Fermionic spinfoam models and TQFTs

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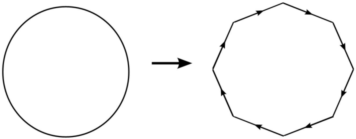

Problems

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- We take the point of view that matter and triangulation independence are crucial!

Induced actions

$$\mathbb{Z} = \int \mathcal{D}\psi \mathcal{D}\overline{\psi}\, e^{i \int \overline{\psi} \not{D} \psi} \qquad \not{D} = \gamma^{\mu}(d_{\mu} - iA_{\mu})$$

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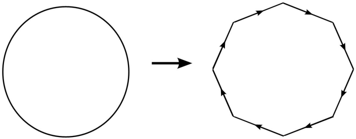
$$= \det(i\not{D}) = e^{\mathrm{t}r \ln i\not{D}}$$
$$= e^{iS_{eff}}$$

- John Barret has suggested that the Standard Model can be induced in this way: arXiv:1101.6078v2 [hep-th]

A one dimensional fermionic TQFT



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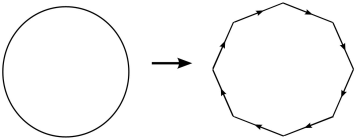

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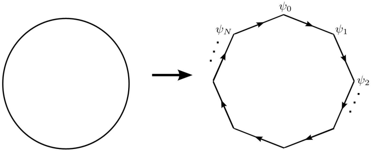

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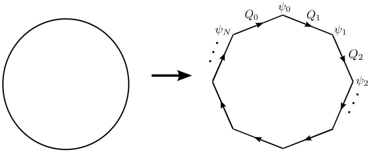

$$\psi(t), \bar{\psi}(t), t \in [0, 2\pi] \qquad \psi_i, \bar{\psi}_i, i = 1..N$$

$$A(t) \qquad Q_i = \mathcal{P} e^{i \int A dt}$$

A one dimensional fermionic TQFT

$$\hat{\mathbb{Z}} = \int \prod_{i=1}^{N} d\psi_i d\bar{\psi}_i \; e^{\sum_{i=1}^{N} \bar{\psi}_i(\psi_i - Q_{i+1}\psi_{i+1})}$$

$$\psi_{N+1} = \psi_1 \quad Q_{N+1} = Q_1$$

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- $\hat{\mathbb{Z}}$ is triangulation independent - a topological invariant!

Action

What is the significance of this theory? It is a discretisation of a one dimensional Dirac theory,

$$\hat{S} = -i \sum_{i=1}^{N} \bar{\psi}_i \left(\psi_i - Q_{i+1} \psi_{i+1} \right)$$
$$= i \Delta t \sum_{i=1}^{N} \bar{\psi}_i \left(\frac{Q_{i+1} \psi_{i+1} - \psi_i}{\Delta t} \right) \quad \Delta t = \frac{2\pi}{N}$$

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$$\lim_{\Delta t\to 0} i\left(\frac{Q_{i+1}\psi_{i+1}-\psi_i}{\Delta t}\right) = \not{D}_t\psi(t)$$

$$\lim_{\Delta t\to 0}\Delta t\sum_{i=1}^{N} = \int_0^{2\pi} dt$$

$$\lim_{\Delta t\to 0}\hat{S} = \int dt\overline{\psi}\not{D}\psi$$

Continuum theory

One can calculate the partition function of the continuum theory exactly

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Naturally, one would like to try do something similar in higher dimensions. This is the subject of current investigation.

Thanks!