School of
Mathematical
Sciences

UNIVERSITY OF Southampton

Junior Challenge ’21

Year 8 or below

Illustrations by Theo Chaddock

Rules

1) The Junior Challenge ’21 should be attempted on your own time at home.
2) Your entry must be your own work, though of course you may ask for help on how to get started or for the meanings of unfamiliar words.
3) Entries without any working out at all or written on this sheet **will not be marked**.
4) It is possible to win a prize or certificate even if you have not completed all of the questions, so hand in your entry even if it is not quite finished.
5) You must write **your name and school neatly on every page.**

Either you or your maths teacher needs to submit a pdf scan of your entry by 23rd April using one of the submission forms at https://sites.google.com/view/southamptonmathchallenge/home Teachers can submit batches of multiple students’ entries, either as a combined pdf or multiple pdfs, using the batch submission form.

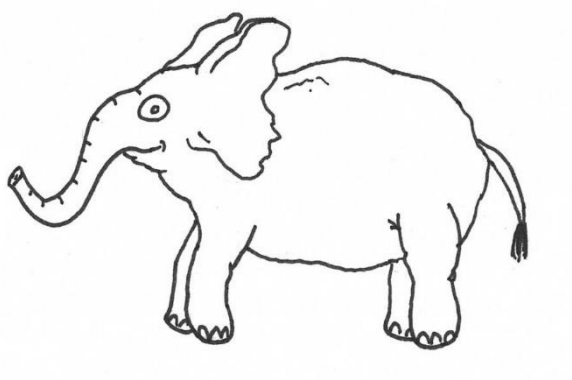
If you and your teacher do not have access to a scanner, you can scan your entry using a smart phone, following the instructions at the end of this document. Please use your phone’s scan function rather than submitting high-resolution photos.

If you’re unable to submit your entry electronically, please have your teacher or parents contact us at math4all@soton.ac.uk

A virtual Prize-Giving Evening will be held online in June.

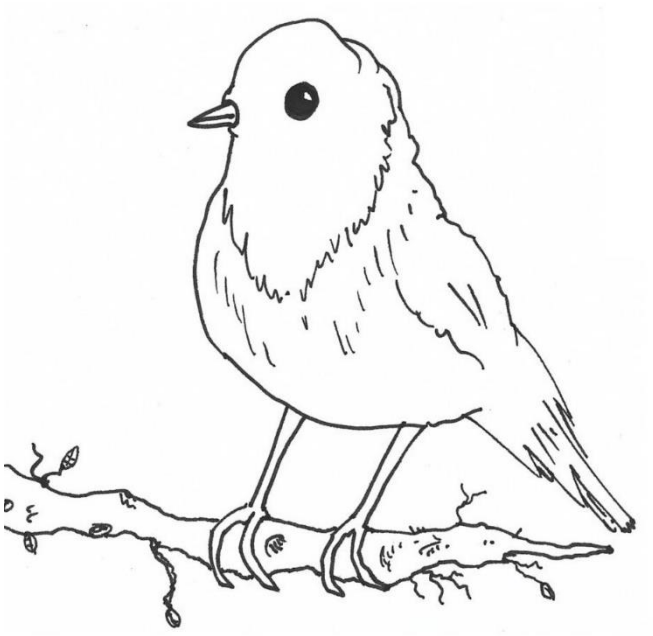
We hope that you enjoy the questions.

1. The Elephants in the Zoo



A zoo has an equal number of African elephants and Asian elephants. The African elephants cost £11 per day to feed and the Asian elephants cost £9 per day to feed. The total daily bill for feeding all of the zoo's elephants is £1000.

How many elephants does the zoo have?



2. Twitching Points

John, Sarah and Michael go birdwatching. They make it a competition by allocating a different number of points for each type of bird they spot. A sparrow is worth 1 point, a robin is worth 2 points and a wren 4. After the birdwatching, this is what they say:

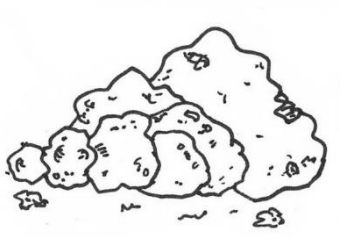
John - "I saw 2 sparrows and at least one of every bird. In total, I got 16 points."

Sarah - "I saw no sparrows and the same number of robins as John."

Michael - "I got 20 points, the same number of wrens as John, double his number of sparrows and double his number of robins."

Find the minimum number of wrens Sarah would need to see to win the competition and the maximum number she could see but still come third in the competition.

