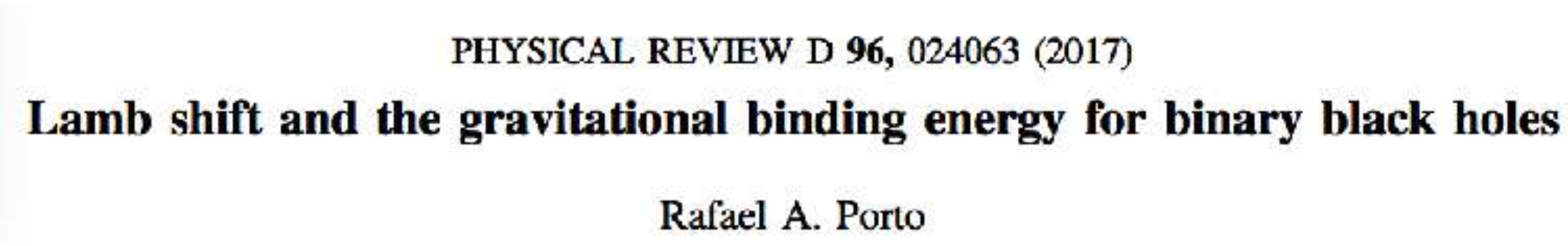
EFT approach to GW physics PM



	0PN	1PN	2PN	3PN	4PN	5PN	6PN	7PN	
1PM	$(1$	$+v^2$	$+v^4$	$+v^6$	$+v^8$	$+v^{10}$	$+v^{12}$	$+v^{14}$	$+\cdots)G$
2PM		$(1$	$+v^2$	$+v^4$	$+v^6$	$+v^8$	$+v^{10}$	$+v^{12}$	$+\cdots)G^2$
3PM			$(1$	$+v^2$	$+v^4$	$+v^6$	$+v^8$	$+v^{10}$	$+\cdots)G^3$
4PM				$(1$	$+v^2$	$+v^4$	$+v^6$	$+v^8$	$+\cdots)G^4$
5PM					$(1$	$+v^2$	$+v^4$	$+v^6$	$+\cdots)G^5$

$$E^{\text{4PN}} = -\frac{\mu c^2 x}{2}\left\{1 + \left(-\frac{3}{4} - \frac{\nu}{12}\right)x + \left(-\frac{27}{8} + \frac{19}{8}\nu - \frac{\nu^2}{24}\right)x^2 + \left(-\frac{675}{64} + \left[\frac{34445}{576} - \frac{205}{96}\pi^2\right]\nu - \frac{155}{96}\nu^2 - \frac{35}{5184}\nu^3\right)x^3 + \left(-\frac{3969}{128} + \left[-\frac{123671}{5760} + \frac{9037}{1536}\pi^2 + \frac{896}{15}\gamma_E + \frac{448}{15}\boxed{\ln(16x)}\right]\nu + \left[-\frac{498449}{3456} + \frac{3157}{576}\pi^2\right]\nu^2 + \frac{301}{1728}\nu^3 + \frac{77}{31104}\nu^4\right)x^4\right\}$$

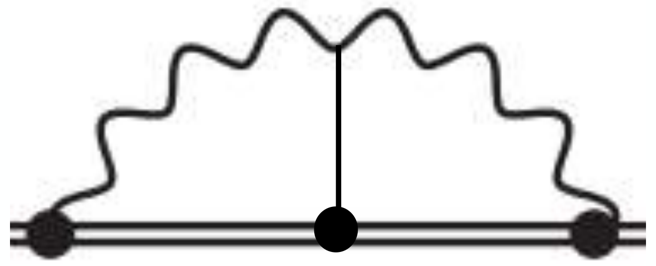
$\nu \sim m_2/m_1$
$x \sim (v/c)^2$



PHYSICAL REVIEW D **96**, 024063 (2017)

Lamb shift and the gravitational binding energy for binary black holes

Rafael A. Porto



$$W_{\text{tail}}[\boldsymbol{x}_a^{\pm}] = \frac{2G_N^2 M}{5}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\,\omega^6\, I_-^{ij}(-\omega)I_+^{ij}(\omega)\left[-\frac{1}{(d-4)_{\text{UV}}} - \gamma_E + \log\pi \boxed{-\log\frac{\omega^2}{\mu^2} + \frac{41}{30}} + i\pi\,\text{sign}(\omega)\right].$$

Lamb Shift!
“log+5/6”

dissipative term

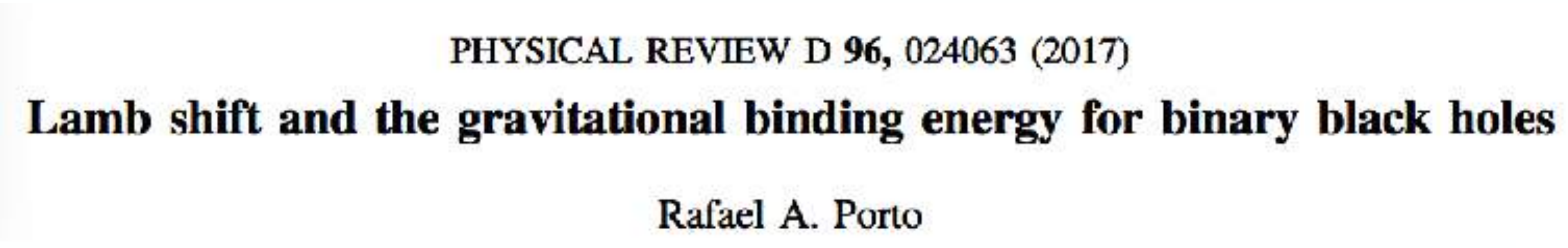
EFT approach to Atomic physics

Adding up 'near' and 'far' zone contributions in NRQED:

IR/UV cancelation NO ambiguities!

Universal log in binding energy

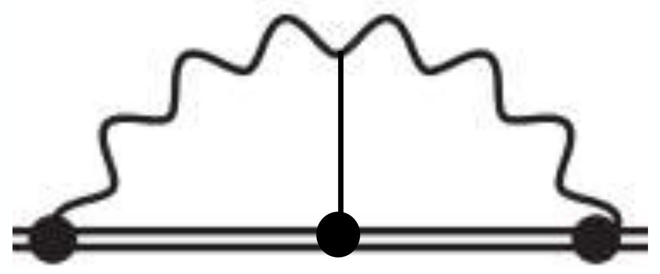
$$
\begin{aligned}
\delta E_{n,\ell} &= (\delta E_{n,\ell})_{US} + (\delta E_{n,\ell})_{cv} + \cdots \\
&= \frac{2\alpha_e}{3\pi}\left[\frac{5}{6}e^2\frac{|\psi_{n,\ell}(\boldsymbol{x}=0)|^2}{2m_e^2} - \sum_{m\neq n,\ell}\left\langle n,\ell\left|\frac{\boldsymbol{p}}{m_e}\right|m,\ell\right\rangle^2 (E_m-E_n)\underbrace{\log\frac{2|E_n-E_m|}{m_e}}_{\log\alpha_e}\right] + \\
&\quad + \frac{4\alpha_e^2}{3m_e^2}\left(\frac{1}{\epsilon_{\mathrm{UV}}} - \frac{1}{\epsilon_{\mathrm{IR}}}\right)|\psi_{n,\ell}(\boldsymbol{x}=0)|^2 .
\end{aligned}
$$



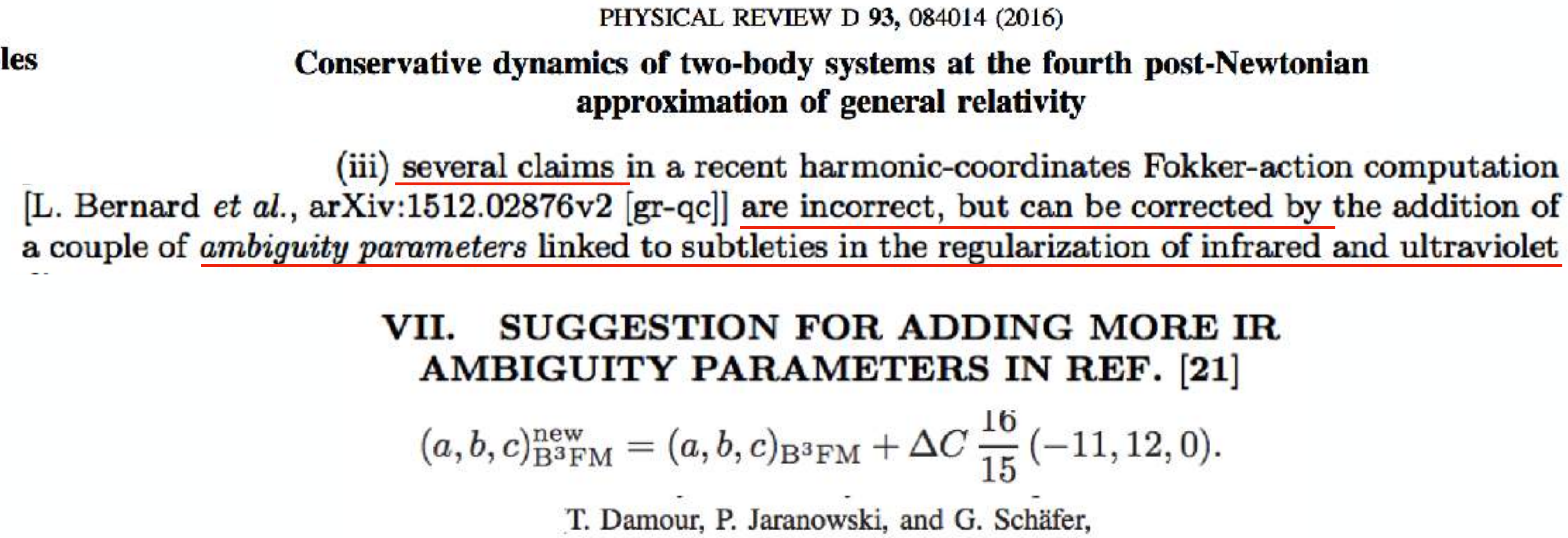
PHYSICAL REVIEW D **96**, 024063 (2017)

Lamb shift and the gravitational binding energy for binary black holes

Rafael A. Porto



Meanwhile in the 'traditional' approach…



PHYSICAL REVIEW D **93**, 084014 (2016)

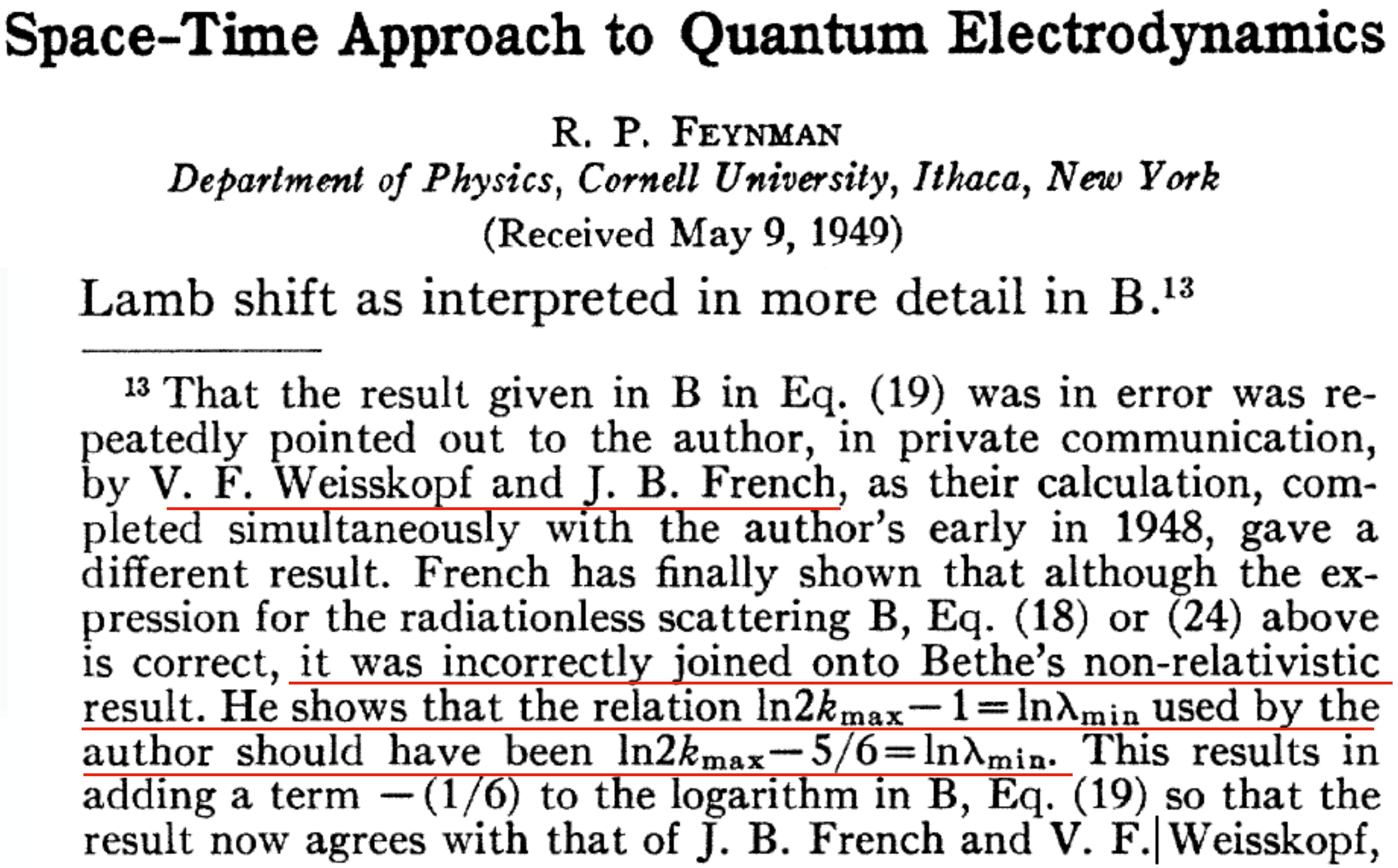
Conservative dynamics of two-body systems at the fourth post-Newtonian approximation of general relativity

(iii) several claims in a recent harmonic-coordinates Fokker-action computation [L. Bernard *et al.*, arXiv:1512.02876v2 [gr-qc]] are incorrect, but can be corrected by the addition of a couple of *ambiguity parameters* linked to subtleties in the regularization of infrared and ultraviolet

VII. SUGGESTION FOR ADDING MORE IR AMBIGUITY PARAMETERS IN REF. [21]

$$(a,b,c)^{\mathrm{new}}_{\mathrm{B^3FM}} = (a,b,c)_{\mathrm{B^3FM}} + \Delta C\,\frac{16}{15}(-11,12,0).$$

T. Damour, P. Jaranowski, and G. Schäfer,



Space-Time Approach to Quantum Electrodynamics

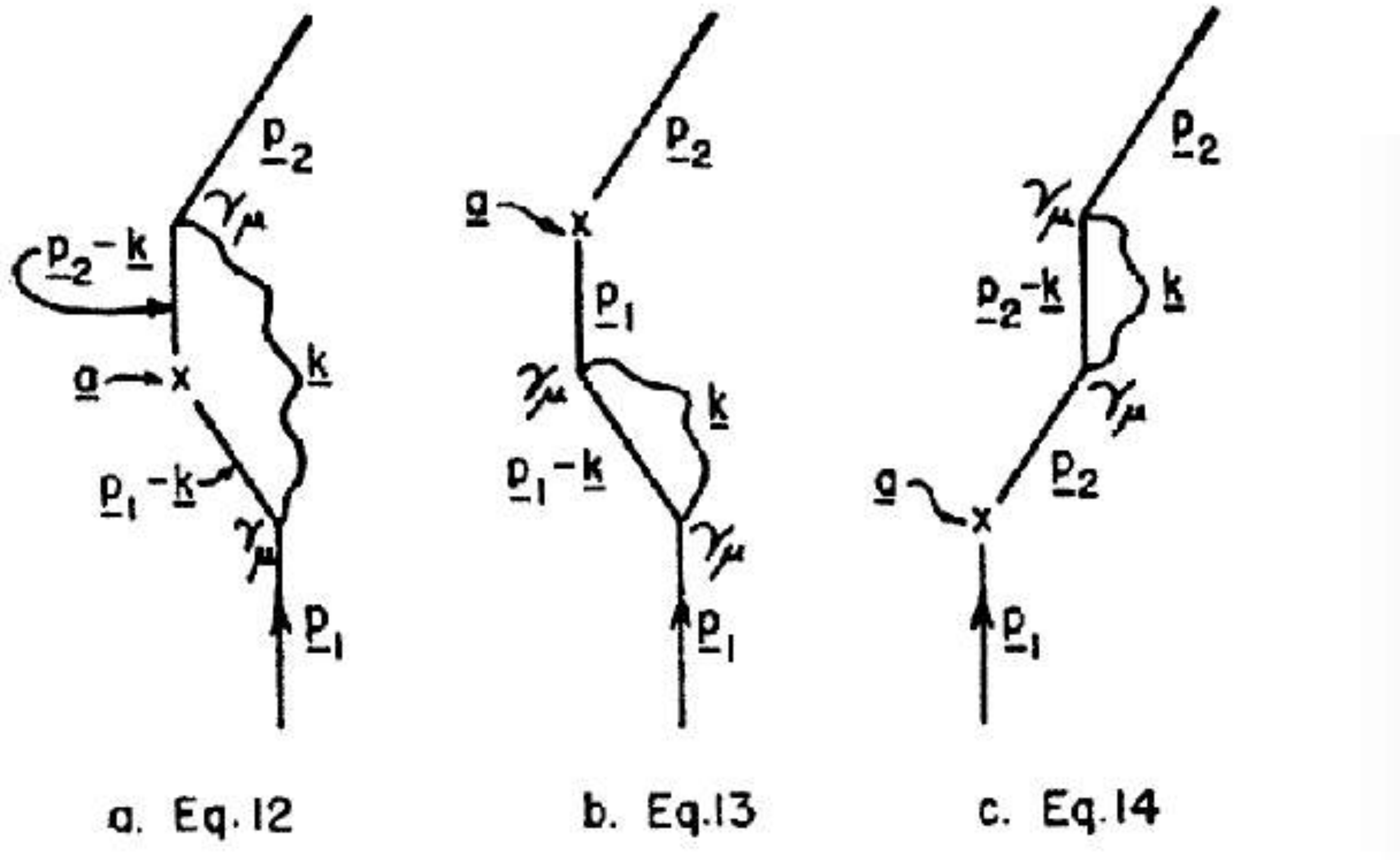
R. P. Feynman
Department of Physics, Cornell University, Ithaca, New York

(Received May 9, 1949)

Lamb shift as interpreted in more detail in B.[13]

---

[13] That the result given in B in Eq. (19) was in error was repeatedly pointed out to the author, in private communication, by V. F. Weisskopf and J. B. French, as their calculation, completed simultaneously with the author's early in 1948, gave a different result. French has finally shown that although the expression for the radiationless scattering B, Eq. (18) or (24) above is correct, it was incorrectly joined onto Bethe's non-relativistic result. He shows that the relation $\ln 2k_{\mathrm{max}}-1=\ln\lambda_{\mathrm{min}}$ used by the author should have been $\ln 2k_{\mathrm{max}}-5/6=\ln\lambda_{\mathrm{min}}$. This results in adding a term $-(1/6)$ to the logarithm in B, Eq. (19) so that the result now agrees with that of J. B. French and V. F. Weisskopf,



a. Eq.12 b. Eq.13 c. Eq.14

H. A. Bethe, The electromagnetic shift of energy levels, Phys. Rev. **72**, 339 (1947).
F. J. Dyson, The electromagnetic shift of energy levels, Phys. Rev. **73**, 617 (1948).
J. B. French and V. F. Weisskopf, The electromagnetic shift of energy levels, Phys. Rev. **75**, 1240 (1949).
N. M. Kroll and W. E. Lamb, On the self-energy of a bound electron, Phys. Rev. **75**, 388 (1949).

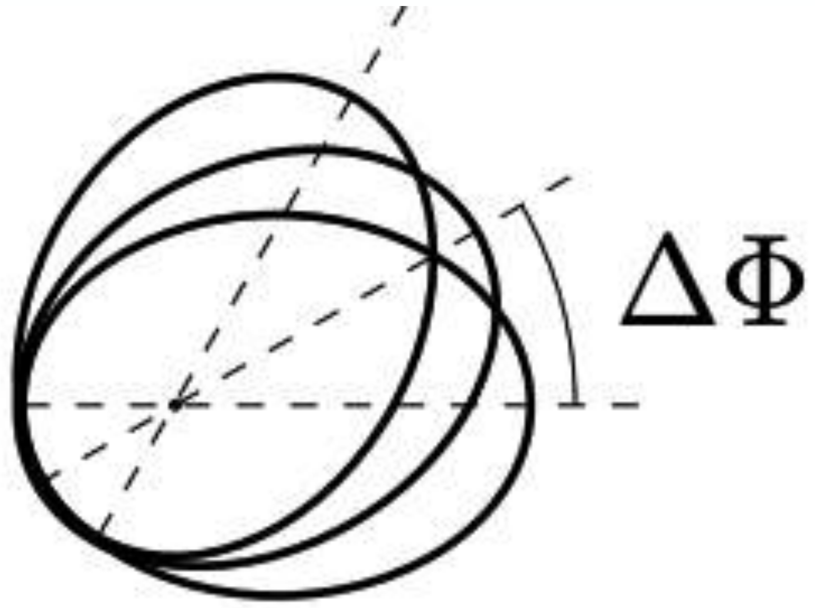
Even Feynman made a similar mistake!

