School of Mathematical Sciences

UNIVERSITY OF Southampton

Senior Challenge ’22

Year 10 or below

Illustrations by Theo Chaddock & Peter Ackerley

Rules

1) Senior Challenge ’22 should be attempted on your own time at home.
2) Your entry must be your own work, though of course you may ask for help on how to get started or for the meanings of unfamiliar words.
3) Entries without any working out at all or written on this sheet **will not be marked**.
4) It is possible to win a prize or certificate even if you have not completed all of the questions, so hand in your entry even if it is not quite finished.
5) You must write **your name and school neatly on the top of every page**.
6) Please staple your pages together if you are submitting it by mail and not online.

Deadline

Either you or your maths teacher needs to submit your entry by **25th March**, by mail or online.

Submit via mail:

Either you or your maths teacher needs to return your entry by the deadline to this address:

Senior Challenge ’22 Entries,
Dr Sunny Yu,
School of Mathematical Sciences,
University of Southampton,
University Road,
Southampton,
SO17 1BJ

Or: submit via online upload:

Scan your entries as a pdf file and upload at:
https://www.southampton.ac.uk/stag/mathschallenge.page

A Prize-Giving Evening will (hopefully) be held at the University of Southampton on 8th June.
We hope that you enjoy the questions.

If you have any questions, please have your teacher or parents contact us at math4all@soton.ac.uk

1. A Young Girl Reading

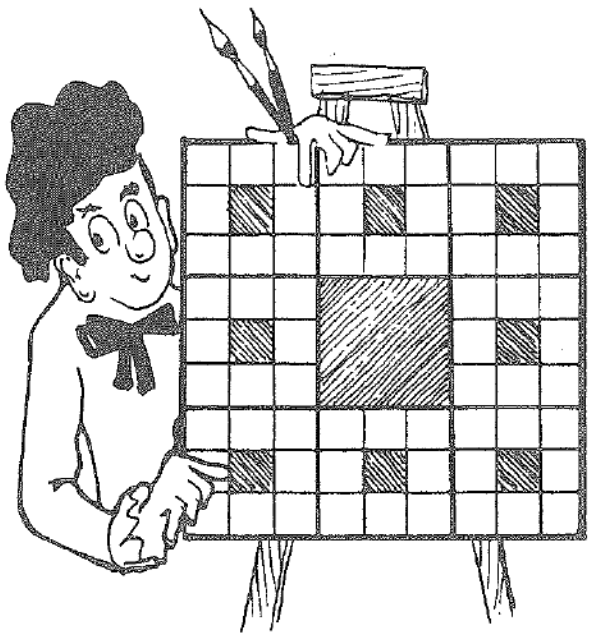
Emily is reading a stapled comic, but 4 of the pages are missing.
Pages 6 and 39 are missing.
Which other pages are missing?
How many pages did the comic have originally?

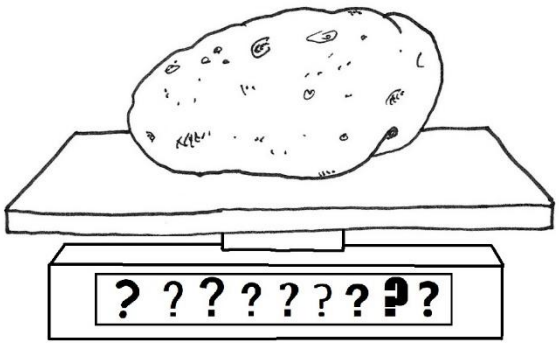
2. Napoleon Crossing the Alps

Napoleon's route through the Alps involved travelling through the Grand St Bernard Pass, a climb of 6km and a descent of 4km. His army travelled twice as fast downhill as it did uphill and the whole journey took 8 hours.
How long did it take him to reach the top?

3. Composition with Red, Blue & Yellow

Ann is a clever painter. Inspired by Mondrian's work, she decides to create a mathematical work of art. She divides a square canvas into 9 equal squares and paints the central one red. She then divides each of the 8 remaining uncoloured squares into 9 equal squares, painting each of the central squares so formed yellow. The remaining uncoloured squares are again each divided into 9, the centres this time being painted blue. This process is repeated using a different colour for each set of central squares until just over half the original area of the canvas has been coloured.
How many different colours have then been used and how many central squares have been painted?





4. **The Potato Eaters**

Vincent sells potatoes. He doesn't sell potatoes by weight or one at a time. He insists on selling them in pairs. His customer Graham wants a potato that weighs exactly 200g.
Vincent tells him, "I only have 3 potatoes left."
"Here they are: A, B and C."
"A & B together weigh 300g, A & C together weigh 500g, B & C together weigh 400g."
"You can have any pair of them."
Which, if any, of the potatoes weighs 200g?
Which pair of potatoes should Graham buy?

5. **When Did You Last See Your Father?**

William replied: "On the first Monday of a certain month last year (2021), I saw him in London, and then, on the Monday after the first Sunday of the month, I saw him in Cardiff.
The following month, we met in Edinburgh on the first Monday of the month and then again in Liverpool on the Monday after the first Sunday of the month."
Work out the actual dates of William and his father's meetings in these cities.
Why could this arrangement of Mondays not have happened the year before last (2020)?

