An efficient iterative method to reduce eccentricity in numerical-relativity simulations of compact binary inspiral

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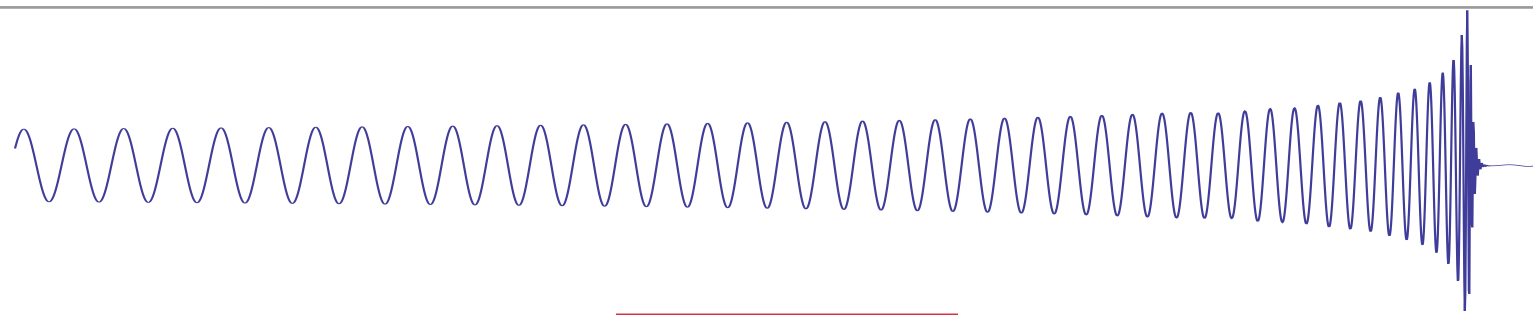

