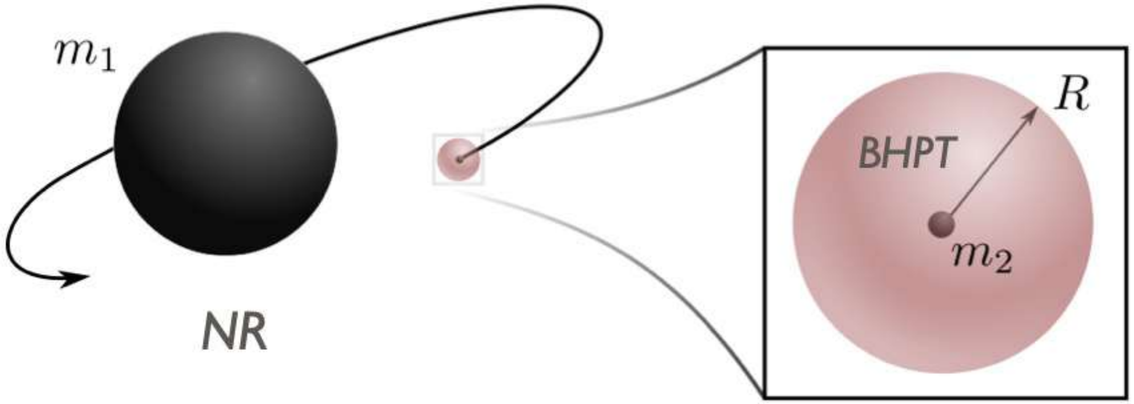
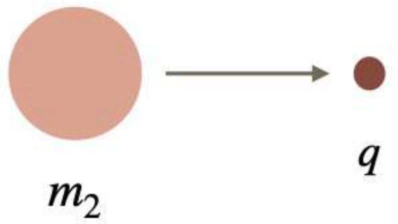
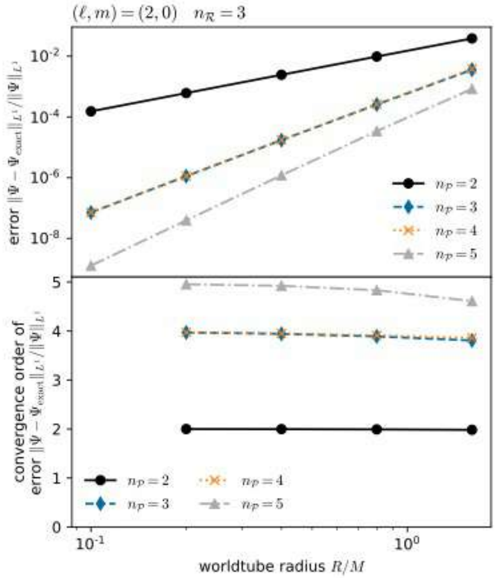
MODELLING BLACK-HOLE BINARIES IN THE INTERMEDIATE-MASS-RATIO REGIME

UNIVERSITY OF Southampton



TOY MODEL





arXiv:2109.03531v1 [gr-qc] 8 Sep 2021

Worldtube excision method for intermediate-mass-ratio inspirals: scalar-field toy model

Mekhi Dhesi,[1] Hannes R. Rüter,[2] Adam Pound,[1] Leor Barack,[1] and Harald P. Pfeiffer[2]

[1]School of Mathematical Sciences and STAG Research Centre, University of Southampton, Southampton, SO17 1BJ, United Kingdom

[2]Max Planck Institute for Gravitational Physics (Albert Einstein Institute), Am Mühlenberg 1, Potsdam 14476, Germany

(Dated: September 9, 2021)

The computational cost of inspiral and merger simulations for black-hole binaries increases in inverse proportion to the square of the mass ratio $q := m_2/m_1 \leq 1$. One factor of q comes from the number of orbital cycles, which is proportional to $1/q$, and another is associated with the required number of time steps per orbit, constrained (via the Courant-Friedrich-Lewy condition) by the need to resolve the two disparate length scales. This problematic scaling makes simulations progressively less tractable at smaller q. Here we propose and explore a method for alleviating the scale disparity in simulations with mass ratios in the intermediate astrophysical range ($10^{-4} \lesssim q \lesssim 10^{-2}$), where purely perturbative methods may not be adequate. A region of radius much larger than m_2 around the smaller object is excised from the numerical domain, and replaced with an analytical model approximating a tidally deformed black hole. The analytical model involves certain *a priori* unknown parameters, associated with unknown bits of physics together with gauge-adjustment terms; these are dynamically determined by matching to the numerical solution outside the excision region. In this paper we develop the basic idea and apply it to a toy model of a scalar charge in a circular geodesic orbit around a Schwarzschild black hole, solving for the massless Klein-Gordon field in a 1+1D framework. Our main goal here is to explore the utility and properties of different matching strategies, and to this end we develop two independent implementations, a finite-difference one and a spectral one. We discuss the extension of our method to a full 3D numerical evolution and to gravity.

I. INTRODUCTION

All gravitational-wave signals reported so far by the LIGO-Virgo Collaboration [1, 2] originated from compact-object binaries in which the two components had fairly comparable masses. The most extreme mass disparity to date was observed in GW190814, whose likely source was the coalescence of a 2.50–2.67$M_\odot$ object (either an exceptionally heavy neutron star or an exceptionally light black hole) with a 22.2–24.3$M_\odot$ black hole [3]. Upgrades and future generations of ground-based detectors [4, 5], and especially the planned space-based detector LISA [6], will open up a new window of observation in the low-frequency band of the gravitational-wave spectrum, enabling the detection of signals from ever heavier binary systems, including ones containing intermediate-mass and supermassive black holes. In consequence, it is expected that the detection of high mass ratio events will become routine, and that the catalogue of detected binary sources will extend to include a broad range of mass ratios—potentially down to $\sim 1{:}10^6$ with LISA [7–9].

In anticipation of this remarkable expansion in observational reach, it is important to develop accurate theoretical waveform templates that reliably cover the entire relevant range of mass ratios. Standard Numerical Relativity (NR) methods [10] work well for mass ratios in the range $0.1 \lesssim q := m_2/m_1 \leq 1$ (see e.g. [11]). However, simulations become progressively less tractable at smaller q, and few numerical simulations have been performed at $q < 0.1$ so far. The root cause is a problematic scaling of the required simulation time with q. Fundamentally, one expects the required simulation time to grow in proportion to q^{-2}, where one factor of q^{-1} is associated with the number of in-band orbital cycles, and the second factor q^{-1} comes from the Courant-Friedrich-Lewy (CFL) stability limit on the time step of the numerical simulation, constrained by the need to spatially resolve the small object. The state of the art in small-q NR is represented by the recent simulations performed at RIT of the last 13 orbital cycles prior to merger of a black-hole binary system with $q = 1/128$ [12, 13]. Such simulations remain extremely computationally expensive.

For *extreme* mass ratios (say, $q \lesssim 10^{-4}$), it is more natural to apply an alternative treatment based on black-hole perturbation theory. Here, the field equations are formally expanded in powers of q, and the orbital dynamics are described in terms of a point-particle inspiral trajectory on the fixed geometry of the large black hole. In the limit $q \to 0$, the trajectory is geodesic. Back reaction from the small object's self-field, which drives the slow inspiral, is accounted for order-by-order in q, in what is known as the gravitational self-force (GSF) approach [14, 15]. GSF is currently the only viable method for modelling astrophysical extreme-mass-ratio inspirals (EMRIs), in which a compact object orbits a massive black hole in a galactic nucleus. Development continues towards an accurate model of EMRI waveforms suitable for signal identification and interpretation with LISA [16–21].

The *intermediate* range of mass ratios, say $10^{-4} \lesssim q \lesssim 10^{-1}$, poses a unique modelling challenge. *A priori*,

arXiv:2109.03531v1

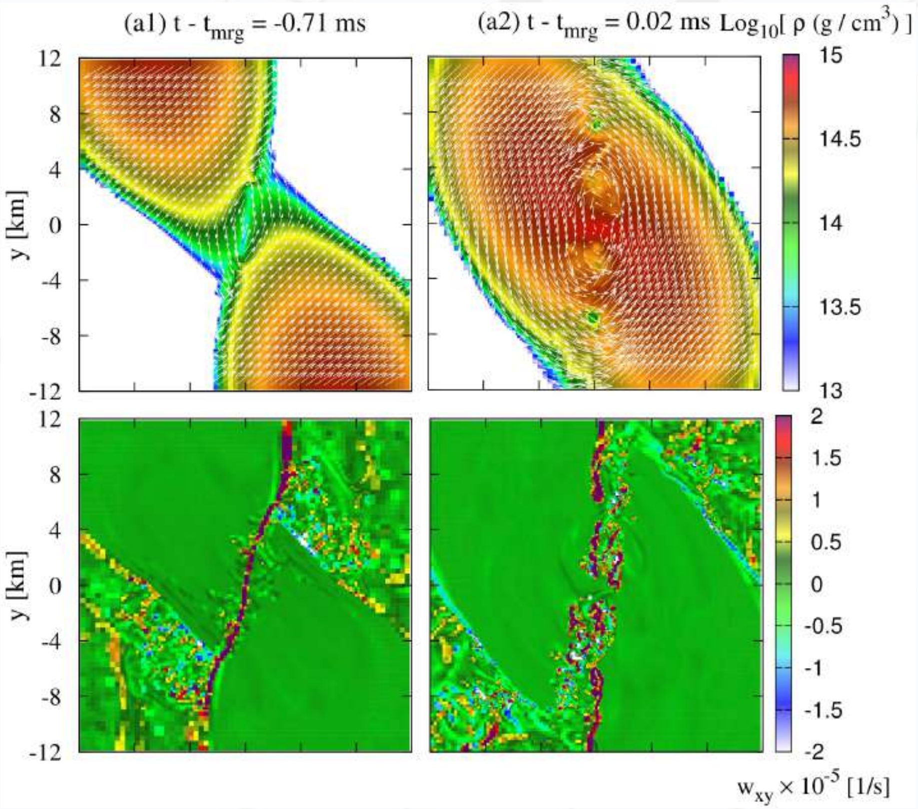
Supervisors: Leor Barack & Adam Pound

Relativistic dissipative hydrodynamics and turbulence

General relativistic hydrodynamics is an invaluable tool for understanding neutron star astrophysics and the modelling of binary neutron star mergers. Typically, in merger simulations neutron stars are modelled as ideal fluids, but we know there is much more to it...

As turbulence develops in merger simulations:

- We need to care about features we can't resolve: new formulation of a general relativistic Large-Eddy-simulation scheme. See [Phys. Rev. D 104 (2021)] .
- Filtering introduces effective dissipative terms. How does this impact on the information we can extract from merger simulations?
- We are investigating the impact of small-scale features on the implementation of bulk-viscosity.

