

Junior Challenge '22

Year 8 or below

Illustrations by Theo Chaddock & Peter Ackerley

Rules

- 1) Challenge '22 should be attempted on your own time at home.
- 2) Your entry must be your own work, though of course you may ask for help on how to get started or for the meanings of unfamiliar words.
- 3) Entries without any working out at all or written on this sheet **will not be marked**.
- 4) It is possible to win a prize or certificate even if you have not completed all of the questions, so hand in your entry even if it is not quite finished.
- 5) You must write **your name and school neatly on the top of every page**.
- 6) Please staple your pages together if you are submitting it by mail and not online.

Deadline

Either you or your maths teacher needs to submit your entry by **25th March**, by mail or online.

Submit via mail:

Either you or your maths teacher needs to return your entry by the deadline to this address:

Challenge '22 Entries,
Dr Sunny Yu,
School of Mathematical Sciences,
University of Southampton,
University Road,
Southampton,
SO17 1BJ

Or: submit via online upload:

Scan your entries as a pdf file and upload at:

<https://www.southampton.ac.uk/stag/mathschallenge.page>

A Prize-Giving Evening will (hopefully) be held at the University of Southampton on 8th June.
We hope that you enjoy the questions.

If you have any questions, please have your teacher or parents contact us at
math4all@soton.ac.uk

1. Café on the Terrace

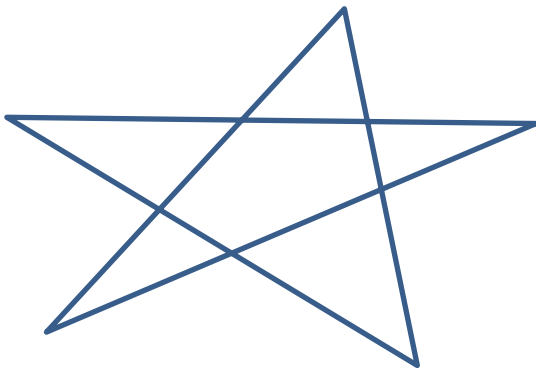
The café's menu lists 3 choices:

Egg & Chips - £3.00

Sausage & Chips - £3.60

Sausage & Mash - £2.70

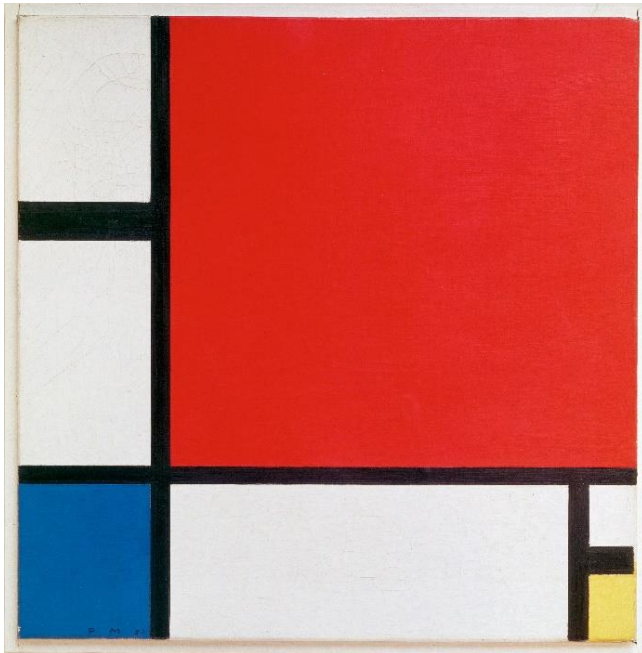
How much would it charge for Egg & Mash?



2. A Starry Night

Ian drew this example of a five-pointed star. He wants to create a computer program to draw five-pointed stars in general. To do this, he needs to know about the relationships between the internal acute angles at the points of the star.

What is the sum of the angles at its five points (in degrees)? Justify your answer.



3. Composition with Red, Blue & Yellow.

The image on the left shows this famous Mondrian painting.

The rectangle in the top right is red, the one in the bottom left is blue and the one in the bottom right is yellow. The remaining ones are white.

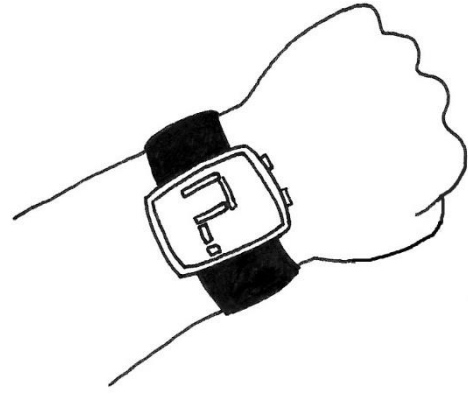
How many rectangles are there altogether in the painting?