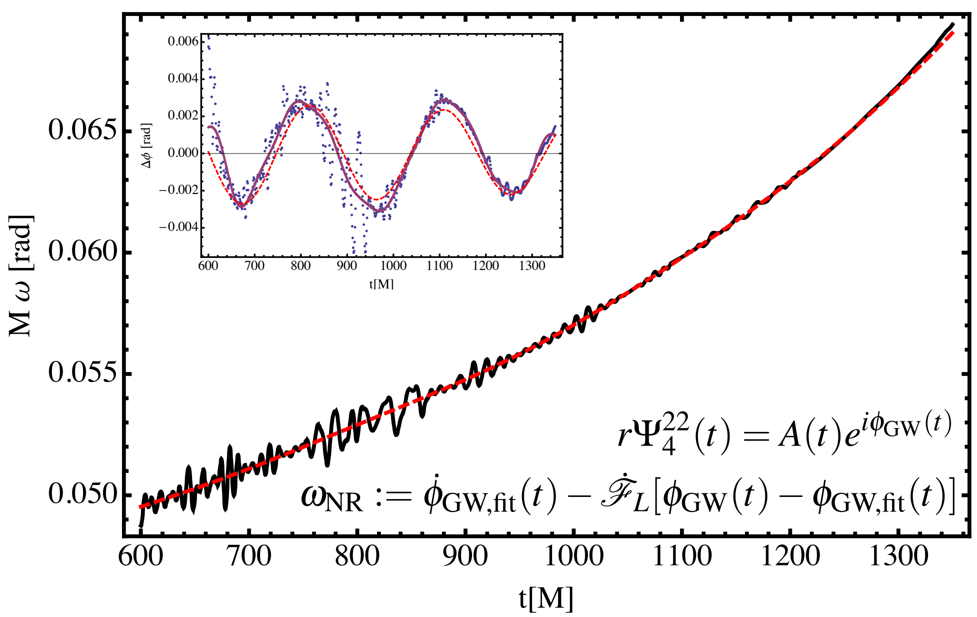

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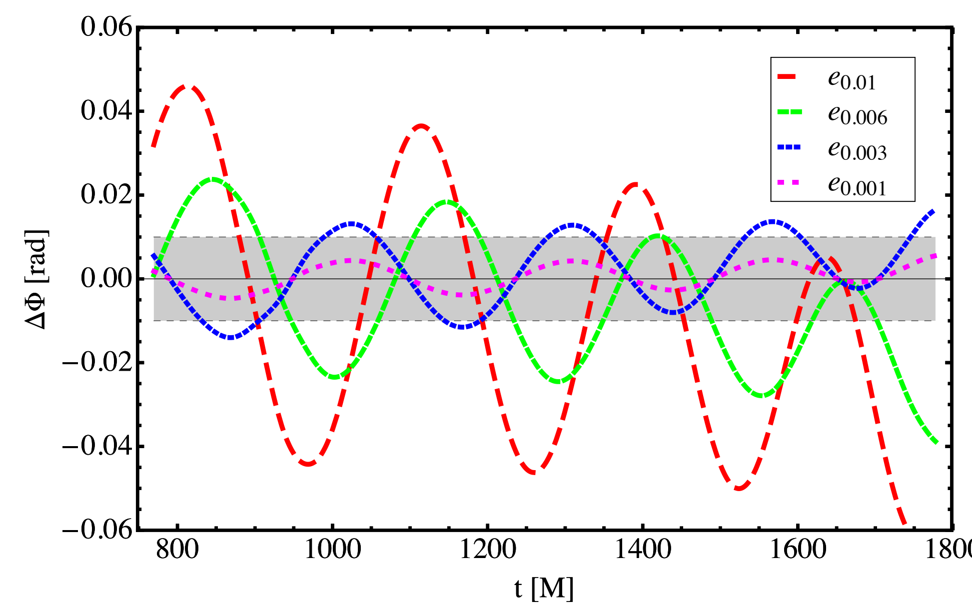
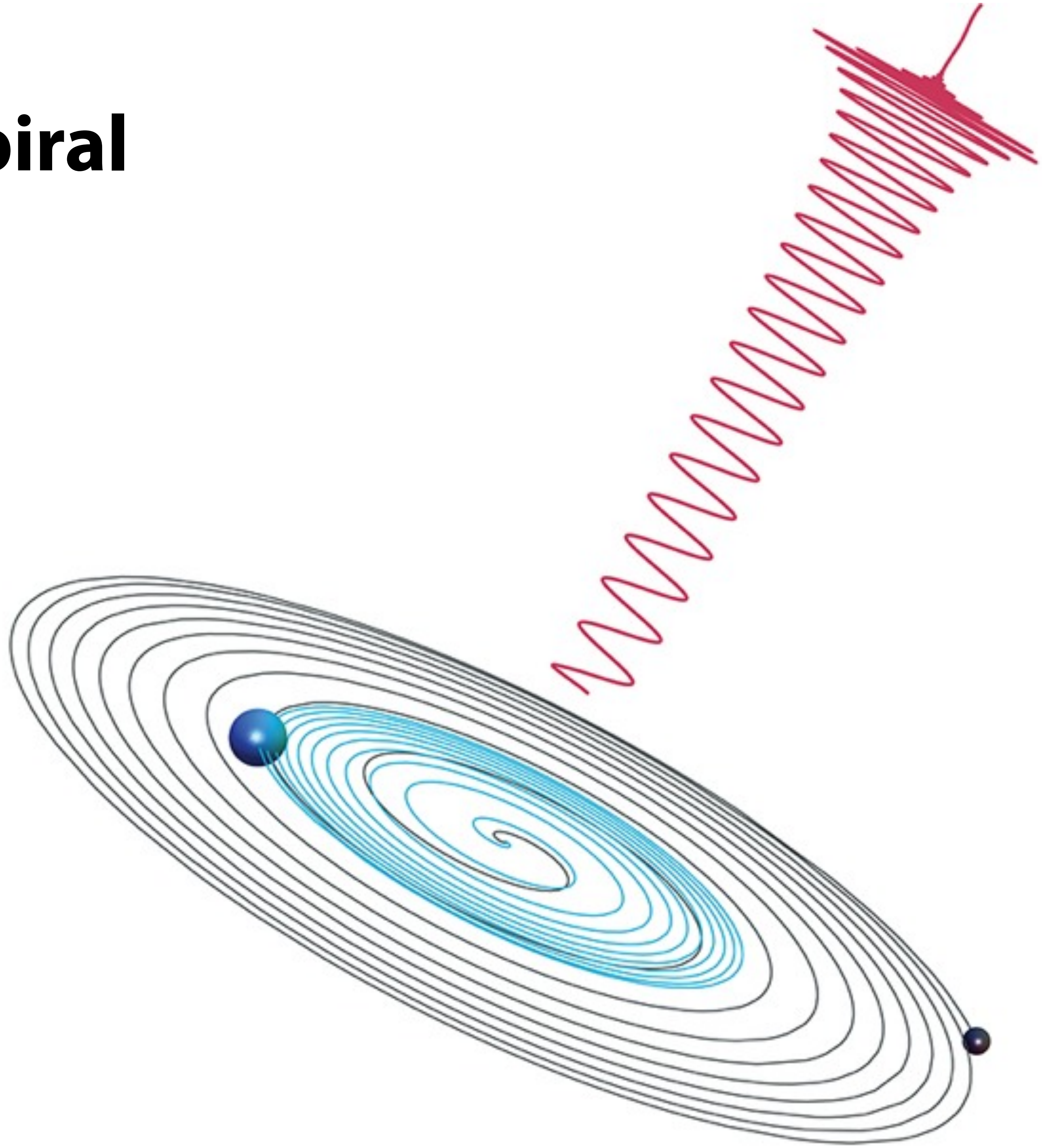