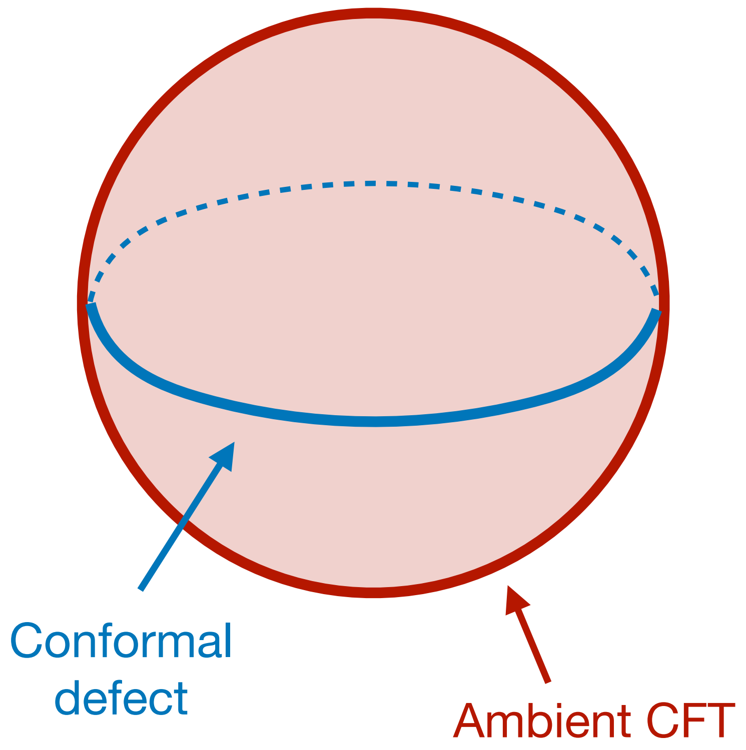


Kiuchi et *al*, [Phys. Rev. D 92 (2015)]

Boundaries and Defects in CFT

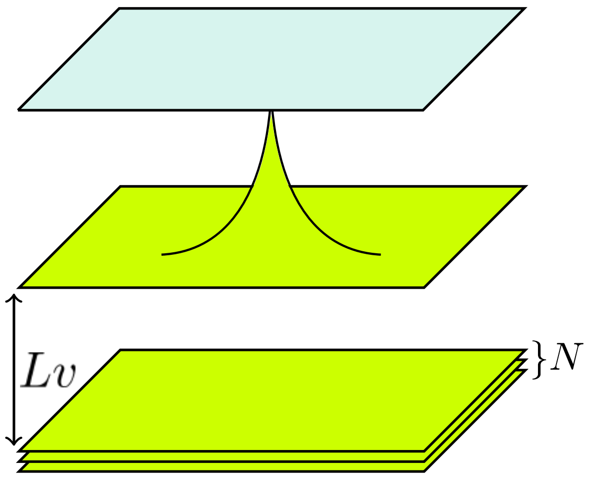
Adam Chalabi

Defect CFT



Strings and branes

Holography

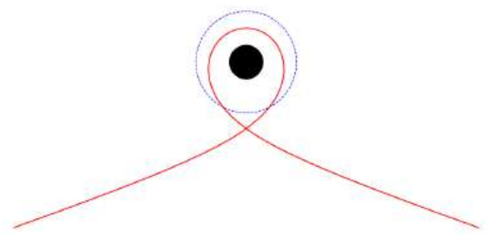


c) Screened Wilson line

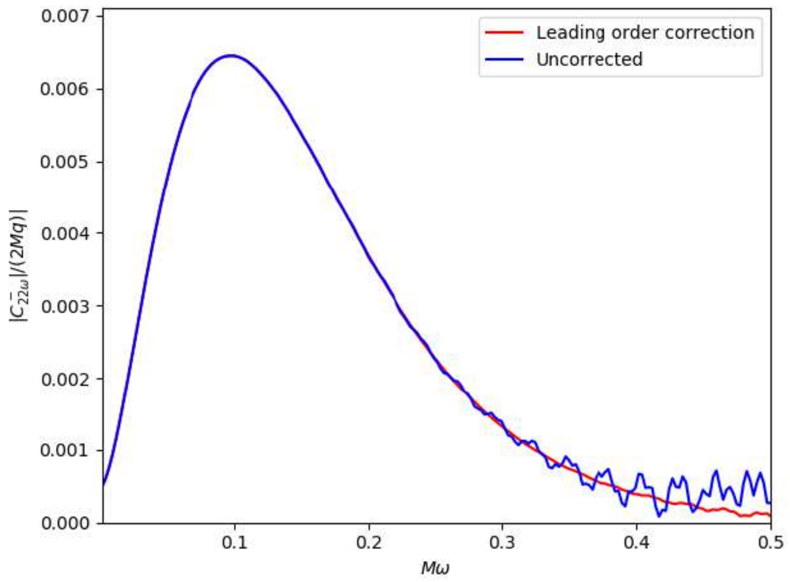
Self-force in hyperbolic scattering: frequency-domain approach (Chris Whittall, supervisor Leor Barack)

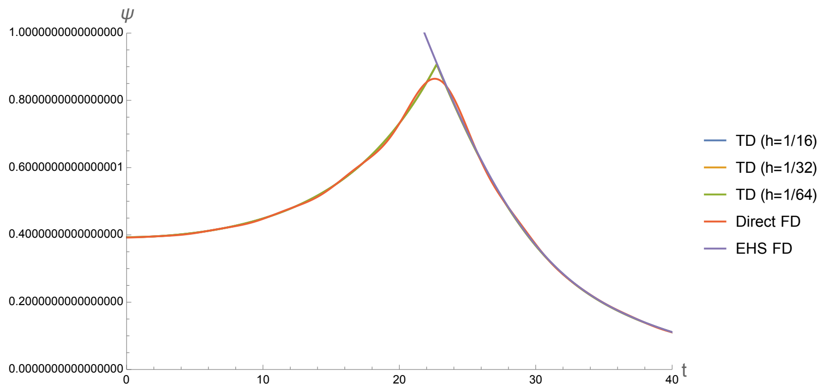
Scatter angle:

$$\delta\varphi = \delta\varphi^{(0)} + \epsilon\delta\varphi^{(1)} + \ldots \quad (1)$$

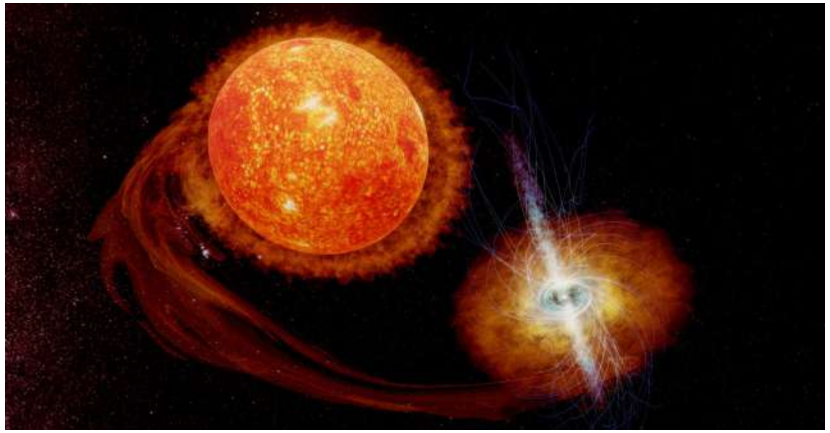


Challenges: continuous spectrum, EHS failure, slowly convergent radial integrals.

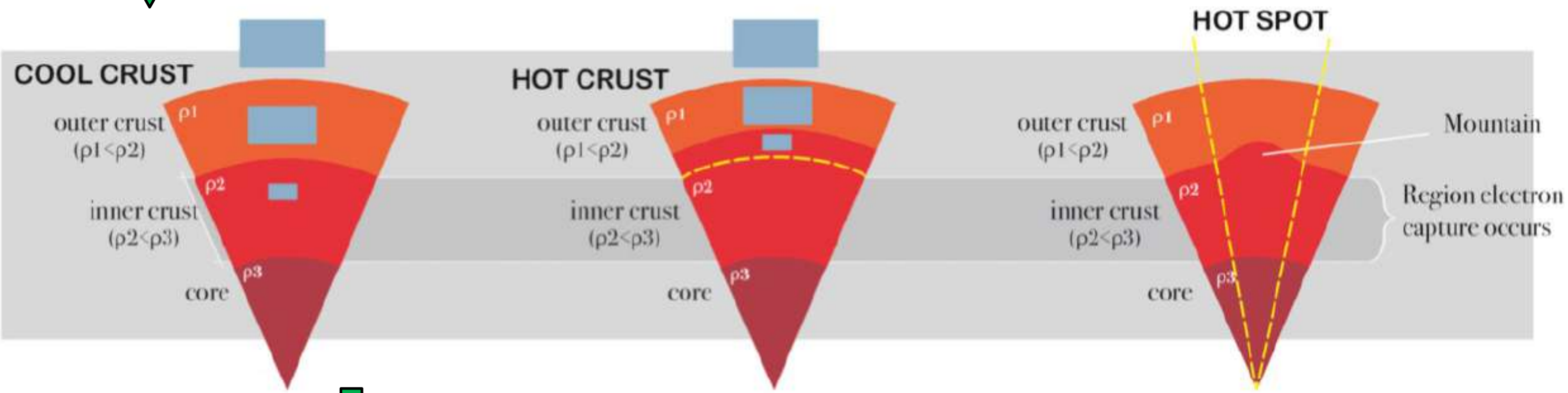




Progress: successful comparison to time-domain field (Olly Long). Self-force calculations underway.



