Cosmic censorship and the collapse of a scalar field in cylindrical symmetry

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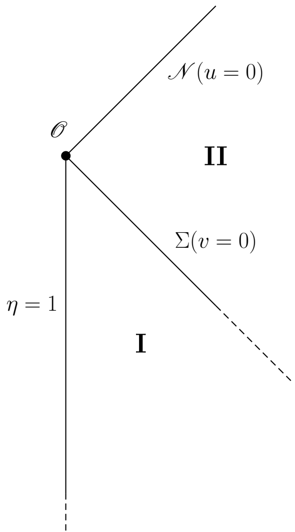
Outline

- The model
- Summary of solutions to the past of the past null cone of the origin Σ
- Field equations
- Solutions to the future of the past null cone of the origin

Conformal diagram of the spacetime



The Model

- The line element for cylindrical symmetry with self-similarity in double null coords with $\eta = v/u$

$$ds^2 = -|u|^{-1} e^{2\gamma(\eta)+2\phi(\eta)} dudv + |u| e^{2\phi(\eta)} S^2(\eta) d\theta + |u| e^{-2\phi(\eta)} dz^2. \quad (1)$$

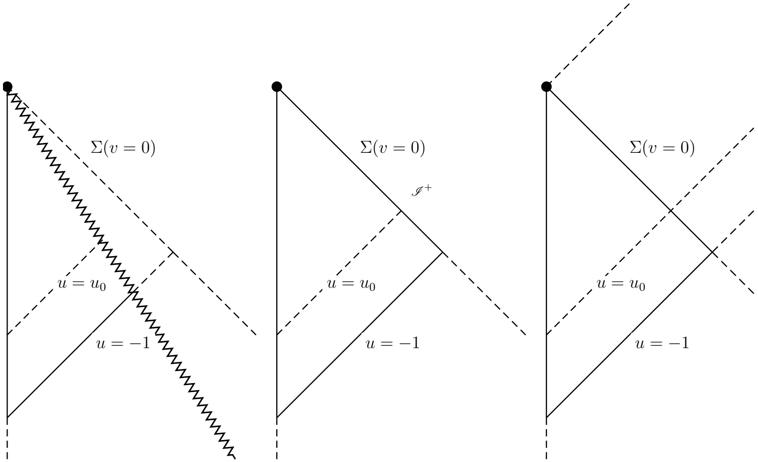
- The energy-momentum tensor for self-interacting scalar field:

$$T_{ab} = \nabla_a \psi \nabla_b \psi - \frac{1}{2} g_{ab} \nabla^c \psi \nabla_c \psi - g_{ab} V(\psi) \quad (2)$$

$$\psi = F(\eta) + \frac{k}{2} \ln |u| \qquad V(\psi) = V_0 e^{-\frac{2}{k}\psi}$$

- Different values of V_0 and k represent different matter models
- Regular axis provides initial data surface

Summary of solutions in region I



$V_0 > 0, k^2 \geq 2$ $V_0 < 0, k^2 \geq 2$ $k^2 < 2$

Field equations in region **II** for $V_0 < 0$

Field equations for $V_0 < 0$

$$x_1'(t) = x_1 - x_2 - x_1^2 \tag{3a}$$

$$x_2'(t) = Lx_2\left(\frac{1}{2} - x_3\right) \tag{3b}$$

$$x_3'(t) = \frac{1}{2}x_3 + \frac{1}{2}x_1 - \frac{2}{k^2}x_2 - x_1x_3 \tag{3c}$$

$$\lim_{t\to-\infty} e^{-Lt/2}(x_1, x_2, x_3) = (a_1, b_1, c_1) \tag{3d}$$

- L, a_1, b_1, c_1 depend on k^2
- Σ is a singular point, and fixed point, of the equations

Three possible outcomes

Case I

$\lim_{t\to\infty} \vec{x} = (1, 0, 1)$

- Corresponds to naked singularity

Case II

$\lim_{t\to\infty} \vec{x} = \left(1/2 - \sqrt{|\lambda|}/2, k^2/8, 1/2\right)$

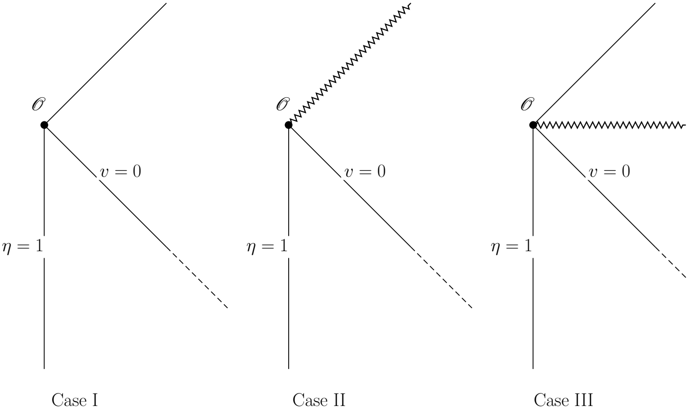
- Corresponds to censored singularity

Case III

$|\vec{x}| \to \infty$ in finite time

- Corresponds to censored singularity

The three pictures



Some results

Lemma 1

Suppose there exists $t_1 > -\infty$ *such that* $x_1(t_1) = 0$. *Then there exists* $t_2 > t_1$ *such that* $\lim_{t\to t_2} x_1 = -\infty$.