Binary black-hole inspiral

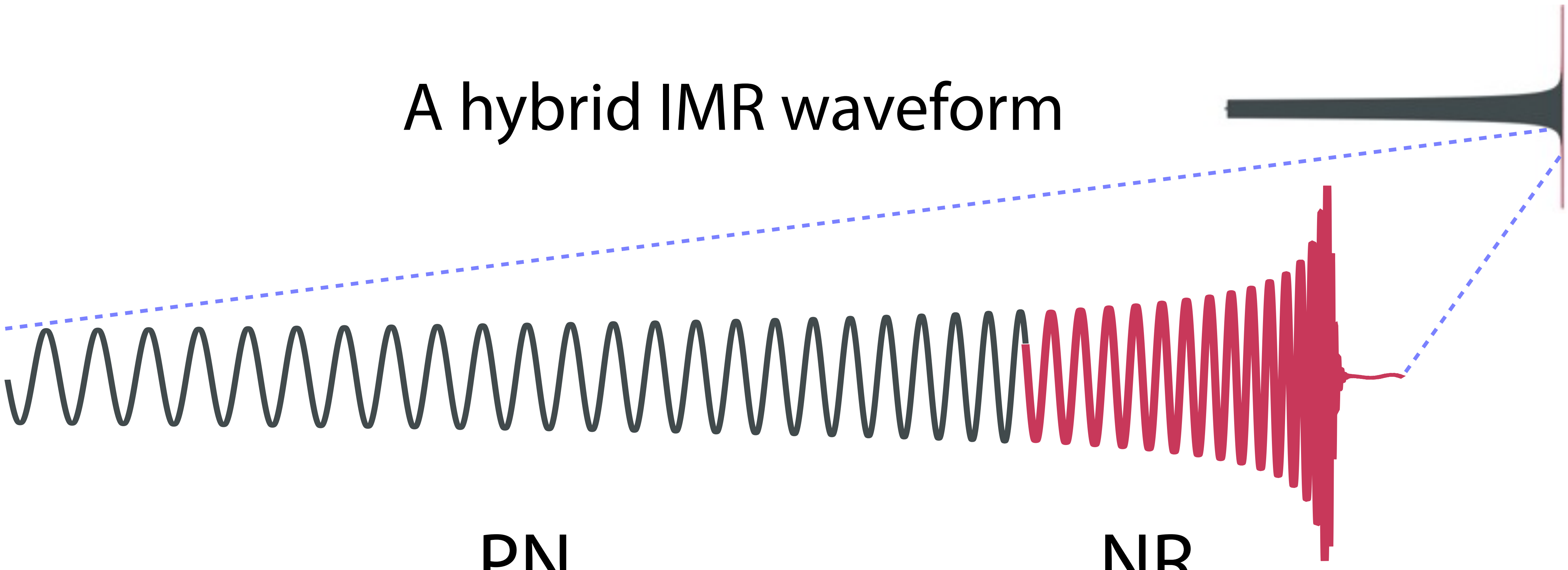
- Binary black-holes (BBHs) inspiral under the emission of gravitational-waves (GW).
- We need to know the GW signal over the BBH parameter space (mass-ratio, spins).
- Post-Newtonian theory (PN) can be used as long as the holes are far enough apart.
- Numerical relativity (NR) simulations are necessary for the last ~10-20 orbits and the merger.