6. **Mona Lisa**

Lisa moans and complains a lot about her drink being flat.
In fact, her local restaurant manager, Ken, considered banning her for making spurious complaints, but then wondered if he should put her forward for a quality-control job with the fizzy-drinks company.
As evidence, he put together a table showing the last 100 drinks she has bought at the restaurant.

	Lisa Complained	Lisa Didn't Complain	Total
Drink was flat	5	1	6
Drink was not flat	3	91	94
Total	8	92	100

If Lisa says a drink is flat, how likely is she to be correct?
How good is Lisa at detecting whether or not a drink is flat?
Would Lisa be good at the quality-control job? Justify your answer.

7. Arrival of the Normandy Train

Claude is waiting to meet the train from Normandy at Gare Saint-Lazare in Paris. He tells Marie the hour of the train's arrival and he tells Jean at which minute it arrives. He also tells them both that the train arrives between 0600 and 1000.

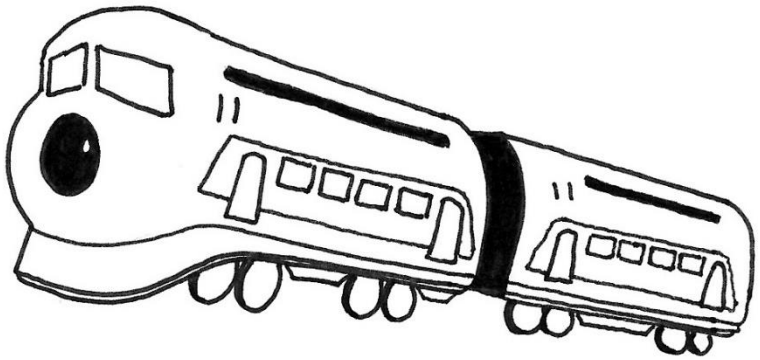
They consult the timetable and find the following services between those times:
0620, 0639, 0650; 0717, 0746; 0825, 0839; 0917, 0925, 0950.

Marie then says, "I don't know when Claude's train arrives, but I'm sure that neither does Jean."

Jean replies, "I didn't know his train, but now I do."

Marie responds, "Now I do as well!"

When is Claude's train and how do you know?



8. The Last Supper

On the last night of Scout camp, Chris, his wife Jo and the 11 other adults sit down for supper along one side of a long rectangular table.

Chris takes the central seat, with six adults on each side of him.

Jo must sit next to Chris.

Alex, Barb and Dave insist on being next to one another (in any order) on the side to Chris's left.

Eddie, Fred and Gareth are fed up with Alex's snoring, so they refuse to be on the same side of Chris as him.

Heather, Iris, Kane, Luke and Michael have no seating preferences.

How many arrangements of the adults are possible?

All of the questions in this year's Challenge have been inspired by famous paintings.
To see a gallery of these works of art, go to www.tinyurl.com/challenges2022gallery

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The Challenge is organised by
the School of Mathematical Sciences outreach team,
University of Southampton

We kindly acknowledge MEM (Mathematical Education on Merseyside) for providing these questions and the concept of the Challenge. http://www.mathsmerseyside.org.uk/

Senior Challenge ’22 Solutions

1. A Young Girl Reading

Hidden assumptions

1. Paper sheets are stacked on top of one another, and stapled down the middle, to create the book.
2. A page is one side of the staples of either the front or back of a stapled sheet of paper.
3. If there is an unnumbered page, like a cover page, then none of the pages comprising that sheet of paper are numbered.
4. The numbering system starts from 1.
5. Missing pages indicate that an entire sheet of paper has been removed from the book.

Deductions

1. Every sheet of paper contains 4 pages.
2. All of the pages comprising a missing sheet are missing.
3. If an odd numbered page is missing, then so is the even numbered page directly following it.
4. If an even numbered page is missing, then so is the odd numbered page directly preceding it.

Solution

1. Page 6 is missing, therefore so is page 5.
2. Page 39 is missing, therefore so is page 40.
3. Only 4 total pages, and therefore one sheet, are missing, and they have all already been accounted for, therefore pages 5,6,39, and 40 comprise a single sheet.
4. There are 4 pages, and therefore 2 sheets, between the missing sheet and the first unnumbered sheet or the end of the book.
5. There are 32 pages in between page 6 and page 39, therefore there are 32/4=8 sheets above the missing sheet.
6. In total, there are therefore 2+8+1=11 sheets in the book, including the missing sheet, but excluding any unnumbered sheets like those containing the front and back cover.
7. There are therefore 11 × 4=44 numbered pages in total.

2. Napoleon Crossing the Alps

6 hours to go uphill (2 hours to go downhill).

Let the uphill speed be s, so then the downhill speed is 2s.
Total time taken = 8 hours = (6/s) + (4/2s).
Solving gives s = 1, so it takes 6 hours to cover the 6km distance.