Accretion-Driven Thermal 'Mountains' in Magnetic Neutron Stars



$$\nabla \cdot \mathbf{F} = \dot{Q}$$

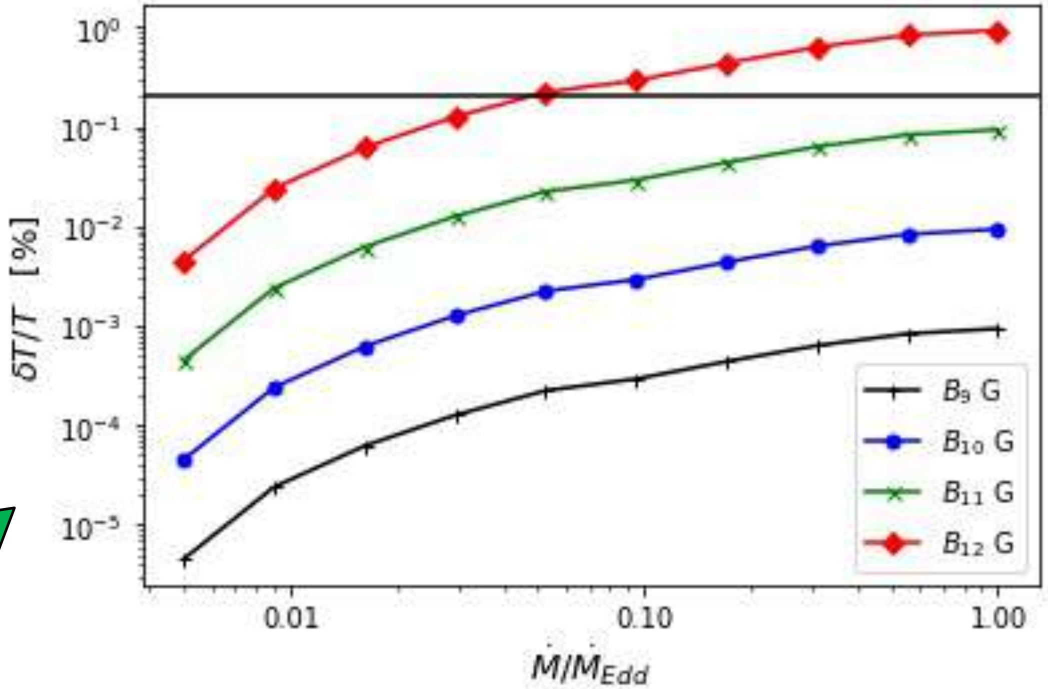
$$\downarrow$$

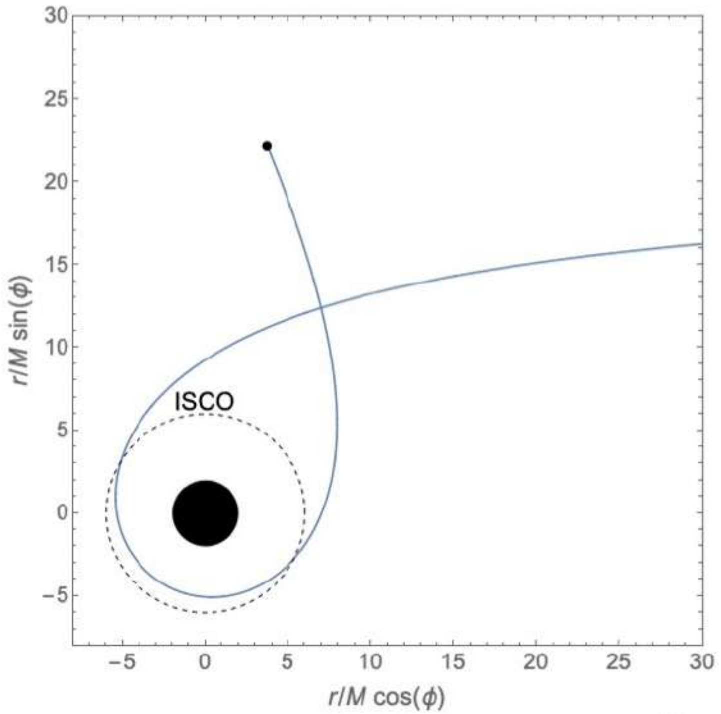
$$\boldsymbol{\nabla} \cdot \delta\mathbf{F} = \frac{\mathrm{d}\dot{\mathrm{Q}}}{\mathrm{dT}} \delta \mathrm{T}_{\mathrm{lm}} \mathrm{Y}_{\mathrm{lm}}$$

$$\mathbf{F} = -\kappa_{\perp}\left[\nabla \mathrm{T} + (\omega_{\mathrm{B}}\tau(\mathrm{T}))^2(\mathbf{b}\cdot\nabla\mathrm{T})\cdot\mathbf{b} + \omega_{\mathrm{B}}\tau(\mathrm{T})(\mathbf{b}\times\nabla\mathrm{T})\right],$$

$$\downarrow$$

$$\delta\mathbf{F} = -\kappa_0\left[\nabla\delta\mathrm{T} + \tilde{\omega}\tau(\mathrm{T})[\mathbf{B}\times\boldsymbol{\nabla}\mathrm{T}_0]\right] - \delta\kappa_{\parallel}\boldsymbol{\nabla}\mathrm{T}_0.$$

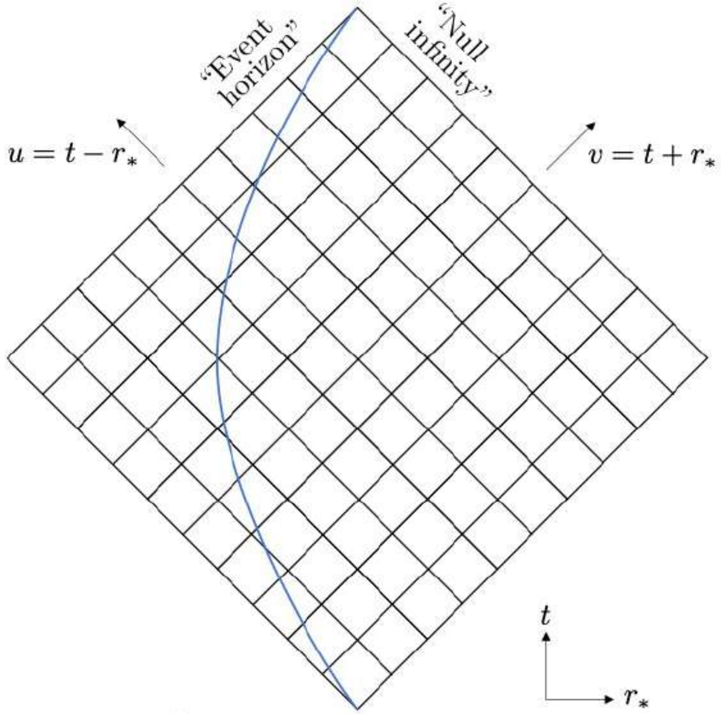


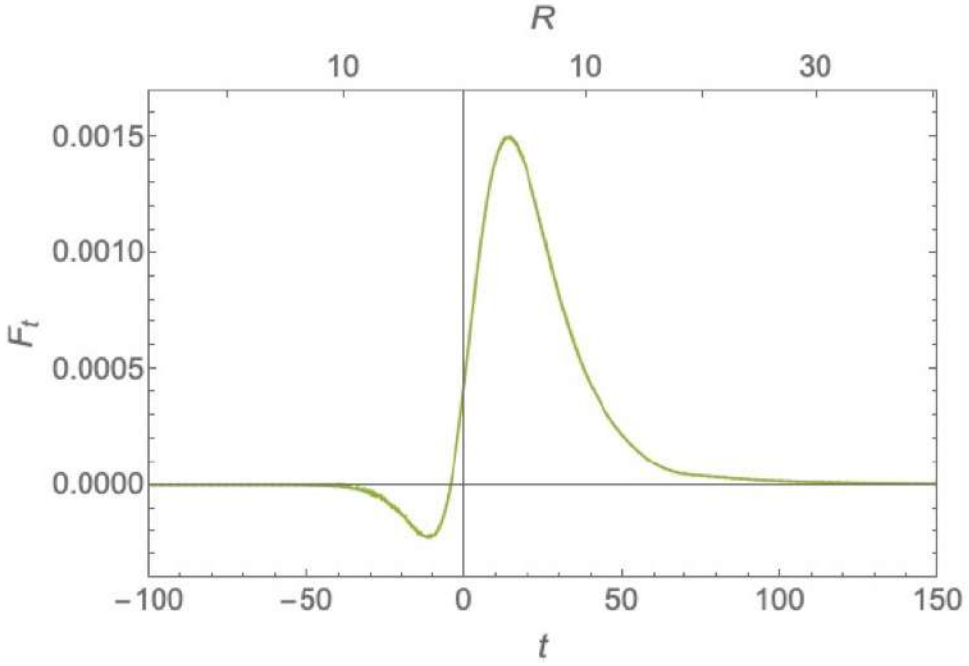


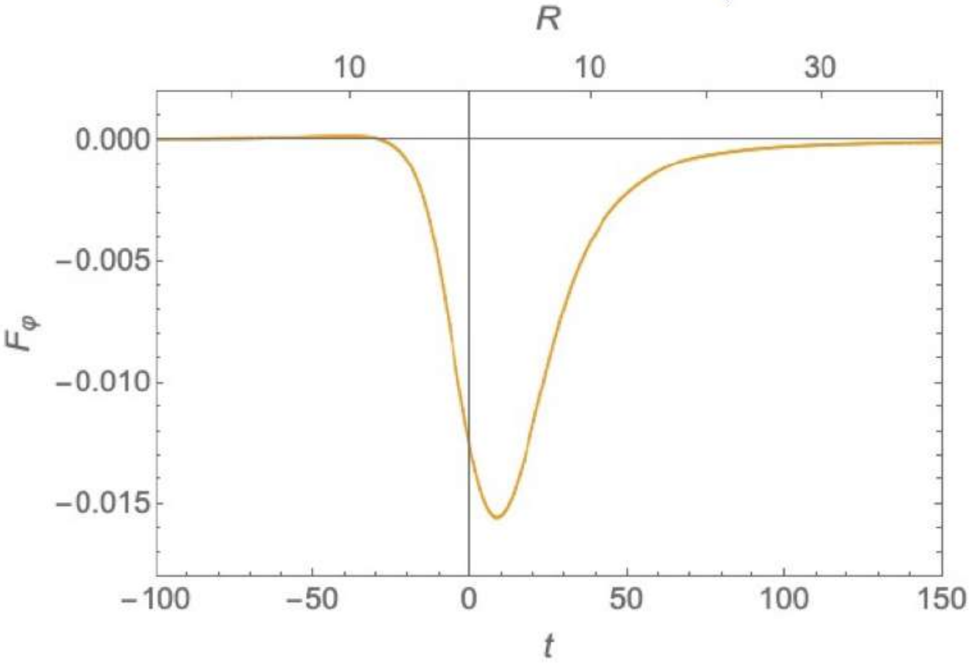
$$\frac{dz^{\alpha}}{d\tau} = F^{\alpha}$$

$$F_{\alpha} = \partial_{\alpha} \sum_{l,m} \psi_{lm}(t,r) Y_{lm}(\theta, \varphi)$$

