University of Southampton

School of Mathematical Sciences

Senior Challenge '23

Year 10 or below

Illustrations by Theo Chaddock & Peter Ackerley

Rules

1) The challenge should be attempted in your own time. Your entry must be **your own work**, though you should ask for the meanings of unfamiliar words.
2) Marks are awarded for reasoning, not only for getting the right answer. It is also possible to win a prize or certificate even if you have not completed all the questions, so hand in your entry even if it is not quite finished.
3) Present your worked solutions **separately on A4 size papers**. Lined papers are recommended, but blank or graph paper are accepted – as long as they are neatly presented. Entries written on this question sheet or without any workings **will not be marked**.
4) Write **your name and school on every page neatly**.
5) Please **staple your pages** together at the top-left corner if submitting your entry by mail.

Deadline

Either you or your teacher needs to submit your entry by **17th March 2023**, by post or online. For submission instructions, see the end of this document or visit the Maths Challenge webpage at www.southampton.ac.uk/stag/mathschallenge.page

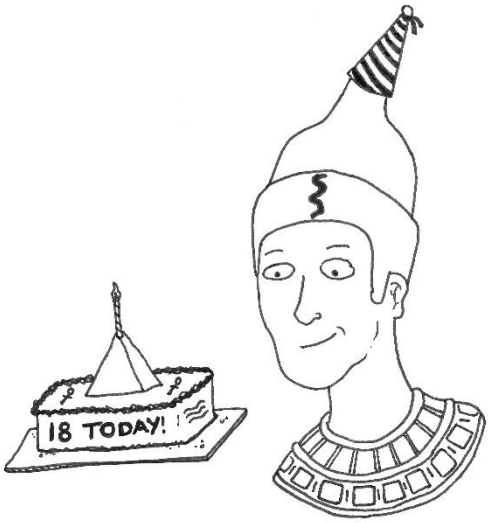
A Prize-Giving Evening will be held at the University of Southampton on 7th June 2023.

We hope that you enjoy the Challenge.
If you have any questions, please ask your teacher or parents to contact us at math4all@soton.ac.uk.

1. Seti's Party

Seti held a party. He says, 'There were 11 people at my party, and everyone had 3 friends there.'

Is this possible? Explain your answer.



2. Egyptian Fractions

The ancient Egyptians used only unit fractions, so they would write ¾ as ½ + ¼.

Find the six ways to write 1 as the sum of 4 distinct unit fractions.

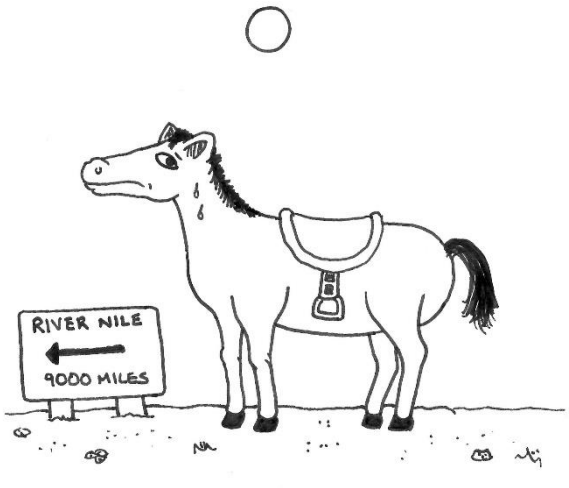
Write 1 as a sum of 5 distinct unit fractions, without using ½.

3. A Horse with No Name

Father and son make a 64km journey through the desert, starting at 4pm. They only have one horse (with no name), which can travel at a steady 8km/hour carrying one of them. Father can maintain a steady walking pace of 3km/hour and son a pace of 4km/hour. They alternately ride and walk. Each one ties the horse up after riding a certain distance, then walks ahead leaving the horse for the other's arrival.

At the half-way mark, they meet and stop for a three-hour nap and to feed the horse. After the nap, they resume the same pattern.

What time do they arrive at their destination?

