School of Mathematical Sciences

UNIVERSITY OF Southampton

Challenge '20

Year 8 or below

Illustrations by Theo Chaddock

Rules

1) Challenge '20 should be attempted at home in your own time.
2) Your entry must be your own work, though of course you may ask for help on how to get started or for the meanings of unfamiliar words.
3) Entries without any working out at all or written on this sheet **will not be marked**.
4) It is possible to win a prize or certificate even if you have not completed all of the questions, so hand in your entry even if it is not quite finished.
5) Please make sure that you staple your pages together and you must write **your name and school neatly on every page.**

Either you or your maths teacher needs to return your entry by 18th March to this address:

Challenge '20 Entries
Dr Adam Pound
School of Mathematical Sciences
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University Road
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SO17 1BJ

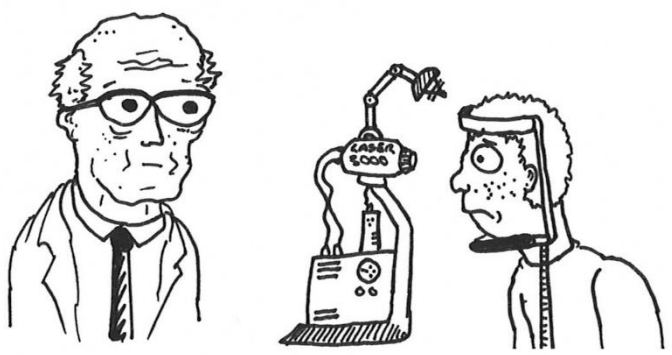
A Prize-Giving Evening will be held at the University of Southampton on 3rd June.
We hope that you enjoy the questions.

1. Barry's Blocks

Barry has 4 wooden identically sized and shaped blocks. 2 are blue, 1 is red and 1 is green.

How many distinct ways can Barry arrange the 4 blocks in a row?

Barry's friend Billie is colour-blind, and cannot distinguish between red and green. How many of Barry's distinct arrangements would Billie see as different?



2. Peter's Patients

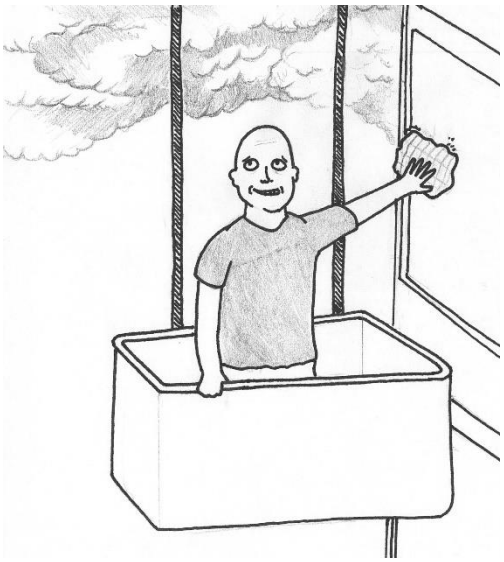
Peter the optician sees 16 patients one Tuesday. 4 are neither short nor long sighted. The total number of short sighted patients is two more than the total number of long sighted patients. How many patients can be short sighted, how many can be long sighted and how many could require varifocal lenses?

3. Light Work

Oscar has 6 tubular neon lights in order to spell out the word VISION to promote his lighting business. It costs £2 to add a corner to a tube, £3 to wire the ends of two tubes together (or the two ends of one tube). It is free to add smooth curves or to stretch the tubes.

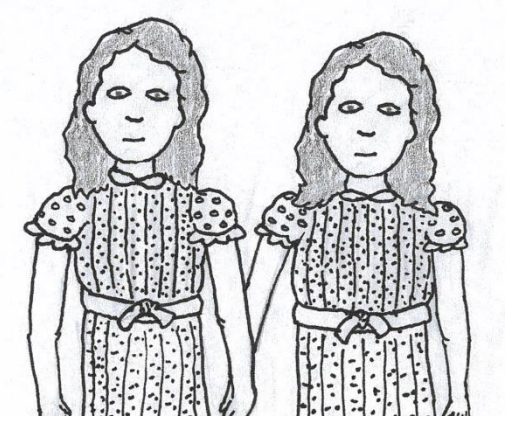
How much does it cost Oscar to spell the word VISION?