Goal: build waveform models

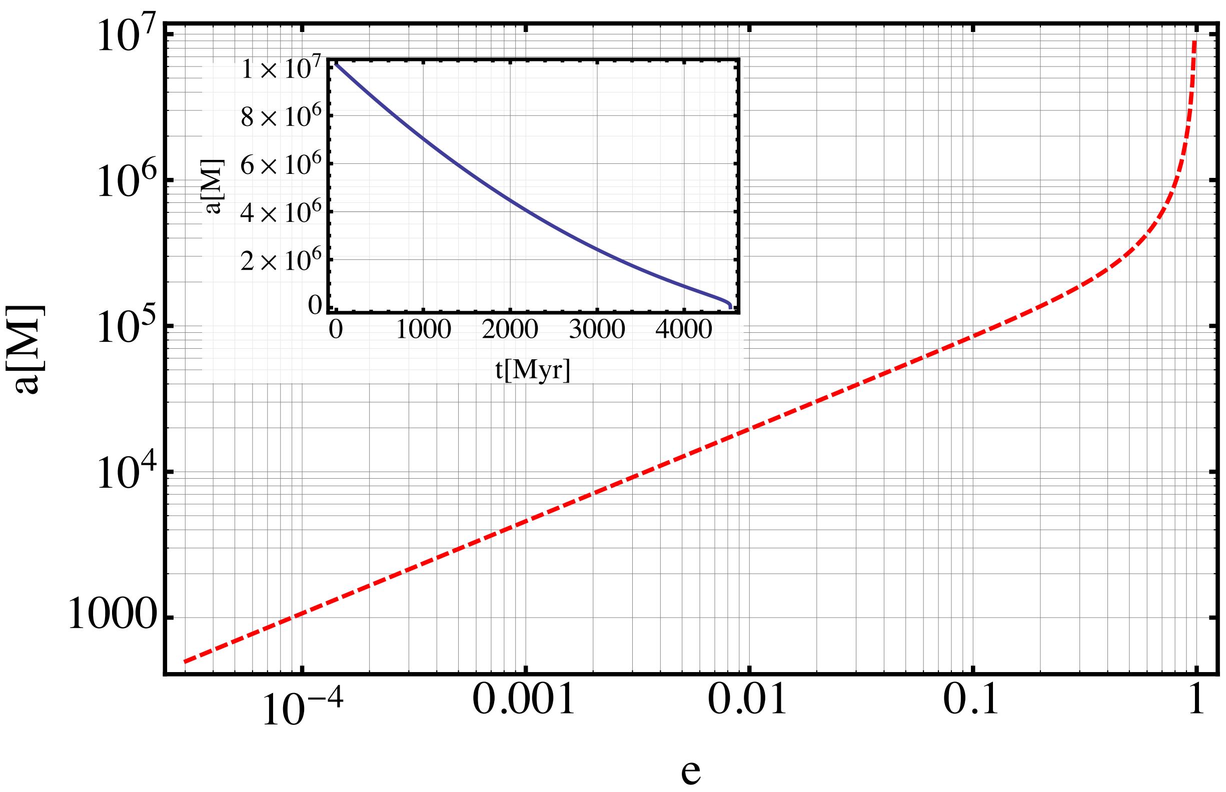
- A large-scale effort is under way to produce models of GW signals from the late inspiral, merger and ringdown of BBHs calibrated against large numbers of numerical relativity (NR) simulations.

- These waveform models will be essential to locate and interpret black-hole-binary GW signals in the data from second-generation laser-interferometric detectors such as Advanced LIGO.



Why eccentricity is low

- A typical length-scale for the initial separation of stellar-mass BHs in a BBH is the size of a supergiant (~ 0.1-10 AUs).
- These BBHs will in general have large initial eccentricities due to natal kicks from supernova explosions.
- The orbit of isolated solar-mass BH-binaries circularizes before GWs can be detected [Peters].
- Detection: possible once the GW enters the frequency band of ground based detectors.
- Therefore, the most pressing need is for models of binaries that undergo non-eccentric inspiral.



