Phase Transition and Anisotropic Deformations of Neutron Star Matter

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Based on a forthcoming paper with Bernard M. A. G. Piette, Durham University

Neutron Stars

- Large mass: 1-2 Solar masses.
- Small radius: 10-15km
- Modelling requires knowledge of:
  - General Relativity,
  - Dense neutron matter - Effective theory for QCD.
- In this talk we show how we can combine GR with the Skyrme model of neutron matter to produce a good neutron star model.

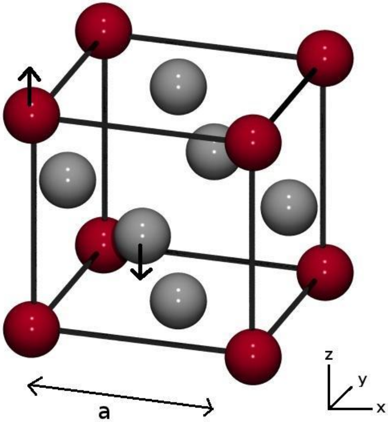
The Skyrme Model

The Skyrme Lagrangian

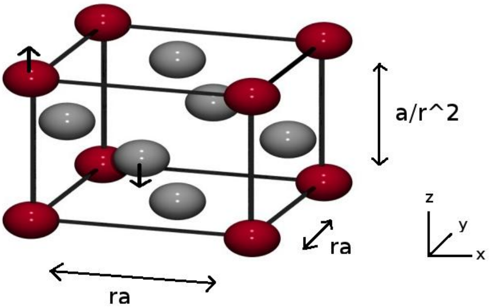
- $\frac{F_\pi^2}{16} Tr(\nabla_\mu U \nabla^\mu U^{-1}) + \frac{1}{32e^2} Tr[(\nabla_\mu U)U^{-1}, (\nabla_\nu U)U^{-1}]^2$

- The Skyrme field, $U(\mathbf{x}, t)$, is a $SU(2)$ valued scalar field.
- $U(\mathbf{x}) \rightarrow \mathbb{I}$ as $|\mathbf{x}| \rightarrow \infty$.
- $U : S^3 \mapsto S^3$ has homotopy group $\pi_3(S^3) = \mathbb{Z}$.
- Topological charge $\leftrightarrow$ baryon number.
- An approximate low energy effective field theory for QCD.
- Successful in modelling small nuclei.
- Model large astrophysical objects such as neutron stars with $B \approx 10^{57}$?

The Skyrme Crystal



The Skyrme Crystal



- Skyrmion size in the radial direction $\lambda_r \propto \frac{a}{r^2}$.
- Skyrmion size in the tangential direction $\lambda_t \propto ra$.

The Skyrme Crystal Energy Dependence

- $E = E(\lambda_r, \lambda_t)$

The TOV Equation

The Stress Tensor

- $T^{\mu}_{\nu} = \mathrm{diag}(\rho(r), p_r(r), p_\theta(r), p_\phi(r))$

Tangential Stresses

- $p_\theta(r) = p_\phi(r) = p_t(r)$

The Metric

- $ds^2 = e^{\nu(r)}dt^2 - e^{\lambda(r)}dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta d\phi^2$

The TOV Equation

- $\frac{dp_r}{dr} = -(\rho + p_r)\left(\frac{m(r)+4\pi r^3 p_r}{r(r-2m)}\right) + \frac{2}{r}(p_t - p_r)$

The Equations Of State

The Equations Of State

- $p_r = p_r(\rho)$ and $p_t = p_t(\rho)$

  - Neutron star temperature ≈ 0.1keV, experimental α-particle excitation energy ≈ 23.3MeV $\implies$ zero temperature assumption.

Calculating The Equations Of State

- $p_r = -\frac{1}{\lambda_t^2}\frac{\partial E}{\partial \lambda_r}$ and $p_t = -\frac{1}{\lambda_r}\frac{\partial E}{\partial \lambda_t^2}$

Calculating The Mass Density

- $\rho = \frac{E}{c^2\lambda_r\lambda_t^2}$

Physical Properties

Boundary Conditions

- $m(r) \to 0$ as $r \to 0 \implies p_t(0) = p_r(0)$

- Radius of the star, R at $p_r(R) = 0$.
- Exterior vacuum Schwarzschild metric can always be matched to our metric if $p_r(R) = 0$.