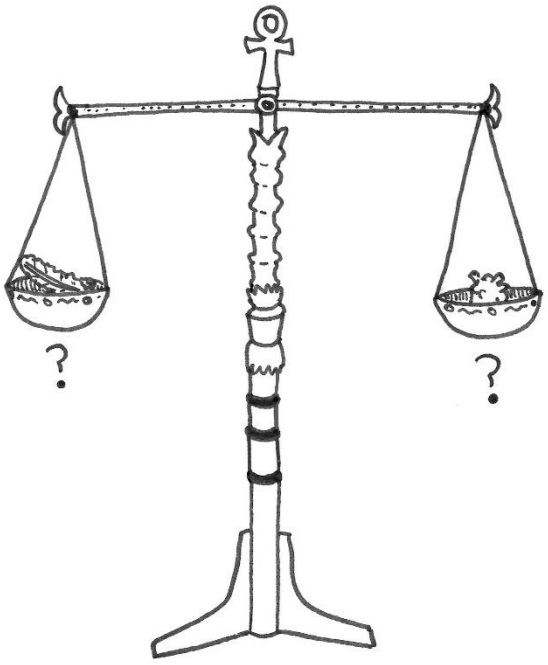


4. Time is Running Out

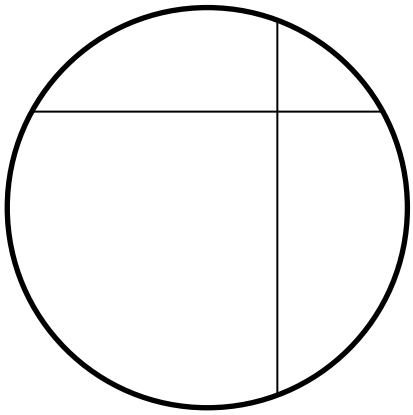
Cleopatra and her brother, Ptolemy, each have cylindrical water clocks (so they drain uniformly), but their clocks have different heights and diameters.
Cleopatra's clock drains from full down to empty in 4 hours and Ptolemy's does so in 5 hours. After two hours of draining, the water height in both clocks is the same.
What fraction of the height of Cleopatra's clock is the height of Ptolemy's?
After each has been draining for three hours, what fraction of the water height of Ptolemy's clock is equal to the water height of Cleopatra's?

5. Getting to the Heart of the Matter

Somehow, Anubis has managed to lose the feather of Maat! (I guess even gods aren't perfect!)
In the important funerary ceremony, a person's heart is balanced against this feather. If it is lighter, it means that they are without sin (pure of heart) and are entitled to join Osiris in the field of reeds for their afterlife. Otherwise, they are eaten by the part-crocodile-part-lion-part-hippo god Ammut and just disappear.



8 people have just arrived to be judged.
Fortunately, it is known that only one of them is pure of heart, whilst the remaining 7 are sinners, and have heavier hearts (but all sinners' hearts weigh the same as each other).
Help Anubis identify the lightest of the 8 hearts, using just the hearts and a balance. But beware! Ammut grows impatient: you may only use the balance twice!



6. Reed the Question

At a temple to Sekhmet, there is a circular reed bed to be planted with 4 different types of reed, one in each of the four sections, as shown here. The radius is 360cm and there are two strings crossing at right angles of lengths 560cm and 640cm.

Find out how far from the centre of the circle the crossing point is.

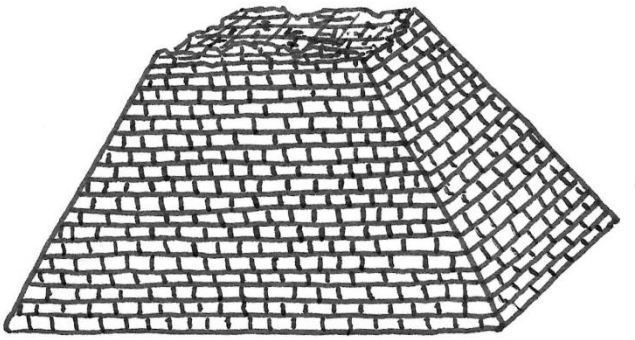
7. The Unfinished Pyramid

Khufu is building a pyramid of height 135 cubits and with a square base of side length 212 cubits. He knows that the formula for the volume of a pyramid of height h and base length b is $\frac{1}{3}hb^2$.

What is the volume of the rock needed to build it (assuming it is solid)?