$$\boxed{\psi_{,uv} + V(r)\psi = \delta(r - R(t))}$$

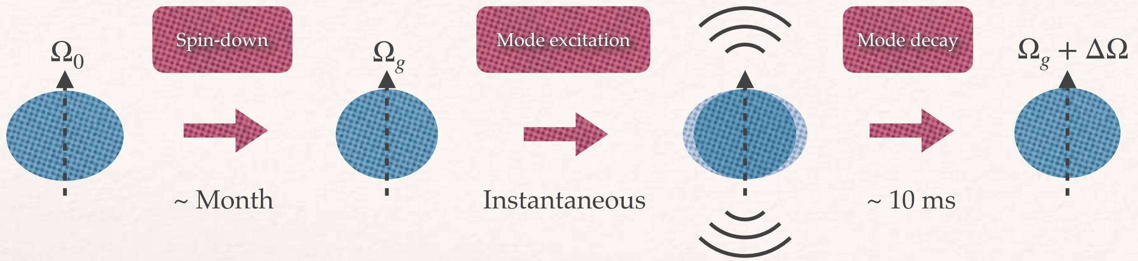




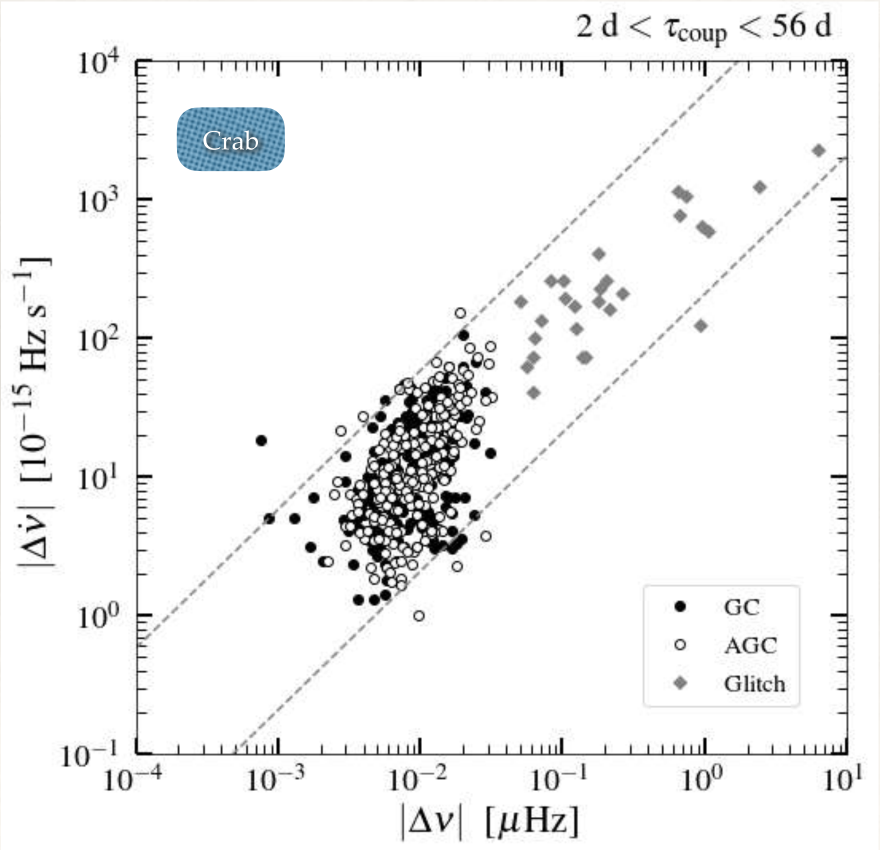


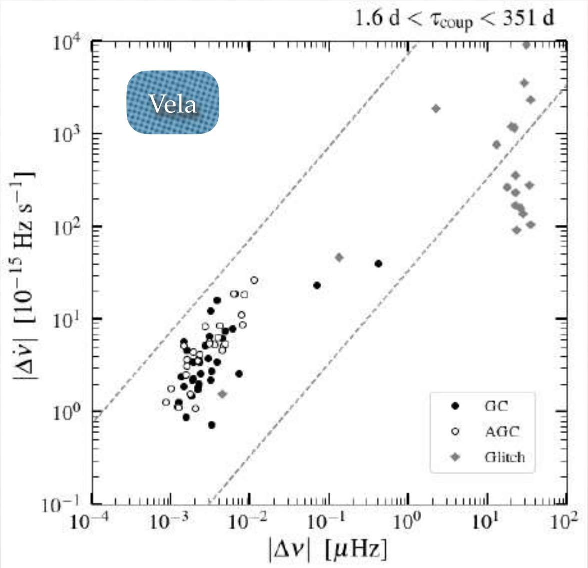
Modelling small glitch-like events as neutron star f-mode oscillations - Garvin Yim

Model outline

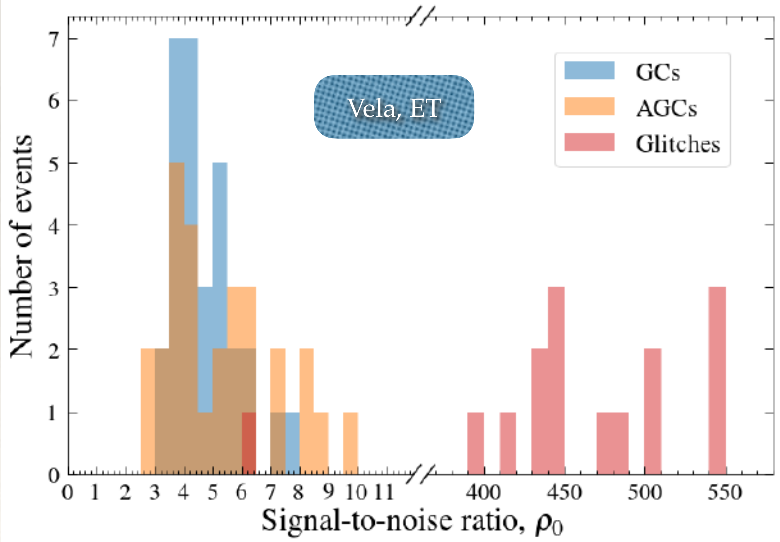


Electromagnetic observations





Gravitational wave observations



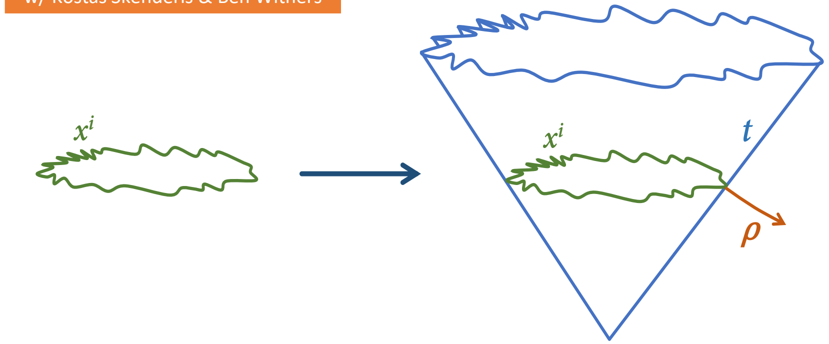
UNIVERSITY OF Southampton

Enrico Parisini

AdS/CFT

w/ Kostas Skenderis & Ben Withers

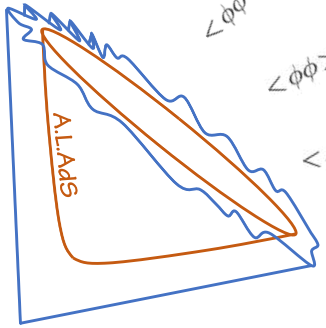
The **ambient space**



Ambient scalars

↓

Weyl-invariant objects



$$<\phi\phi>_\Delta = \frac{1}{X_{12}^\Delta}\left(c_0 + c_2 X_{12} I^{(2)} +\right.$$

$$<\phi\phi>_\Delta = \frac{1}{X_{12}^\Delta}\left(c_0 + c_2 X_{12} I^{(2)} + c_4 X_{12}^2 I^{(4)} + \ldots\right)$$

$$<\phi\phi>_\Delta = \frac{1}{X_{12}^\Delta}\left(c_0 + c_2 X_{12} I^{(2)} + c_4 X_{12}^2 I^{(4)} + \ldots\right)$$

$I^{(k)}$ universal building blocks of holographic correlators

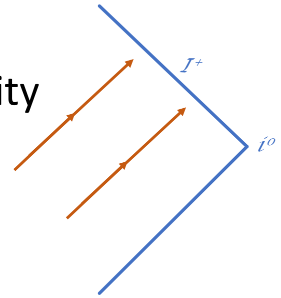
Flat holography

w/ Federico Capone

- Bondi expansion with $\Lambda \neq 0$ in general d

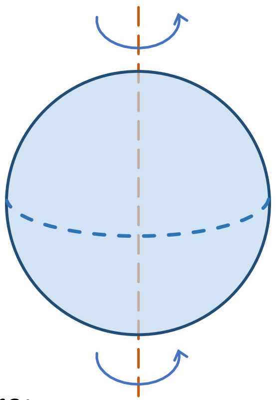
  Holographic dictionary ⟶ *Anomaly in Bondi gauge*

- Relation between spatial and null infinity *in asympt.flat spaces*



Fluid/gravity

w/ Jay Armas, …



- Equatorial QNMs:

  In the ocean, particular waves at the equator from geometry of the sphere

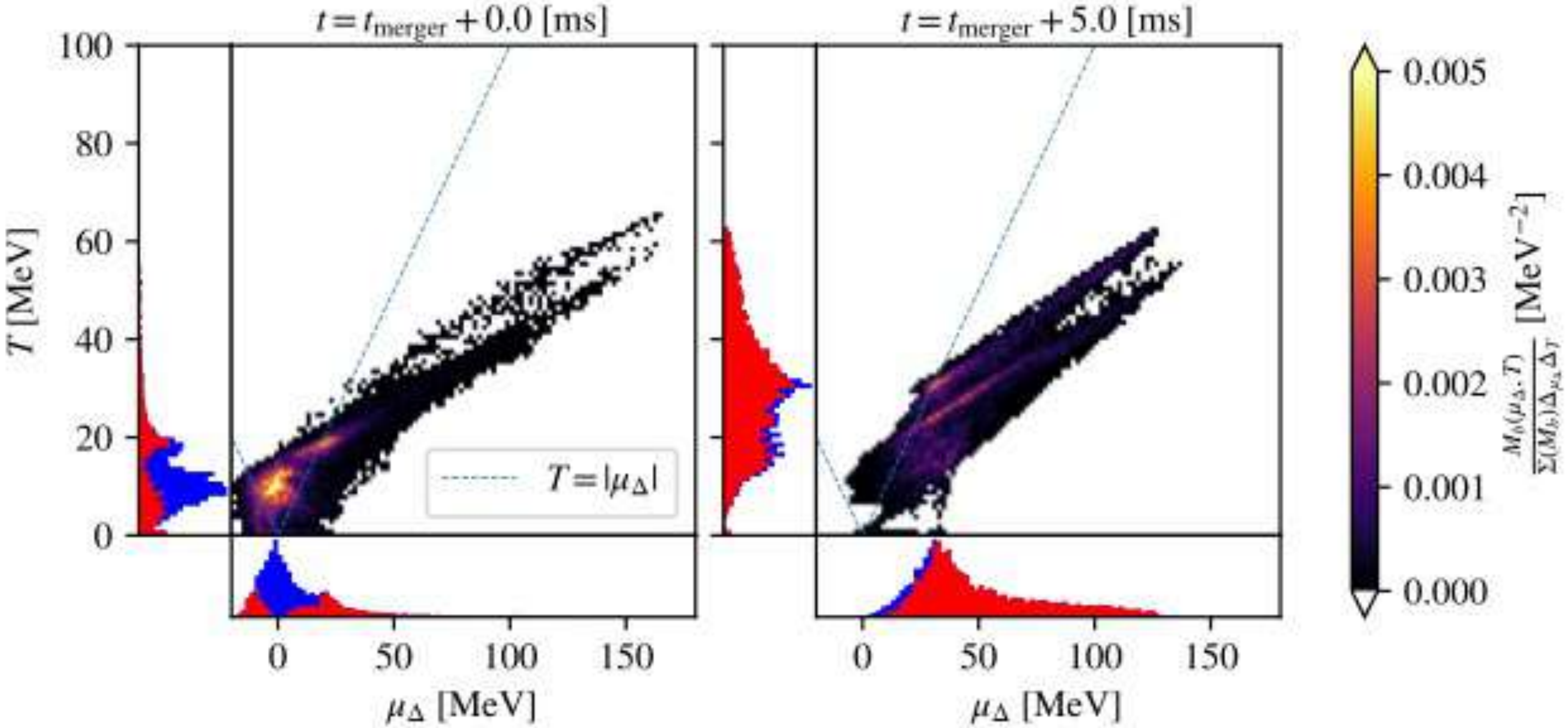
  Extend that technique to relativistic fluids on a sphere:

  » *via fluid/gravity, such modes are related to QNMs of Kerr-AdS BHs*

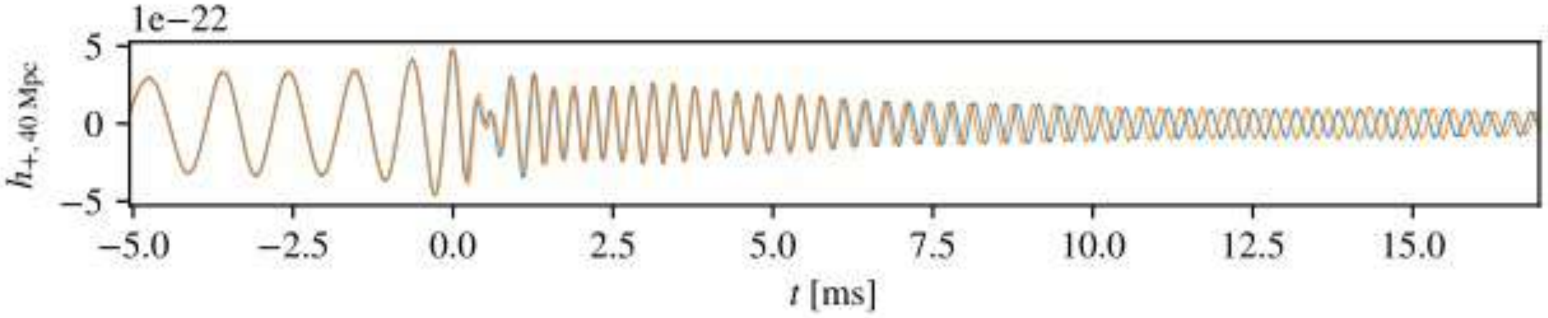
  » *can it work if we add elasticity? NS?*

Urca reactions: $p + e \to n + \nu_e, n \to p + e + \overline{\nu}_e$

$\Rightarrow \mu_n = \mu_p + \mu_e$, let $\mu_\Delta = \mu_n - \mu_p - \mu_e$.