Lemma 2

Suppose there exists t_1 *such that* $x_1(t_1), x_3(t_1) < 1/2$ *and* $x'(t_1), x_3'(t_1) < 0$. *Then there exists* $t_2 > t_1$ *such that* $x_1(t_2) = 0$.

...

Lemma 7

Suppose that for $1 < k^2 < 4/3$ *there exists some* t_1 *such that*
$x_3'(t_1), x_3''(t_1), x_3'''(t_1), u_1'''(t_1) < 0$, *and* $Lx_3(t_1)/2 < x_1(t_1) < 2x_2(t_1)$,
$x_1'(t_1) < 2x_2'(t_1)$, $x_1''(t_1) < 2x_2''(t_1)$.
Then there exists some time $t_2 > t_1$ *such that* $x_1(t_2) = 0$.

Formal series solutions

$$x_1 = \sum_{n=1}^{\infty} a_n e^{nLt/2} \qquad x_2 = \sum_{n=1}^{\infty} b_n e^{nLt/2} \qquad x_3 = \sum_{n=1}^{\infty} c_n e^{nLt/2} \tag{4}$$

Recurrence relations for coefficients

$$a_n = \left(\frac{nL}{2} - 1\right)^{-1} \left(-b_n - \sum_{j=1}^{n-1} a_j a_{n-j}\right), \tag{5a}$$

$$b_n = -\frac{2}{n-1} \sum_{j=1}^{n-1} b_j c_{n-j}, \tag{5b}$$

$$c_n = \left(\frac{nL}{2} - \frac{1}{2}\right)^{-1} \left(\frac{a_n}{2} - \frac{2b_n}{k^2} - \sum_{j=1}^{n-1} a_j c_{n-j}\right). \tag{5c}$$

Field equations in region II for $V_0 < 0$

Field equations for $V_0 < 0$

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Theorem 8

Suppose that $|a_j|, |b_j|, |c_j| \leq \kappa\delta^{j-1}/j^2$ *for all* $j < n_*$, *for some* n_*, κ, δ. *Suppose further that*

$$n_* > \frac{2}{L}\left(1 + \frac{2}{k^2} + \frac{\kappa}{\delta} s_{n_*}\right), \tag{7}$$

where

$$s_{n_*} = 2H_{2,n_*-1} + \frac{4}{n_*} H_{1,n_*-1}. \tag{8}$$

Then $|a_{n_*}|, |b_{n_*}|, |c_{n_*}| \leq \kappa\delta^{n_*-1}/n_*^2$.

Corollary 9

If theorem 8 holds for some κ, δ, n_*, *then it holds for* κ, δ *and all* $n > n_*$.

Uniform convergence & truncation error

Theorem 10

Let $\tau = e^{Lt/2}$. Suppose that there exist κ, δ such that $|a_n|, |b_n|, |c_n| \leq \kappa\delta^{n-1}/n^2$ for all $n \geq 1$.
Then the series (4) converge uniformly on $\tau \in [0, 1/\delta)$.

Lemma 11

Suppose that there exist κ, δ such that $|a_n|, |b_n|, |c_n| \leq \kappa\delta^{n-1}/n^2$ for all $n \geq 1$, and that the series (4) are approximated by the first N terms. Then the truncation errors e_i for these approximations satisfy the bound

$$e_i \leq \frac{\kappa}{N^2\delta}\left(\frac{(\delta\tau)^{N+1}}{1-\delta\tau}\right). \tag{9}$$

Numerical data

Let

$$m = \max_{j\in[1,N]} \{j^2(|a_j|, b_j|, |c_j|)/\kappa\delta^{j-1}\} \qquad (10)$$

k^2	δ	κ	N	m	τ_1	$\bar{x}_1(\tau_1)$	$e(\tau_1) <$
0.01	10^6	10^5	412	0.796	9×10^{-7}	-2.4×10^{-5}	10^{-24}
0.02	10^5	$25,000$	224	0.792	9×10^{-6}	-3.04×10^{-4}	10^{-14}
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0.32	90	80	80	0.846	0.008	-0.0527	10^{-13}
0.33	80	80	80	0.976	0.01	-0.0832	10^{-9}
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Thanks for listening.
Any questions?