Footnote13 (without any #12!)

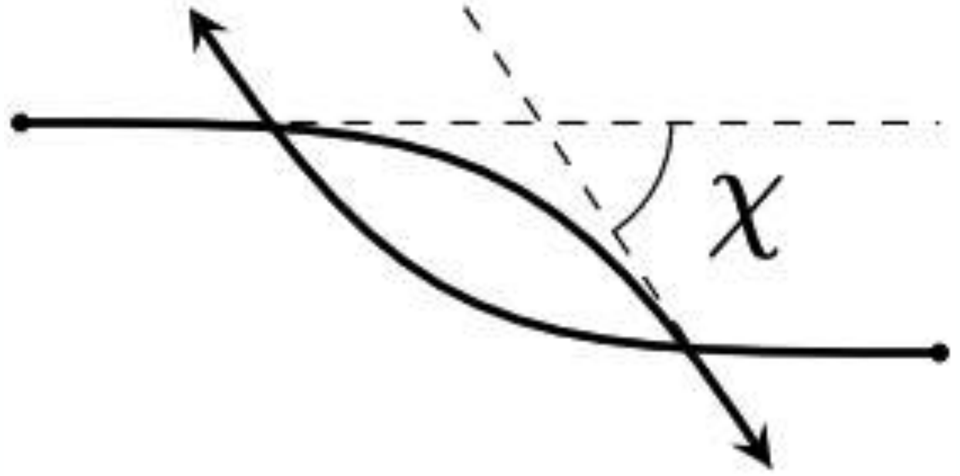
Discovery Potential =

Precise Theoretical Predictions

- Part I: Bound

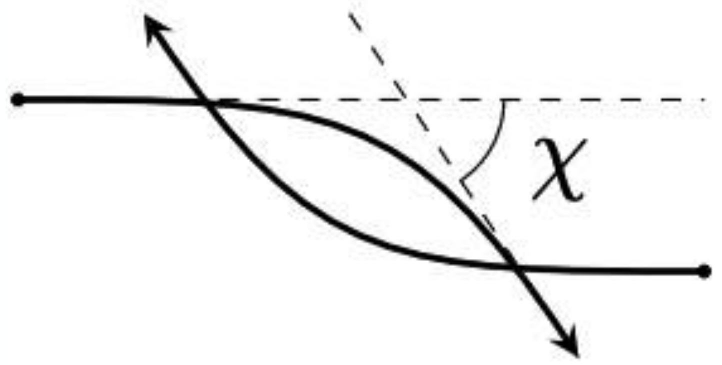


- **Part II: Boundary2Bound**



Kalin **RAP**
2006.01184
Liu **RAP** Yang
2102.10059

EFT approach to GW physics *PM*



$$e^{iS_{\rm eff}[x_a]} = \int \mathcal{D}h_{\mu\nu}\, e^{iS_{\rm EH}[h]+iS_{\rm GF}[h]+iS_{\rm pp}[x_a,h]}\,,$$

$$\Delta p^\mu_a = -\eta^{\mu\nu}\int_{-\infty}^{+\infty} \mathrm{d}\tau_a \frac{\partial \mathcal{L}_{\rm eff}}{\partial x^\nu_a}(x_a(\tau_a))$$

$$\Delta S^{ab}_A = \int_{-\infty}^{+\infty} d\tau \{S^{ab}_A, \mathcal{L}_{\rm eff}\}$$

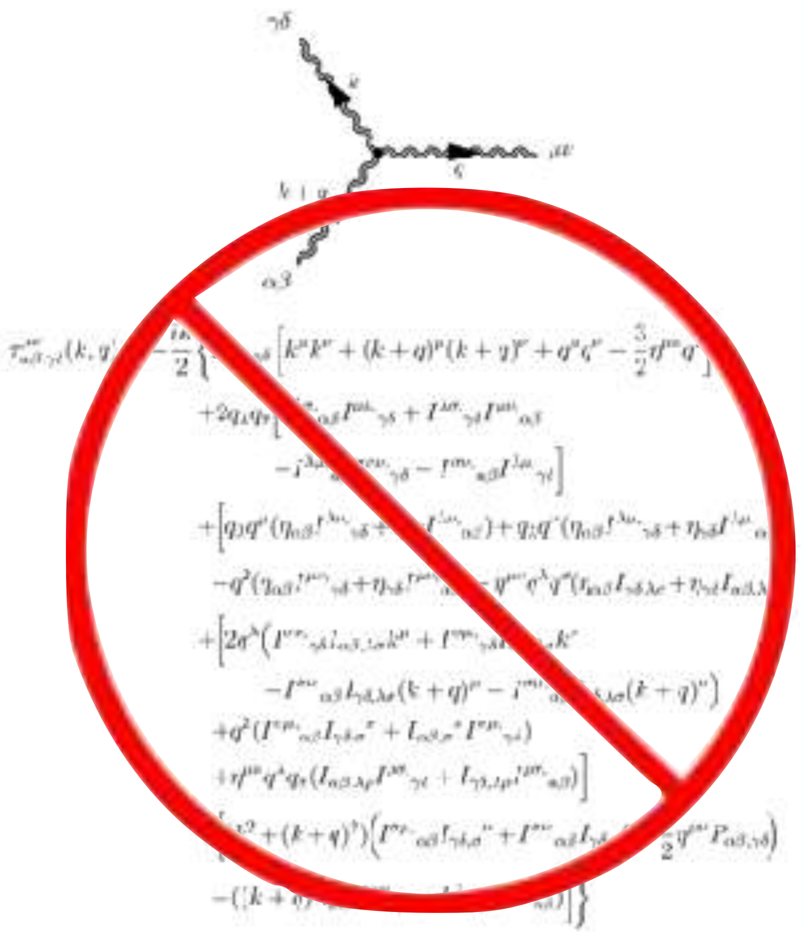
Potential Modes
($k_0 \ll |\boldsymbol{k}|$)

Radiation Modes
($k_0 \sim |\boldsymbol{k}|$)

Radiation-Reaction
(Cons + Dissip)

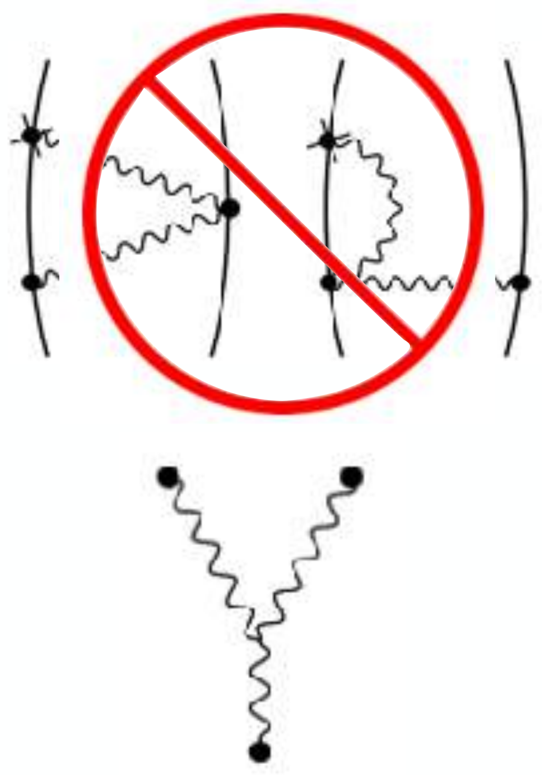
(classical) *"Soft"* Region

Simplified Feynman rules through proper time GF and total derivatives (but no field redef.)



$$S_{\rm pp} = -\sum_a \frac{m_a}{2}\int \mathrm{d}\tau_a\, g_{\mu\nu}(x_a(\tau_a))v^\mu_a(\tau_a)v^\nu_a(\tau_a)\,.$$

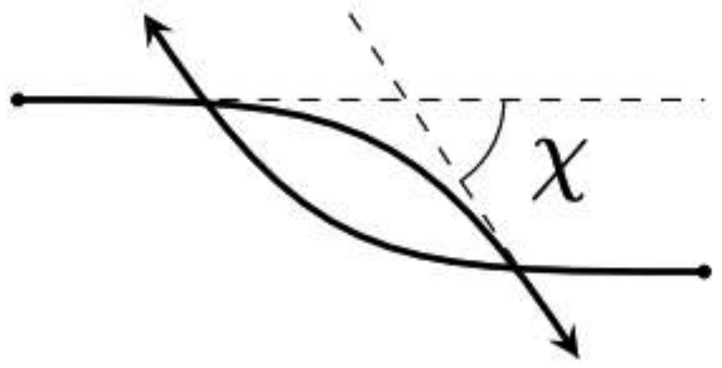
$$M_{\rm Pl}\mathcal{L}_{hhh} = -\frac{1}{2}h^{\mu\nu}\partial_\mu h^{\rho\sigma}\partial_\nu h_{\rho\sigma} + \frac{1}{2}h^{\mu\nu}\partial_\rho h\partial^\rho h_{\mu\nu} - \frac{1}{8}h\partial_\rho h\partial^\rho h + h^{\mu\nu}\partial_\nu h_{\rho\sigma}\partial^\sigma h_\mu{}^\rho - h^{\mu\nu}\partial_\sigma h_{\nu\rho}\partial^\sigma h_\mu{}^\rho + \frac{1}{4}h\partial_\sigma h_{\nu\rho}\partial^\sigma h^{\nu\rho}\,.$$



Lots of redundancy — No need to panic!

Kalin **RAP**
2006.01184
Kalin Liu **RAP**
2007.04977

EFT approach to GW physics *PM*



$$e^{iS_{\rm eff}[x_a]} = \int \mathcal{D}h_{\mu\nu}\, e^{iS_{\rm EH}[h]+iS_{\rm GF}[h]+iS_{\rm pp}[x_a,h]},$$

Potential Modes
$(k_0 \ll |\boldsymbol{k}|)$

Radiation Modes
$(k_0 \sim |\boldsymbol{k}|)$

Radiation-Reaction
(Cons + Dissip)

(classical) *"Soft"* Region

IR/UV finite!

Differential Equations
b.c. from **<u>entire region</u>**

$$(\gamma \to 1)$$

$$\partial_x \vec{h}(x,\epsilon) = \mathbb{M}(x,\epsilon)\,\vec{h}(x,\epsilon)$$

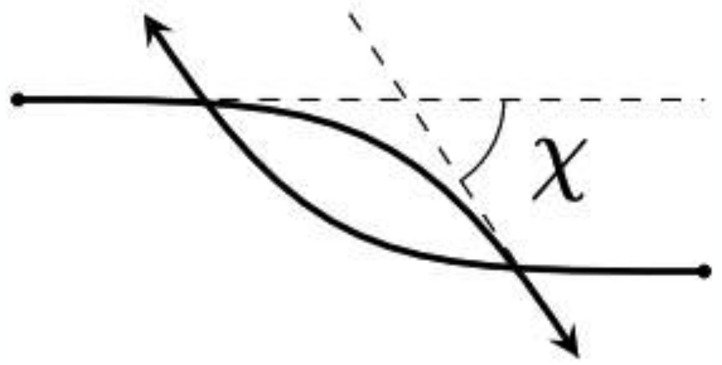
Single scale!

PM =
differential equations
PN boundary conditions

$$\gamma = \frac{1+x^2}{2x}, \quad \gamma \equiv u_1 \cdot u_2$$

Kalin **RAP**
2006.01184
Kalin Liu **RAP**
2007.04977

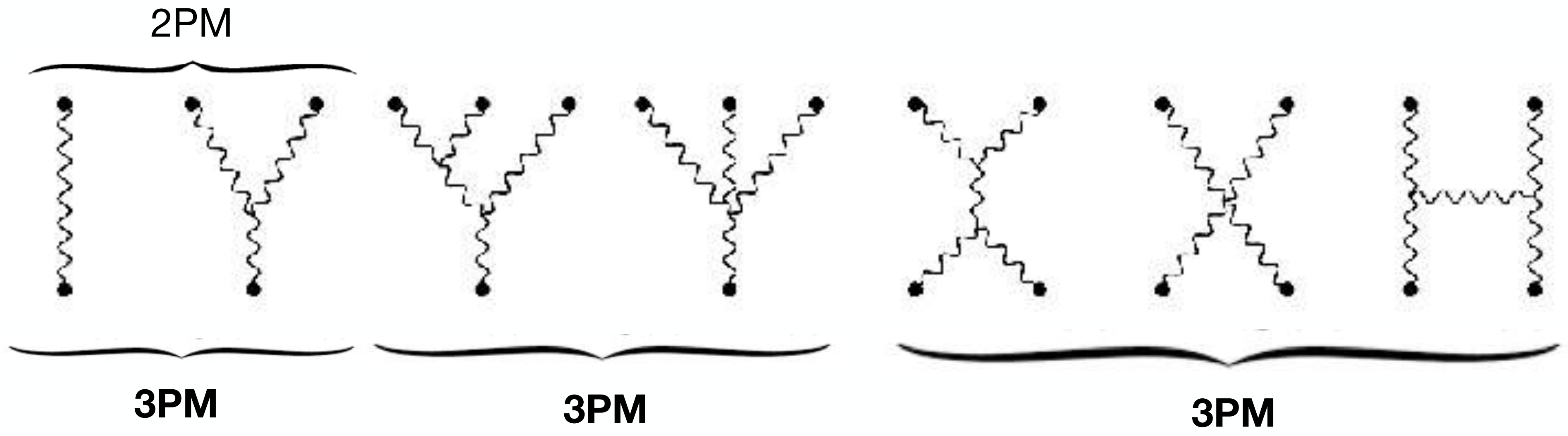
EFT approach to GW physics *PM*



$$e^{iS_{\rm eff}[x_a]} = \int \mathcal{D}h_{\mu\nu}\, e^{iS_{\rm EH}[h]+iS_{\rm GF}[h]+iS_{\rm pp}[x_a,h]}\,,$$

Enough for conservative to NNLO →

Potential Modes
$(k_0 \ll |\boldsymbol{k}|)$

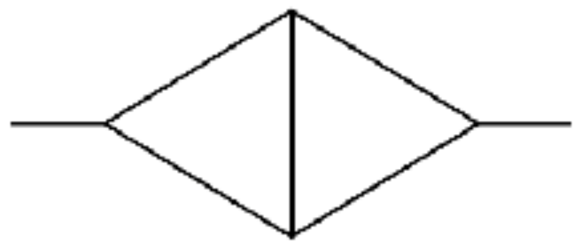


log x

$$\Delta^{(3)}p_1^\mu = \frac{G^3 b^\mu}{|b^2|^2}\left(\frac{16m_1^2m_2^2(4\gamma^4-12\gamma^2-3)\sinh^{-1}\sqrt{\frac{\gamma-1}{2}}}{(\gamma^2-1)} - \frac{4m_1^2m_2^2\gamma(20\gamma^6-90\gamma^4+120\gamma^2-53)}{3(\gamma^2-1)^{5/2}} - \frac{2m_1m_2(m_1^2+m_2^2)(16\gamma^6-32\gamma^4+16\gamma^2-1)}{(\gamma^2-1)^{5/2}}\right)$$
$$+\frac{3\pi}{2}\frac{(2\gamma^2-1)(5\gamma^2-1)}{(\gamma^2-1)^2}\frac{G^3M^2\mu}{|b^2|^{3/2}} \times \left((\gamma m_2+m_1)u_2^\mu - (\gamma m_1+m_2)u_1^\mu\right).$$

Differential Equations
b.c. from **potentials**

PN Kite



$$\partial_x \vec{h}(x,\epsilon) = \epsilon\, \mathbb{M}(x)\, \vec{h}(x,\epsilon)$$

canonical to NNLO!

Can be solved in terms of Polilogarithms

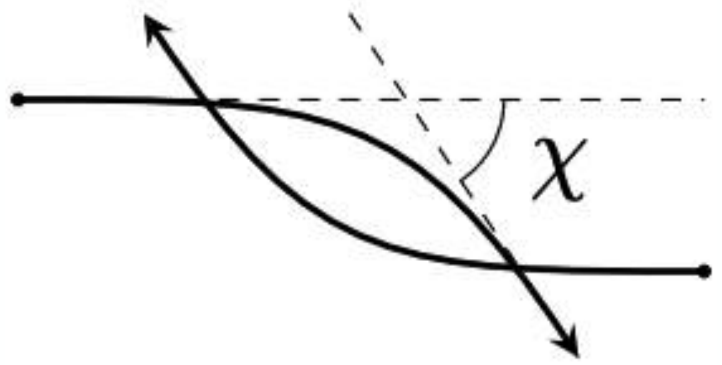
$$\gamma = \frac{1+x^2}{2x}, \qquad \gamma \equiv u_1 \cdot u_2$$

Dlapa Kalin Liu **RAP**

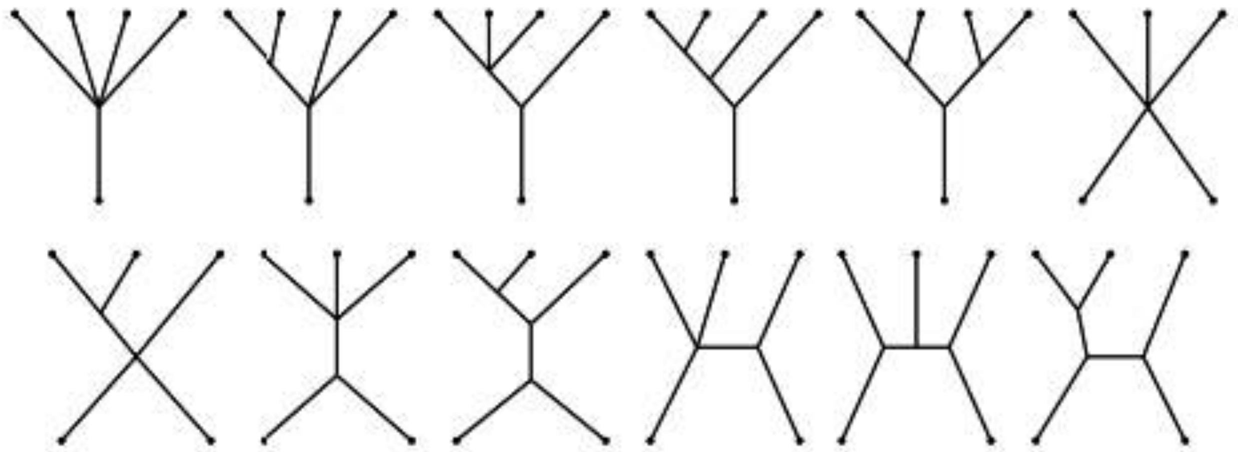
2106.08276

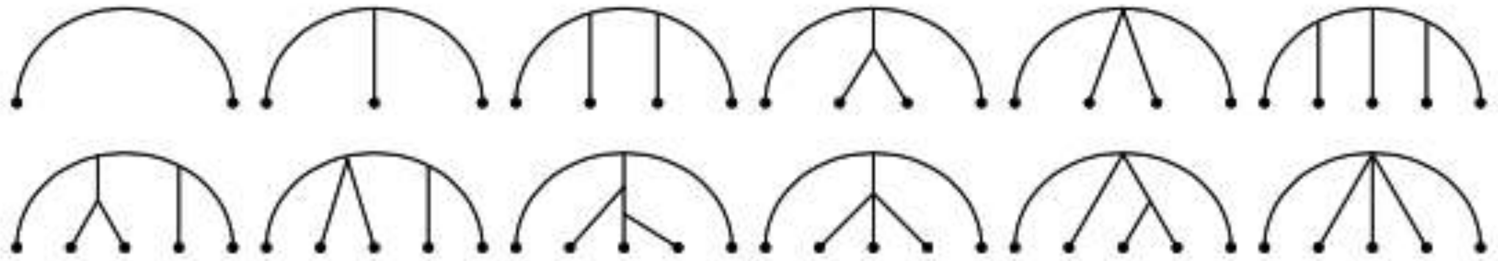
2112.11296

EFT approach to GW physics *PM*



$$e^{iS_{\text{eff}}[x_a]} = \int \mathcal{D}h_{\mu\nu}\, e^{iS_{\text{EH}}[h]+iS_{\text{GF}}[h]+iS_{\text{pp}}[x_a,h]},$$





Potential Modes

$(k_0 \ll |\boldsymbol{k}|)$

Radiation Modes

$(k_0 \sim |\boldsymbol{k}|)$

IR/UV finite!

$$\frac{\chi^{(4)}_{b\,(\text{pot})}(\gamma)}{\pi\Gamma} = \chi_s(x) + \nu\left(\frac{\chi_{2\epsilon}(x)}{2\epsilon} + \chi_p(x)\right)$$

$$\frac{\chi^{(4)}_{b\,(\text{rad})}(\gamma)}{\pi\Gamma} = \nu\left(-\frac{\chi_{2\epsilon}(x)}{2\epsilon}(1-x)^{-4\epsilon} + \chi_t(x)\right)$$

Bethe logarithm

Differential Equations

b.c. from **potentials**

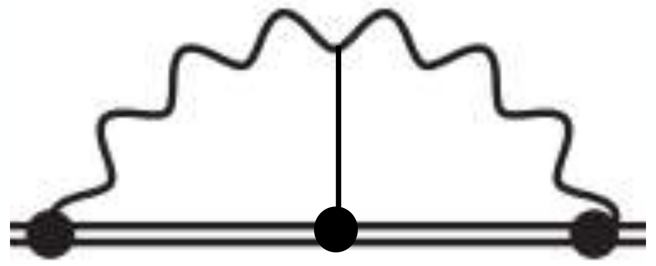
and radiation

$$\partial_x \vec{h}(x,\epsilon) = \mathbb{M}(x,\epsilon)\,\vec{h}(x,\epsilon)$$

Not Canonical!

