A Space-Time Foam Contribution to the Universe's Initial Perturbations

Michael L. Fil'chenkov

Yuri P. Laptev

Institute of Gravitation and Cosmology,
Peoples' Friendship University of Russia, Moscow

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Introduction

Fluctuations at the early Universe's de Sitter stage give rise to initial perturbations being observed via cosmic microwave background (CMB) temperature anisotropy. The latter was shown to depend on the potential barrier in the tunneling approach for the Universe's birth from nothing [1]. Loop quantum cosmology (LQC) corrections to the pre-de-Sitter potential energy have proved to be of importance for the initial density perturbations. Thus, the space-time foam described by LQC should be taken into account, while estimating the initial density perturbations in the early Universe.

[1] I. Dymnikova, M. Fil'chenkov. Quantum birth of a hot universe, Phys. Lett. B 545 (2002) 214-220.

Loop Quantum Cosmology

In the eighties quantum geometrodynamics was generalized to quantizing space itself. This approach is called quantum geometry or loop quantum gravity. Ashtekar variables have been introduced as follows

$$\hat{E}^a \psi = i \frac{\delta \psi}{\delta A_a}, \tag{1}$$

where $\hat{E}^a$ is the operator connected with a triad, A_a the gauge field and Wilson's loops $\gamma = \oint A_a d\gamma^a$ to which the operators $\hat{\gamma}$ acting on a vacuum correspond. They generate a so-called spin network forming space. The geometrical quantities (volumes and areas) are operators.

Their eigenvalues are quantized. For the area we have

$$S_j = 8\pi l_{pl}^2 \sum_i \sqrt{j_i\left(j_i+1\right)}, \qquad (2)$$

where i are integers, j halfintegers.

In the framework of this approach Hawking's temperature has been evaluated. In the 21st century loop quantum cosmology has been elaborated to succeed in eliminating singularities.

A LQC factor correcting the inverse volume is given by the formula

$$D_j(q) = \left(\frac{8}{77}\right)^6 q^{3/2}\{7[(q+1)^{11/4} - |q-1|^{11/4}] - 11q[(q+1)^{7/4} - \mathrm{sgn}(q-1)|q-1|^{7/4}]\}^6, \qquad (3)$$

where $q = \left(\frac{a}{a_*}\right)^2$, $a_* = \sqrt{\frac{\gamma j}{3}} l_{pl}$, $\gamma = \frac{\ln 2}{\pi\sqrt{3}}$. $D_j = 1$ for $a \gg a_*$ and

$$D_j = \left(\frac{12}{7}\right)^6 \left(\frac{a}{a_*}\right)^{15} \text{ for } a \ll a_*. \qquad (4)$$

Quantum Pre-de-Sitter Universe

In quantum geometrodynamics (QGD) Friedmann's quantum Universe is describable in terms of Wheeler-DeWitt's minisuperspace model:

$$\frac{d^2\psi}{da^2}-\frac{1}{l_{pl}^4}\left(ka^2-\frac{8\pi G\varepsilon a^4}{3c^4}\right)\psi=0, \qquad (5)$$

where a is the scale factor, $k=0,\pm1$ model parameter.

Consider a multicomponent medium:

$$\varepsilon=\varepsilon_0\sum_n B_n\left(\frac{r_0}{a}\right)^n, \qquad (6)$$

where ε_0 is de Sitter's vacuum energy density, $n=3(1+w)$, $\sum_n B_n=1$,

B_n is a contribution of the n-th component with equation of state $p = w\varepsilon$ to the total energy density on de Sitter's horizon r_0. Restrict ourselves to de Sitter's vacuum $(n=0)$, strings $(n=2)$ and radiation $(n=4)$.

Wheeler – DeWitt's equation is reducible to Schrödinger's type equation

$$\frac{d^2\psi}{da^2} - \frac{2m_{pl}}{\hbar^2}\left[U(a) - E\right]\psi = 0, \qquad (7)$$

where

$$U(a) = \frac{m_{pl}c^2}{2l_{pl}^2}\left[\left(k - B_2\right)a^2 - \frac{B_0 a^4}{r_0^2}\right] \qquad (8)$$

is the potential energy and

$$E = \frac{m_{pl} c^2}{2} \left(\frac{r_0}{l_{pl}} \right)^2 B_4 \qquad (9)$$

is the total energy being quantized near the potential energy minimum:

$$E = \hbar \omega_{QDG} \left(n + \frac{1}{2} \right), \qquad (10)$$

where $n = 0, 1, 2, \ldots,$

$$\omega_{QGD} = \frac{m_{pl} c^2}{\hbar} \sqrt{k - B_2} \qquad (11)$$

is the oscillator frequency in terms of QGD.