Which capital letters of the alphabet is it possible to make with a single tube using the Arial font? Justify your answer.



4. Window Cleaning

A skyscraper has 100 floors, numbered 0 (ground floor) to 99, each with one window. A team of 10 inefficient window cleaners are hired to clean the windows. They work from bottom to top and, when each cleaner has cleaned their highest floor, they leave. The 1st cleaner cleans every 10th window (so her first window is on the 9th floor), the 2nd cleans every 9th window, the 3rd cleans every 8th window and so on until the 9th cleans every 2nd window. The final cleaner has an attack of vertigo and can't work. Which floors will still have dirty windows?



5. Spot the Difference

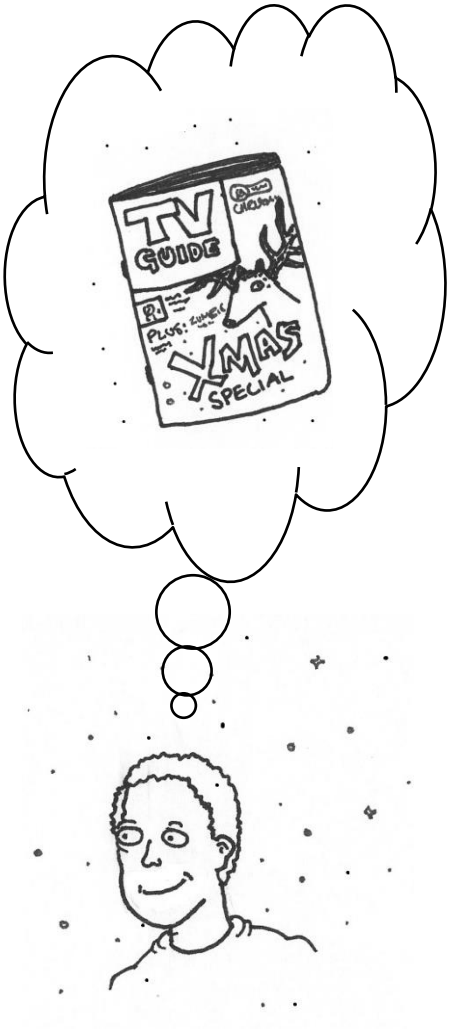
Twins can be identical boys, identical girls, non-identical boys, non-identical girls or one boy and one girl.
What are the possible combinations with triplets?
What are the possible combinations with quadruplets?

6. Schedule Shenanigans

Nick the TV scheduler is writing the Christmas Eve schedule from 6pm to midnight. These are the shows to be broadcast:

- 3 fifteen-minute News bulletins; there must be at least 2½ hours between bulletins
- 'Night of the Zombie Reindeer', a 2½-hour film which must be shown after the 9pm watershed
- a 30-minute astronomy programme – 'Stars on TV'
- a 60-minute Christmas special of 'The Liver Robins'
- the Christmas episode of Eastdale Street (30 mins)
- a 45-minute sci-fi programme (Snowmen in Space).

His wife, Mary, an avid Eastdale Street fan, has asked him to ensure that it doesn't start before 8pm so she can be home in time to watch it live.
Meanwhile, their young daughter Holly has asked him to put The Liver Robins on early enough for her to watch it before her mum puts her to bed when she gets in.
Assuming no advert breaks, how could he fit it all in?