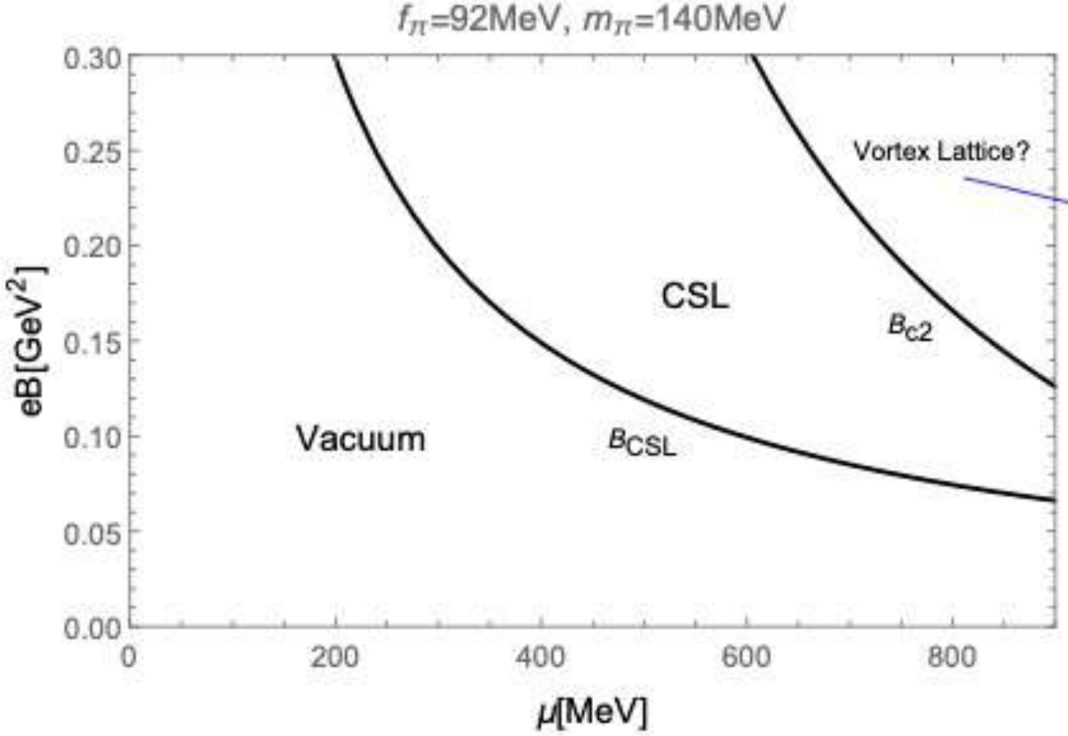


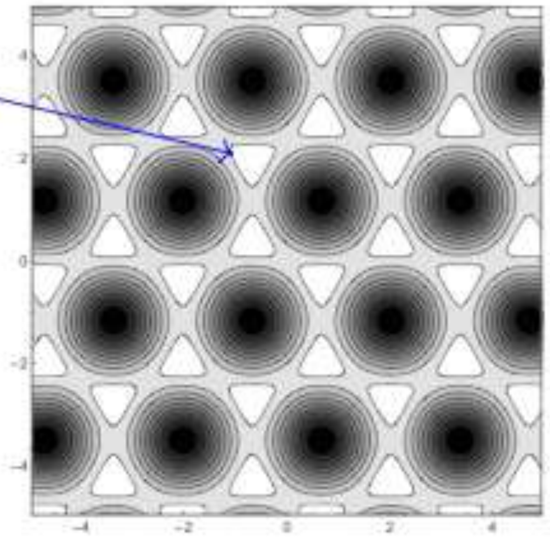
Compare equilibrium and non-equilibrium simulations:



Chiral anomaly induces charged pion vortex lattice

$$\mathcal{L} = \frac{f_\pi^2}{4}\mathrm{Tr}\left[(D_\mu\Sigma)^\dagger D^\mu\Sigma\right] + \frac{m_\pi^2 f_\pi^2}{4}\mathrm{Tr}\left[\Sigma + \Sigma^\dagger\right] + \text{WZW term}$$



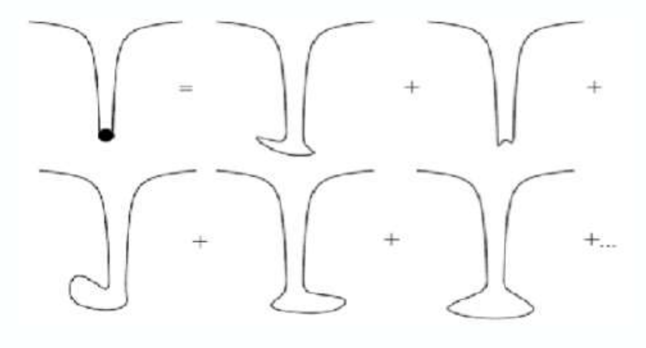


How do black hole microstates look like in the full quantum gravity description?

- Semi-classical expectation: GR breaks down when $\mathcal{R} \gg 1/l_p^2$: our description of BH is accurate until a neighbourhood of the singularity

VS

- Fuzzball proposal: in String Theory, BH microstates present strong quantum gravity effects already at the horizon scale.



Conjecture motivated by explicit construction of "simple" microstates: how generic are these features?
Are these features stable against perturbations?

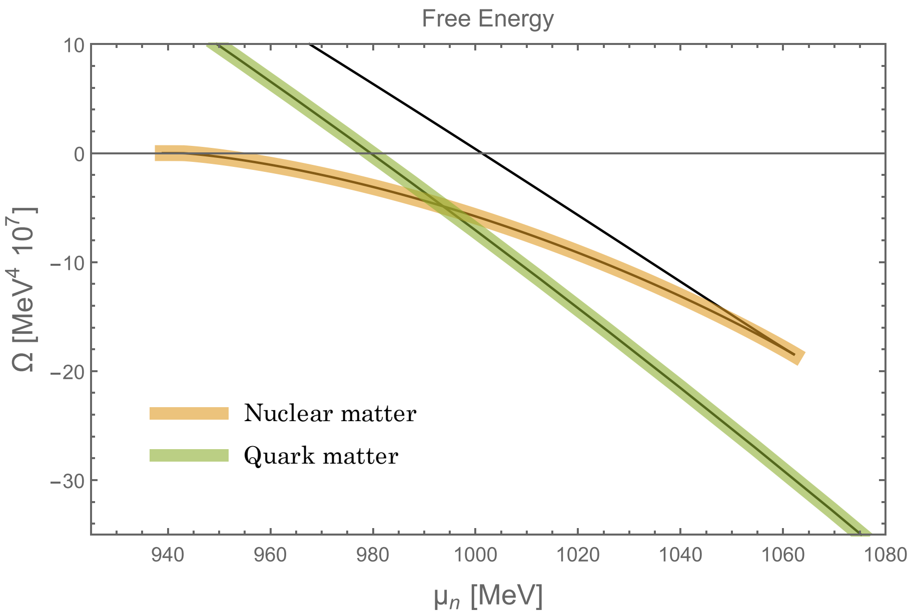
Families of extremal microstates have a non-linear instability.
Modelling it with the addition of shockwave on the microstate and computing its backreaction

- We have constructed new classes of BH microstates
- We have shown that they share the features conjectured by the fuzzball proposal

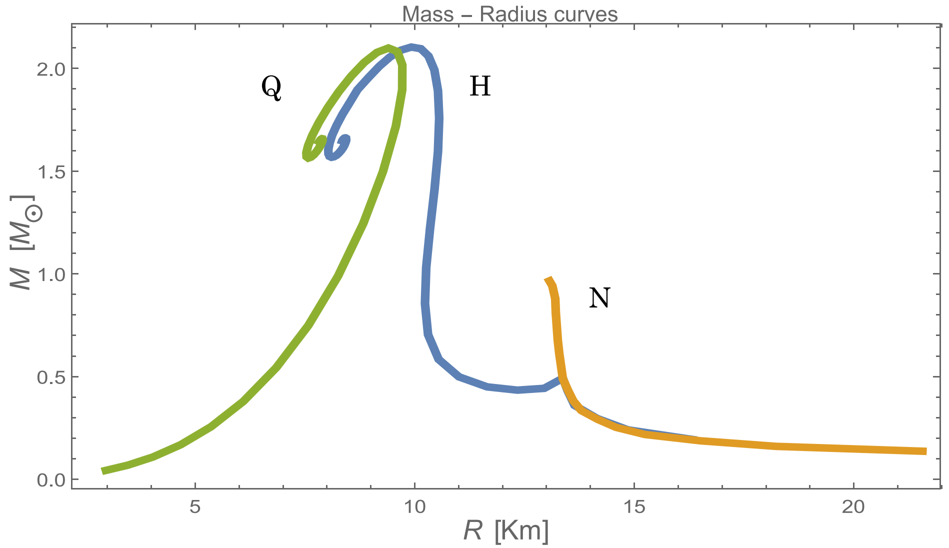
[Chakrabarty, SR, Turton, to appear]

Quark–Hadron transition in compact stars

- **Chirally symmetric** model fitted to nuclear matter at saturation.
- **Nucleonic** and **hyperonic** d.o.f. interacting via **meson** exchange.
- Spontaneous restoration of chiral symmetry at high densities.



$$\mathcal{L} = \bar{\psi}_i(\gamma^\mu \partial_\mu + \gamma^0 \mu_i - m_i)\psi_i + \frac{1}{2}\partial_\mu \sigma\, \partial^\mu \sigma - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} - \frac{1}{4}\phi_{\mu\nu}\phi^{\mu\nu} - \frac{1}{4}\rho_{\mu\nu}\rho^{\mu\nu} + \mathcal{L}_{\mathrm{Int}} + \cdots$$

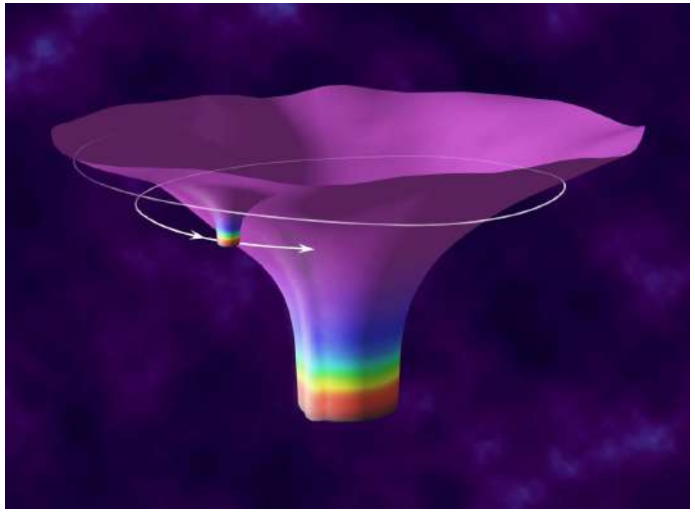


- **Chiral symmetry** restoration, **strangeness** charge and the correct conformal limit for the speed of sound $\boldsymbol{c_s^2} \sim \mathbf{1/3}$ → "Quark matter".
- Spinodal regions constrain the Mass – Radius curve.