Total Gravitational Mass

- $M_G = m(R) = m(\infty) = \int_0^R 4\pi r^2 \rho(r) dr$

Stars Made of Isotropically Deformed Skyrme Crystal

Isotropic Skyrme Crystal

- $\lambda_t(r) = \lambda_r(r)$

- We find results up to a baryon number of 2.61×10^{57}, equivalent to 1.49 solar masses.

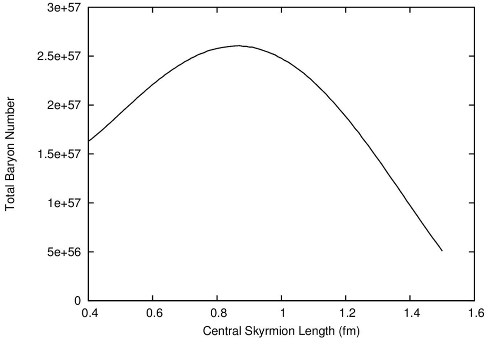
Theorem

- For a given total baryon number, if there is a locally isotropic, stable (minimum energy) solution to the generalised TOV equation with mass M, then all locally anisotropic solutions will have a mass greater than or equal to M.

Stars Made of Isotropically Deformed Skyrme Crystal

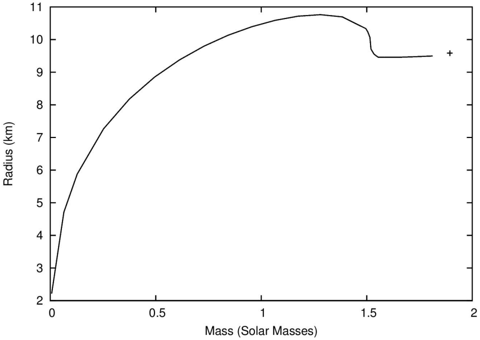
Central Skyrmion Length

- $\lambda_t(r = 0) = \lambda_r(r = 0) = L(r = 0)$



Stars Made of Ansotropically Deformed Skyrme Crystal

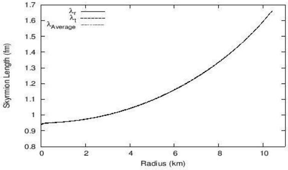
- We find results from energy minimisation up to a baryon number of 3.25×10^{57}, equivalent to 1.81 solar masses.
- The maximum mass solution is found at a baryon number of 3.41×10^{57}, equivalent to 1.90 solar masses.



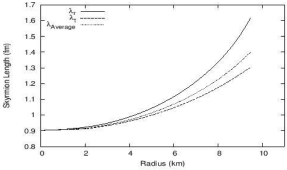
Stars Made of Ansotropically Deformed Skyrme Crystal

- Allowing anisotropically deformed Skyrme crystal solutions we have increased the maximum mass by 28% from the maximum mass found in the isotropic case.
- So we should not take isotropic deformation of matter as an assumption.
- The recent discovery of a 1.97 ± 0.04 solar mass neutron star, the highest neutron star mass ever determined, makes our result of a maximum mass of 1.90 solar masses very encouraging.
- Including the effects of rotation into our model will increase the maximum mass found, by up to 2% for a star with a typical spin period.

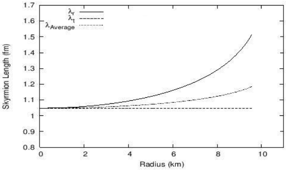
Neutron Star Configurations



(a) Isotropic Skyrme Crystal: $B = 2.60 \times 10^{57}$, $M = 1.49$ solar masses.



(b) Anisotropic Skyrme Crystal: $B = 3.00 \times 10^{57}$, $M = 1.69$ solar masses.



(c) Maximum Mass Anisotropic Skyrme Crystal: $B = 3.41 \times 10^{57}$, $M = 1.90$ solar masses.

Conclusions

- The Skyrme model is an approximate low energy effective field theory for QCD.
- We have used a Skyrme crystal to construct neutron star configurations.
- We have found masses up to 1.90 solar masses, and found appropriate radii.
- There is a phase transition between stars composed of isotropically and anisotropically deformed matter at a critical mass of 1.49 solar masses.
- By allowing anisotropically deformed Skyrme crystal configurations the maximum mass is 28% more than the maximum mass in the isotropically deformed case.