Shattering of the Neutron Star Crust

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see C. Gundlach, I. Hawke, and SJE, CQG **29** 015055 (2012)

The Physics Problem

- NS crust is only small fraction of total mass
- BUT, crustal modes are qualitatively different
- AND, crustal modes are at much lower frequencies
- Several models have shown that the crust can contribute to observable behavior, (ie. Pulsar glitches, Ruderman 1969 and GRB's, Blaes et al. 1989)

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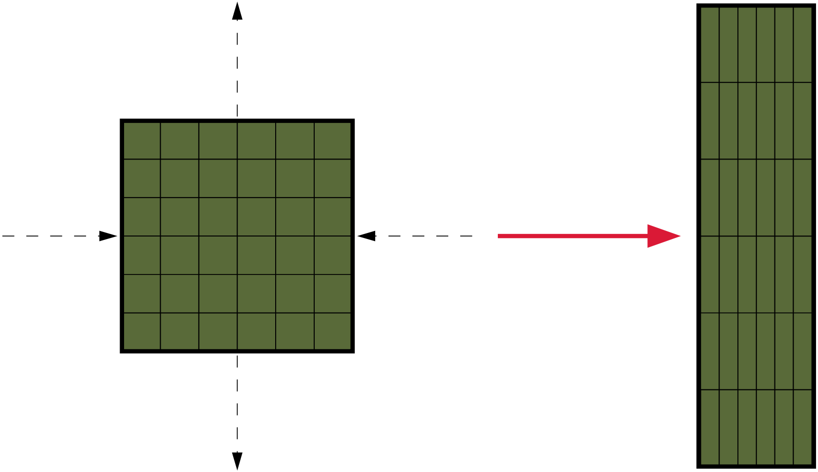
How does shattering due to tidal forcing affect the NS and the merger system as a whole?

Plan

What do we need to do the simulation?

- Elasticity
- Interfaces
- Shattering

Elasticity: Relationship with relaxed state



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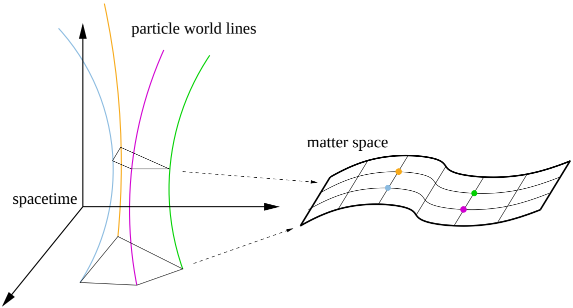
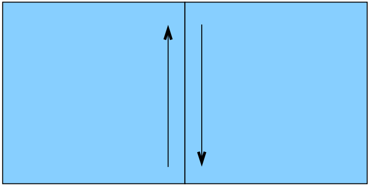
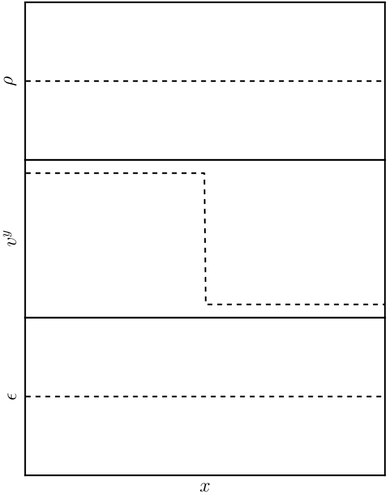


- Use two manifolds and map between them (Carter and Quintana, 1972)
- Derivatives of map: *configuration gradient*
- Use commutation of partial derivatives to write evolution equation and constraint for configuration gradient