10 $M_\odot$ equal-mass BBH
(Newtonian + Quadrupole approximation)

Initial semi-major axis and eccentricity:
a_0 = 1 AU
e_0 = 0.98

Coalescence time:
Tc ~ 4.5 Gyr (~ 30% age of the universe)

Eccentricity at NR starting separations (~ 10-15M):
e ~ 10^{-7}

Numerical relativity simulations of BBHs

- BBH initial data: need to specify the initial momenta of the BHs.
- Can estimate initial parameters for quasi-circular inspiral of BBHs from post-Newtonian (PN) theory.
- For high mass-ratios and/or high spins PN initial parameters lead to eccentricities ~ 0.01 or higher and need to be decreased.
- We present the first systematic procedure to reduce eccentricity for aligned-spin BBH simulations performed using the ‘moving-puncture method’, which is the most common in the field.
- Instead of using the gauge dependent orbital motion this general method is based on the gauge invariant *GW signal*.

The basic idea

- Start with a short NR simulation that exhibits eccentricity, and a non-eccentric PN/EOB evolution of the same system.
- Adjust the initial momenta in the PN/EOB evolution until it exhibits eccentricity oscillations that agree with those in the NR waveforms, in both *amplitude* and *phase*.
- The inverse adjustment is then applied to the NR initial momenta, and a new NR simulation performed, and the process repeated.

Key quantities

Select an approximate PN/EOB model $\omega_{\mathrm{M}}(p_r, p_t; t)$ of the GW frequency as a function of the initial radial and tangential momenta (p_r, p_t).

Choose initial momenta (p_r^0, p_t^0) for a first NR simulation, such that $e_M(p_r^0, p_t^0) = 0$. Then $e_{\mathrm{NR}}(p_r^0, p_t^0) > 0$.

Define the GW and model frequency residuals relative to the quasi-circular model $\omega_{\mathrm{M}}(t) := \omega_{\mathrm{M}}(p_r^0, p_t^0; t)$

$$\mathscr{R}^i(t) = \omega_{\mathrm{NR}}^i(t) - \omega_{\mathrm{M}}(t)$$

$$\mathscr{R}_{\mathrm{M}}^{\lambda}(t) := \mathscr{R}_{\mathrm{M}}(\lambda_r, \lambda_t; t) = \omega_M(\lambda_r p_r^i, \lambda_t p_t^i; t) - \omega_{\mathrm{M}}(t)$$

A single iteration step

Choose the momentum scale factors (λ_r, λ_t) so that

$$\mathscr{R}^{\lambda}_{\mathrm{M}}(t) \approx \mathscr{R}^{0}(t)$$

with agreement in both the *amplitude* and *phase* of the residuals.