4. The Night Watch

Charlie wakes up in the night and looks at his 12-hour digital watch.

The time he sees can be read as a three-digit number. He remarks: "The digits add to six and, if you divide the number by six, the answer is a prime."

What time could it be?



5. Dogs Playing Poker



Barry and Kerrie are discussing their pets during a poker game. Barry tells Kerrie that he has at least one dog at home.

When Kerrie asks what breed, he answers cryptically: "If I have a Labrador but not a Staffie, I also have a Collie. I either have both a Collie and a Staffie or neither. If I have a Collie, then I also have a Labrador." What breed or breeds does Barry have at home?

6. Whistler's Mother

Anna Whistler was asked how old she was when her son painted her. She replied in the form of a riddle:

If to my age there added be,
Half of it, a third of it and three times three,
Six score and ten, the sum you'll see,
So tell me, please, what age I be?



7. The Birth of Venus



Venus and her cousin Ursula recently celebrated their birthdays on the same Sunday in December 2021. Ursula is now four times as old as Venus was when Ursula was the same age as Venus is now. Venus is a teenager. How old are Venus and Ursula? On which day of the week was Venus born?

8. The Money Changer and his Wife

In a particular country, the coins come in the denominations 1, 15 and 50 schillings. Whilst visiting this country, Peter bought a book for his wife, Rachel, paying with several (more than 2) coins. He paid with the minimum number of coins needed to make that amount.

His change contained one more coin than the handful of coins with which he paid, but again it was the minimum number of coins needed to make that amount.

What are the eight lowest possible prices for the book?

What is the lowest price of the book where the coins Peter pays with is a sensible set of coins to have handed over?

All of the questions in this year's Challenge have been inspired by famous paintings. To see a gallery of these works of art, go to www.tinyurl.com/challenges2022gallery

The Challenge is organised by
the School of Mathematical Sciences outreach team,
University of Southampton

We kindly acknowledge MEM (Mathematical Education on Merseyside) for providing these questions and the concept of the Challenge.
<http://www.mathsmerseyside.org.uk/>

Junior Challenge '22 Solutions

1. Café on the Terrace

By comparing the price of Sausage & Chips to that of Sausage & Mash, we can see that Chips cost 90p **more** than Mash.

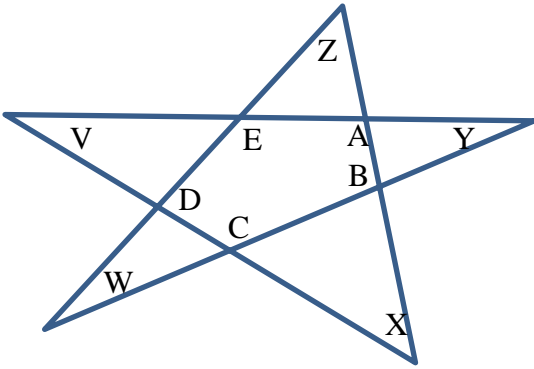
Egg and Mash should therefore cost 90p **less** than Egg & Chips, so it should **cost £2.10**.

NB It is not possible to fix the price of the individual items.

2. A Starry Night

The sum of the angles is 180°.

Labelling the five angles in the pentagon A through E and the angles at the points V through Z:



The angles adjacent to A are both $180 - A$.

Each triangle contains 180° . This means the five triangles total 900°

This means we have the equation:

$$\begin{aligned} 900 &= 2(180 - A) + 2(180 - B) + 2(180 - C) + \\ & 2(180 - D) + 2(180 - E) + V + W + X + Y + Z \\ 900 &= 1800 - 2(A + B + C + D + E) + V + W + X + Y \\ & \quad + Z \end{aligned}$$

$A + B + C + D + E = 540$ so

$$\begin{aligned} 900 &= 1800 - 2 \times 540 + V + W + X + Y + Z \\ V + W + X + Y + Z &= 900 - (1800 - 1080) = 900 - 720 = 180 \end{aligned}$$

3. Composition with Red, Blue and Yellow

We have the following combinations to make rectangles:

1. 7 individual rectangles.
2. Yellow + one above,
3. Yellow + one above + 1 to left
4. Yellow + one above +1 to left + blue
5. Yellow + one above +1 to left + red
6. Blue plus one to right
7. Blue plus one above
8. Blue plus 2 above
9. Red plus 2 to left
10. 2 whites in top left
11. All together

We get 17 different rectangles in total

4. The Night Watch

We assume that night-time starts at 10pm (times between 7pm and 10pm will in any case be excluded later). Times between midnight and 1am are written as 12:xx, so are not 3 digits.