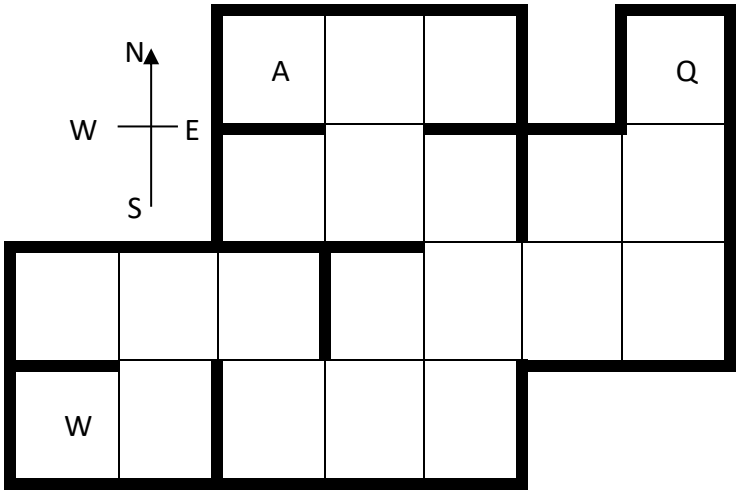


Ryan Pippin, the presenter of 'Stars on TV', gets embroiled in an academic scandal and the programme needs to be pulled from the schedule. It is to be replaced with 3 ten-minute programmes called 'Santa Watch'. These need to be spread as evenly as possible through the night, whilst keeping the news bulletins at least 2½ hours apart.
How can the schedule be rearranged?

7. Second Sight

Zelda the psychic has minimal psychic powers. When she tries to 'see' which card Hilda is holding, she gets the colour correct 100% of the time. She can also tell if it is a picture card (King, Queen, Jack) or not. Being a romantic, when a card is a Heart, her chances of correctly identifying the face value are doubled; however, she hates gardening, so, when a card is a spade, her chances of finding the right face value are halved. What is the probability that she guesses correctly if Hilda is holding the Ten of Diamonds, the Jack of Clubs, the Queen of Hearts or the Ace of Spades?

a



8. Go With the Flo

Florence Nightingale, born in 1820, is moving around a hospital, and is trying to avoid the administrator who is a terrible bore. It is a dark and stormy night, and she won't go outside.
On the map on the left, each square represents a room. Thin lines are walls with a door in them, thick lines are walls without a door in them.
Each time Florence moves into an adjacent room, the administrator also moves into an adjacent room; however, he is superstitious and so he does not re-enter a room he has just left, although he can return to a room later.
She starts on the ward (W), whilst he starts in the admin office (A).
How might she be able to reach her quarters (Q) without being caught by him?

The competition in Hampshire is organised by
the School of Mathematical Sciences Outreach Team, University of Southampton

With kind acknowledgement to MEM (Mathematical Education on Merseyside) for providing these questions and the concept of the Challenge Competition. http://www.mathsmerseyside.org.uk/

Challenge '20 Solutions

1. Barry's Blocks

Consider laying the blocks down one at a time.

If the first block is blue, there are 3 choices for the second block, 2 for the third, 1 for the fourth; so 3x2x1 = 6 arrangements.

If the first block isn't blue, but the second is, there are 2 choices for the first block, 2 for the third, 1 for the fourth; so 2x2x1 = 4 arrangements.

If neither the first nor second block is blue, there are 2 choices for the first block, 1 for the second, 1 for the third and fourth; so 2 arrangements.

So there are a total of 6+4+2 = 12 ways of arranging the blocks.

Explicitly, the 12 arrangements are

BBRG, BBGR
BRBG, BGBR
BRGB, BGRB
RBGB, GBRB
RBBG, GBBR
RGBB, GRBB

Billie sees only 6 different arrangements, as he sees each of the above pairs as only one arrangement.

2. Peter's Patients

There are 12 long- and/or short-sighted patients. Let x be the number of patients who are only short sighted, y the number who are only long sighted, and z be the number who are both short and long sighted. Then

$$x + y + z = 12 \quad \text{and} \quad x + z = y + z + 2$$

Eliminating z from the second equation gives $y = x - 2$
Substituting that into the first equation gives $z = 14 - 2x$

Since z must be greater than or equal to zero, x must be less than or equal to 7. Since y must be greater than or equal to zero, x must be greater than or equal to 2. So x can take 6 possible values, with y and z determined by the equations above.

Short only (x)	Long only (y)	Varifocal (z)
7	5	0
6	4	2
5	3	4
4	2	6
3	1	8
2	0	10

3. Light Work

V and N between them have three corners, so these letters add £6.
O requires wiring, so another £3.
All other modifications are free, so the total cost is £9.

I can be made with no modifications.

C, J, S, and U can be made by bending.

L, M, N, V, W, Z can be made by creating corners.