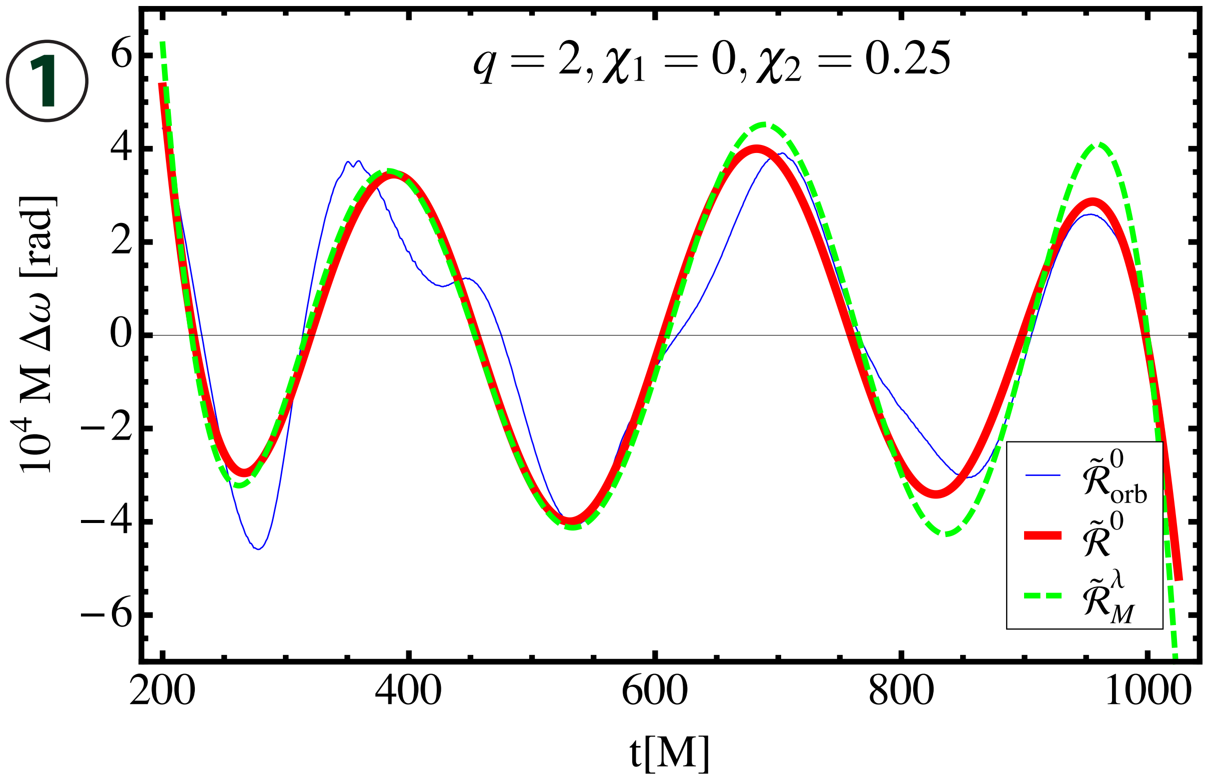
Produce updated initial momenta for the next NR simulation

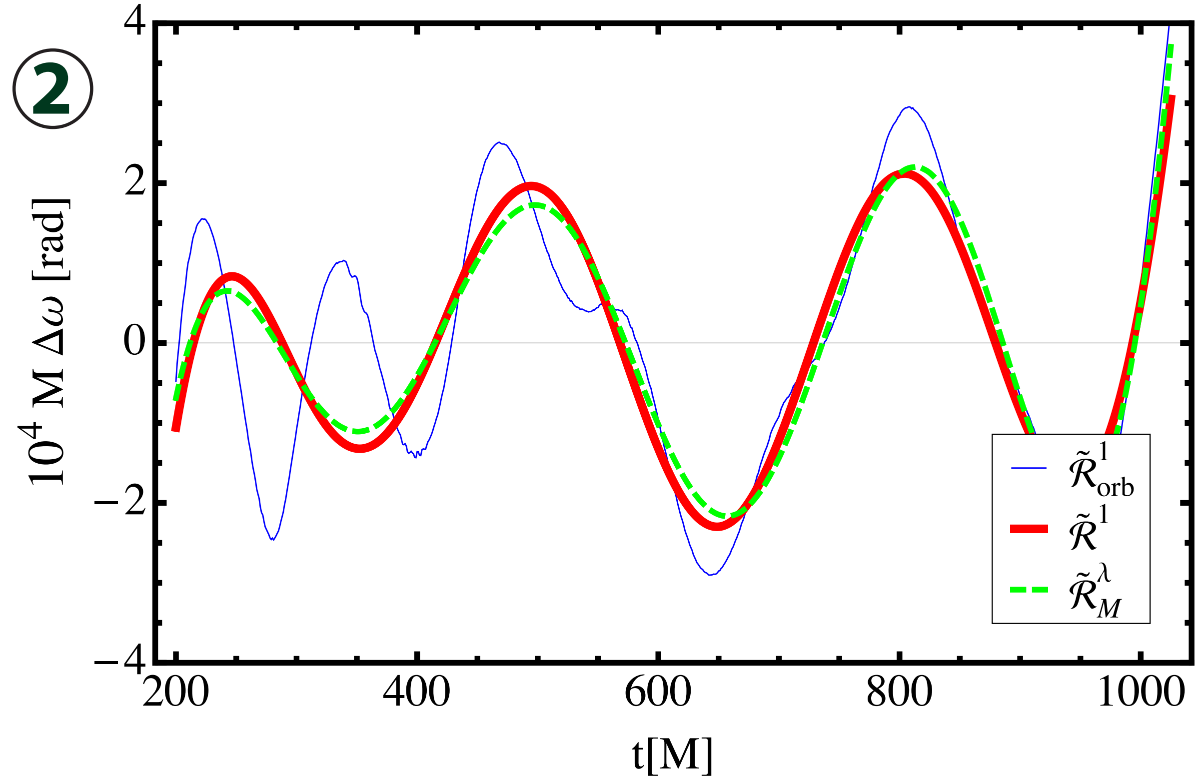
$$p_r^1 = p_r^0/\lambda_r^0$$
$$p_t^1 = p_t^0/\lambda_t^0$$