How much rock will have been used when the as-yet-unfinished pyramid reaches half the height of the final pyramid?

You may assume that the top of the unfinished pyramid is a flat platform parallel to the base.



The Challenge is organised by
the School of Mathematical Sciences outreach team,
University of Southampton.

www.soton.ac.uk/maths/outreach/index.page.

We gratefully acknowledge MEM (Mathematical Education on Merseyside) for providing these questions and the concept of the Challenge.

www.mathsmerseyside.org.uk/

Instructions

Please note each school's submission should **include a marksheet**, which can be found at the Maths Challenge webpage: www.southampton.ac.uk/stag/mathschallenge.page

Instructions for students:

1) Follow the rules provided on the front of the question sheets.
2) Once you have finished the Challenge, return your entry to a designated person at your school. Entries should be submitted by the designated person whenever possible.
3) If for some reason you are unable to return your entry to a designated person, ask your parent or guardian to submit your entry and a completed Excel marksheet via email to math4all@soton.ac.uk, with your name in the email subject: "**Maths Challenge 23 submissions: *Student Name***". Please do not send high-resolution photos; instead follow the scanning instructions on the next page.

Instructions for schools/teachers:

1) Once you have received your students' entries, please **fill in the marksheets** (one each for the Junior and Senior Challenge separately).
2) Send the marksheets as Excel files to math4all@soton.ac.uk, with your school name in the email subject: "**Maths Challenge 23 submissions: *School Name***".
3) For the actual entries, you can submit them by post or online:
    a. By post (preferred): Physical entries should be mailed to the address below.

Junior/Senior Challenge '23 Entries
Dr Sunny Yu
School of Mathematical Sciences
University of Southampton
Southampton, SO17 1BJ

    b. Online: In your email containing your marksheet, let us know that you would like to submit your students' entries electronically. We will provide a link for you to upload your files. Entries should be in the form of PDF scans.

Instructions for scanning your entry using a smart phone

If you are submitting an entry electronically, it should be in PDF format. Make sure you include all the pages and that all your writing (including your name and the name of your school) is clearly visible. If you do not have access to a flatbed scanner, then follow the instructions below.

Scanning PDF files with an Android phone

We suggest you use one of three scanner apps: (a) Google Drive, (b) Adobe Scan, (c) Microsoft Lens. These are all free to use and can be downloaded from the Google Play store. We provide detailed instructions for using Google Drive.

Google Drive: You probably already have this on your phone. If not, it is downloadable from play.google.com/store/apps/details?id=com.google.android.apps.docs
Note that this is the full Google Drive app, not the standalone Sheets, Docs, or Slides apps.

- In the Google Drive app, click on the plus sign in the bottom-right corner and then click on “Scan”.
- Tap the large white button to scan a page. After taking the scan, click “OK” if you are happy with the scan or “Retry” if not. After clicking OK, click the plus sign on the bottom left **to add another page to the same PDF**. Please do this rather than submitting a separate PDF file for each page! When you have scanned all your pages, click “Save” to save the PDF file.

Scanning pdf files with an iPhone or iPad

For an iPhone, three options are (a) Notes, (b) Adobe Scan, and (c) Microsoft Lens. The first is built into your iPhone (as long as it is not too old) while the other two are free and can be downloaded from the App Store. We provide detailed instructions for using the Notes app.

Notes: This is the simplest in that Notes is already installed on your iPhone/iPad.

- Open Notes on your iPhone or iPad.
- Create a new note or tap on an existing one to add a document to it.
- Tap the camera button at the bottom of the screen or above the keyboard.
- Tap Scan Documents.
- Line up the document you want to scan.
- Tap the shutter button if the scanner doesn't automatically scan the document. Repeat this step for every page you want to scan.
- Tap Save **after** you've scanned all the pages of your entry. The button will have a count of how many pages you scanned.

Senior Challenge '23 Solutions

1. Seti's Party

It is impossible.

Counting 3 friends for each of the 11 people, we get 33.

However, we have counted each pair of friends twice, so 33 would have to be 2 x number of pairs which is impossible as 33 is odd.

