Superfluid Instability of r-Modes in Differentially Rotating Neutron Stars

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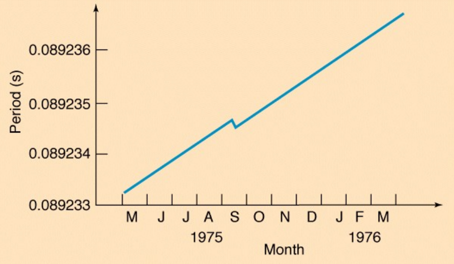
Outline

1. Introduction and Motivation
2. Possible Causes of Neutron Star Glitches
3. Two Stream Instabilities
4. Differential Rotation and r-Modes Instabilities
5. Conclusions and the Future

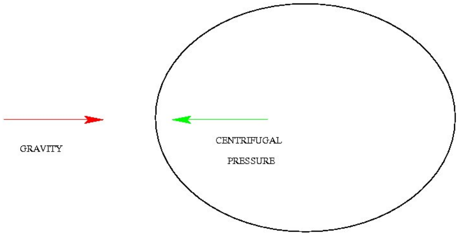
Neutron Star Glitches

- New born neutron stars spin very rapidly.
- Generally they are slowed monotonically, predominantly by magnetic braking.
- Periodically they ‘glitch’; that is they increase their rotation rate very rapidly.
- What causes this?



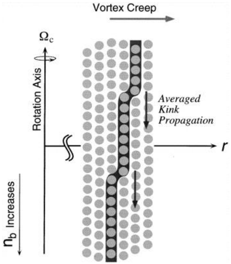
Possible Causes of Glitches - Starquakes

- Young neutrons stars are oblate, partially maintained by pressure from rotation.
- Crust is frozen into shape.
- As they slow, the pressure reduces.
- Under gravity the crust cracks and reforms in a less oblate shape.
- This reduces moment of inertia and increases spin rate.



Possible Causes of Glitches - Vortex Unpinning

- Rotation causes quantised vortices in neutron superfluid in core, extending to that in crust.
- Pinned to crust via superfluid neutrons in inner crust.
- When pinned, vortices angular momentum undiminished, so acts as reservoir as rest of the star slows.
- If vortices become unpinned, angular momentum is rapidly transferred to rest of star until they repin.
- A trigger mechanism for this unpinning?



The Two-Stream Instability

- 2004 paper. Comer, Andersson and Prix.
- Described how in a linear model with two component superfluid, there exists a critical velocity above which the flow becomes prone to instabilities.
- Somewhat analogous to Kelvin-Helmholtz instability.
- Generic to all multi-component superfluid systems.

Differential Rotation of Superfluid Core

- Can we use variant of two-stream instability as a trigger mechanism for unpinning?
- On a large scale the vortex array appears as bulk rotation.
- Assume protons in core are slowed more than neutrons - differential rotation.
- So we have two rotating fluids with differing angular velocities.
- We assume solid rotation for mathematical simplicity.

Forming the Problem - 1

- Consider rotating star with observed angular velocity Ω^i.
- Assume that proton component of core rotates at same rate as crust but neutron components is slowed less. So

$$\Omega_p^i = \Omega^i$$

$$\Omega_n^i = (1 + \Delta)\,\Omega^i$$

- Δ is small and positive.

Forming the Problem - 2

- We also define a relative velocity between the two components

$$w^i_{\mathrm{pn}} = v^i_{\mathrm{p}} - v^i_{\mathrm{n}}$$

where

$$v^i_{\mathrm{x}} = \Omega_{\mathrm{x}} \hat{e}^i_{\varphi}$$

- Assume incompressible fluids

$$\nabla_i \delta v^i_{\mathrm{x}} = 0$$

- Assume harmonic perturbations $\sim \exp(i\omega t)$

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Forming the Problem - 3

- We assume the perturbed momenta of the two components satisfy

$$\delta p_i^{\mathrm{x}} = \delta v_i^{\mathrm{x}} + \varepsilon_{\mathrm{x}} \delta w_i^{\mathrm{yx}}$$

where ε_{x} represents the entrainment.