Alternative solution

Let the time to go uphill be x and the time to go downhill be y, therefore, x+y=8. The uphill speed is 6/x and the downhill speed is 4/y, therefore 4/y=2 × 6/x. Solving for x, we get **x=6**.

3. Composition with Red, Blue & Yellow

Looking at the area **not** painted:

After red, 8/9 remains unpainted with one red square
After yellow, $8/9 \times 8/9$ remains unpainted with 8 yellow squares
After blue, $8/9 \times 8/9 \times 8/9 = (8/9)^3$ remains unpainted with 8x8 blue squares
After "green", $(8/9)^4 \approx (0.624)$ remain unpainted, with 8^3 'green' squares
After "pink", $(8/9)^5 \approx (0.555)$ remain unpainted, with 8^4 'pink' squares
After "orange", $(8/9)^6 \approx (0.493)$ remain unpainted with 8^5 'orange' squares

Looking at the area **that is** painted:

After red, 1/9 is painted with one red square
After yellow, $1/9 + 8/9^2$ is painted with 8 yellow squares
After blue, $1/9 + 8/9^2 + 8^2/9^3$ is painted with 8x8 blue squares
After "green", $1/9 + 8/9^2 + 8^2/9^3 + 8^3/9^4 \approx (0.376)$ is painted, with 8^3 'green' squares
After "pink", $1/9 + 8/9^2 + 8^2/9^3 + 8^3/9^4 + 8^4/9^5 \approx (0.445)$ is painted, with 8^4 'pink' squares
After "orange", $1/9 + 8/9^2 + 8^2/9^3 + 8^3/9^4 + 8^4/9^5 + 8^5/9^6 \approx (0.507)$ is painted with 8^5 'orange' squares

NB "colours" are their choice.

So **6 colours** are required.
There will then be $1+8+8^2+8^3+8^4+8^5$ = **37449 coloured squares**

4. The Potato Eaters

Add all the weights together and divide by two to get the weight of A+B+C = 600g.

Now since A+B=300, C must weigh 300. A+C=500, so B = 100 and B+C=400, so A=200.

So he can buy either **A and B** or **A and C**.

5. When Did You Last See Your Father?

City	Date
London	Monday 1st February
Cardiff	Monday 8th February
Edinburgh	Monday 1st March
Liverpool	Monday 8th March

If the first Monday of the month is not after the first Sunday of the month, it must be the 1st of the month.

This means the first Monday after the first Sunday must be the 8th.

For this to be the case 2 months in a row, the first month must be February and the second must be March.

This could not happen in a leap year, as 1st March would be a different day of the week to the 1st February.

6. Mona Lisa

She is correct 5 times out of 8 times that she complains. So the inferred likelihood for being correct is **5/8=0.625**

Of the 100 drinks bought, she complained about 5 of the 6 drinks that were flat, and did not complain about 91 of the 94 drinks that were not flat. Therefore, **96% of the drinks** are correctly identified.

Since she only missed 1 flat drink, this means that she is very effective at telling whether or not a drink is flat, **so she would be good at the quality-control job,** assuming that the important thing is that she catches most of the flat drinks.

7. Arrival of the Normandy Train

It cannot be 06xx or 07xx, because of Marie's certainty that Jean doesn't know the departure time, and both those hours contain a minute that occurs only in that hour (0620 and 0746), which would otherwise give Jean a chance of knowing already.

That leaves us with 08xx and 09xx. The fact that Jean now knows the time tells us that it's not 0825 or 0925, as having been told xx25 would be insufficient information for her.

Since this gives Marie what she needs, then it can only be 08xx that she was told, because she'd still have a choice of two 09xx times and then would not know which of those it was.

Therefore, Claude is on the 0839 train.

8. The Last Supper

Case 1: Jo sits on Chris's left side.

Consider A, B and D as one 'block', because they must sit adjacent to one another. Their block can be seated in 3 positions and their order within the block doesn't matter, so there are 3! = 6 ways of arranging them for each seating position, giving 18 possible arrangements.

E, F and G must sit to Chris's right side. Six seats are vacant, so they can be arranged in 6 × 5 × 4 = 120 ways.

Finally, the remaining 5 adults don't care where they sit, so can be arranged in 5! = 120 ways.

Thus, with Jo on Chris's left, there are 18 × 120 × 120 = 259, 200 possibilities.

Case 2: Jo sits on Chris's right side.

Again, consider A, B and D as one 'block'. Their block can be seated in 4 positions and their order within the block doesn't matter, so there are 3! = 6 ways of arranging them for each seating position, giving 24 possible arrangements.

E, F and G must sit to Chris's right side. Five seats are vacant, so they can be arranged in 5 × 4 × 3 = 60 ways.

Finally, the remaining 5 adults don't care where they sit, so can be arranged in 5! = 120 ways.

Thus, with Jo on Chris's right, there are 24 × 60 × 120 = 172,800 possibilities.

Adding these together, we find 432,000 ways to arrange the adults.