Dr. Andreas Schmitt and Savvas Pitsinigkos

SECOND-ORDER GRAVITATIONAL SELF-FORCE IN A HIGHLY REGULAR GAUGE

SAM UPTON
SUPERVISOR: ADAM POUND



Credit: NASA

- Extreme-mass-ratio inspiral (EMRI) – compact object $m \sim 1\text{–}10^2 M_\odot$ spirals into supermassive black hole $M \sim 10^5\text{–}10^7 M_\odot$ constantly radiating gravitational waves
  - Will be detectable by the Laser Interferometer Space Antenna (LISA) launching in 2034
- Solve motion of system through perturbative method – gravitational self-force
  - Treat small object as source of perturbations: $\hat{g}_{\mu\nu} = g_{\mu\nu} + \epsilon h^1_{\mu\nu} + \epsilon^2 h^2_{\mu\nu} + \cdots$, where $\epsilon = m/M$
  - Motion of small object given by $\frac{D^2 z^\alpha}{d\tau^2} = \epsilon f_1^\alpha + \epsilon^2 f_2^\alpha + \cdots$ – self-force, f_n^α, accelerates object from geodesic motion
    - Can write as a geodesic in effective metric, $\tilde{g}_{\mu\nu} = g_{\mu\nu} + \epsilon h^{R1}_{\mu\nu} + \epsilon^2 h^{R2}_{\mu\nu} + \cdots$, governed by $\frac{\tilde{D}^2 z^\alpha}{d\tilde{\tau}^2} = O(\epsilon^3)$. Here, $h^R_{\mu\nu} = h_{\mu\nu} - h^S_{\mu\nu}$, is the field the small object 'sees' and $h^S_{\mu\nu}$ describes object's multipole structure

Problem

- Need to solve Einstein equations through second order
  - $\delta G^{\mu\nu}[h^1] = 8\pi T_1^{\mu\nu}$ – here $T_1^{\mu\nu}$ is stress-energy of point particle
  - $\delta G^{\mu\nu}[h^2] = 8\pi T_2^{\mu\nu} - \delta^2 G^{\mu\nu}[h^1, h^1]$
- BUT! Second equation not well defined in a generic gauge!
- To see, $\delta^2 G^{\mu\nu}[h,h] \sim h\partial^2 h + \partial h \partial h$ and $h^1 \sim \frac{1}{r}$, so $\delta^2 G^{\mu\nu}[h^1,h^1] \sim \frac{1}{r^4}$
- Not integrable nor well-defined as a distribution at worldline, $r = 0$

Solution (SDU & Pound, 2101.11409)

- Found a "highly regular gauge" in which most singular part of $\delta^2 G^{\mu\nu}[h^1, h^1] \sim \frac{1}{r^2}$
- Second-order EFEs now well defined when including worldline
  - Allows rigorous derivation of second-order stress-energy tensor
  - In fact, $\epsilon T_1^{\mu\nu} + \epsilon^2 T_2^{\mu\nu} = \tilde{T}^{\mu\nu}$, where $\tilde{T}^{\mu\nu}$ is the stress-energy tensor of a point particle in the effective metric!
  - We name this the *Detweiler stress-energy tensor*

SCC conjecture: Generically, one cannot extend beyond $r = r_-$ as a *suitably regular* manifold.

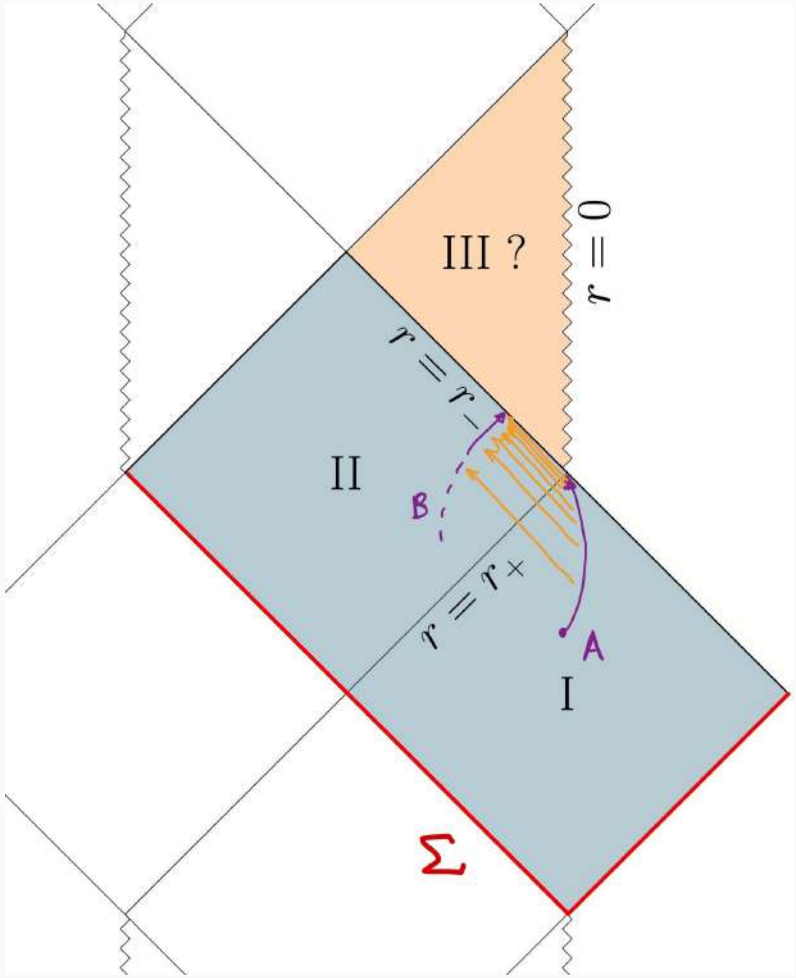
$\underline{\Lambda = 0}$: SCC respected.

$\underline{\Lambda > 0}$: SCC respected if there exists a quasi-normal mode ω with

$$\beta \equiv -\frac{\mathrm{Im}(\omega)}{\kappa_-} \leq \frac{1}{2}$$

where κ_- is the surface gravity at r_-.

$d = 4$	$d > 4$
Kerr-dS	**?**
RNdS	RNdS



We considered: Myers-Perry-dS† $\Longrightarrow$ **SCC respected!**

†(equal angular momenta, in odd dimensions)

Chapman-Enskog Expansion Applied to the Israel-Stewart Formalism for Near-Ideal Hydrodynamics

Goal: realistic NS merger simulations including non-ideal effects: heat fluxes and viscosities

$$\partial_t v_y = 2\eta \partial_x^{(2)} v_y - 4\eta^2 \tau_\pi \partial_x^{(4)} v_y.$$

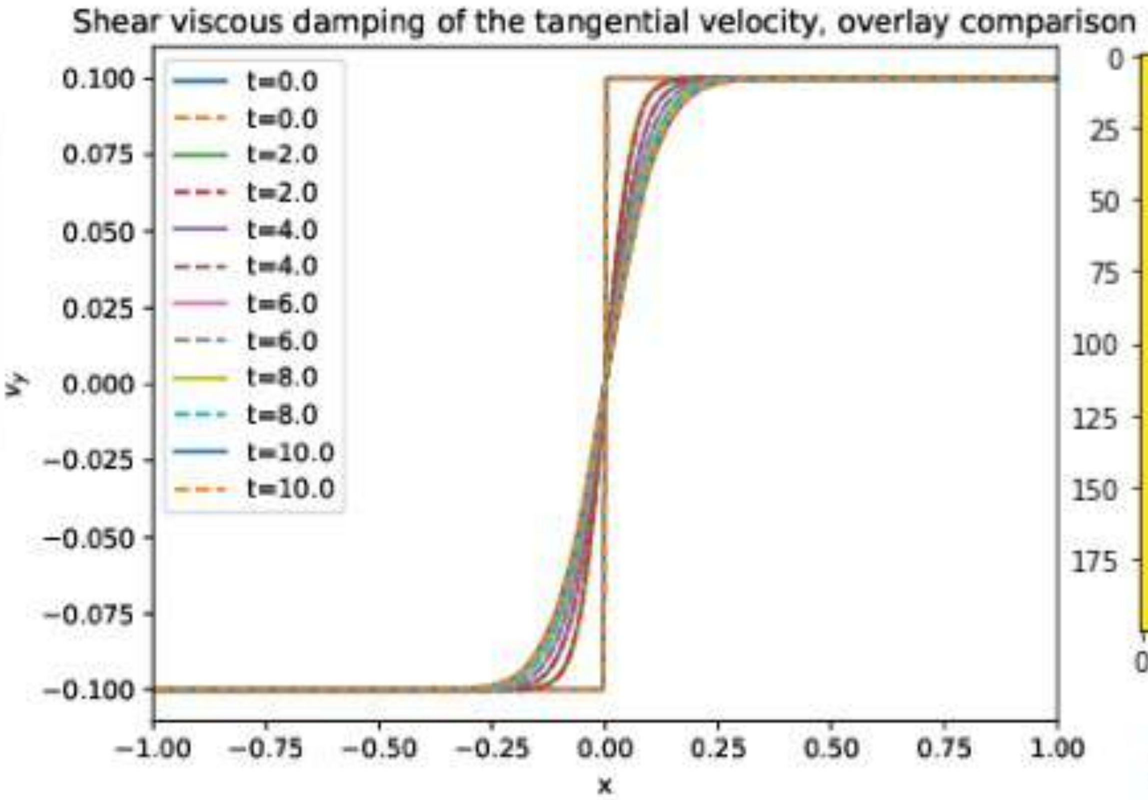
Coupled, Stiff Source

$$\partial_t T + \partial_i q^i = 0,$$
$$\partial_t q_j = \frac{1}{\tau_q}(q_{j,\mathrm{NS}} - q_j)$$