Elasticity: Shear Stresses

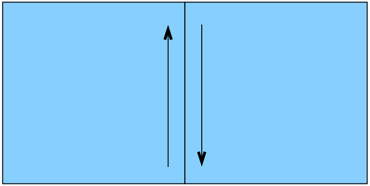
Perfect Fluid

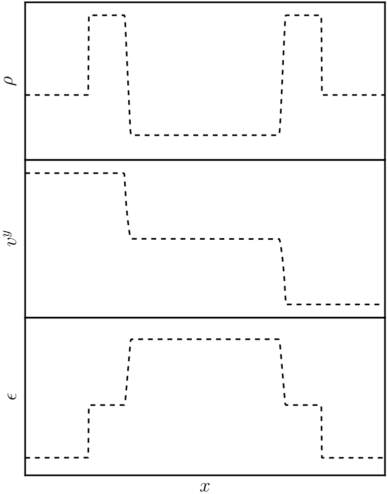




Elasticity: Shear Stresses

Elastic Solid





Elasticity: Shear Stresses

- Need to include shear stresses
- Add anisotropic stress term to stress-energy tensor (Karvolini and Samuelsson, 2003)
- More general, but still no heat flow

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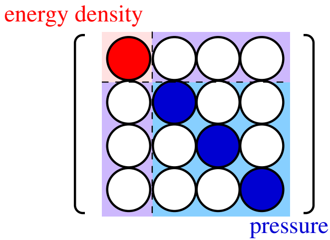

$$T^{ab} = (e + p)u^a u^b + pg^{ab}$$

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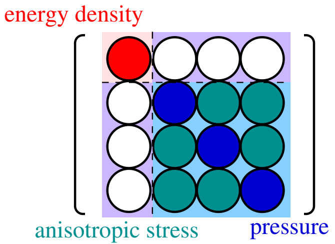
Elastic Material



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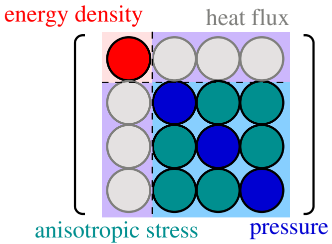
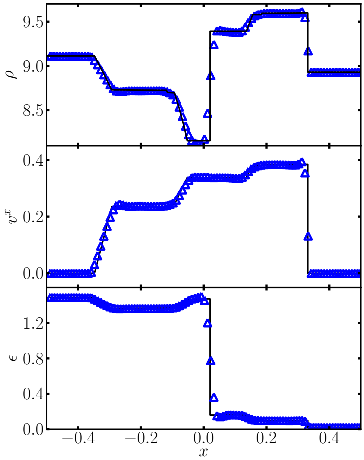
General



Elasticity Code

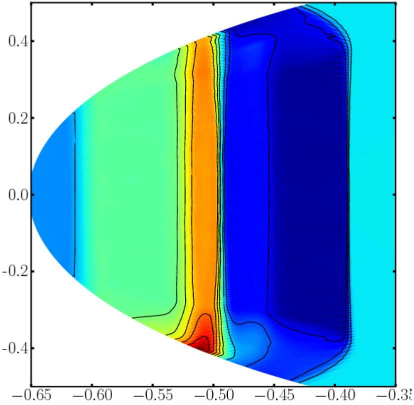
- 3D variables on 1D or 2D grid with planar symmetry
- Cartesian or cylindrical Minkowski metric
- Newtonian version of code from $v \ll c$
- Test using Riemann problems

Newtonian Elasticity Results



- Can reproduce published Newtonian exact Riemann solutions (Barton et al, 2009)
- Relativistic code results approach Newtonian results in Newtonian limit

2D Cylindrical Coordinates



Formalism works for curved coordinates

- Riemann test in 2D cylindrical coordinates
- Ring rotor in 2D cylindrical coordinates

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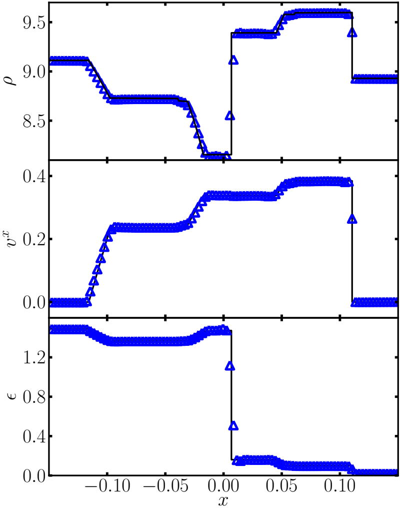