"Tail effect"

(scattering off the geometry)



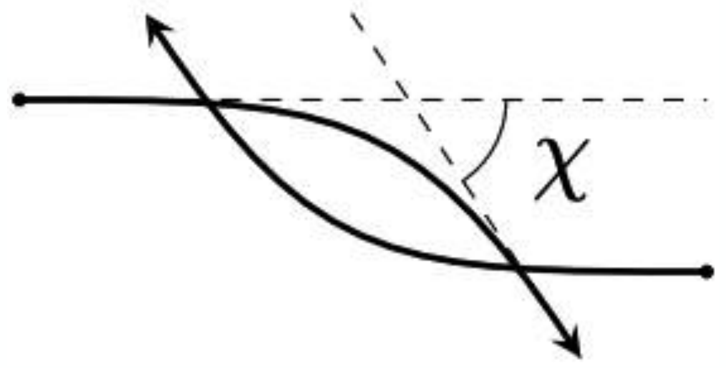
Introduces elliptic integrals!

$$\gamma = \frac{1+x^2}{2x}, \quad \gamma \equiv u_1 \cdot u_2$$

$$\mathrm{K}(x^2) = \int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{1-x^2\sin^2\theta}} \qquad \mathrm{E}(x^2) = \int_0^{\frac{\pi}{2}} d\theta\,\sqrt{1-x^2\sin^2\theta}$$

Dlapa Kalin Liu **RAP**
2106.08276
2112.11296

EFT approach to GW physics *PM*



Combined result includes logarithms, dilogarithms and elliptic integrals of the first and second kind

$\log v$

$$\frac{\chi^{(4)}_{b\,(\text{comb})}}{\pi\Gamma} = \chi_s + \nu\Big(\chi_c(x) + \boxed{2\chi_{2\epsilon}(x)\log(1-x)}\Big)\,,$$

Bethe logarithm

$$\chi_s(x) = \frac{105h_1(x)}{128\left(x^2-1\right)^4}\,,$$

$$\chi_{2\epsilon}(x) = -\frac{3h_2(x)\log(x)}{32x\left(x^2-1\right)^5} + \frac{3h_3(x)\log\left(\frac{x+1}{2}\right)}{32x^2\left(x^2-1\right)^2} + \frac{h_4(x)}{64x^2\left(x^2-1\right)^4}\,,$$

$$\begin{aligned}\chi_c(x) = & -\frac{21h_6(x)\mathrm{E}^2\left(1-x^2\right)}{8\left(x^2-1\right)^4} + \frac{3h_7(x)\mathrm{K}\left(1-x^2\right)\mathrm{E}\left(1-x^2\right)}{8\left(x^2-1\right)^4} - \frac{15h_8(x)\mathrm{K}^2\left(1-x^2\right)}{16\left(x^2-1\right)^4} - \frac{h_{16}(x)\log\left(x^2+1\right)}{32x^3\left(x^2-1\right)^4} \\ & + \frac{3h_{19}(x)\mathrm{Li}_2\left(-\frac{(x-1)^2}{(x+1)^2}\right)}{128x^4\left(x^2-1\right)^2} + \frac{\pi^2h_{35}(x)}{512(x-1)^3x^4(x+1)^5} + \frac{3h_{36}(x)\log^2(2)}{16x^2\left(x^2-1\right)^2} + \frac{3h_{37}(x)\log(2)\log(x)}{8\left(x^2-1\right)^5} - \frac{3h_{38}(x)\log(2)\log(x+1)}{16x^2\left(x^2-1\right)^2} \\ & + \frac{3h_{39}(x)\log(2)}{16x^2\left(x^2-1\right)^4} + \frac{3h_{40}(x)\log^2(x)}{256x^4\left(x^2-1\right)^8} - \frac{3h_{41}(x)\log(x)\log(x+1)}{128x^4\left(x^2-1\right)^5} + \frac{h_{42}(x)\log(x)}{64x^3\left(x^2-1\right)^7} - \frac{3h_{43}(x)\log^2(x+1)}{2x\left(x^2-1\right)^2} \\ & + \frac{h_{44}(x)\log(x+1)}{32x^3\left(x^2-1\right)^4} + \frac{3h_{45}(x)\left(\mathrm{Li}_2\left(\frac{x-1}{x}\right)-\mathrm{Li}_2(-x)\right)}{128(x-1)^3x^4(x+1)^5} - \frac{3h_{46}(x)\mathrm{Li}_2\left(\frac{x-1}{x+1}\right)}{64(x-1)^2x^4} + \frac{h_{47}(x)}{384x^3\left(x^2-1\right)^6\left(x^2+1\right)^7}\,.\end{aligned}$$

h(x)'s are polynomials in the variable x

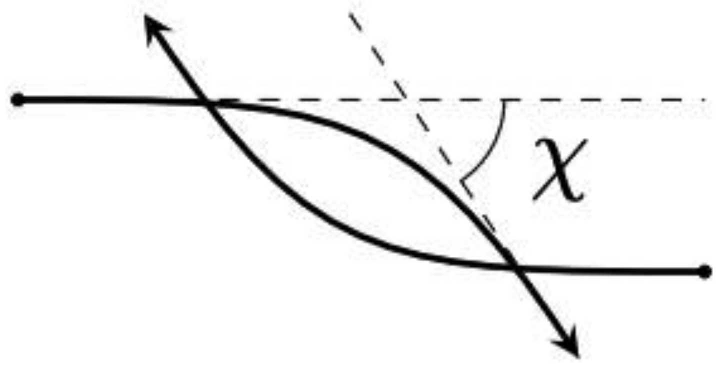
$$\gamma = \frac{1+x^2}{2x}, \quad \gamma \equiv u_1 \cdot u_2$$

Dlapa Kalin Liu **RAP**

2106.08276

2112.11296

EFT approach to GW physics *PM*



$$\frac{\chi_{b\,(\mathrm{comb})}^{(4)}}{\pi\Gamma} = \chi_s + \nu\Big(\chi_c(x) + 2\chi_{2\epsilon}(x)\log(1-x)\Big)\,,$$

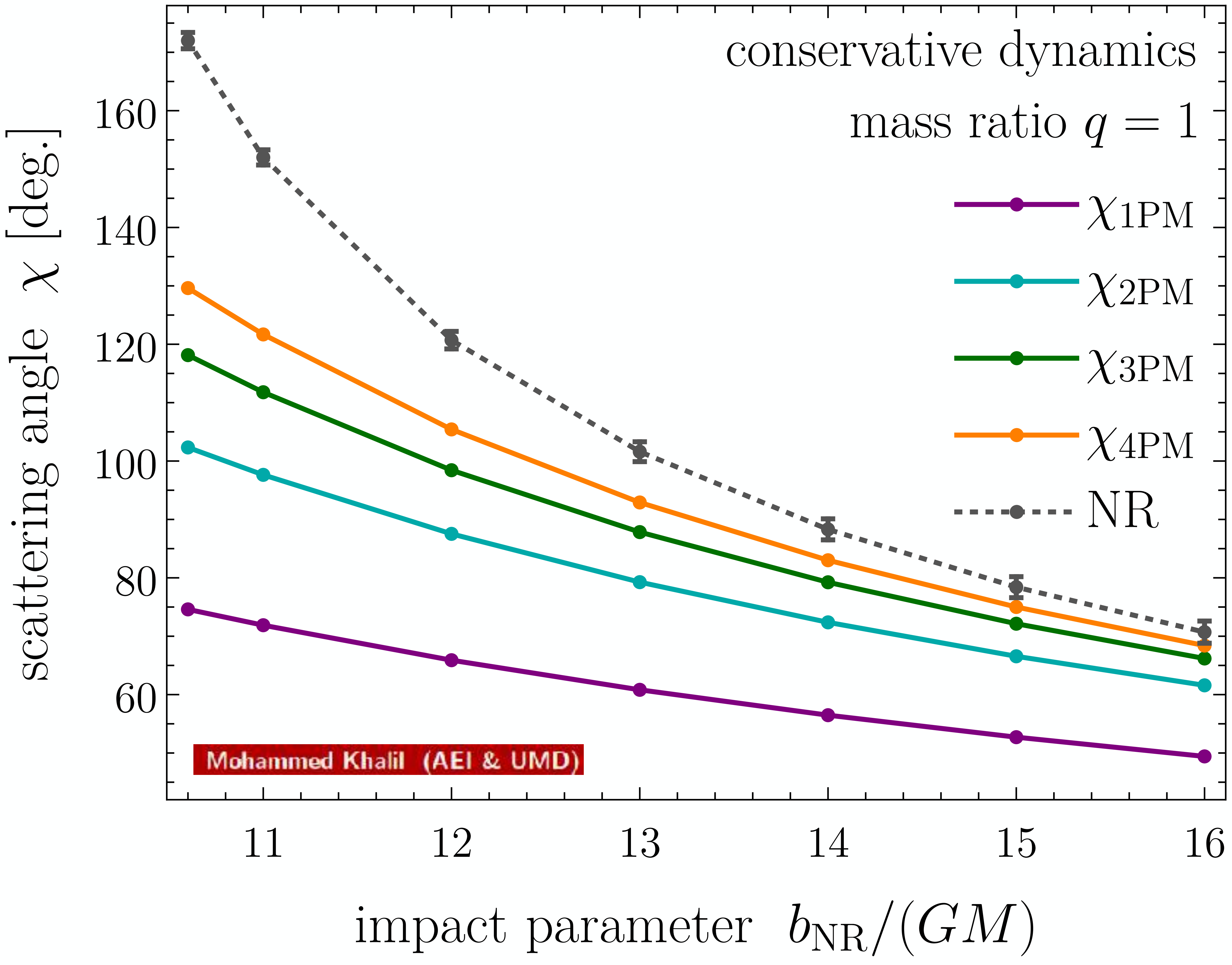
$$\chi_s(x) = \frac{105h_1(x)}{128\left(x^2-1\right)^4}\,,$$

$$\chi_{2\epsilon}(x) = -\frac{3h_2(x)\log(x)}{32x\left(x^2-1\right)^5} + \frac{3h_3(x)\log\left(\frac{x+1}{2}\right)}{32x^2\left(x^2-1\right)^2} + \frac{h_4(x)}{64x^2\left(x^2-1\right)^4}\,,$$

$$\begin{aligned}\chi_c(x) = &-\frac{21h_6(x)\mathrm{E}^2\left(1-x^2\right)}{8\left(x^2-1\right)^4} + \frac{3h_7(x)\mathrm{K}\left(1-x^2\right)\mathrm{E}\left(1-x^2\right)}{8\left(x^2-1\right)^4} - \frac{15h_8(x)\mathrm{K}^2\left(1-x^2\right)}{16\left(x^2-1\right)^4} - \frac{h_{16}(x)\log\left(x^2+1\right)}{32x^3\left(x^2-1\right)^4}\\ &+ \frac{3h_{19}(x)\mathrm{Li}_2\left(-\frac{(x-1)^2}{(x+1)^2}\right)}{128x^4\left(x^2-1\right)^2} + \frac{\pi^2h_{35}(x)}{512(x-1)^3x^4(x+1)^5} + \frac{3h_{36}(x)\log^2(2)}{16x^2\left(x^2-1\right)^2} + \frac{3h_{37}(x)\log(2)\log(x)}{8\left(x^2-1\right)^5} - \frac{3h_{38}(x)\log(2)\log(x+1)}{16x^2\left(x^2-1\right)^2}\\ &+ \frac{3h_{39}(x)\log(2)}{16x^2\left(x^2-1\right)^4} + \frac{3h_{40}(x)\log^2(x)}{256x^4\left(x^2-1\right)^8} - \frac{3h_{41}(x)\log(x)\log(x+1)}{128x^4\left(x^2-1\right)^5} + \frac{h_{42}(x)\log(x)}{64x^3\left(x^2-1\right)^7} - \frac{3h_{43}(x)\log^2(x+1)}{2x\left(x^2-1\right)^2}\\ &+ \frac{h_{44}(x)\log(x+1)}{32x^3\left(x^2-1\right)^4} + \frac{3h_{45}(x)\left(\mathrm{Li}_2\left(\frac{x-1}{x}\right)-\mathrm{Li}_2(-x)\right)}{128(x-1)^3x^4(x+1)^5} - \frac{3h_{46}(x)\mathrm{Li}_2\left(\frac{x-1}{x+1}\right)}{64(x-1)^2x^4} + \frac{h_{47}(x)}{384x^3\left(x^2-1\right)^6\left(x^2+1\right)^7}\,.\end{aligned}$$

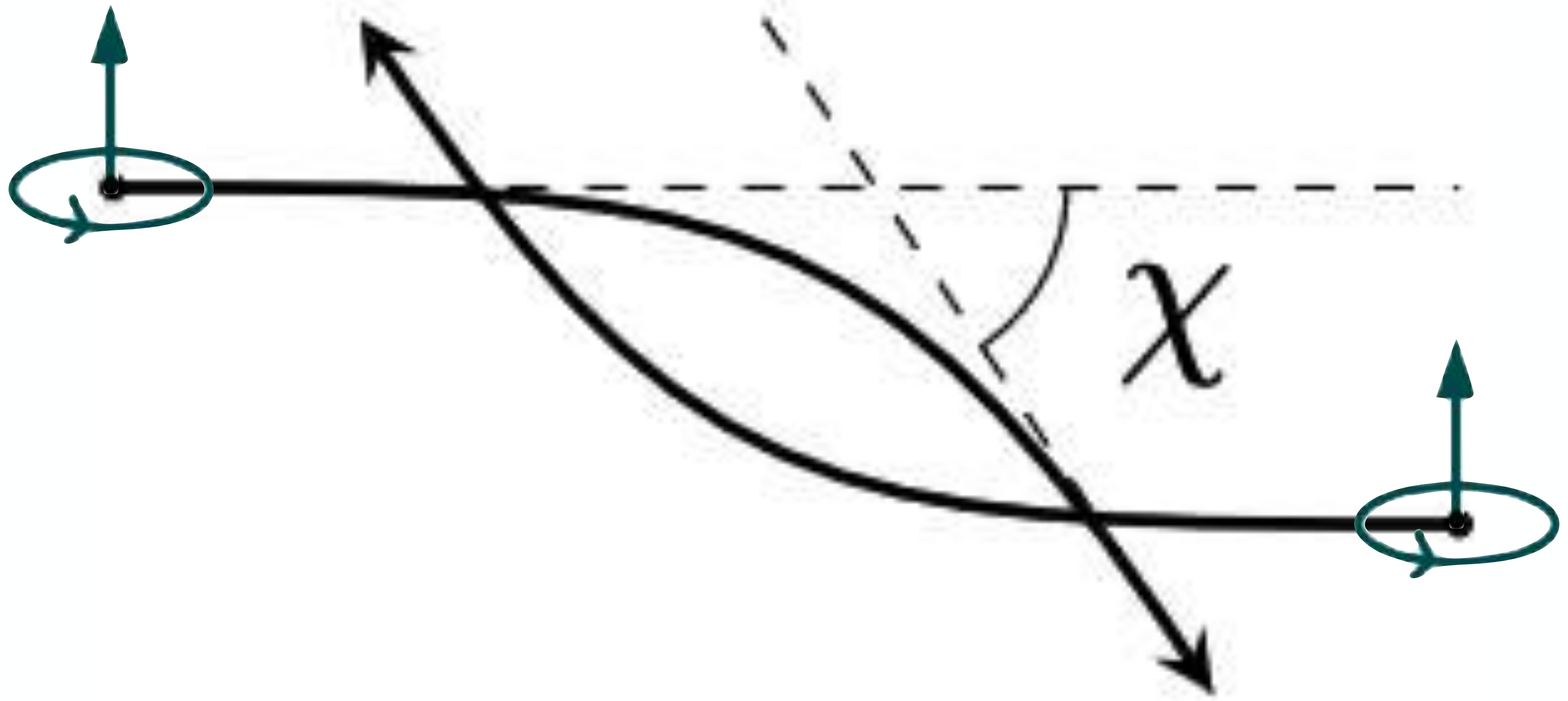
Comparison with numerical simulations

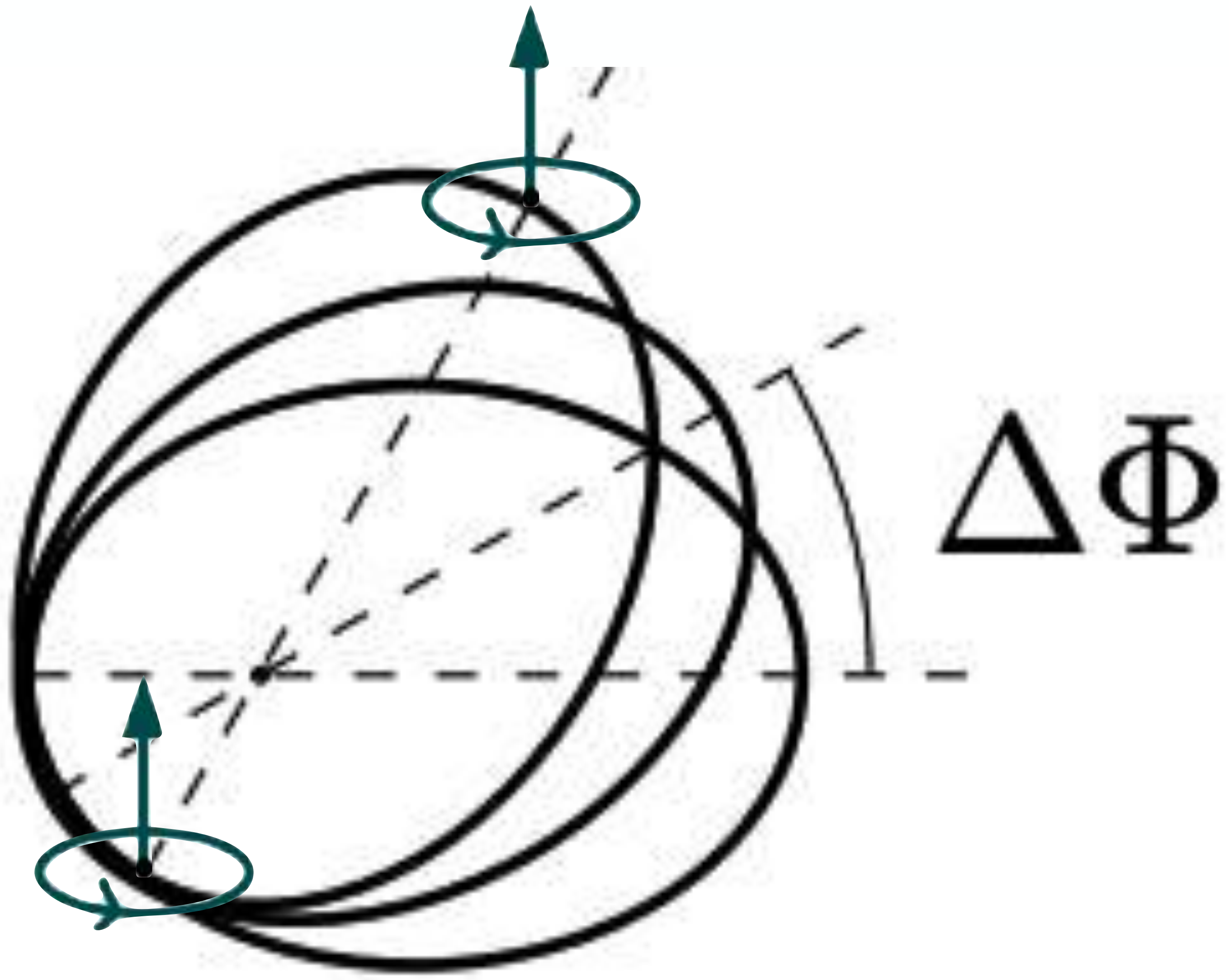
(M. Khalil et al., to appear)



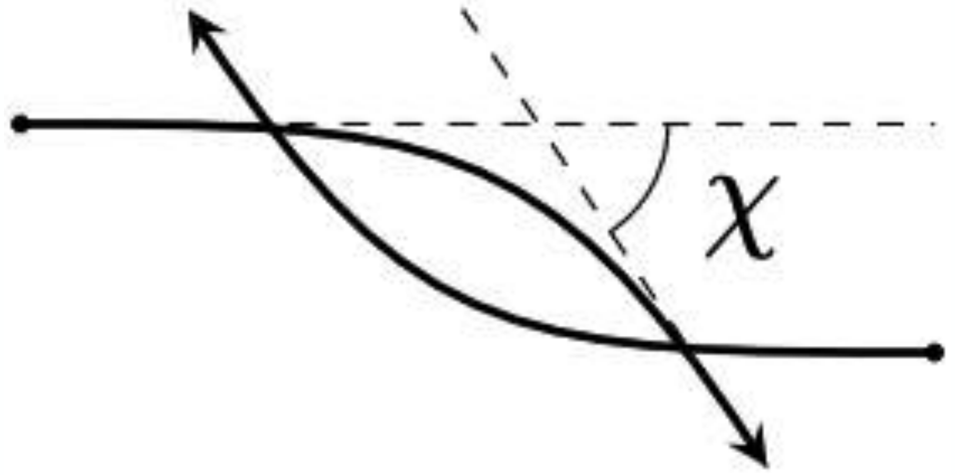
Kalin **RAP**
1910.03008
1911.09130

How do we compute *bound* observables from *boundary* data?