2. Egyptian Fractions

4 unit fractions solutions are:

$$\frac{1}{2}+\frac{1}{3}+\frac{1}{7}+\frac{1}{42}$$

$$\frac{1}{2}+\frac{1}{3}+\frac{1}{8}+\frac{1}{24}$$

$$\frac{1}{2}+\frac{1}{3}+\frac{1}{9}+\frac{1}{18}$$

$$\frac{1}{2}+\frac{1}{3}+\frac{1}{10}+\frac{1}{15}$$

$$\frac{1}{2}+\frac{1}{4}+\frac{1}{5}+\frac{1}{20}$$

$$\frac{1}{2}+\frac{1}{4}+\frac{1}{6}+\frac{1}{12}$$

5 unit fractions without ½ is

$$\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{20}$$

3. A Horse With No Name

Let the father walk x km and ride $(32-x)$ km to halfway.

Then the time taken is: $\frac{x}{3}+\frac{(32-x)}{8}$ hours.

The son rides x km and walks $(32-x)$ km, so his time is: $\frac{x}{8}+\frac{(32-x)}{4}$ hours. Since they meet at half way, both times are the same:

$$\frac{x}{3}+\frac{(32-x)}{8}=\frac{x}{8}+\frac{(32-x)}{4}$$
$$8x+3(32-x)=3x+6(32-x)$$
$$5x+96=192-3x$$
$$8x=96$$
$$x=12.$$

Time taken to half way is: 12/3 + (32 − 12)/8 = 6.5 hours.

Total time for journey is: 6.5 + 3 + 6.5 = 16 hours.

They arrive at 0800 or 8:00am the next day.

4. Time is Running Out

Let C_1 and P_1 be the heights of Cleopatra's and Ptolemy's clocks, respectively.
1/2 C_1=3/5 P_1,
so **$P_1 = 5/6\ C_1$.**

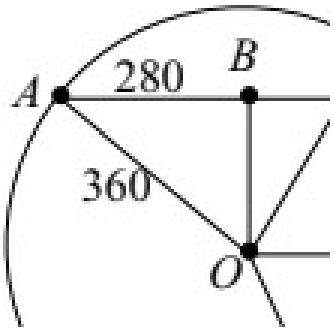
Let C_2 and P_2 be the respective water heights of Cleopatra's and Ptolemy's clocks after 3 hours.
P_2=2/5 P_1 = 2/5 (5/6 C_1) = 1/3 C_1
C_2=1/4 C_1
Therefore, **$C_2 = 3/4\ P_2$.**

5. Getting to the Heart of the Matter

8 Hearts (A through H):
Put hearts ABC on one side of the balance and DEF on the other. (W1)
If they balance, put G on one side and H on the other, the lighter of these two is the pure heart. (W2)
If they do not balance, then if ABC is the lighter side, put A and B on either side of the balance. (W2)
If they balance, C is the pure heart, if not it is the lighter of A and B.
Similarly if DEF is the lighter side.

6. Reed the Question

OB and OD are perpendicular to the two strings, so that B and D are the midpoints of the strings.
Thus AB is half the length of one string, hence 280 cm, and CD is half the length of the other string, hence 320 cm.



Using right-angled triangle ABO we find
$OB^2 = 360^2 - 280^2 = 40^2(9^2 - 7^2) = 40^2 \times 32 = 51{,}200$
and using right-angled triangle CDO we find
$OD^2 = 360^2 - 320^2 = 40^2(9^2 - 8^2) = 40^2 \times 17 = 27{,}200$

Note also that OBED is a rectangle, so that BE = OD.

We now get $OE^2 = BE^2 + OB^2 = OD^2 + OB^2$
$= 402 \times 32 + 40^2 \times 17 = 40^2\,(17 + 32) = 40^2 \times 7^2 = 280^2$
So OE = 280 cm.

7. The Unfinished Pyramid

$\frac{1}{3}hb^2 = \frac{1}{3} \times 135 \times 212^2 = \mathbf{2,022,480}$ **cubic cubits**

Using similar triangles, the missing part of the pyramid's dimensions are:

135: 212 => 67.5: 106 cubits

The volume of the missing part is then

$\frac{1}{3}hb^2 = \frac{1}{3} \times 67.5 \times 106^2 = 252{,}810$ cubic cubits

So the built part is 2,022,480 – 252,810 **= 1,769,670 cubit cubits**.