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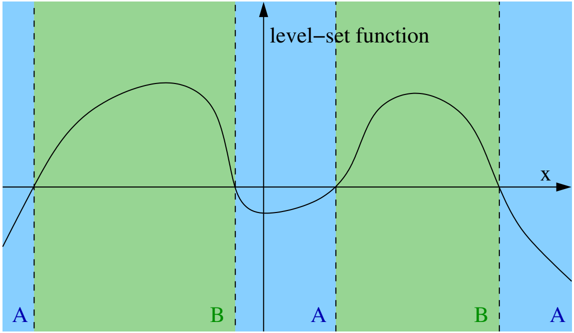
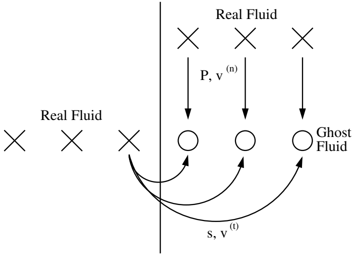
Interfaces: Where is the interface?



- Use a *level-set function* to track the interface
- Positive in cells filled by one material, negative for other material, zero at interface
- Advected along with material

Interfaces: What happens at the interface?



Fedkiw et al, 1999—Ghost fluid method (GFM):

- Continuous across contact: p, $v^{(n)}$
- Discontinuous across contact: s, $v^{(t)}$
- Calculate ρ from s and p

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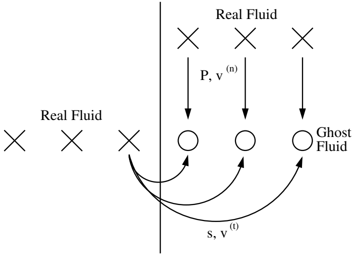

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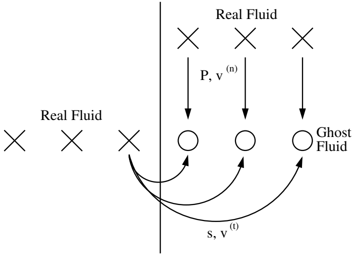

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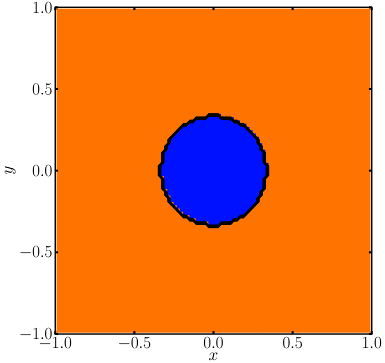
Progress: Fluid interfaces in 1D – can reproduce results of published Newtonian and special relativistic tests

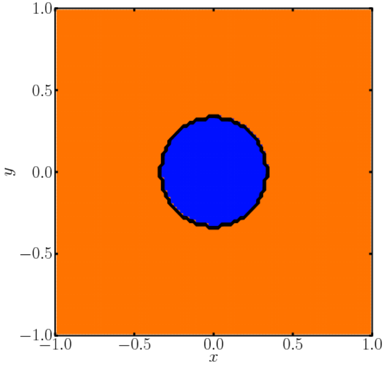
Shattering

- *Shatter* = instantaneous relaxation
- Relaxed state occurs when matter-space metric is proportional to spacetime metric pushed forward onto matter space
- SO, to shatter, reset variables to relaxed state (matter-space metric or configuration gradient)



Shattering

- 2D homogeneous anisotropic initial data, then shatter a circular region at the center
- Encountered no numerical problems



Conclusions

- **Want** to find out what happens when part of the crust shatters due to tidal forcing in a binary merger system
- **Need** elasticity in GR, interfaces, and shattering
- **Have** elasticity code, shattering with no problems so far, and interfaces for fluids
- **Still to come:** solid-fluid interface, combine components