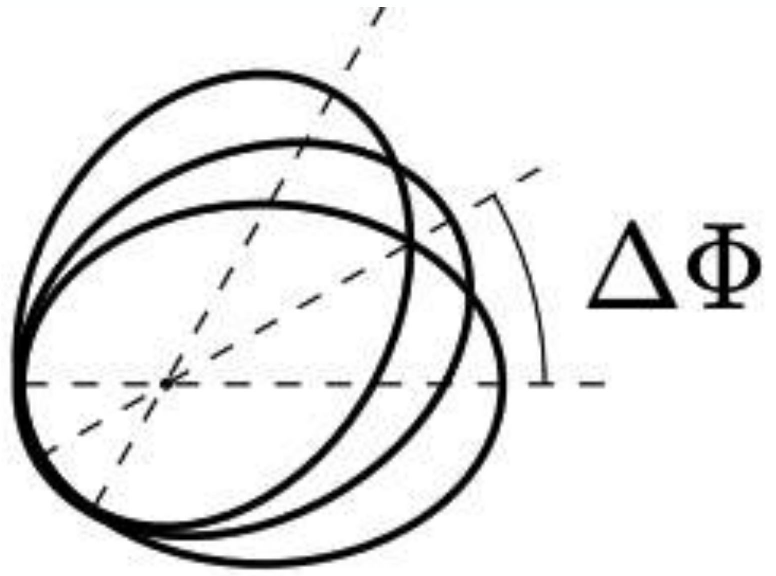




How do we compute *bound* observables from *boundary* data?



Conservative effects



Gravitational interaction is UNIVERSAL!

Conservative 3PM Hamiltonian

ZB, Cheung, Roiban, Shen, Solon, Zeng (2019)

The O(G^3) 3PM Hamiltonian: $H(\boldsymbol{p},\boldsymbol{r}) = \sqrt{\boldsymbol{p}^2 + m_1^2} + \sqrt{\boldsymbol{p}^2 + m_2^2} + V(\boldsymbol{p},\boldsymbol{r})$

$$V(\boldsymbol{p},\boldsymbol{r}) = \sum_{i=1}^{3} c_i(\boldsymbol{p}^2)\left(\frac{G}{|\boldsymbol{r}|}\right)^i,$$

Newton in here

$$c_1 = \frac{\nu^2 m^2}{\gamma^2\xi}\left(1-2\sigma^2\right), \qquad c_2 = \frac{\nu^2 m^3}{\gamma^2\xi}\left[\frac{3}{4}\left(1-5\sigma^2\right) - \frac{4\nu\sigma\left(1-2\sigma^2\right)}{\gamma\xi} - \frac{\nu^2(1-\xi)\left(1-2\sigma^2\right)^2}{2\gamma^3\xi^2}\right],$$

$$\begin{aligned} c_3 = \frac{\nu^2 m^4}{\gamma^2\xi}\Bigg[&\frac{1}{12}\left(3-6\nu+206\nu\sigma-54\sigma^2+108\nu\sigma^2+4\nu\sigma^3\right) - \frac{4\nu\left(3+12\sigma^2-4\sigma^4\right)\operatorname{arcsinh}\sqrt{\frac{\sigma-1}{2}}}{\sqrt{\sigma^2-1}} \\ &- \frac{3\nu\gamma\left(1-2\sigma^2\right)\left(1-5\sigma^2\right)}{2(1+\gamma)(1+\sigma)} - \frac{3\nu\sigma\left(7-20\sigma^2\right)}{2\gamma\xi} - \frac{\nu^2\left(3+8\gamma-3\xi-15\sigma^2-80\gamma\sigma^2+15\xi\sigma^2\right)\left(1-2\sigma^2\right)}{4\gamma^3\xi^2} \\ &+ \frac{2\nu^3(3-4\xi)\sigma\left(1-2\sigma^2\right)^2}{\gamma^4\xi^3} + \frac{\nu^4(1-2\xi)\left(1-2\sigma^2\right)^3}{2\gamma^6\xi^4}\Bigg], \end{aligned}$$

$$m = m_A + m_B, \qquad \mu = m_A m_B/m, \qquad \nu = \mu/m, \qquad \gamma = E/m,$$
$$\xi = E_1E_2/E^2, \qquad E = E_1 + E_2, \qquad \sigma = p_1\cdot p_2/m_1 m_2,$$

Kalin **RAP**
1910.03008
1911.09130

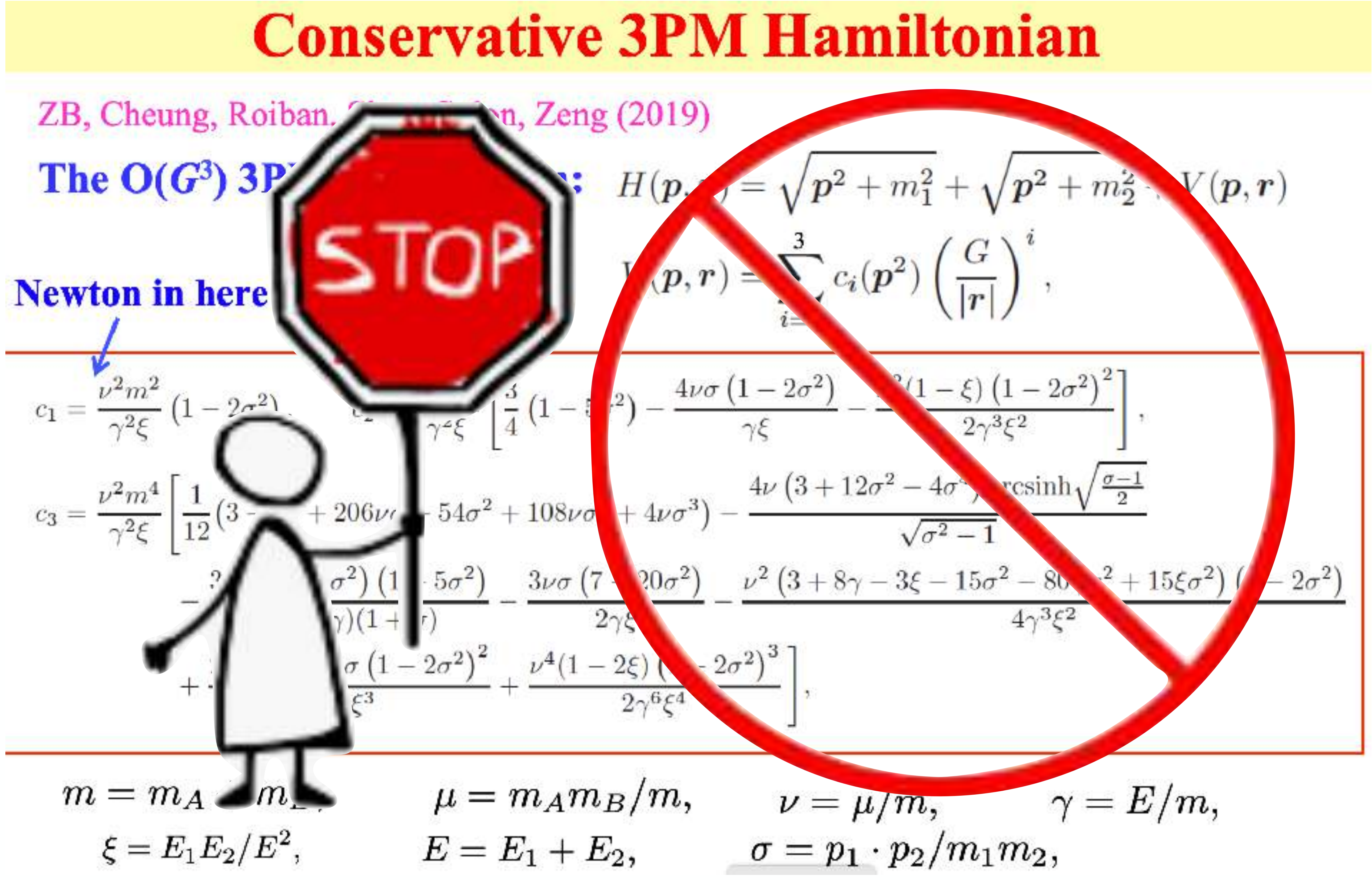
How do we compute *bound* observables from *boundary* data?

Conservative effects

Gravitational interaction is UNIVERSAL!

BUT:

Do we really need a Hamiltonian?



Conservative 3PM Hamiltonian

ZB, Cheung, Roiban.
n, Zeng (2019)

The O(G^3) 3P

Newton in here

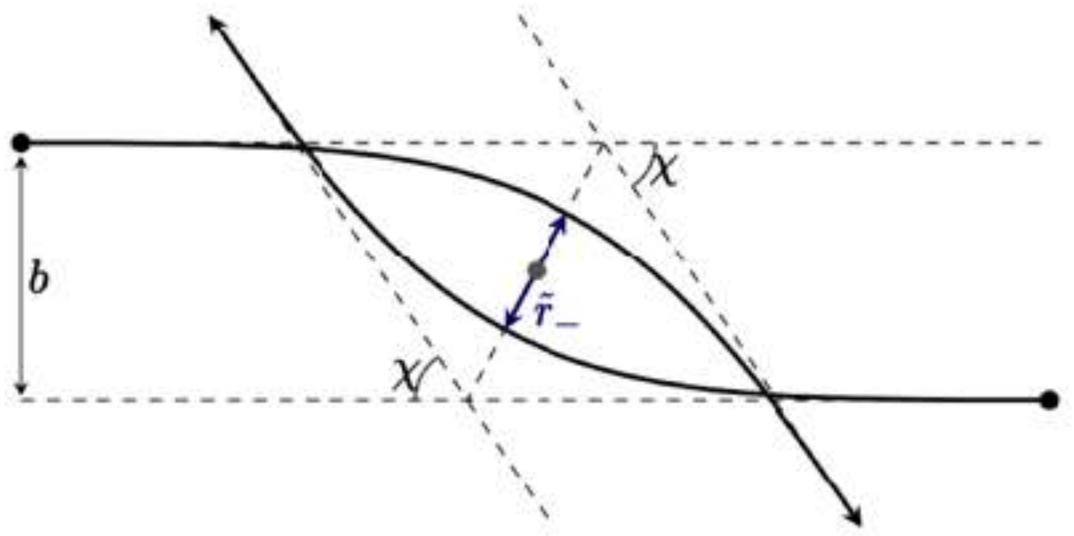
$$c_1 = \frac{\nu^2 m^2}{\gamma^2 \xi}\left(1-2\sigma^2\right)$$

$$m = m_A \qquad \mu = m_A m_B/m, \qquad \nu = \mu/m, \qquad \gamma = E/m,$$

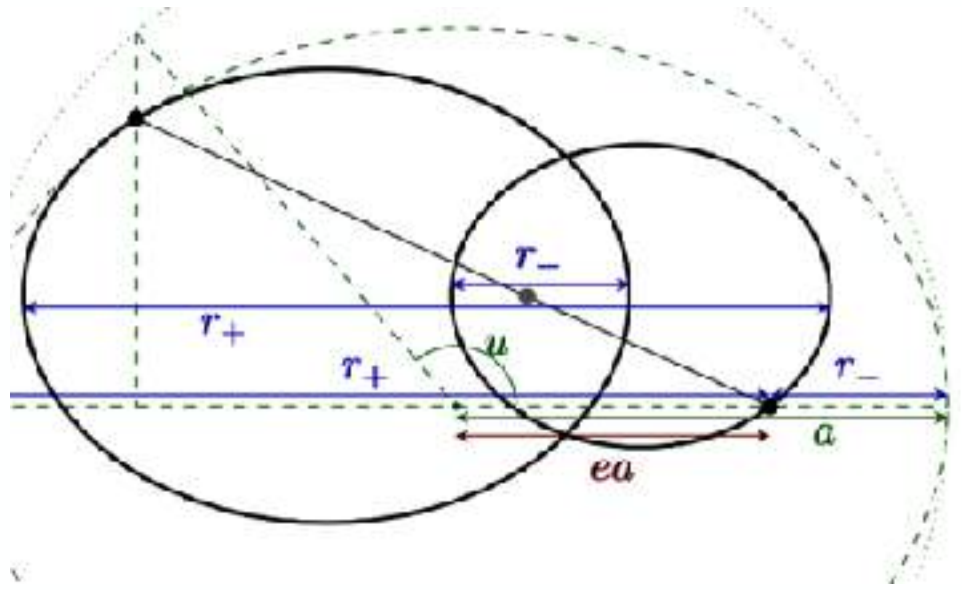
$$\xi = E_1 E_2/E^2, \qquad E = E_1 + E_2, \qquad \sigma = p_1 \cdot p_2/m_1 m_2,$$

Kalin **RAP**
1910.03008
1911.09130

B2B correspondence



Conservative effects



$$\underbrace{\frac{1}{\pi}\int_{\tilde{r}_-(J,\mathcal{E})}^{\infty}}_{\text{Scattering angle}}\frac{J}{r^2\sqrt{\boldsymbol{p}^2(\mathcal{E},r)-J^2/r^2}}\mathrm{d}r\,;$$

$$\underbrace{\frac{1}{\pi}\int_{r_-(J,\mathcal{E})}^{r_+(J,\mathcal{E})}}_{\text{Periastron advance}}\frac{J}{r^2\sqrt{\boldsymbol{p}^2(\mathcal{E},r)-J^2/r^2}}\mathrm{d}r$$

$$\begin{aligned} r_-(J,\mathcal{E}) &= \tilde{r}_-(J,\mathcal{E}) \qquad J>0,\ \mathcal{E}<0\,. \\ r_+(J,\mathcal{E}) &= \tilde{r}_-(-J,\mathcal{E}) \qquad J>0,\ \mathcal{E}<0\,, \end{aligned}$$

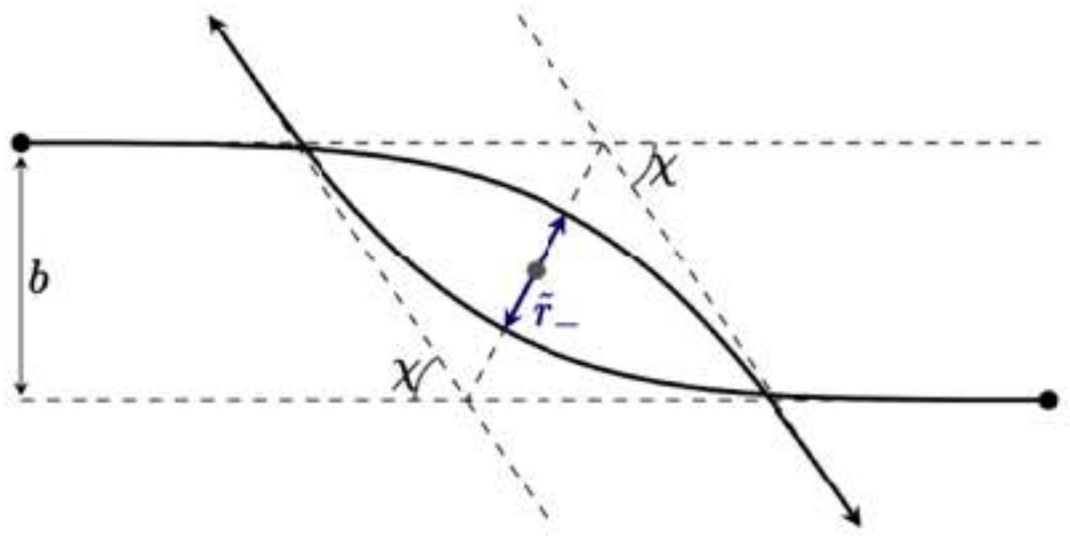
endpoints related by analytic continuation!

The most exciting phrase
to hear in science,
the one that heralds
new discoveries, is not
"EUREKA!"
but, "that's funny..."

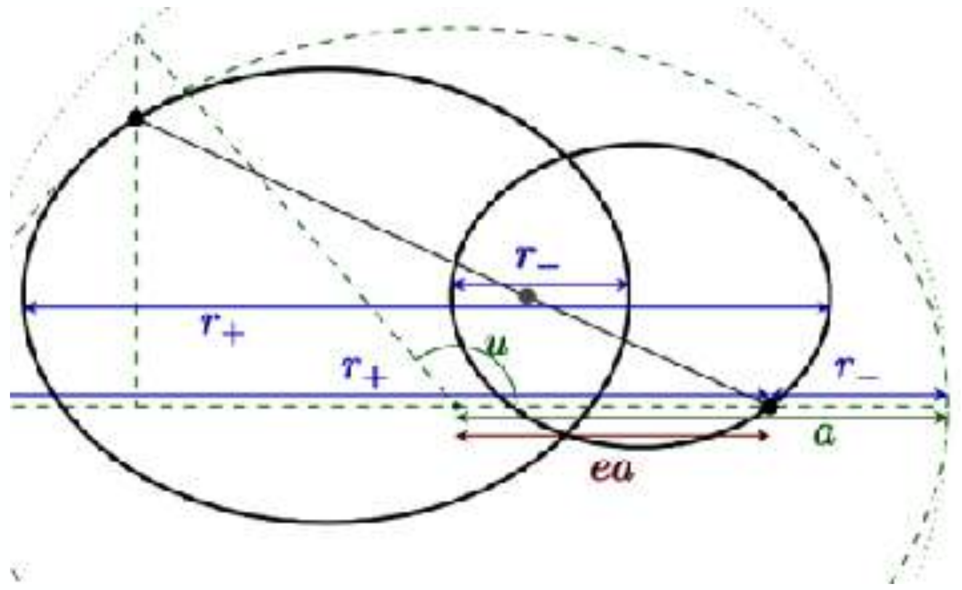
—Isaac Asimov

Kalin **RAP**
1910.03008
1911.09130

B2B correspondence



Conservative effects



$$\underbrace{\frac{1}{\pi}\int_{\tilde{r}_-(J,\mathcal{E})}^{\infty}}_{\textbf{Scattering angle}} \frac{J}{r^2\sqrt{\boldsymbol{p}^2(\mathcal{E},r)-J^2/r^2}}\mathrm{d}r\,;$$

$$\underbrace{\frac{1}{\pi}\int_{r_-(J,\mathcal{E})}^{r_+(J,\mathcal{E})}}_{\textbf{Periastron advance}} \frac{J}{r^2\sqrt{\boldsymbol{p}^2(\mathcal{E},r)-J^2/r^2}}\mathrm{d}r$$

$$\boxed{\Delta\Phi(J,\mathcal{E}) = \chi(J,\mathcal{E}) + \chi(-J,\mathcal{E})}$$

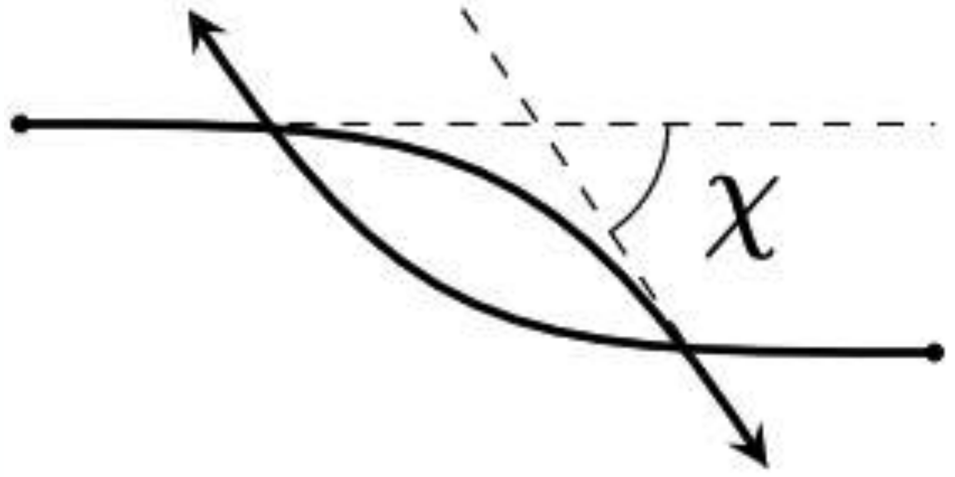
LOOP AROUND INFINITY!