3. Going Up

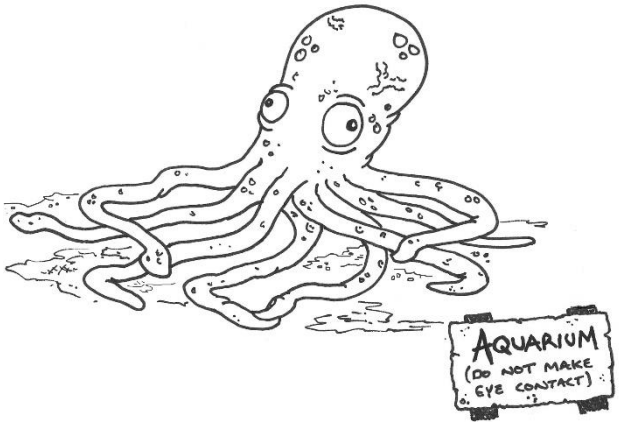


Ginoni the goat is at the bottom of a series of 7 boulders, each higher than the last. She wants to reach the top, where there is a huge patch of grass that she'd like to graze. Being an athletic goat, she sometimes bounds up two boulders at a time; otherwise, she just jumps up one.

Ginoni can use both one-boulder and two-boulder leaps within an ascent.

Starting at the bottom, in how many ways could she reach the grass at the top?

Investigate what happens for different numbers of boulders and explain the pattern of how many different ways she can ascend.



4. Aquarium Query.

Barney Akles runs an aquarium. In his collection, he has cyclops starfish (T) (one eye, five legs), blue octopuses (C) (2 eyes, 8 legs) and red lobsters (L) (2 eyes, 10 legs). (For the purpose of this question, we are using 'legs' to cover all limbs, arms, tentacles, etc.)

In each of 3 tanks, there are a prime number of animals with some or none of each species.

Work out how many starfish, lobsters and octopuses could be in each tank:

Botany Bay Tank – 175 legs and 41 eyes

Sargasso Sea Tank – 92 legs and 22 eyes

La Manche Tank – 46 legs and 10 eyes

5. Felines on the Farm

There are 8 mice on a farm at dawn on 1st March.

The population of mice doubles at 1am on the 1st of each month.

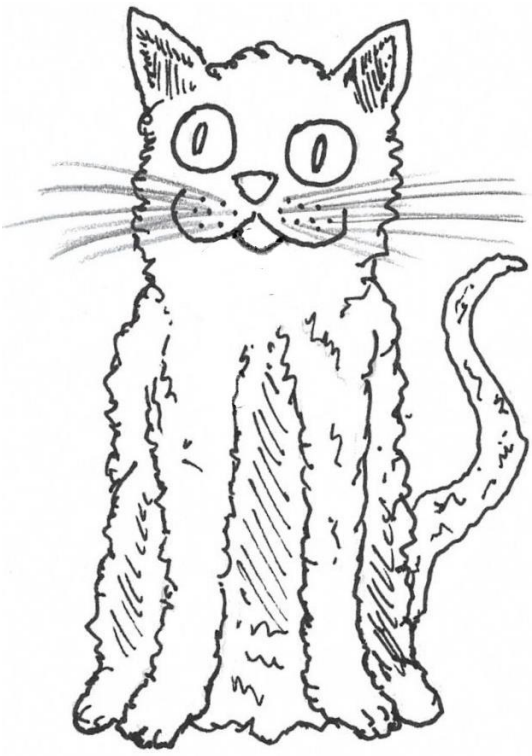
On the morning of 1st August, Linda and Richard, the farmers, each buy a cat.

For the first 2 months, Linda's cat catches 3 mice at noon each day; this then drops to 2 mice at noon each day until there are no mice left.

At sunset on the last day of each month, Richard's cat catches a third of the population of mice (to the nearest mouse).

Calculate the population of mice at dawn on the 1st of each month.

On which date does Linda's cat catch the final mouse?