The possible times starting at 1am, where the 3 digits add to 6 are: 1:05, 1:14, 1:23, 1:32, 1:41, 1:50, 2:04, 2:13, 2:22, 2:31, 2:40, 3:03, 3:12, 3:21, 3:30, 4:02, 4:11, 4:20, 5:01, 5:10, 6:00

Asking that the number also be divisible by 6 reduces these possibilities to: 1:14, 1:32, 1:50, 2:04, 2:22, 2:40, 3:12, 3:30, 4:02, 4:20, 5:10, 6:00

Asking that the answer after division by 6 is prime reduces the possibilities to: **1:14, 2:22, 4:02**

5. Dogs Playing Poker

There are 3 statements:

1. If I have a Labrador but not a Staffie, I also have a Collie.
2. I either have both a Collie and a Staffie or neither.
3. If I have a Collie then I also have a Labrador.

Since he has at least 1 dog, the 2nd statement gives us 3 possibilities:

- a. just a Labrador,
- b. a Collie and a Staffie,
- c. or all 3.

Statement 1 excludes option a.

Statement 3 excludes option b.

This leaves the option that Barry has all three breeds.

6. Whistler's Mother

Six score and 10 is 130. Let the mother's age be a . The mother's claim, as an equality, is

$$130 = a + \frac{1}{2}a + \frac{1}{3}a + 9.$$

Equivalently,

$$121 = \frac{6 + 3 + 2}{6}a = \frac{11}{6}a.$$

Solving, we get **$a = 66$**

Example of worded reasoning:

To have added a half, a third and 9 (3x3), the age must be divisible by 6, and the result has been roughly doubled. Six score and ten is 130. Half of this is 65, but this is not divisible by 6.

Try 60 => 60+20+30+9=119 so no good

Try 66 => 66+33+22+9=130 **so 66 is the answer.**

7. Birth of Venus

Let U = Ursula now and V = Venus now. Set $X = U - V$.

X is the difference in ages, and **also** the difference to the previous date: because Venus is now what Ursula was then, so the difference in ages is the same as the number of years which have passed since then.

So $U - X =$ Ursula then, $V - X =$ Venus then

	Ursula	Venus
Now	$V + X$	V
Then	V	$V - X$

$$V + X = 4(V - X) \Rightarrow V + X = 4V - 4X \Rightarrow 5X = 3V$$

So V must be a multiple of 5, which since she is a teenager means 15.

$$\text{So } 45 = 5X \Rightarrow X = 9$$

	Ursula	Venus
Now	<u>24</u>	<u>15</u>
Then	15	9

A Sunday in December 2021 => Tuesday in 2017 => Thursday in 2013 => Saturday in 2009 => Monday in 2005
=> **Tuesday in 2006**

8. The Money Changer and his Wife

There must be at least 3 coins.

Pay With	Total Paid	No. Paid	No. Change	Change is	Total Change	Book Cost
15+1+1	17	3	4	1+1+1+1	4	13
15+15+1	31	3	4	1+1+1+1	4	27
15+15+15	45	3	4	1+1+1+1	4	41
50+1+1	52	3	4	1+1+1+1	4	48
50+1+1	52	3	4	15+1+1+1	18	34
50+1+1	52	3	4	15+15+1+1	32	20
50+1+1	52	3	4	15+15+15+1	46	6
15+15+15+15	60	4	5	1+1+1+1+1	5	55
OR 50+50+1	101	3	4	15+15+15+1	46	55

The **lowest sensible cost for the book is 41**, as the amount paid with for the other scenarios makes little sense – you wouldn't pay for 13 with 17 etc.

Note that the prices are all 6 mod 7

The table above in order of book cost:

Pay With	Total Paid	No. Paid	No. Change	Change is	Total Change	Book Cost
50+1+1	52	3	4	15+15+15+1	46	6
15+1+1	17	3	4	1+1+1+1	4	13
50+1+1	52	3	4	15+15+1+1	32	20
15+15+1	31	3	4	1+1+1+1	4	27
50+1+1	52	3	4	15+1+1+1	18	34
15+15+15	45	3	4	1+1+1+1	4	41
50+1+1	52	3	4	1+1+1+1	4	48
15+15+15+15	60	4	5	1+1+1+1+1	5	55
OR 50+50+1	101	3	4	15+15+15+1	46	55