The most exciting phrase
to hear in science,
the one that heralds
new discoveries, is not
"**EUREKA!**"
but, "**that's funny...**"

—Isaac Asimov

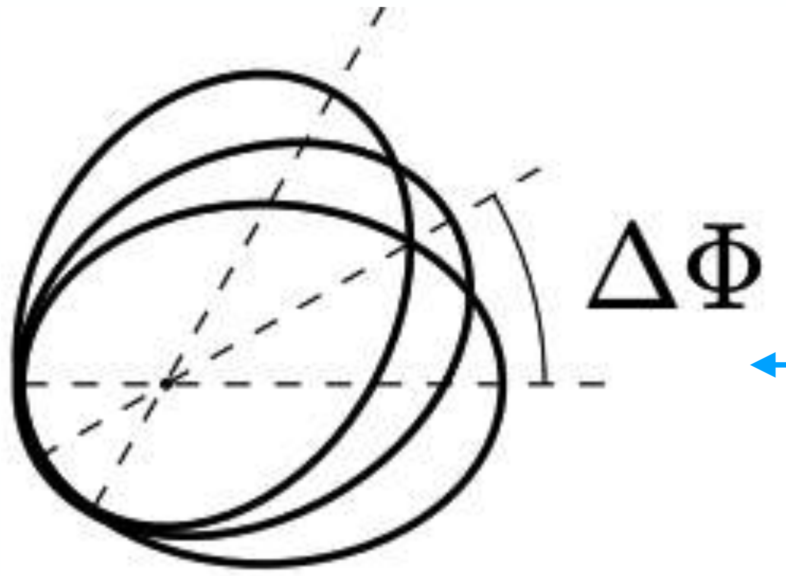
Kalin **RAP**
1910.03008
1911.09130

B2B correspondence



Conservative effects

$$\Delta\Phi(J,\mathcal{E}) = \chi(J,\mathcal{E}) + \chi(-J,\mathcal{E})$$



Analytic continuation

$\mathcal{E} < 0$

At the level of the radial action:

$$i_r^{bound}(j,\mathcal{E}) = i_r^{unbound}(j,\mathcal{E}) - i_r^{unbound}(-j,\mathcal{E})$$

Central object for the **bound** problem:

$$\delta S_r(J,\mathcal{E},m_a) = -\left(1+\frac{\Delta\Phi}{2\pi}\right)\delta J + \frac{\mu}{\Omega_r}\delta\mathcal{E} - \sum_a \frac{1}{\Omega_r}\left(\langle z_a\rangle - \frac{\partial E(\mathcal{E},m_a)}{\partial m_a}\right)\delta m_a$$

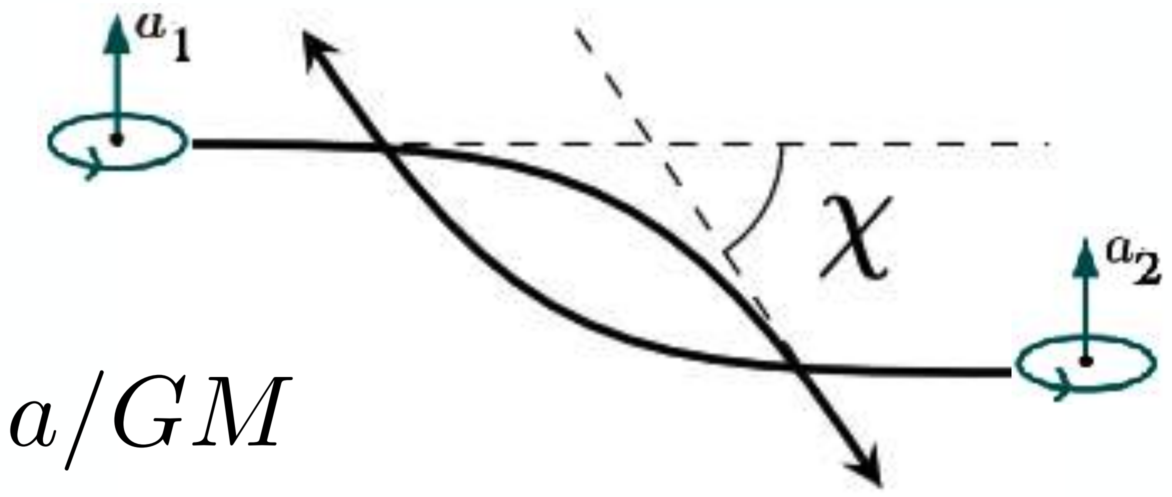
ALL conservative observables!

Kalin **RAP**
1910.03008
1911.09130

Liu **RAP** Yang
2102.10059

B2B correspondence

valid for (planar) aligned-spin

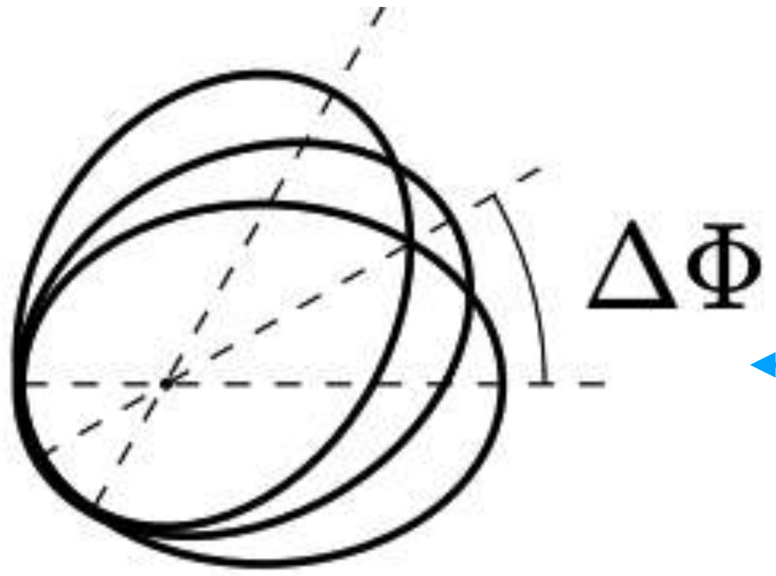


$\tilde{a} = a/GM$

Conservative effects

$$\boxed{\Delta\Phi(J,\mathcal{E}) = \chi(J,\mathcal{E}) + \chi(-J,\mathcal{E})}$$

J total (canonical) angular momentum



Analytic continuation

$\mathcal{E} < 0$

At the level of the radial action:

$$i_r^{(\text{bound})}(\mathcal{E} < 0, \ell, \tilde{a}_\pm) = i_r^{(\text{unbound})}(\mathcal{E} < 0, \ell, \tilde{a}_\pm) - i_r^{(\text{unbound})}(\mathcal{E} < 0, -\ell, -\tilde{a}_\pm)\,,$$

Central object for the **bound** problem:

$$\boxed{\delta S_r(J,\mathcal{E},m_a) = -\left(1 + \frac{\Delta\Phi}{2\pi}\right)\delta J + \frac{\mu}{\Omega_r}\delta\mathcal{E} - \sum_a \frac{1}{\Omega_r}\left(\langle z_a\rangle - \frac{\partial E(\mathcal{E},m_a)}{\partial m_a}\right)\delta m_a}$$

ALL conservative observables!

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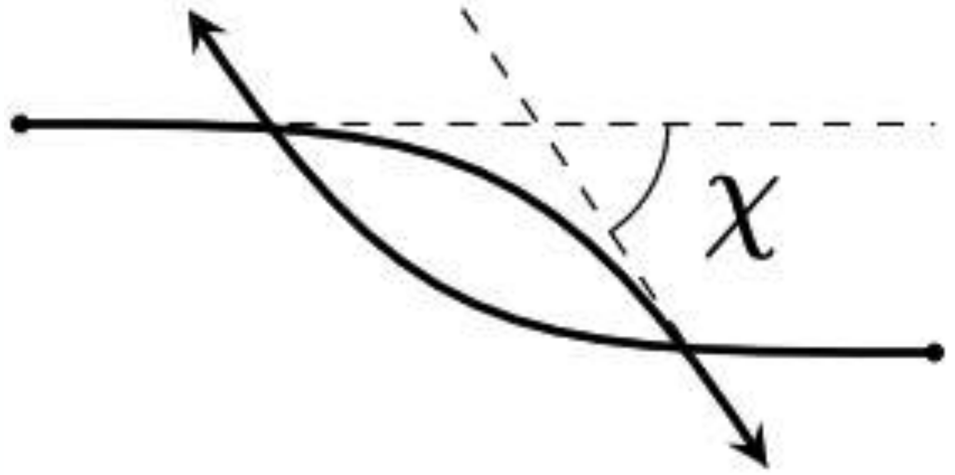
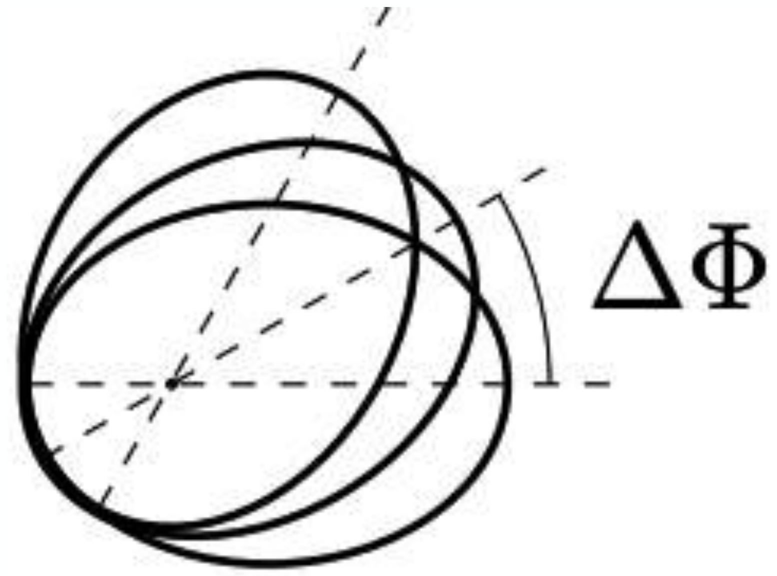
1910.03008
1911.09130

B2B correspondence



Conservative effects

↔



✓ 1PN ✓ 2PN ✓ 2PN

$$\left(\frac{\Delta\Phi}{2\pi}\right)_{\text{2-loop}} = \frac{3}{j^2} + \frac{3(35-10\nu)}{4j^4} + \frac{3}{4j^2}\left(10-4\nu+\frac{194-184\nu+23\nu^2}{j^2}\right)\mathcal{E}$$

✓ 3PN

$$+\frac{3}{4j^2}\left(5-5\nu+4\nu^2+\frac{3535-6911\nu+3060\nu^2-375\nu^3}{10j^2}\right)\mathcal{E}^2$$

✓ 4PN

$$+\frac{3}{4j^2}\left((5-4\nu)\nu^2+\frac{35910-126347\nu+125559\nu^2-59920\nu^3+7385\nu^4}{140j^2}\right)\mathcal{E}^3$$

$$+\frac{3}{4j^2}\left((5-20\nu+16\nu^2)\,\frac{\nu^2}{4}\right)\mathcal{E}^4+\cdots,$$

ONE-LOOP EXACT! $\frac{\Delta\Phi}{2\pi}=\frac{\widetilde{M}_2 G^2}{2J^2}$

$$E = M(1+\nu\mathcal{E})$$

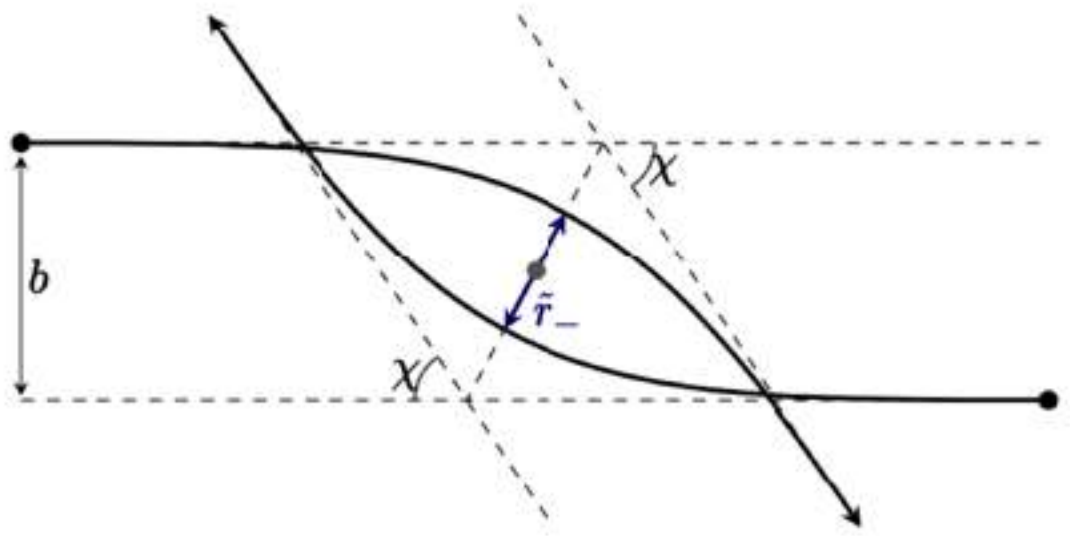
$$\Gamma \equiv E/M = \sqrt{1+2\nu(\gamma-1)},$$

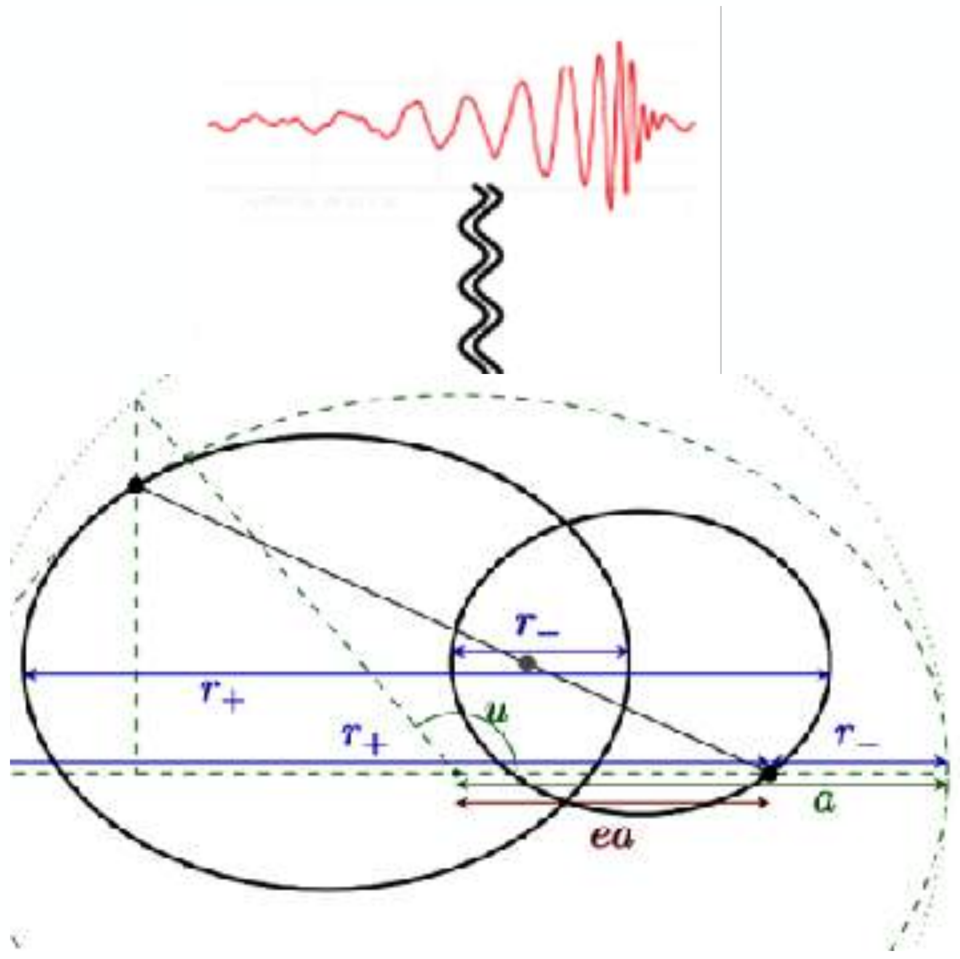
$$\chi_j^{(2)} \propto \widetilde{M}_2 = \frac{3M^2\mu^2}{2}\left(\frac{5\gamma^2-1}{\Gamma}\right)$$

Cho Kalin **RAP**
2112.03976

B2B correspondence

Radiative effects?!





$$r_-(J,\mathcal{E}) = \tilde{r}_-(J,\mathcal{E}) \qquad J>0,\, \mathcal{E}<0\,.$$

$$r_+(J,\mathcal{E}) = \tilde{r}_-(-J,\mathcal{E}) \qquad J>0,\, \mathcal{E}<0\,,$$

$$\Delta E_{\text{hyp}}(J,\mathcal{E}) = \int_{-\infty}^{+\infty} dt \frac{dE}{dt}$$

$$\Delta E_{\text{ell}}(J,\mathcal{E}) = \oint dt \frac{dE}{dt}$$

$$2\int_{\tilde{r}_-}^{+\infty} \frac{dr}{\dot{r}} \frac{dE}{dt}(r,J,\mathcal{E})$$

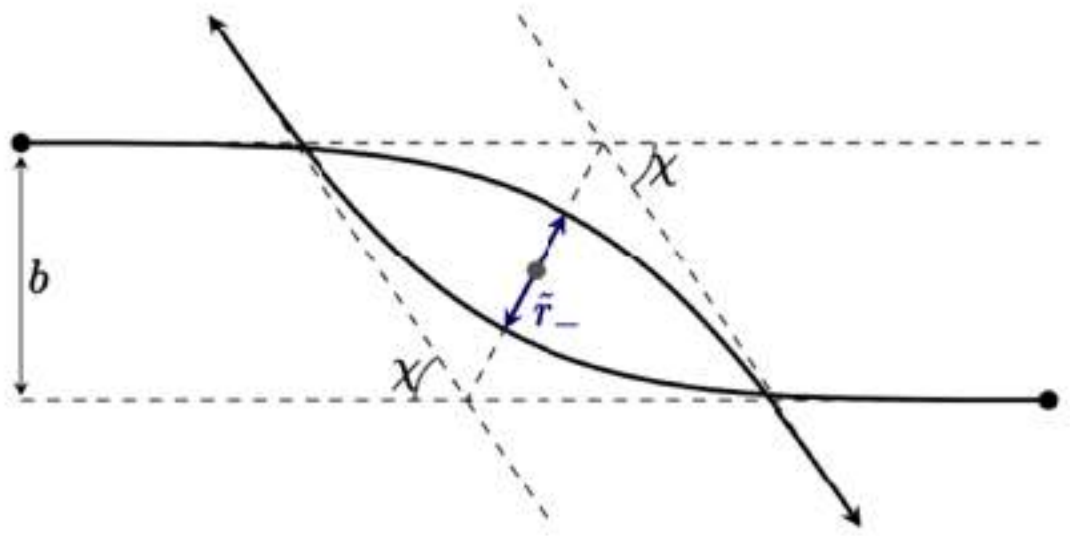
$$2\int_{r_-}^{r_+} \frac{dr}{\dot{r}} \frac{dE}{dt}(r,J,\mathcal{E})$$

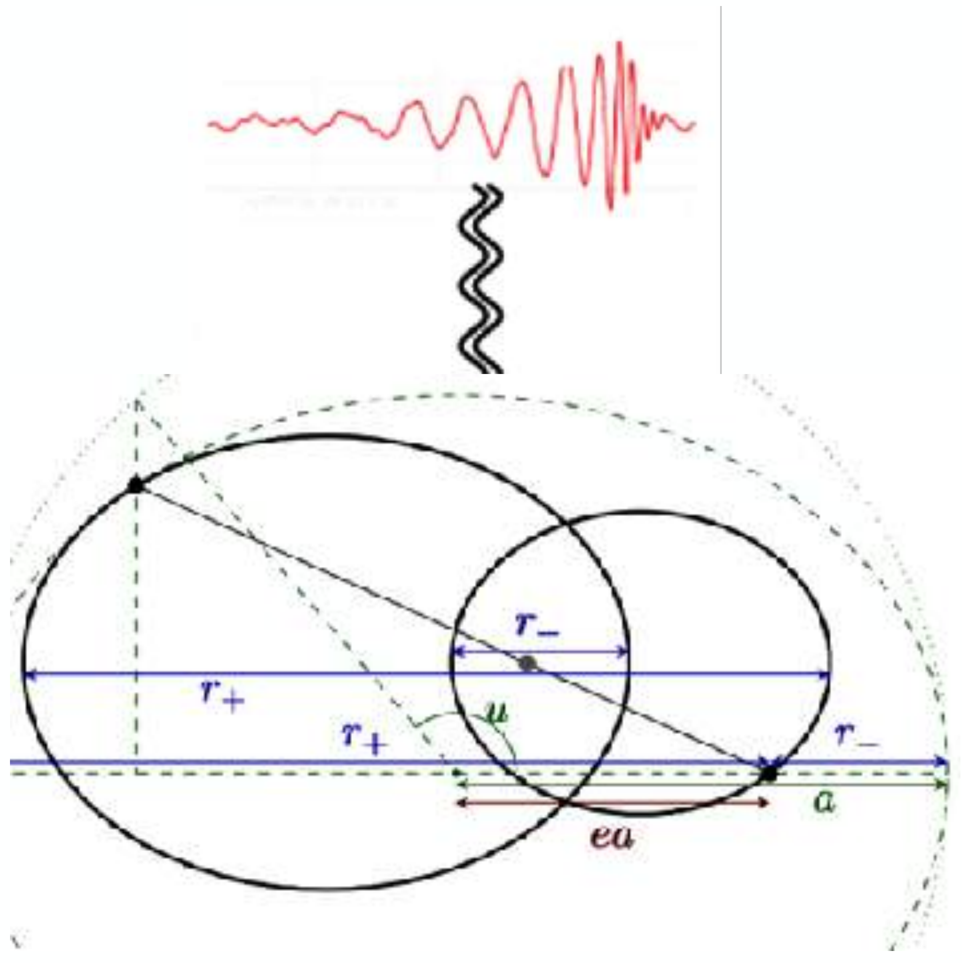
Aligned-spin configurations

Adiabatic Approx.