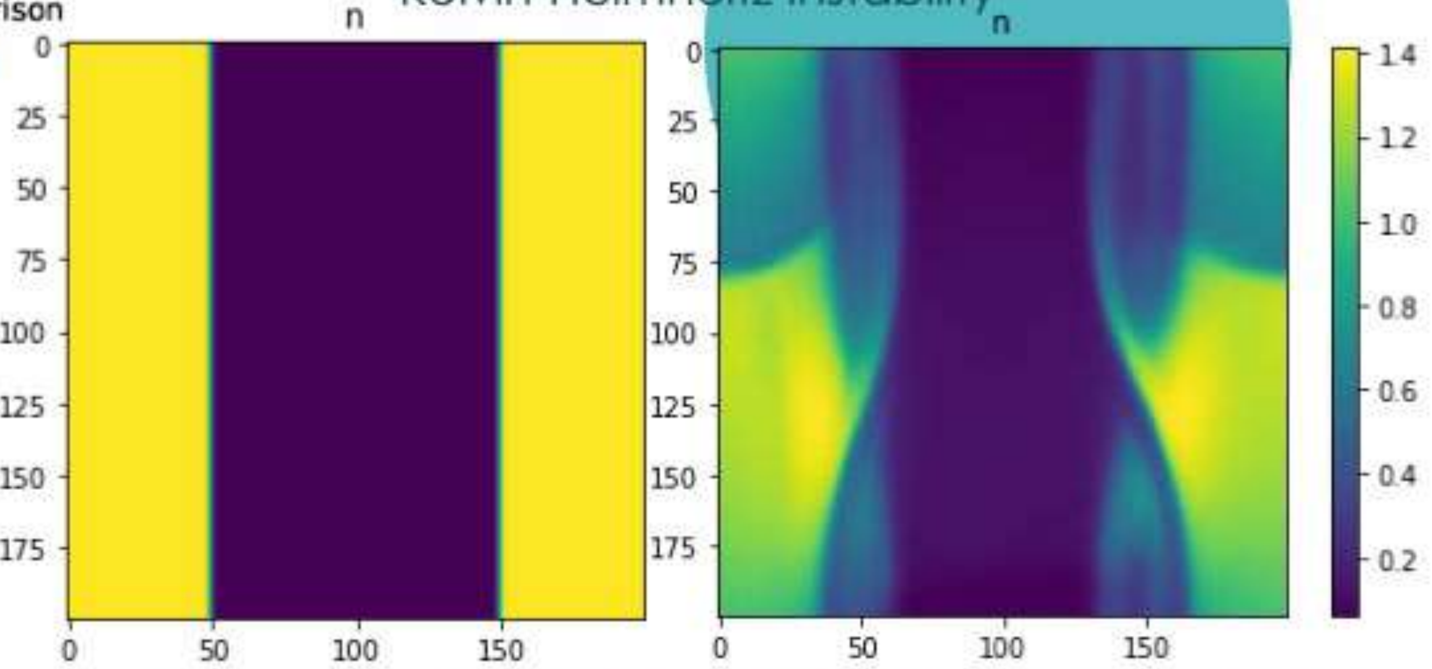
$$q_j = q_{j,NS} + \tau_q q_1 + \mathcal{O}(\tau_q^2)$$

Single, Non-Stiff Equation

$$\partial_t T = \kappa \left[\partial_x^{(2)} T + \tau_q \kappa \partial_x^{(4)} T + \mathcal{O}(\tau_q^2)\right]$$



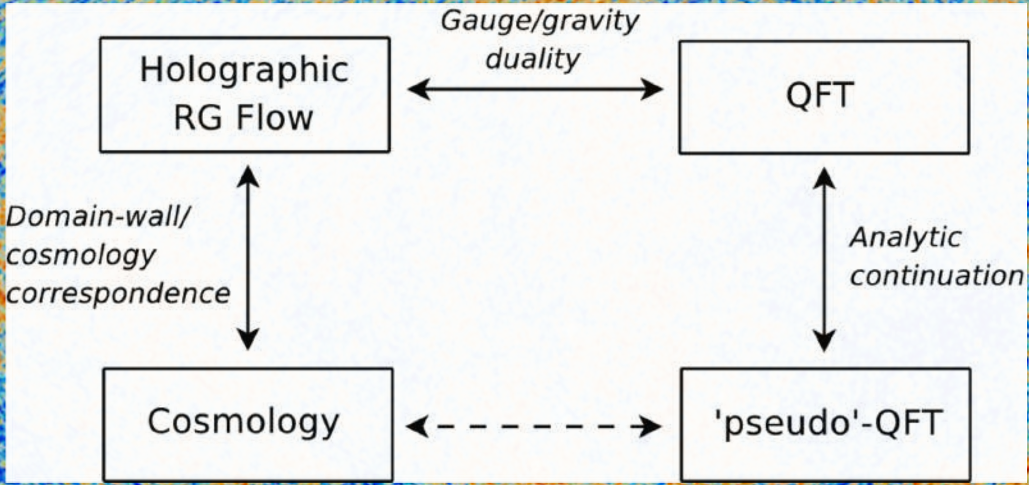
Kelvin-Helmholtz Instability



Reduced System Size + Adaptive Model Refinement → Faster, Explicit-Only Simulations

Simulations of Quantum Field Theories that are holographically dual to the early universe allow us to fit the CMB Power spectrum!

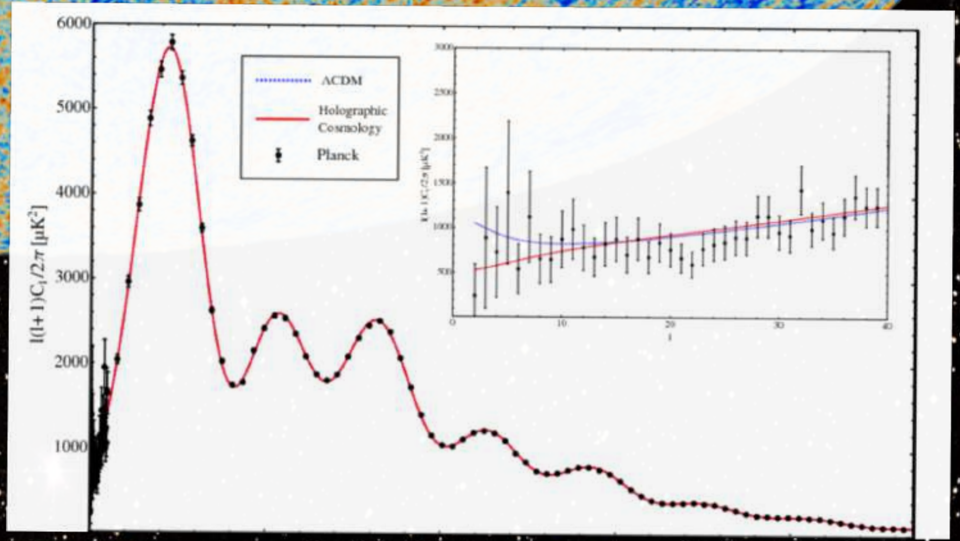
$$\langle T_{ij}(\mathbf{q_1})T_{kl}(\mathbf{q_2})\rangle = (2\pi)^3\delta^3(\mathbf{q_1}+\mathbf{q_2})[A(q)\Pi_{ijkl} + B(q)\pi_{ij}\pi_{kl}],$$



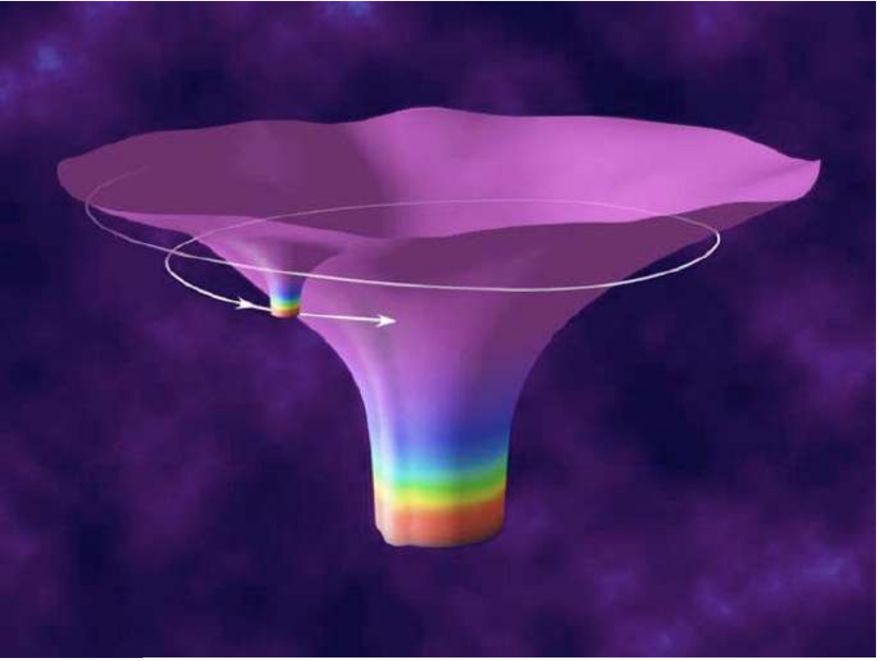
$$\Delta_S^2(q) = -\frac{q^3}{16\pi^2 ImB(-iq)}, \qquad \Delta_T^2(q) = -\frac{2q^3}{\pi^2 ImA(-iq)},$$

Previous perturbative calculations show Holographic Cosmology is competitive with ΛCDM in explaining the CDM Power Spectrum where pertubation theory is valid.

Simulation on the lattice will allows us to compare the theories at all momentum scales.