D, G, and O can be made by bending (D, G, and O), creating a corner (D and G), and wiring ends (D and O).

B, P, R can be made by creating a corner (all three), bending (all three), and wiring ends (only B). But this is only possible if a small gap is allowed between the vertical side and the middle horizontal line in each case.

4. Window Cleaning

First renumber all floors by adding 1 to each of them, so they run from 1 to 100. The first cleaner cleans all floors that are divisible by 10, the second cleans all floors that are divisible by 9, and so on down to the ninth cleaner, who cleans all floors that are divisible by 2. This means the floors with dirty windows are the ones not divisible by any number between 2 and 10; these are the prime numbers 1, 11, 13, 17, 19, 23, 29, etc.

Subtracting 1 from each of these to return to the original numbering, we have that the floors with dirty windows are

0, 10, 12, 16, 18, 22, 28, 30, 36, 40, 42, 46, 52, 58, 60, 66, 70, 72, 78, 82, 88, 96.

5. Spot the Difference

Triplets - 10 possible combinations

3 identical (id) boys, 3 id girls,
2 id boys and a boy, 2 id boys and a girl,
2 id girls and a boy, 2 id girls and a girl,
3 non-identical (nid) boys, 3 nid girls,
2 nid boys and a girl, 2 nid girls and a boy

Quadruplets - 20 possible combinations

4 id boys, 4 id girls,
3 id boys and a boy, 3 id boys and a girl, 3 id girls and a boy, 3 id girls and a girl,
2 id boys and 2 id boys, 2 id boys and 2 nid boys, 2 id boys and 2 id girls,
2 id boys and 2 nid girls, 2 id boys a boy and a girl,
2 id girls and 2 id girls, 2 id girls and 2 nid girls, 2 id girls and 2 nid boys,
2 id girls a boy and a girl,
4 nid boys, 4 nid girls,
3 nid boys and a girl, 3 nid girls and a boy,
2 nid boys and 2 nid girls

6. Schedule Shenanigans

Here is one example solution. Initial schedule on left, revised on right.

Time	Show		Time	Show
1800	News		1800	News
1815	Stars on TV		1815	Santa Watch
1845	The Liver Robins		1825	The Liver Robins
			1925	Snowmen in Space
1945	Snowmen in Space		2010	Eastdale Street
2030	Eastdale Street		2040	Santa Watch
2100	News		2050	News
2115	Night of the Zombie Reindeer		2105	Night of the Zombie Reindeer
			2335	Santa Watch
2345	News		2345	News
0000	Midnight		0000	Midnight

7. Second Sight

In each case, she has a 1/2 chance of getting the suit right.

If the card is a picture card, she has a 1/3 chance of getting the value right (2/3 if it's a Heart, 1/6 if it's a Spade).

If it's not a picture card, she has a 1/10 chance of getting the value right (1/5 if it's a Heart, 1/20 if it's a Spade).

10 of Diamonds: 1/2 x 1/10 = 1/20

Jack of Clubs: 1/2 x 1/3 = 1/6

Queen of Hearts: 1/2 x 2/3 = 1/3

Ace of Spades: 1/2 x 1/20 = 1/40

8. Go With the Flo

First note that the administrator can take multiple paths. But since he doesn't enter a room he's just left, he won't enter the rooms labelled with an "x" (or if he does, he'll be stuck there all night, and Florence will easily walk to Q). So his first four movements, labelled A1 through A4 are fixed.

Similarly, there's no good reason for Florence to enter *. So her first five moments, labelled F1 through F5, are fixed.

The crucial point is then the three shaded squares. If the administrator enters either of the orange squares on his fifth movement, then Florence needs to enter the other one on her sixth. Because the administrator can't immediately backtrack to the room he just left (A4), he will have to move to F5 while she moves to A4, making it impossible for him to catch her.

If the administrator instead moves to the green square on his fifth movement, then by the time Florence reaches that square (on her eighth movement), he will be in Q (a possibility we'll ignore because it's unacceptable behaviour on his part!) or he'll be in either the square directly above or directly to the right of the green square. If he's to the right, Florence can move up; if he's above, Florence can move to the right. It's then impossible for him to catch her.