Cho Kalin **RAP**
2112.03976

B2B correspondence





valid for (planar) aligned-spin

Radiative effects

$$\longleftrightarrow$$

$$r_-(J,\mathcal{E}) = \tilde{r}_-(J,\mathcal{E}) \qquad J>0,\, \mathcal{E}<0\,.$$
$$r_+(J,\mathcal{E}) = \tilde{r}_-(-J,\mathcal{E}) \qquad J>0,\, \mathcal{E}<0\,,$$

$$\Delta E_{\rm hyp}(J,\mathcal{E}) = \int_{-\infty}^{+\infty} dt \frac{dE}{dt} \quad \longleftrightarrow \quad \Delta E_{\rm ell}(J,\mathcal{E}) = \oint dt \frac{dE}{dt}$$

$$2\int_{\tilde{r}_-}^{+\infty} \frac{dr}{\dot{r}}\frac{dE}{dt}(r,J,\mathcal{E}) \qquad \boxed{\frac{dE}{dt}(r,J,\mathcal{E}) = \frac{dE}{dt}(r,-J,\mathcal{E})} \qquad 2\int_{r_-}^{r_+} \frac{dr}{\dot{r}}\frac{dE}{dt}(r,J,\mathcal{E})$$

Similar to radial action: **Loop-around!**

$$\boxed{\Delta E_{\rm ell}(J,\mathcal{E}) = \Delta E_{\rm hyp}(J,\mathcal{E}) - \Delta E_{\rm hyp}(-J,\mathcal{E})} \quad \mathcal{E}<0$$

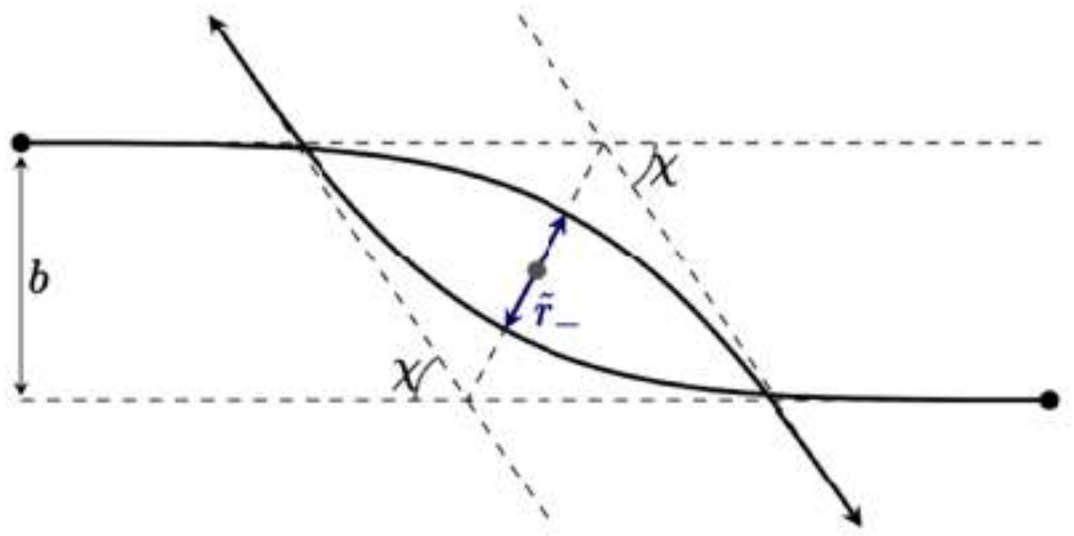
AND MORE!

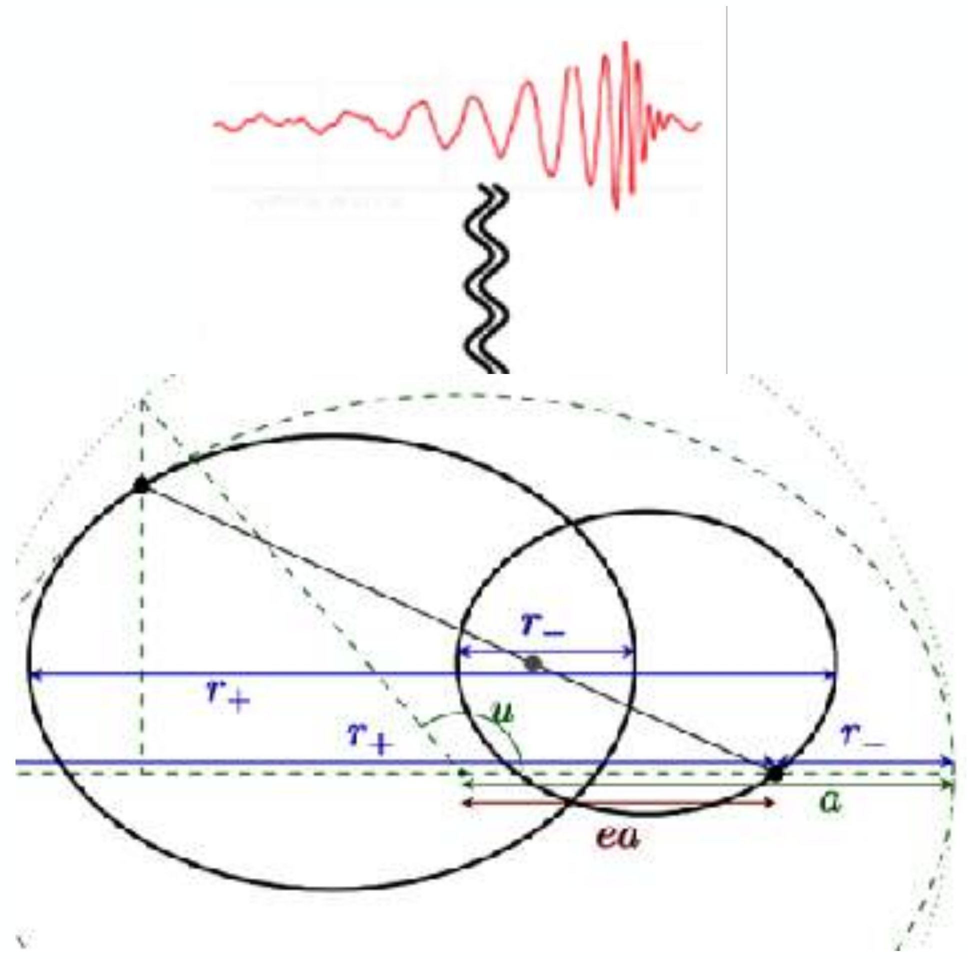
Aligned-spin configurations

Adiabatic Approx.

Cho Kalin **RAP**
2112.03976

B2B correspondence





valid for (planar) aligned-spin

Radiative effects

$\longleftrightarrow$

$$\begin{aligned} r_-(J,\mathcal{E}) &= \tilde{r}_-(J,\mathcal{E}) \qquad J>0,\, \mathcal{E}<0\,. \\ r_+(J,\mathcal{E}) &= \tilde{r}_-(-J,\mathcal{E}) \qquad J>0,\, \mathcal{E}<0\,, \end{aligned}$$

$$\Delta \overset{J}{\cancel{E}}_{\rm hyp}(J,\mathcal{E}) = \int_{-\infty}^{+\infty} dt\, \frac{d\overset{J}{\cancel{E}}}{dt} \quad \longleftrightarrow \quad \Delta \overset{J}{\cancel{E}}_{\rm ell}(J,\mathcal{E}) = \oint dt\, \frac{d\overset{J}{\cancel{E}}}{dt}$$

$$2\int_{\tilde{r}_-}^{+\infty} \frac{dr}{\dot{r}} \frac{d\overset{J}{\cancel{E}}}{dt}(r,J,\mathcal{E}) \qquad \boxed{\frac{dJ}{dt}(r,J,\mathcal{E}) = -\frac{dJ}{dt}(r,-J,\mathcal{E})} \qquad 2\int_{r_-}^{r_+} \frac{dr}{\dot{r}} \frac{d\overset{J}{\cancel{E}}}{dt}(r,J,\mathcal{E})$$

Similar to radial action: **Loop-around!**

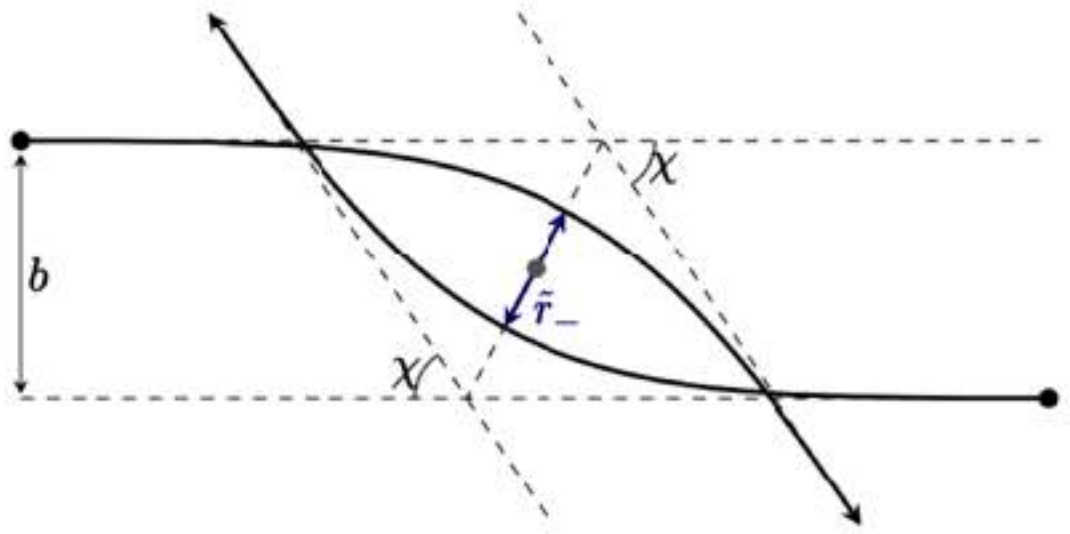
$$\boxed{\Delta J_{\rm ell}(J,\mathcal{E}) = \Delta J_{\rm hyp}(J,\mathcal{E}) + \Delta J_{\rm hyp}(-J,\mathcal{E})} \quad \mathcal{E}<0$$

AND MORE!

Sign flip
Similar to periastron to angle

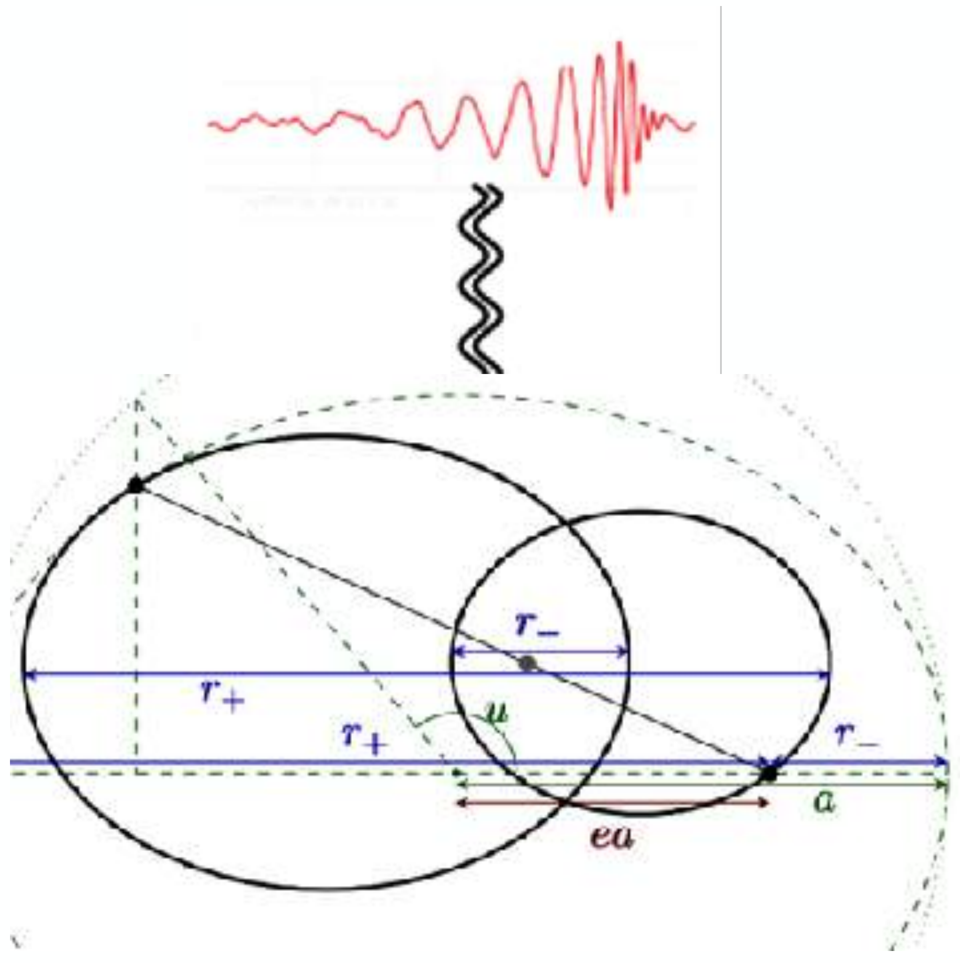
Cho Kalin **RAP**
2112.03976

B2B correspondence



Radiative effects

$$\cos^{-1}\left(\frac{1}{e}\right)+\cos^{-1}\left(-\frac{1}{e}\right)=\pi$$



$$e=\frac{r_+-r_-}{r_++r_-}$$

$$\Delta E_{\rm hyp}(j,\mathcal{E})=\frac{M\,\nu^2}{15}\left[\frac{850\sqrt{2}\sqrt{\mathcal{E}}}{j^6}+\frac{2692\sqrt{2}\mathcal{E}^{3/2}}{3j^4}+\left(\frac{850}{j^7}+\frac{1464\mathcal{E}}{j^5}+\frac{296\mathcal{E}^2}{j^3}\right)\cos^{-1}\left(-\frac{1}{e}\right)\right.$$
$$+\frac{\sqrt{2}\mathcal{E}^{5/2}(2506431-3009160\nu)}{105(1+2\mathcal{E}j^2)j^4}+\frac{\mathcal{E}^{3/2}(182337-140480\nu)}{3\sqrt{2}(1+2\mathcal{E}j^2)j^6}-\frac{7\sqrt{\mathcal{E}}(-5763+3220\nu)}{2\sqrt{2}(1+2\mathcal{E}j^2)j^8}$$
$$-\frac{2\sqrt{2}\mathcal{E}^{7/2}(-89907+156380\nu)}{35(1+2\mathcal{E}j^2)j^2}+\left(\frac{\mathcal{E}\left(\frac{33885}{2}-15900\nu\right)}{j^7}+\frac{\mathcal{E}^2\left(\frac{46617}{7}-10464\nu\right)}{j^5}\right.$$
$$\left.+\frac{\frac{40341}{4}-5635\nu}{j^9}+\frac{\mathcal{E}^3\left(\frac{4786}{7}-888\nu\right)}{j^3}\right)\cos^{-1}\left(-\frac{1}{e}\right)$$

$$\Delta E_{\rm ell}(j,\mathcal{E})=\frac{M\,\nu^2}{15}\left[\frac{850\pi}{j^7}+\frac{1464\mathcal{E}\pi}{j^5}+\frac{296\mathcal{E}^2\pi}{j^3}+\frac{\mathcal{E}^2\pi}{j^5}\left(\frac{46617}{7}-10464\nu\right)\right.$$
$$+\frac{7\pi(5763-3220\nu)}{4j^9}+\frac{15\mathcal{E}\pi(2259-2120\nu)}{2j^7}+\frac{\mathcal{E}^3\pi}{j^3}\left(\frac{4786}{7}-888\nu\right)$$

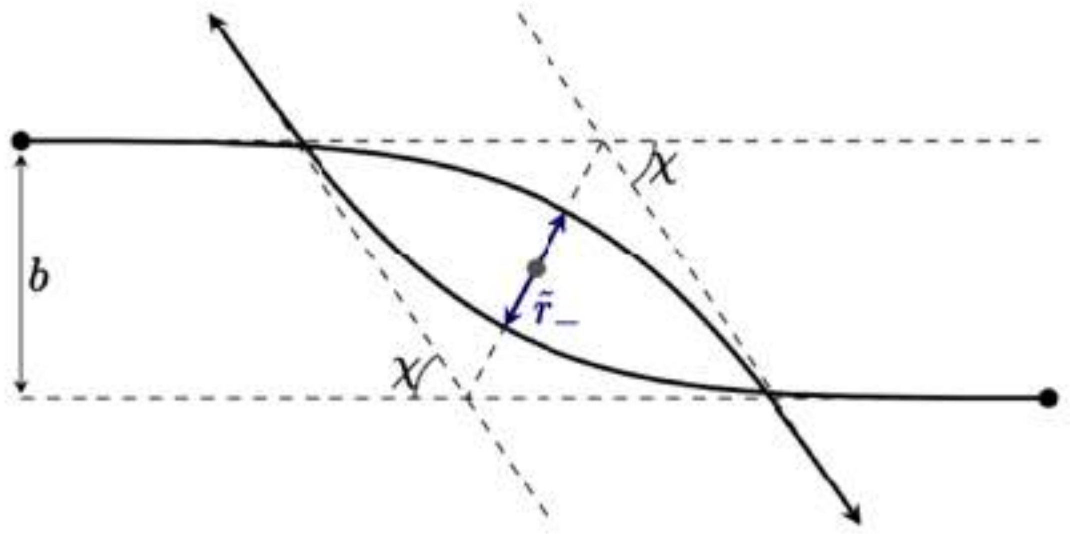
$$\boxed{\Delta E_{\rm ell}(e,\mathcal{E})=\Delta E_{\rm hyp}(e,\mathcal{E})-\Delta E_{\rm hyp}(-e,\mathcal{E})}$$

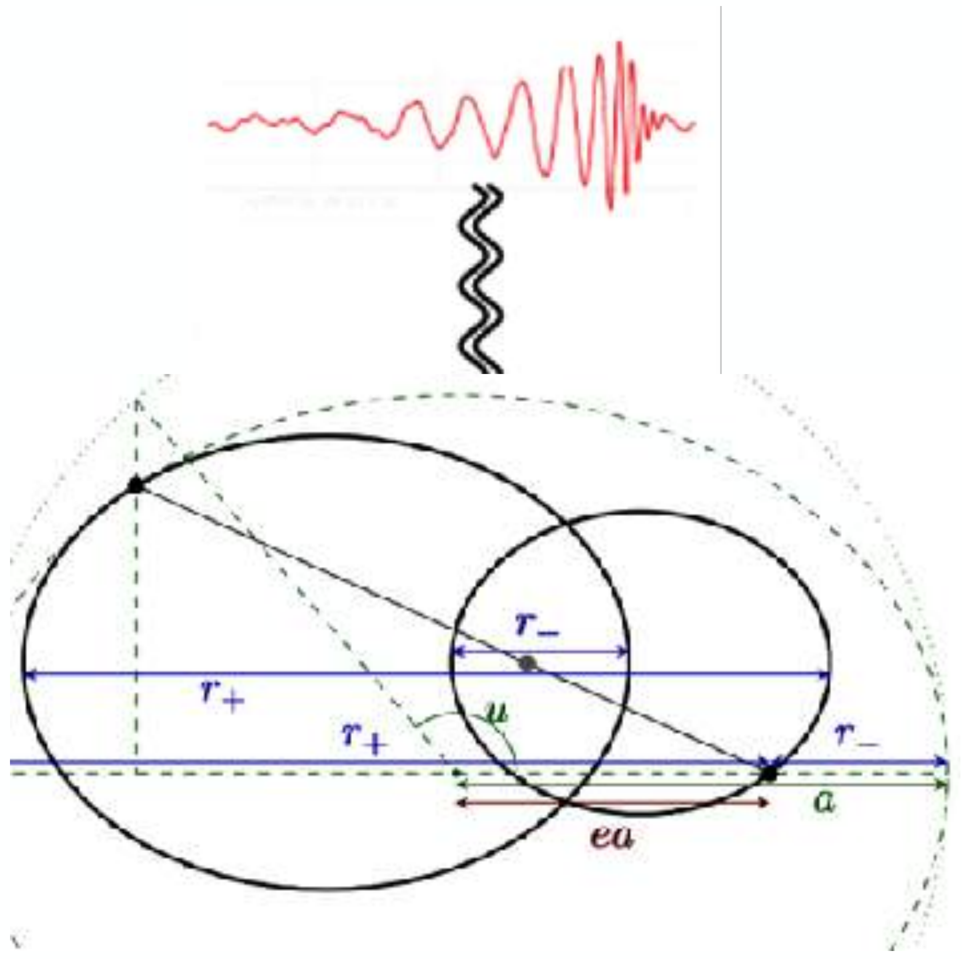
Odd terms survive

Cho Kalin **RAP**
2112.03976

B2B correspondence

Conservative?!