6. Combine Harvesters

Margaret is looking to buy a new combine harvester. She has quotes from 3 companies:

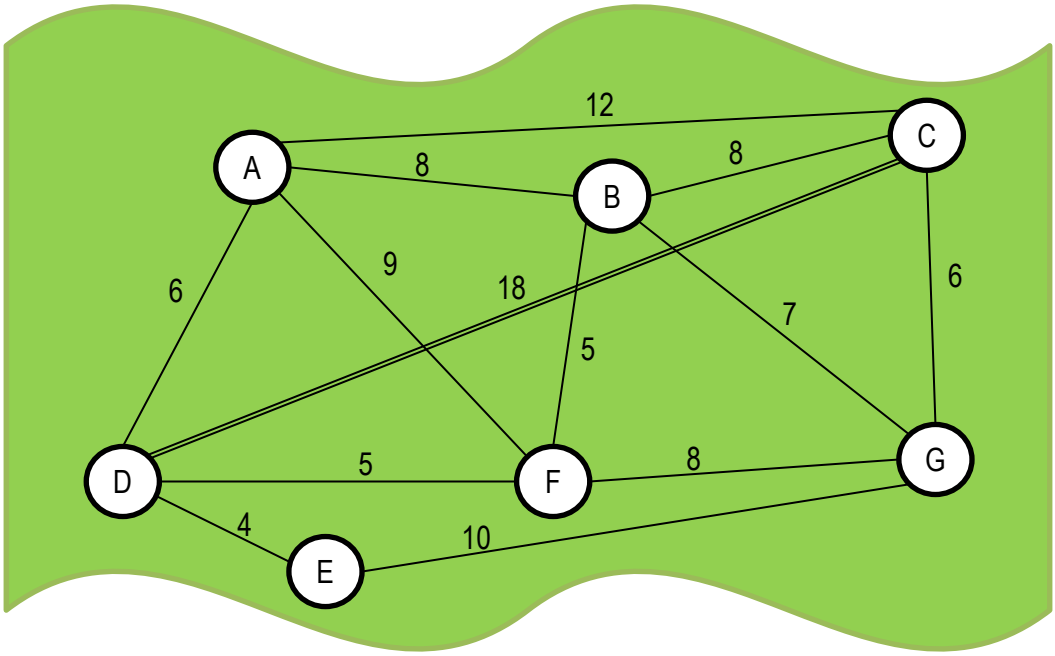
- Kate's Kwik Kombine will collect 85% of a crop in one day, and costs £15,000;
- Elijah's Easy-Crop will collect 94% of a crop in 3 days, and costs £17,000;
- Abi's All-Clear will collect 99% of a crop in 7 days, and costs £18,000.

Margaret will have to pay Keith the driver £150 per day and can sell each percentage point of a crop for £300.

Which machine should she buy?

Explain your answer.

7. Oakdale Woods



Above is a map of Oakdale Woods. Each of the seven clearings in the woods is shown, along with the time in minutes that it takes to walk along each of the woodland paths.
The path between Don's Dell (D) and Cedar Clearing (C) is a disused railway track. The paths which cross it do so on bridges, so you cannot go from one path to another except in a clearing.
The Jones family arrive at Grenville Grove (G) for a walk around the woods.
What is the quickest route they can follow to visit each of the clearings exactly once, before returning to Grenville Grove?

Rajit the Ranger is given the task of litter picking along all of the paths. The rangers are based at Bert's Bunker (B). He starts at 9am. Litter picking takes twice as long as just walking the paths.
Describe a route which means he only has to walk along each path once.
At what time will he get back to Bert's Bunker?

The competition in Hampshire is organised by the School of Mathematical Sciences Outreach Team, University of Southampton

We kindly acknowledge MEM (Mathematical Education on Merseyside) for providing these questions and the concept of the Challenge Competition. http://www.mathsmerseyside.org.uk/

Challenge '21 Solutions

1. Elephants in the Zoo

Let the number of African elephants in the zoo be a.
Let the number of Asian elephants in the zoo be b.
Given $a = b$.

Given the cost per African elephant $c_1 = £11$.
Given the cost per Asian elephant $c_2 = £9$.
Given total cost $a \times c_1 + b \times c_2 = £1000$.

Insert and solve:

$$a \times c_1 + b \times c_2 = £1000 \quad (1)$$

$$a \times £11 + a \times £9 = £1000 \quad (2)$$

$$a \times (£11 + £9) = £1000 \quad (3)$$

$$a = \frac{£1000}{£20} \quad (4)$$

$$a = 50 \quad (5)$$

Total number of elephants = $a + b = 50 + 50 =$ **100**.

2. Twitching Points

Let:
The number of birds seen by each person be p_b, where $p = J$ (John), S (Sarah) or M (Michael), and $b = s$ (sparrow), r (robin) or w (wren).
(E.g., M_s = the number of sparrows that Michael saw.)