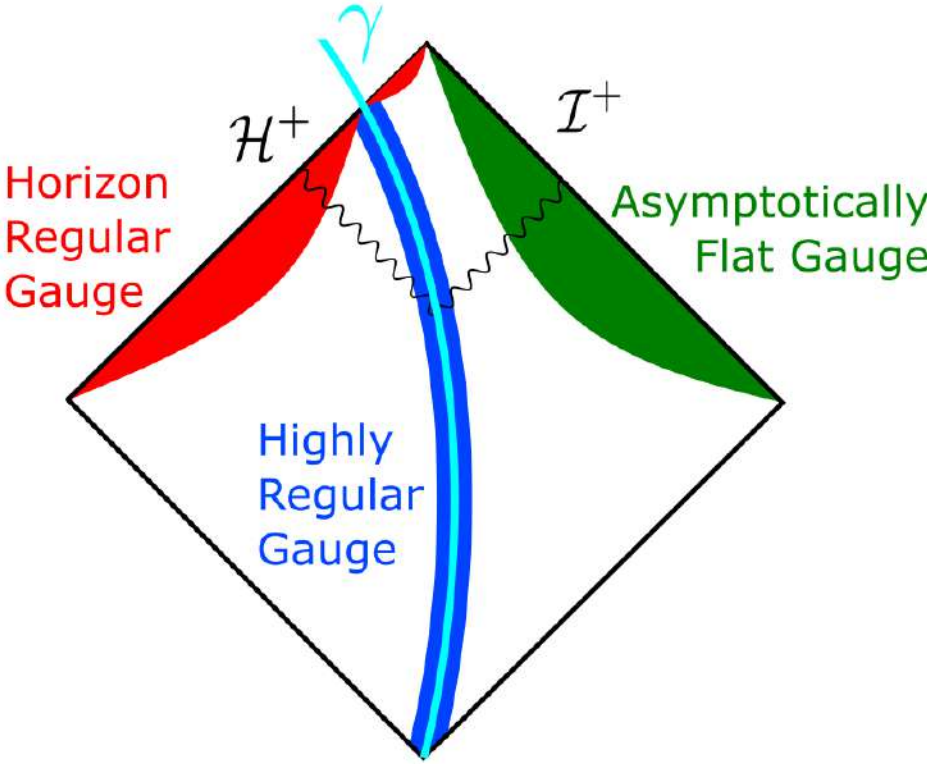
Second-Order Self-Force

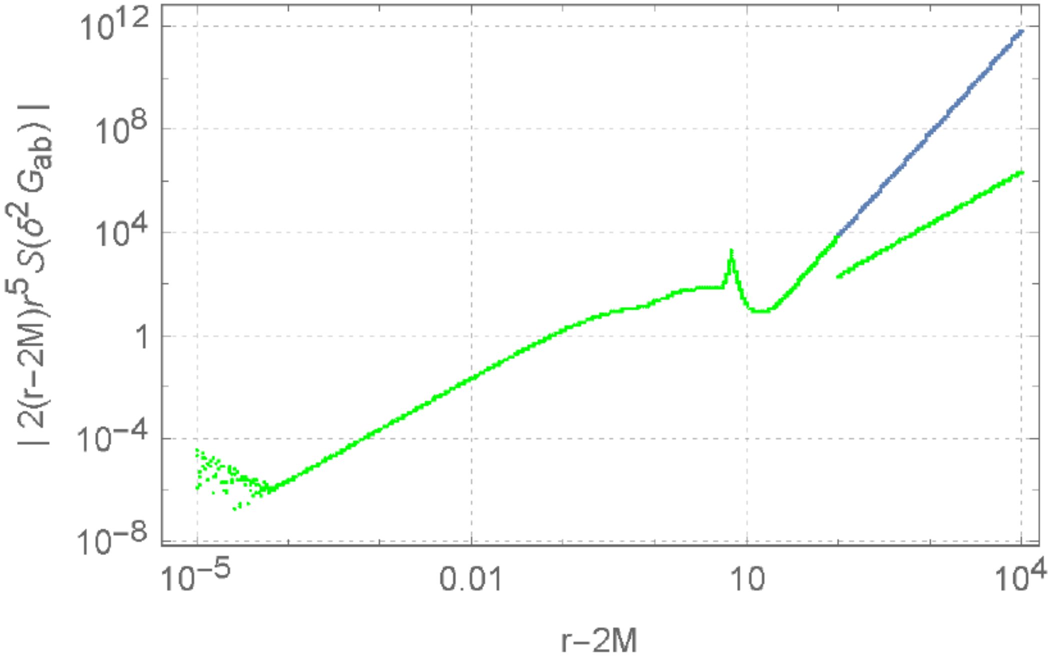


$$g_{\mu\nu} = g_{\mu\nu}^{(0)} + \varepsilon h_{\mu\nu}^{(1)} + \varepsilon^2 h_{\mu\nu}^{(2)} + \ldots$$

$$\varepsilon \sim \frac{m}{M}$$

Andrew Spiers
Supervisor: Adam Pound





$$\mathcal{O}\left[\psi_{4L}^{(2)}\right] = \mathcal{S}[T_{\mu\nu}^{(2)} - \delta G_{\mu\nu}^{(2)}\left[h_{\alpha\beta}^{(1)}\right]]$$

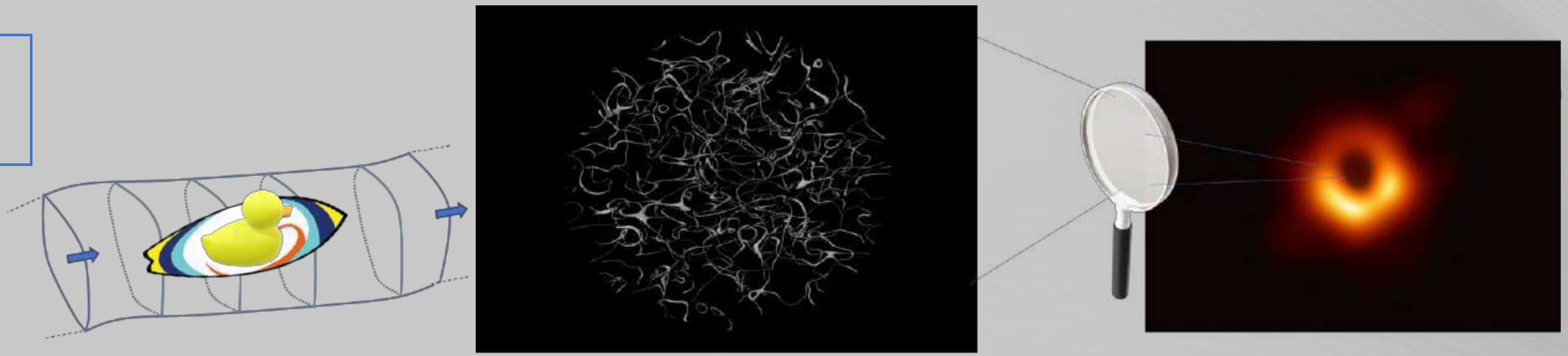
Surfing the string worldsheet inside black holes

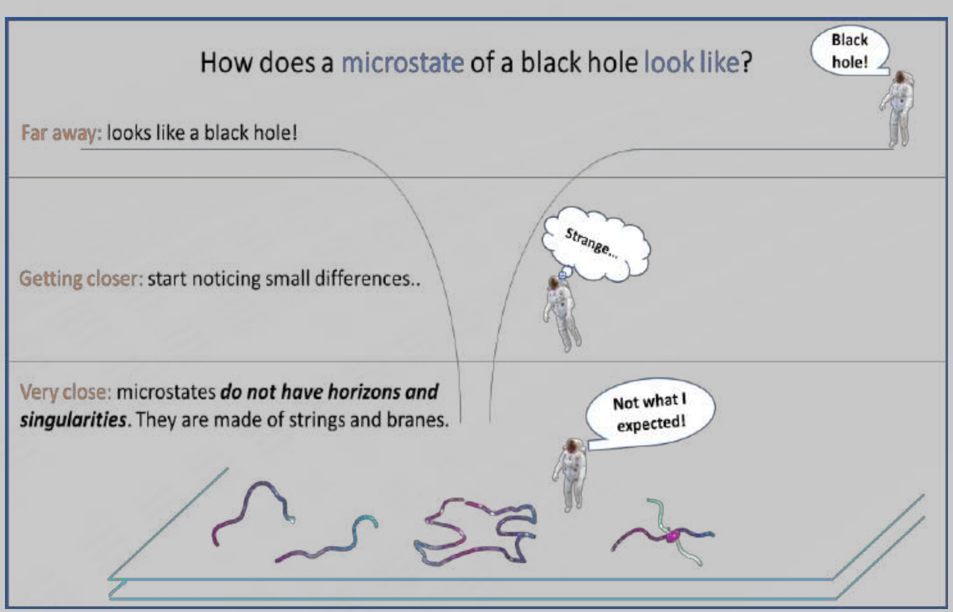
Understanding black holes with String Theory

Fuzzball proposal: [Mathur 2005]
[Skenderis, Taylor 2006, 2007, 2008, …]

Worldsheet description exact in α'
of strings in black hole microstates

[Martinec, Massai 2017]
[Martinec, Massai, Turton 2018,2019,2020]





Use the *power* of string theory
to study **black hole microstates**

Consistency conditions from string theory:

- Smoothness
- No CTCs
- No horizons

[DB, Iguri, Kovensky, Turton 2021]

Compute correlation functions using the worldsheet:

1. Matches holography results present in the literature:
   ““Proof from String Worldsheet””
2. Matching of Hawking radiation from dual D1D5 CFT
3. We *generalize* holographic results (new correlators!)

[Avery, Chowdhury, Mathur 2009]
[Galliani, Giusto, Moscato, Russo 2016]

[DB, Iguri, Kovensky, Turton; TO APPEAR]

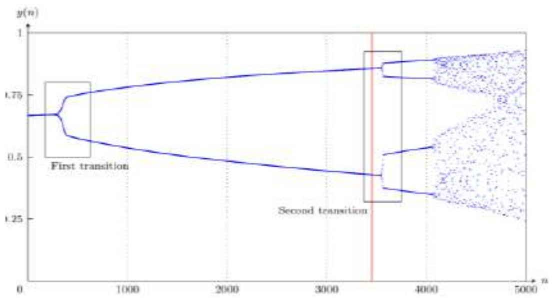
Davide Bufalini
28/10/2021

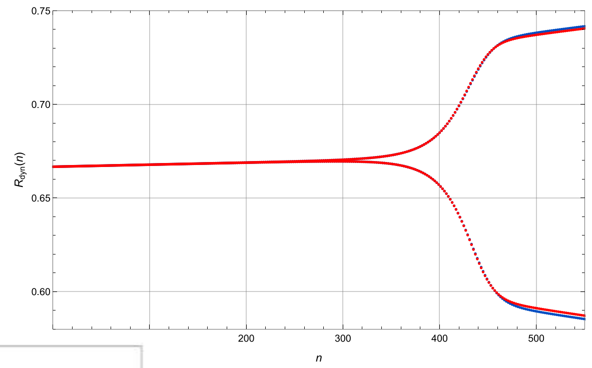
Exponential asymptotics $e^{-\frac{1}{x}}$

1) Logistic map

-> Difference equation
-> Study bifurcations
-> Both static and non-static

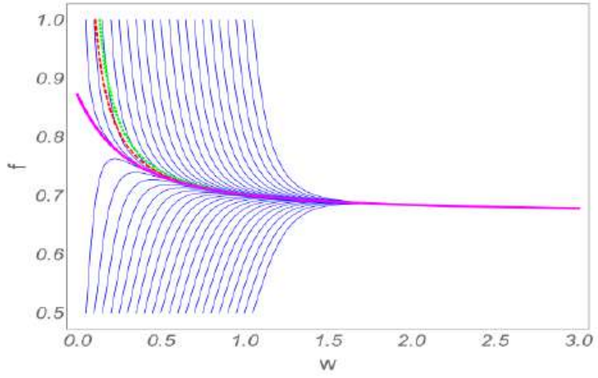
$$Y_{n+1} = (3+\varepsilon)Y_n(1-Y_n)$$
$$Y_{n+1} = (3+n\varepsilon)Y_n(1-Y_n)$$





2) Relativistic late time behaviour

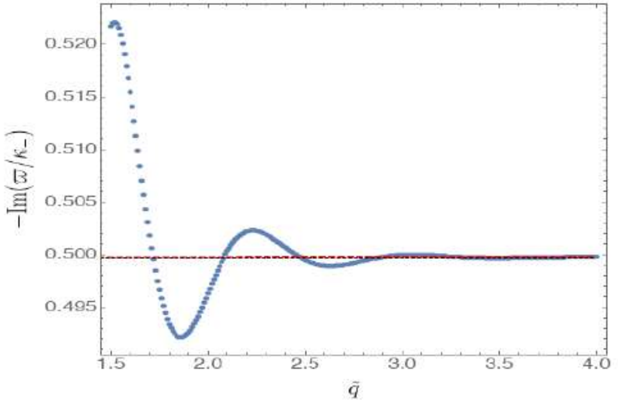
-> Boost-invariant Bjorken flow
-> $T(\tau) \leftrightarrow f(w)$
-> Early time - late time matching



[Heller, Spalinski '15]

3) QNMs in RNdS space

-> QNMs decide about SCC
-> Large-charge asymptotics



[Dias, Reall, Santos '19]