Radiative effects

$$r_-(J,\mathcal{E}) = \tilde{r}_-(J,\mathcal{E}) \qquad J>0,\, \mathcal{E}<0\,.$$
$$r_+(J,\mathcal{E}) = \tilde{r}_-(-J,\mathcal{E}) \qquad J>0,\, \mathcal{E}<0\,,$$

$$\delta\mathcal{S}_r^{bound} = -\frac{1}{2\pi}\oint H_{\text{tail}}dt \qquad \delta\mathcal{S}_r^{unbound} = -\frac{1}{2\pi}\int_{-\infty}^{+\infty} H_{\text{tail}}dt$$

$$i_r^{bound}(j,\mathcal{E}) = i_r^{unbound}(j,\mathcal{E}) - i_r^{unbound}(-j,\mathcal{E})$$

(Local + Non-Local) Conserv.

Valid in the “large-j” limit ONLY

What about the tail Hamiltonian? **Loop around again!**

$$\int_{\tilde{r}_-}^{\infty}\frac{dr}{p_r}H_{\text{tail}} \xleftrightarrow{H_{\text{tail}}(r,\mathcal{E},j) = H_{\text{tail}}(r,\mathcal{E},-j)} \int_{r_-}^{r_+}\frac{dr}{p_r}H_{\text{tail}}$$

Cho Kalin **RAP**
2112.03976

Dlapa Kalin Liu **RAP**
2106.08276
2112.11296

B2B correspondence

Binding energy for circular orbits

$x = (GM\omega)^{2/3} \sim v^2$

$$\begin{aligned}
\epsilon(x) = \Bigg\{ & 1 + \left[-\frac{\nu}{12} - \frac{3}{4}\right] x + \left[-\frac{\nu^2}{24} + \frac{19\nu}{8} - \frac{27}{8}\right] x^2 \qquad \text{(large-eccentricity)} \\
& + \left[-\frac{35\nu^3}{5184} - \frac{155\nu^2}{96} - \frac{5}{576}\left(246\pi^2 - 6889\right)\nu - \frac{675}{64}\right] x^3 \\
& + \Bigg[\frac{77\nu^4}{31104} + \frac{301\nu^3}{1728} + \frac{7\left(2706\pi^2 - 71207\right)\nu^2}{3456} + \frac{7\left(19365\pi^2 - 98756\right)\nu}{23040} - \frac{3969}{128} \\
& + \left(\frac{448\log x}{15} - \frac{271768\zeta_3}{45} + \frac{19576}{135} + \frac{463232\log 2}{45}\right)\nu\Bigg] x^4 \Bigg\} + \cdots .
\end{aligned}$$

NON-LOCAL PART $\longrightarrow$

Mismatch with known result (even different transcendental numbers!):

$$\delta\epsilon = \nu x^5 \frac{56}{135}\left(14559\,\zeta_3 - 329 + 144\gamma_E - 24528\log 2\right) \simeq 10^2\,\nu x^5$$

Cho Kalin **RAP**
2112.03976

Dlapa Kalin Liu **RAP**
2106.08276
2112.11296

B2B correspondence

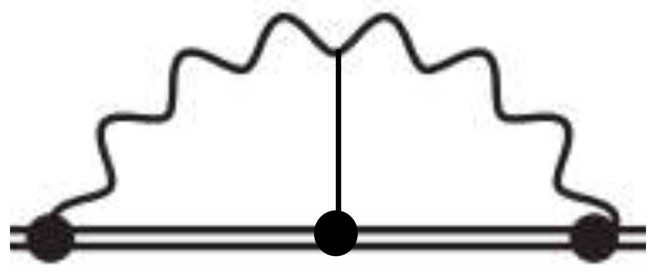
$$\epsilon_{\log x} = \left\{ \frac{448}{15}\nu x^5 + \left[\left(-\frac{224}{5} - \frac{432}{5}\right)\nu^2 + \left(-176 + \frac{1172}{35}\right)\nu\right] x^6 \right. \qquad \text{(large-eccentricity)}$$
$$+ \left[\left(\frac{616}{27} + \frac{792}{5} + \frac{176}{3}\right)\nu^3 + \left(\frac{39776}{45} + \frac{491326}{315} - \frac{1394492}{945}\right)\nu^2\right.$$
$$\left.\left. + \left(-\frac{2638064}{45} + \frac{6032774}{105} + \frac{1138874}{1215}\right)\nu\right] x^7 \right\} \log x \,.$$

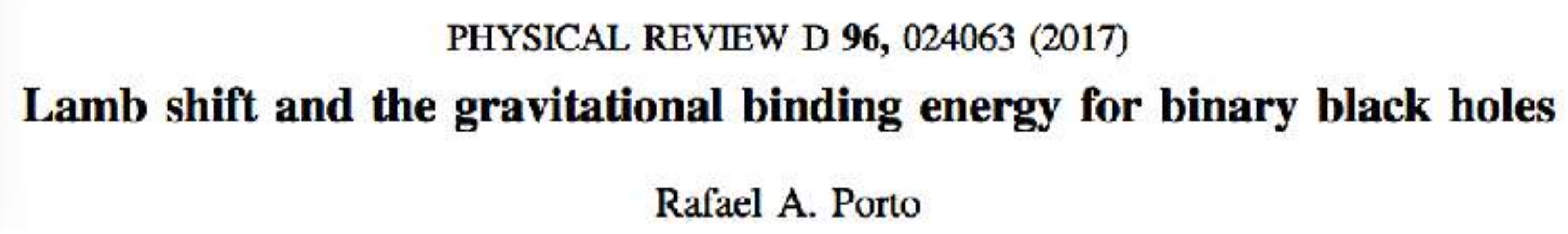
✓

ALL LOCAL + LOGS ARE A PERFECT MATCH!

$$i^{4\mathrm{PM}}_{r(\log)} = -\frac{E}{(2\pi)M^2\nu}\Delta E_{\mathrm{ell}}(j)\log(-\mathcal{E}) \quad \text{(bound)}$$
$$= \frac{2\nu}{3}\frac{(1-\gamma^2)^2}{(\Gamma j)^3}\chi_{2\epsilon}(\gamma)\log(-\mathcal{E}) + \cdots ,$$

$$\chi_{2\epsilon}(x) = -\frac{210\gamma^6 - 552\gamma^5 + 339\gamma^4 - 912\gamma^3 + 3148\gamma^2 - 3336\gamma + 1151}{32(\gamma^2-1)^2}$$
$$+ \frac{3\left(35\gamma^4 + 60\gamma^3 - 150\gamma^2 + 76\gamma - 5\right)}{16(\gamma^2-1)}\log\left(\frac{\gamma+1}{2}\right) - \frac{3\gamma\left(2\gamma^2-3\right)\left(35\gamma^4 - 30\gamma^2 + 11\right)}{32\left(\gamma^2-1\right)^2}\frac{\operatorname{arccosh}(\gamma)}{\sqrt{\gamma^2-1}}$$

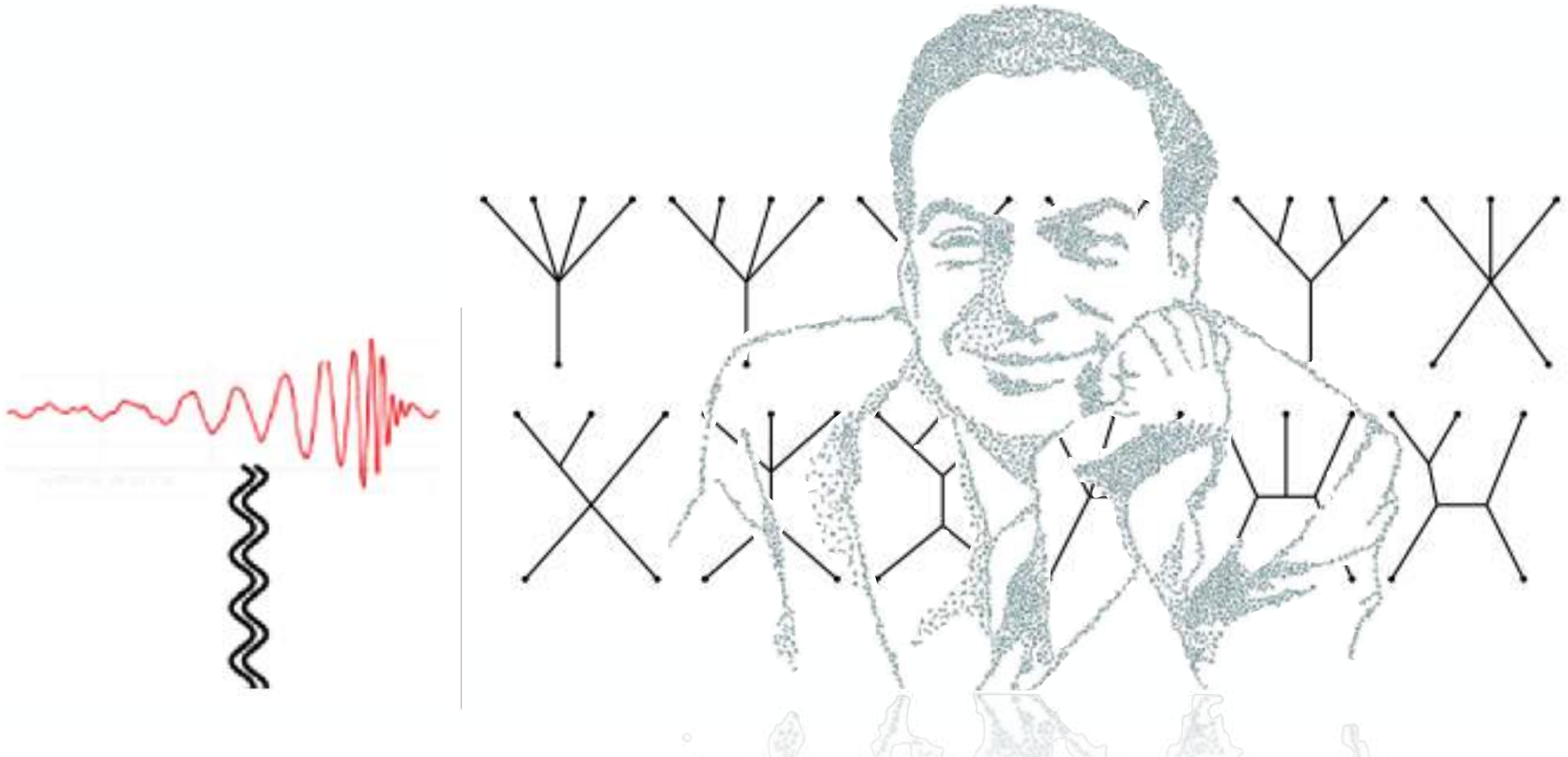


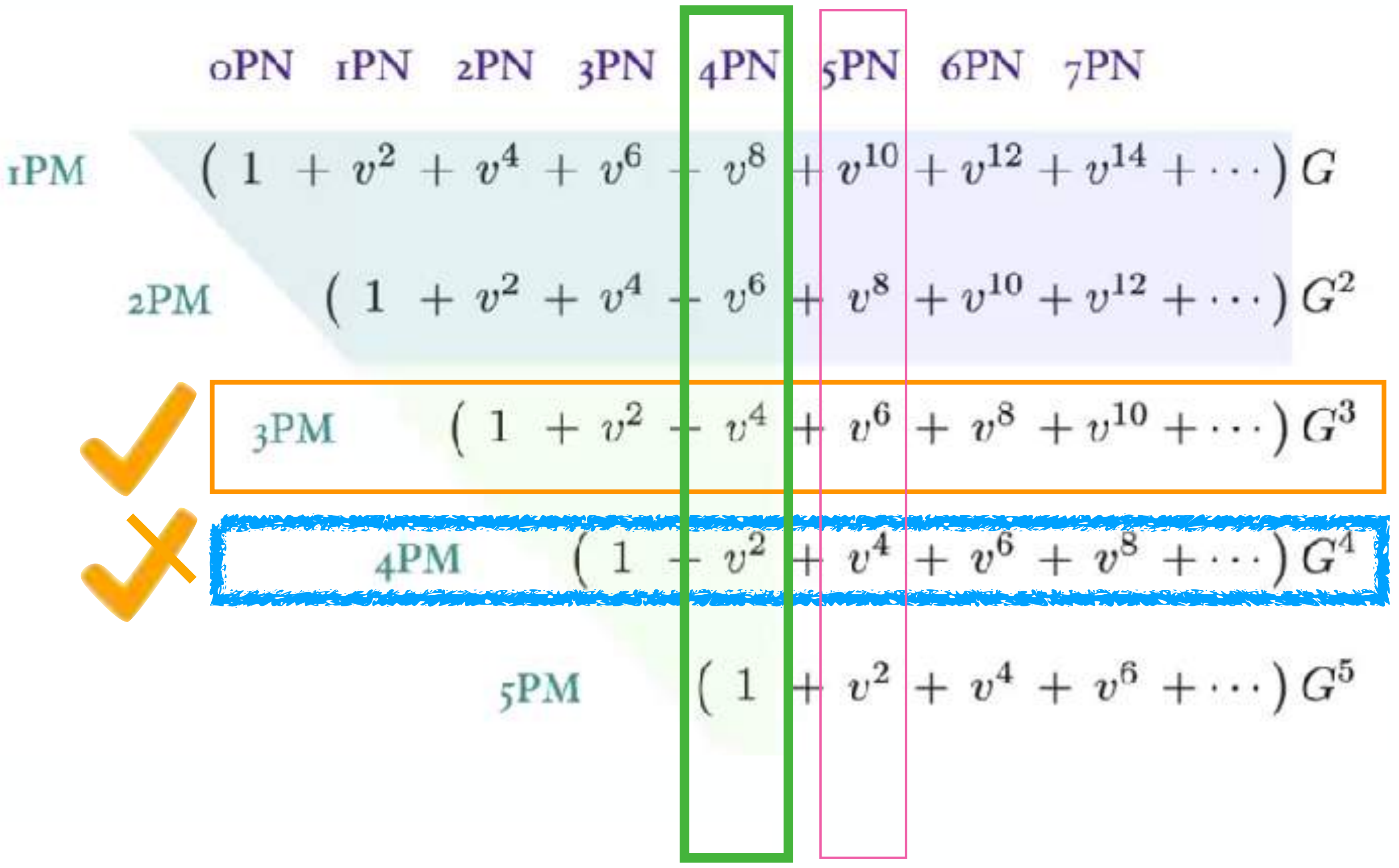


PHYSICAL REVIEW D **96**, 024063 (2017)

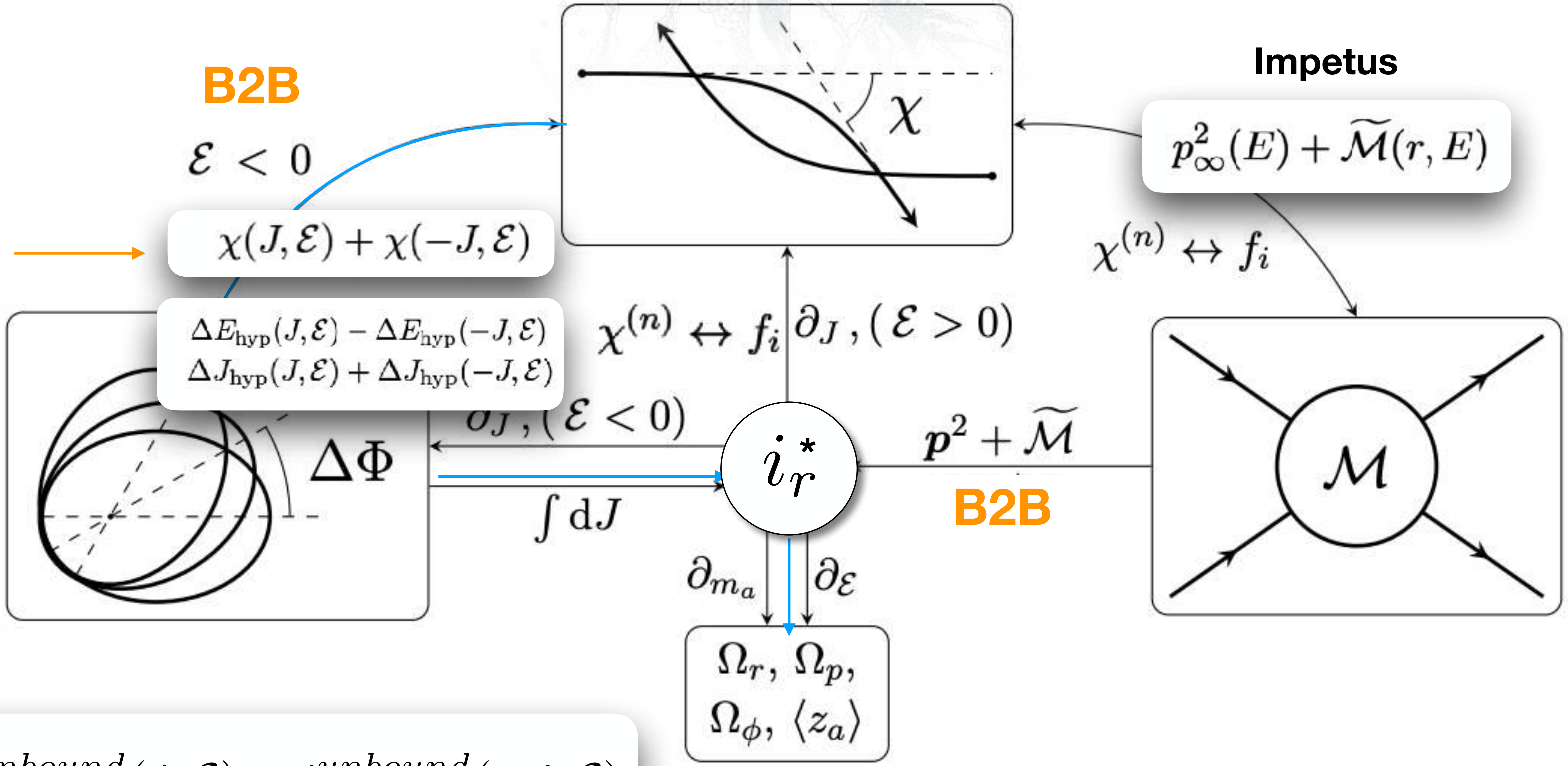
Lamb shift and the gravitational binding energy for binary black holes

Rafael A. Porto





	0PN	1PN	2PN	3PN	4PN	5PN	6PN	7PN	
1PM	$(1$	$+ v^2$	$+ v^4$	$+ v^6$	$+ v^8$	$+ v^{10}$	$+ v^{12}$	$+ v^{14}$	$+ \cdots) G$
2PM		$(1$	$+ v^2$	$+ v^4$	$+ v^6$	$+ v^8$	$+ v^{10}$	$+ v^{12}$	$+ \cdots) G^2$
✓ 3PM			$(1$	$+ v^2$	$+ v^4$	$+ v^6$	$+ v^8$	$+ v^{10}$	$+ \cdots) G^3$
✓✗ 4PM				$(1$	$+ v^2$	$+ v^4$	$+ v^6$	$+ v^8$	$+ \cdots) G^4$
5PM					$(1$	$+ v^2$	$+ v^4$	$+ v^6$	$+ \cdots) G^5$



B2B

$\mathcal{E} < 0$

Impetus

$p_\infty^2(E) + \widetilde{\mathcal{M}}(r, E)$

$\chi(J,\mathcal{E}) + \chi(-J,\mathcal{E})$

$\Delta E_{\rm hyp}(J,\mathcal{E}) - \Delta E_{\rm hyp}(-J,\mathcal{E})$
$\Delta J_{\rm hyp}(J,\mathcal{E}) + \Delta J_{\rm hyp}(-J,\mathcal{E})$

aligned-spins →

B2B

B2B

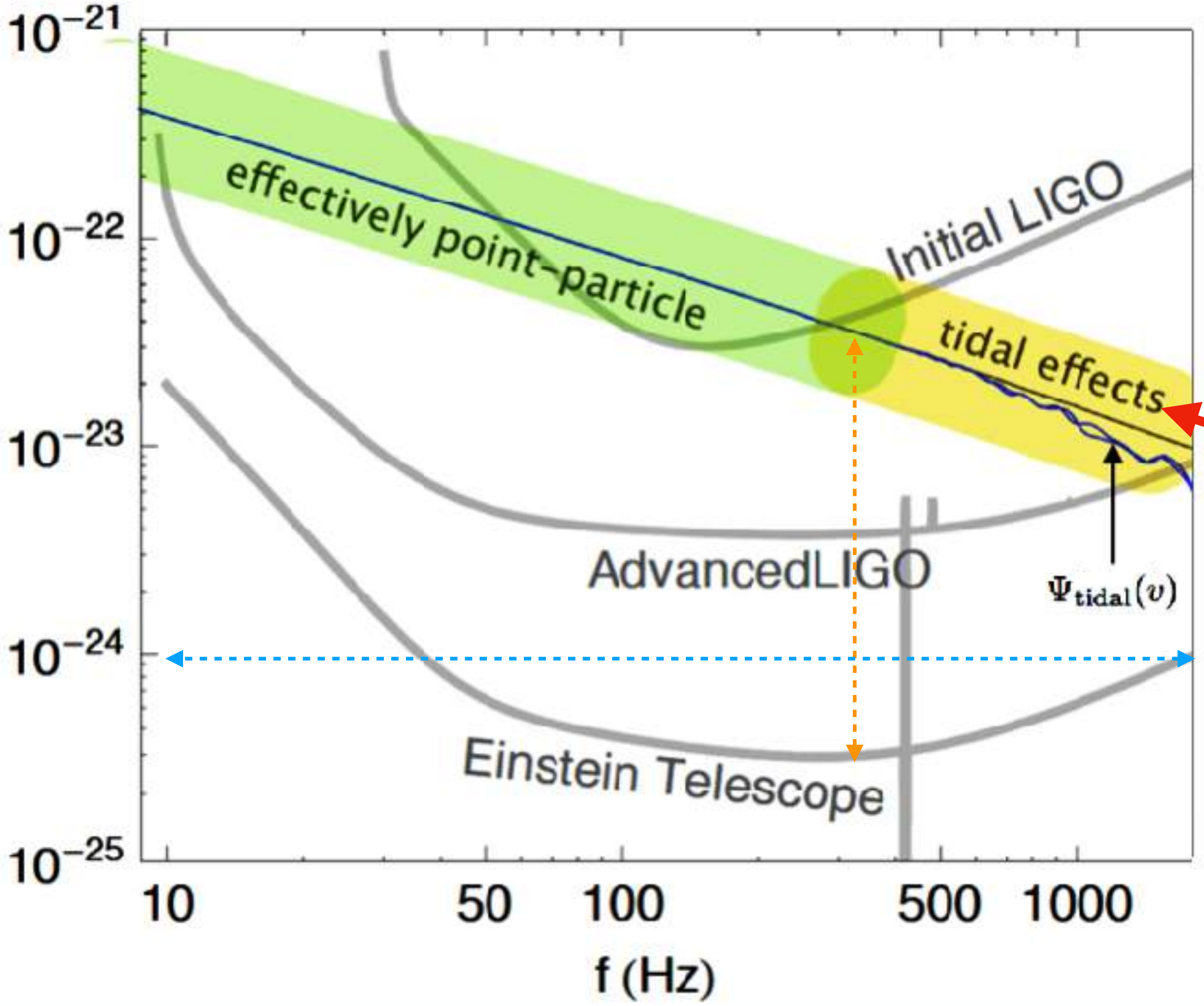
$$i_r^{bound}(j,\mathcal{E}) = i_r^{unbound}(j,\mathcal{E}) - i_r^{unbound}(-j,\mathcal{E})$$

"Waveforms will be far more complex and carry more information than expected. Improved modeling will be needed for extracting the GW's information"

Kip Thorne 'Last 3 minutes' **1993**

20+ years prior to first detection!