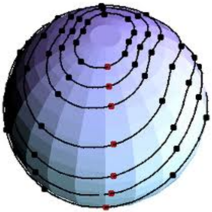
- Then they are governed by the Euler equations

$$\mathcal{E}_i^{\mathrm{x}} = \delta v_{\mathrm{x}}^j \nabla_j p_i^{\mathrm{x}} + \underbrace{i\omega \delta p_i^{\mathrm{x}} + v_{\mathrm{x}}^j \nabla_j \delta p_i^{\mathrm{x}}}_{\text{perturbed momentum}}$$
$$+ \underbrace{\varepsilon_{\mathrm{x}} (\delta w_j^{\mathrm{yx}} \nabla_i v_{\mathrm{x}}^j + w_j^{\mathrm{yx}} \nabla_i \delta v_{\mathrm{x}}^j)}_{\text{entrainment}} + \nabla_i \delta \Psi_{\mathrm{x}} = \delta f_i^{\mathrm{x}}$$

$$\Psi_{\mathrm{x}} = \Phi + \mu_{\mathrm{x}} \tag{5.1}$$

The r-Modes - 1

- Do not consider all possible modes.
- Restrict ourselves to consideration of purely the r-modes.
- These modes are associated with simple velocity fields of the type our model employs.



The r-Modes - 2

- This leads to perturbed velocities of the form

$$\delta v_{\mathrm{x}}^{i} = -\frac{im}{r^2 \sin\theta} U_{\mathrm{x}}^{l} Y_l^m \hat{e}_\theta^i + \frac{1}{r^2 \sin\theta} U_{\mathrm{x}}^{l} \partial_\theta Y_l^m \hat{e}_\varphi^i .$$

- $Y_l^m(\theta, \varphi)$ are the standard spherical harmonics and U_{x}^{l} are the mode velocities.
- We find it convenient to work in sum and difference of the two fluid perturbation velocities.

$$\rho U^l = \rho_{\mathrm{n}} U_{\mathrm{n}}^l + \rho_{\mathrm{p}} U_{\mathrm{p}}^l$$

$$u^l = U_{\mathrm{p}}^l - U_{\mathrm{n}}^l$$

The r-Modes - 3

- A lot of algebra later, we arrive at two relatively simple equations for the amplitude relations.

$$
\begin{aligned}
&[(m+1)\tilde{\kappa} - 2 + \Delta(1-x_{\rm p})(m-1)(m+2)]U^m \\
&-(1-x_{\rm p}-\varepsilon_{\rm p})\Delta x_{\rm p}(m-1)(m+2)u^m = 0
\end{aligned}
$$

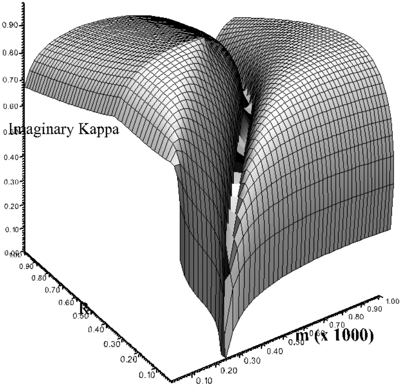
and

$$
\begin{aligned}
&-\left[(m-1)(m+2) + 2\bar{\varepsilon} - m(m+1)(\bar{\mathcal{B}}' + i\bar{\mathcal{B}})\right]\Delta U^m \\
&+\Big\{(1-\bar{\varepsilon})(m+1)\tilde{\kappa} - 2(1-\bar{\mathcal{B}}' + i\bar{\mathcal{B}}) + \Delta x_{\rm p}(m-1)(m+2) \\
&-\bar{\varepsilon}\Delta\left\{[m(m+1)-4]x_{\rm p} + 2\right\} - m(m+1)\Delta x_{\rm p}(1-\bar{\varepsilon})(\bar{\mathcal{B}}' + i\bar{\mathcal{B}}) \\
&+2(1-\varepsilon_{\rm n})(\bar{\mathcal{B}}' - i\bar{\mathcal{B}})\Delta\Big\}u^m = 0,
\end{aligned}
$$

where $\tilde{\kappa}$ is representative of the frequency, $x_{\rm p}$ is the proton fraction, $\bar{\varepsilon} = \varepsilon_{\rm n}/x_{\rm p}$ and $\bar{\mathcal{B}}'$ & $\bar{\mathcal{B}}$ depend on the resistive friction.

RESULTS

- The trench indicates change of sign in Im($\tilde{\kappa}$).
- To the right the sign is negative, indicating growing solutions.
- $\mathcal{R}$ is the resistive friction which governs $\bar{\mathcal{B}}'$ and $\bar{\mathcal{B}}$.



- We can see that there are unstable modes dependent on the resistive friction.

Results and Conclusions

- Are these modes suppressed by shear viscosity?
- High m modes grow more quickly so these are least likely to be suppressed.
- High m modes are local. Higher m, more local
- But shear viscosity grows at shorter scales.
- Competition between these two phenomena.
- There is some range of values at which instability wins.

Discussion

- Can such local modes trigger global unpinning?
- Two possibilities. Both speculation.
- They occur almost simultaneously throughout the star.
- Once unpinning starts, there is a cascade effect. Each unpinned group of vortices displaces an adjoining group.

Thank You