The energy levels have a natural width

$$\Gamma = \frac{2\alpha_{GUT}\hbar\omega_{QGD}^2 n}{m_{pl}c^2}, \qquad (12)$$

where $\alpha_{GUT} = \frac{1}{40}$ is the GUT constant.

Hence for $n \gg 1$

$$\frac{\Delta E}{E} = \frac{2\alpha_{GUT}\hbar\omega_{QGD}}{m_{pl}c^2}. \qquad (13)$$

Loop Quantum Cosmology Corrections to Wheeler – DeWitt's equation

A spatial discreteness in LQC is taken into account multiplying the coefficients B_n by a factor $\left(\frac{n}{6D_j}+\frac{6-n}{6}\right)$.

The potential and total energies respectively take the form:

$$U(a)=\frac{m_{pl}c^2}{2l_{pl}^2}\left[\left(k-\frac{2}{3}B_2\right)a^2-\frac{B_0a^4}{r_0^2}-\frac{B_2a^2}{3D_j}-\frac{2B_4r_0^2}{3D_j}\right], \qquad (14)$$

$$E = \frac{m_{pl} c^2}{2} \left(\frac{r_0}{l_{pl}} \right)^2 B_4 . \tag{15}$$

Due to D_j the potential energy $U(a)$ acquires an additional maximum for small a, playing the role of a wall removing the cosmological singularity. $D_j = 1$ for $a \gg a_*$, and quantum geometry transforms to quantum geometrodynamics.

Thus, the potential energy $U(a)$ has two roots, two maxima and a minimum between them.

The roots, i.e. $U(a) = 0$, at $a_0 = r_0 \sqrt{-\frac{2B_4}{B_2}}$ and $a_1 = r_0 \sqrt{\frac{k - B_2}{B_0}}$.

The maxima:

$$U(a_{\max}) = \frac{2}{39}\left(\frac{7}{12}\right)^6 \left(\frac{r_0}{l_{pl}}\right)^2 \left(\frac{a_*}{a_{\max}}\right)^{15} B_4 m_{pl} c^2 \quad \text{at } a_{\max} = r_0\sqrt{-\frac{30B_4}{13B_2}}; \tag{16}$$

$$U(a_m) = \frac{m_{pl}c^2}{8}\left(\frac{r_0}{l_{pl}}\right)^2 \frac{\left(k - B_2\right)^2}{B_0} \qquad \text{at } a_m = r_0\sqrt{\frac{k-B_2}{2B_0}} \tag{17}$$

The minimum

$$U(a_{\min}) = -\frac{B_2}{6}\left(\frac{7}{12}\right)^6 \left(\frac{a_*}{a_{\min}}\right)^{15} \left(\frac{a_{\min}}{l_{pl}}\right)^2 m_{pl} c^2 \text{ at } a_{\min} = \left[\frac{13}{4}\left(\frac{7}{16}\right)^6\right]^{\frac{1}{15}} a_*. \tag{18}$$

Expanding the potential energy near the minimum, we obtain the oscillator frequency in terms of LQC

$$\omega_{LQC} = \sqrt{-10B_2}\,\frac{m_{pl}c^2}{\hbar} \tag{19}$$

for $k = 0$, $B_2 < 0$.

$$\frac{\Delta E}{E} = \frac{2\alpha_{GUT}\hbar\omega_{LQC}}{m_{pl}c^2}. \tag{20}$$

CMB Temperature Fluctuations as a Test for Quantum Cosmology Models

According to Silk's effect CMB temperature fluctuations

$$\frac{\Delta T}{T} = \frac{1}{3}\frac{\Delta \rho}{\rho}, \qquad (21)$$

where ρ is an initial matter density in the early Universe.

As follows from (9) E is related to the contribution of radiation B_4 to the total energy density.

$$\frac{\Delta E}{E} = \frac{\Delta T}{T}. \tag{22}$$

Hence

$$\frac{\Delta\rho}{\rho} = \frac{6\alpha_{GUT}\hbar\omega}{m_{pl}c^2}, \tag{23}$$

where ω is the oscillator frequency. Thus, considering $\omega = \omega_{QGD}$ for minisuperspace space and $\omega = \omega_{LQC}$ for discrete space, we can estimate the contribution of spatial discreteness to matter density perturbations.

Calculate the ratio

$$\frac{(\Delta\rho/\rho)_{LQC}}{(\Delta\rho/\rho)_{QGD}} = \frac{\omega_{LQG}}{\omega_{QGD}} = \sqrt{10}\,. \tag{24}$$

Thus the spatial discreteness should be taken into account, while estimating the initial density perturbations in the early Universe.

Conclusion

According to C. Rovelli, a spinfoam represents the evolution of a spin network. Because of their foamy structure at the Planck scale, spinfoams can be viewed as a mathematically precise realization of Wheeler's space-time foam. The latter may be of importance for an analysis of the initial density perturbations in the early Universe.