Given:
Points system:
1 point/sparrow, 2 points/robin, 4 points/wren
From John's statement:
$J_s = 2,\ J_r \geq 1,\ J_w \geq 1$
John's total points: $J_s \times 1 + J_r \times 2 + J_w \times 4 = 16$
From Sarah's statement:
$S_s = 0,\ S_r = J_r$
From Michael's statement:
$M_s = 2J_s = 4,\ M_r = 2J_r,\ M_w = J_w$
Michael's total points: $M_s \times 1 + M_r \times 2 + M_w \times 4 = 20$

Solve:
From John's statements:
$2 + 2J_r + 4J_w = 16$
$\Rightarrow \quad J_r + 2J_w = 7$
From Michael's total points:
$(2J_s) \times 1 + (2J_r) \times 2 + (J_w) \times 4 = 20$
$\Rightarrow \quad J_r + J_w = 4$
Together implies:
$J_w = 3,\ J_r = 1$

For Sarah to win, need:
$S_s \times 1 + S_r \times 2 + S_w \times 4 > 20$
$\Rightarrow \quad (0) \times 1 + (1) \times 2 + S_w \times 4 > 20$
$\Rightarrow \quad S_w > 4.5$

∴ Sarah need to spot **5 wrens to win**.

For Sarah to come third, need:

$$S_s \times 1 + S_r \times 2 + S_w \times 4 < 16$$

$$\Rightarrow \quad (0) \times 1 + (1) \times 2 + S_w \times 4 < 16$$

$$\Rightarrow \quad S_w < \frac{14}{4} = 3.5$$

∴ Sarah will still score less than 16 (come third) if she spots **3 wrens**.

3. Going Up

Tabulating the ways of going up 7 boulders:
1 step × 7 times = 1 way
2 steps × 1 time + 1 step × 5 times = 6 ways
2 steps × 2 time + 1 step × 3 times = 10 ways
2 steps × 3 time + 1 step × 1 times = 4 ways
Total **21 ways**.

Investigate:

No. of Boulders	1	2	3	4	5	6	7	8	9
No. of Ways Up	1	2	3	5	8	13	21	34	55

For *n*+1 boulders, add up the number of ways for *n* and *n*–1 boulders.

This is because you require a 1-boulder leap from the n^{th} boulder, or a 2-boulder leaps from the $(n–1)^{th}$ boulder, so the number of ways is the sum of the number of ways of getting to the nth and (n–1)th boulder.

4. Aquarium Query

Note that one lobster is the same total as two starfish.

So, L = 2T.

In the Botany Bay Tank, there are:

175 = 8C + 5T and 41 = 2C + T => 205 = 10C + 5T => 30 = 2C => C = 15.

C	T	L	Total	
15	11	0	26	Not prime
15	9	1	25	Not prime
15	7	2	24	Not prime
15	5	3	23	Prime
15	3	4	22	Not prime
15	1	5	21	Not prime

Therefore, there must be **15 octopuses, 5 starfish and 3 lobsters.**

In the Sargasso Sea Tank, there are:

92 = 8C + 5T and 22 = 2C + T => 110 = 10C + 5T => 18 = 2C => C = 9.

C	T	L	Total	
9	4	0	13	Prime
9	2	1	12	Not prime
9	0	2	11	Prime

Therefore, **9 octopuses and 2 lobsters OR 9 octopuses and 4 starfish.**

In the La Manche Tank. there are:

46 = 8C + 5T and 10 = 2C + T => 50 = 10C + 5T => 4 = 2C => C = 2.

C	T	L	Total	
2	6	0	8	Not prime
2	4	1	7	Prime
2	2	2	6	Not prime
2	0	3	5	Prime

So **2 octopuses, 4 starfish and 1 lobster OR 2 octopuses & 3 lobsters.**

5. Felines on the Farm

	Mice	Linda	Richard
01/03/2020	8		
01/04/2020	16		
01/05/2020	32		
01/06/2020	64		
01/07/2020	128		
01/08/2020	256	93	54
01/09/2020	218	90	43
01/10/2020	170	62	36
01/11/2020	144	60	28
01/12/2020	112	62	17
01/01/2021	66	62	1
01/02/2021	6	6	0
01/03/2021	0		

The last mouse is caught on **3rd February**.

6. Combine Harvesters

Company	Cost	Days	Keith	Percent	Income	Profit
Kate	15000	1	150	85	25500	10350
Elijah	17000	3	450	94	28200	10750
Abi	18000	7	1050	99	29700	10650

Elijah's Easy-Crop is the most profitable.

7. Oakdale Woods

GCBFADEG = 48 minutes (or the reverse)

ABGCDAFGEDFB = 212 minutes
Starting at 09.00 gives **12.32** as time to return.