More 'luminosity/sensitivity' at 'short/long distances'

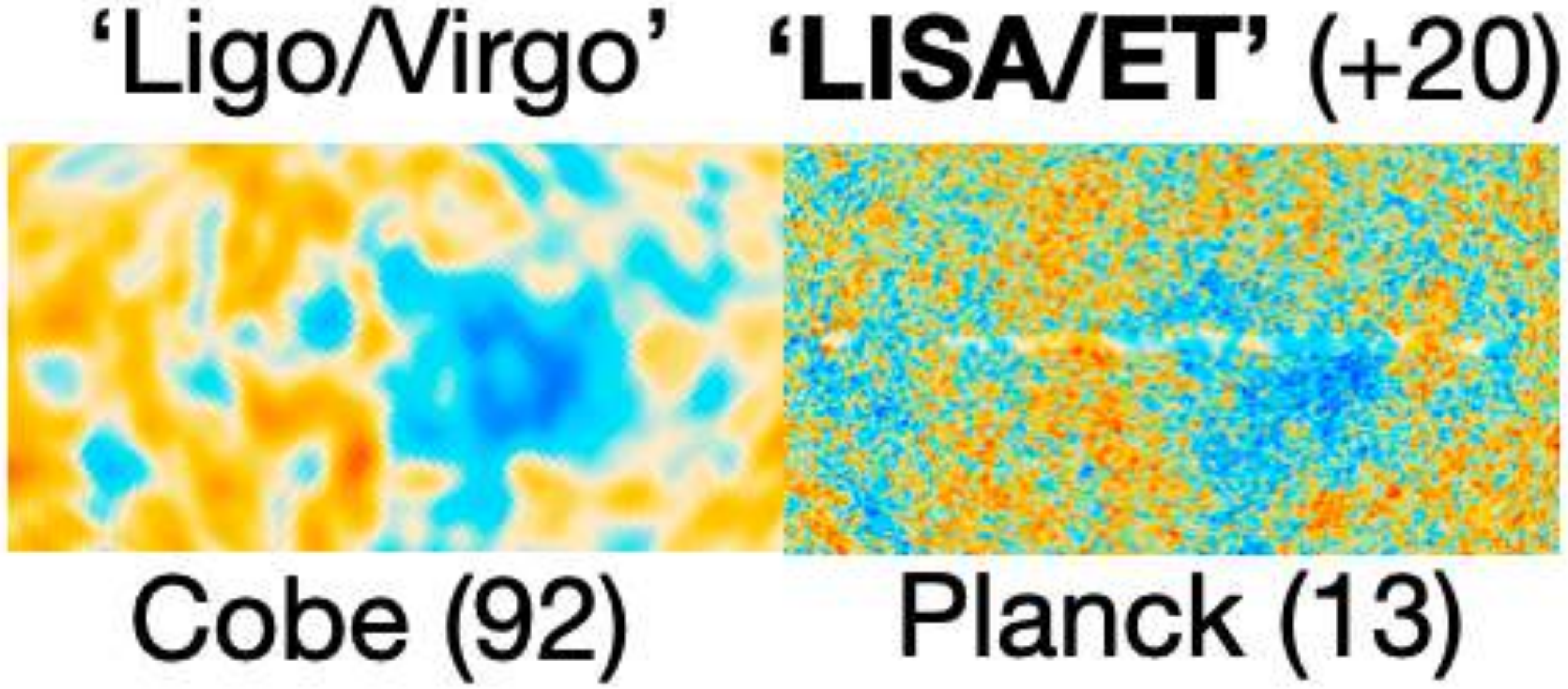


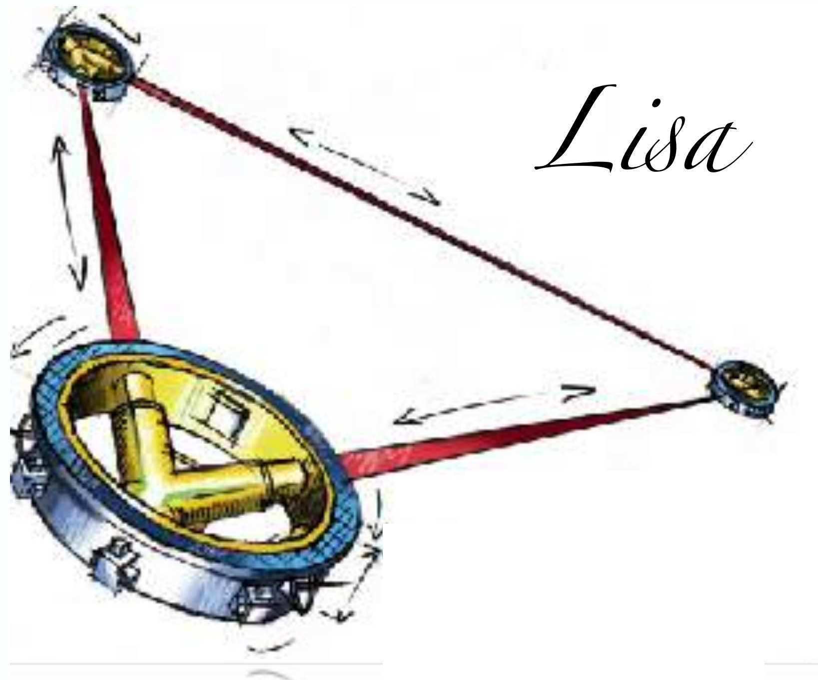
'New Physics Threshold'

- **Energy/Frequency Frontier**
- **Luminosity Frontier**

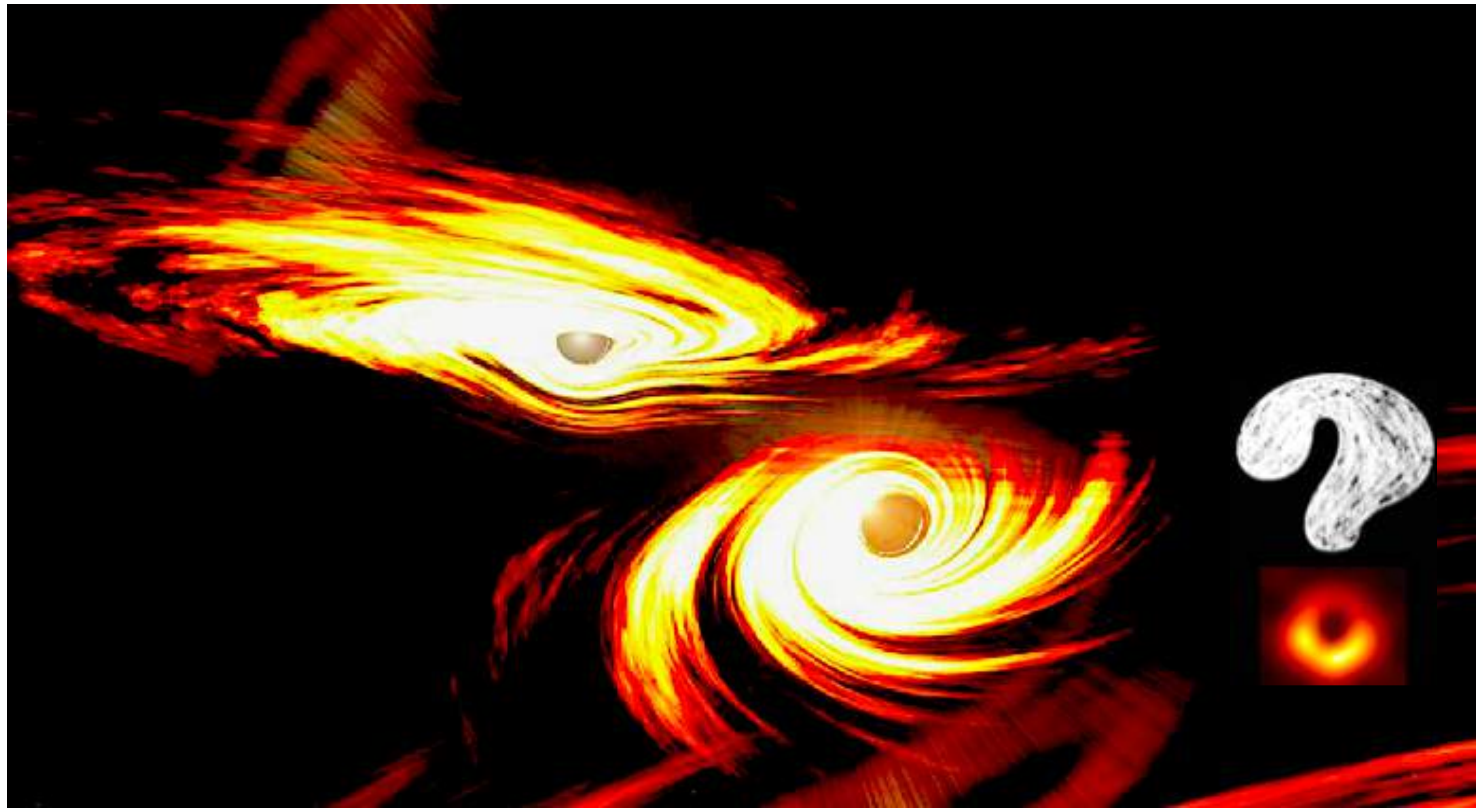
also 2G+!

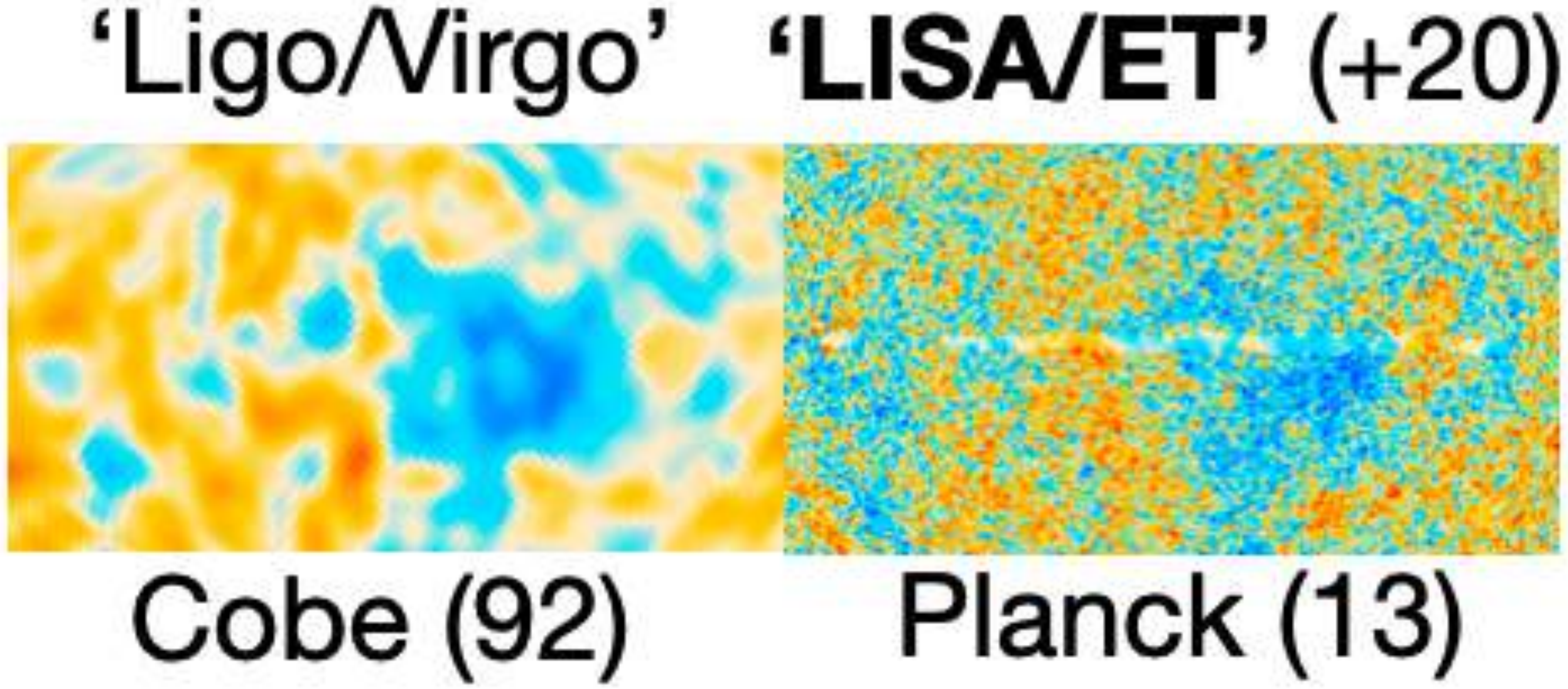
Are we ready for the future?

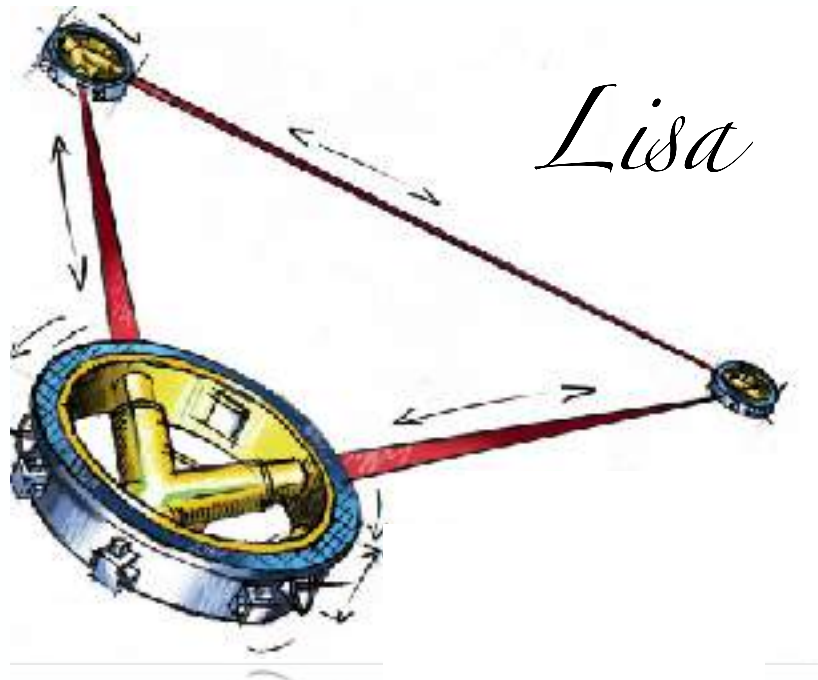




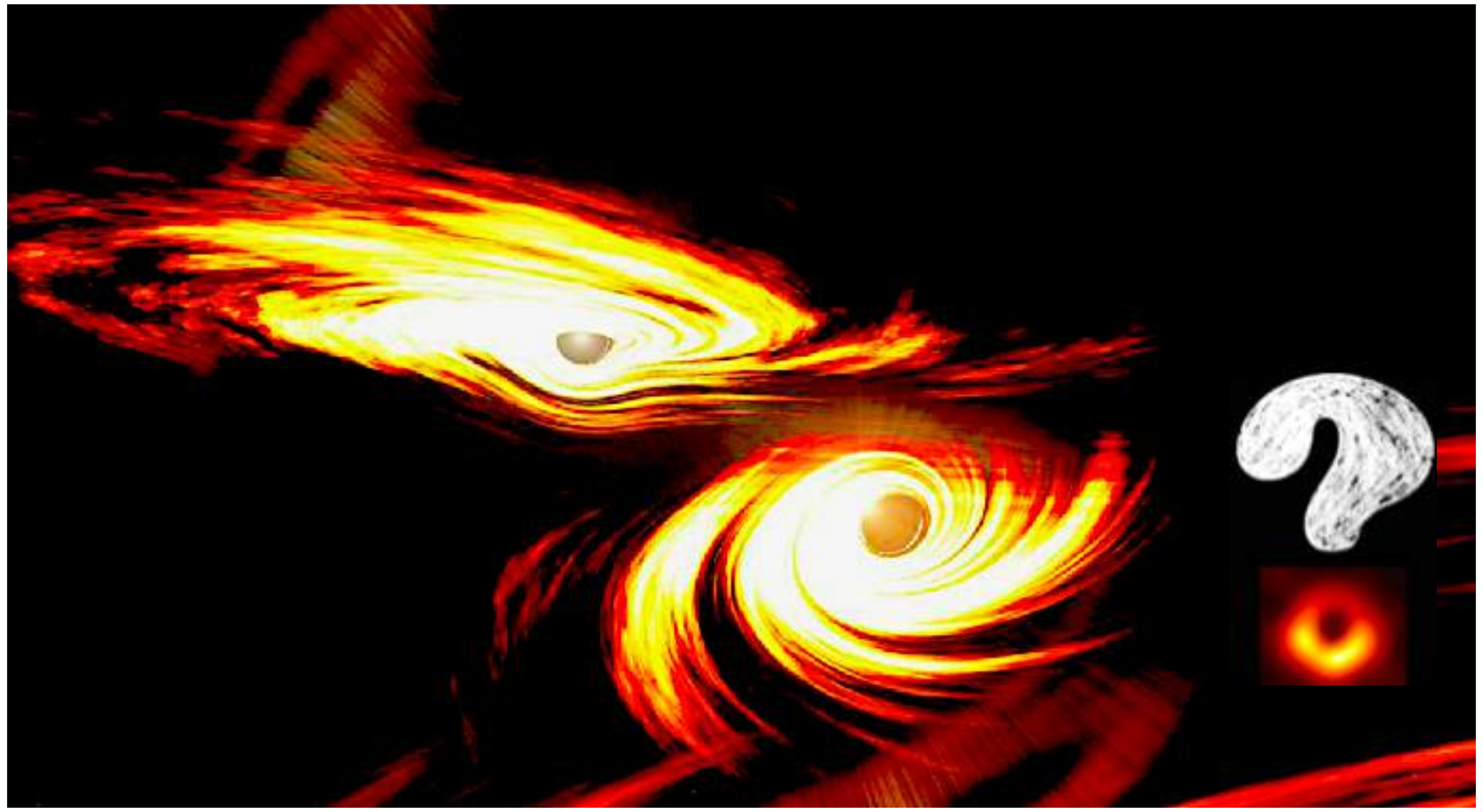
Einstein Telescope







Einstein Telescope



Are we ready for the future?

NYT 1991

Experts Clash Over Project To Detect Gravity Wave

Die Ziet

no.203.078 01.01.203X

EinsTein reloaded!

New era of foundational investigations established through GW Precision Data.

New particles discovered!

Black Holes unveiled!

Origin of structure uncovered!

Thank you!

*This project has received funding from the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme (grant agreement No 817791).