with the expectation that $e^1_{\mathrm{NR}} < e^0_{\mathrm{NR}}$.

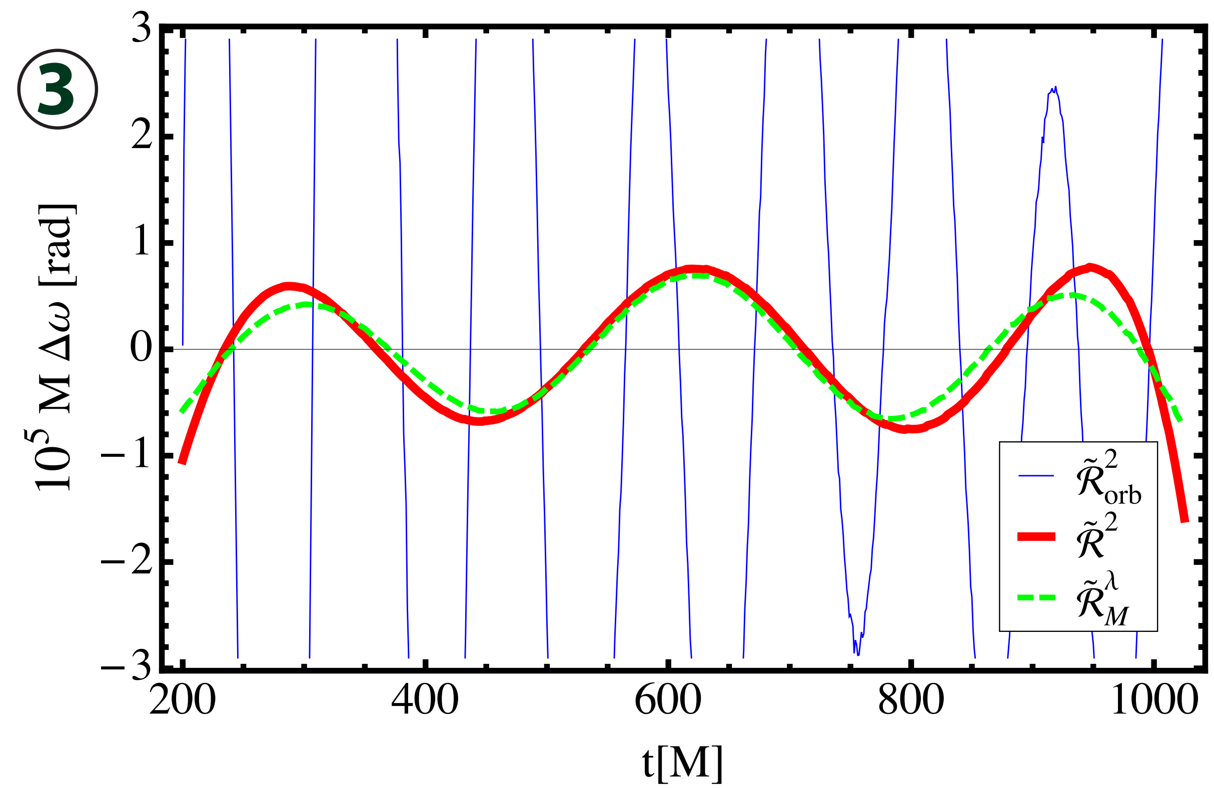
Iterate until eccentricity is below a desired target.

Eccentricity reduction example



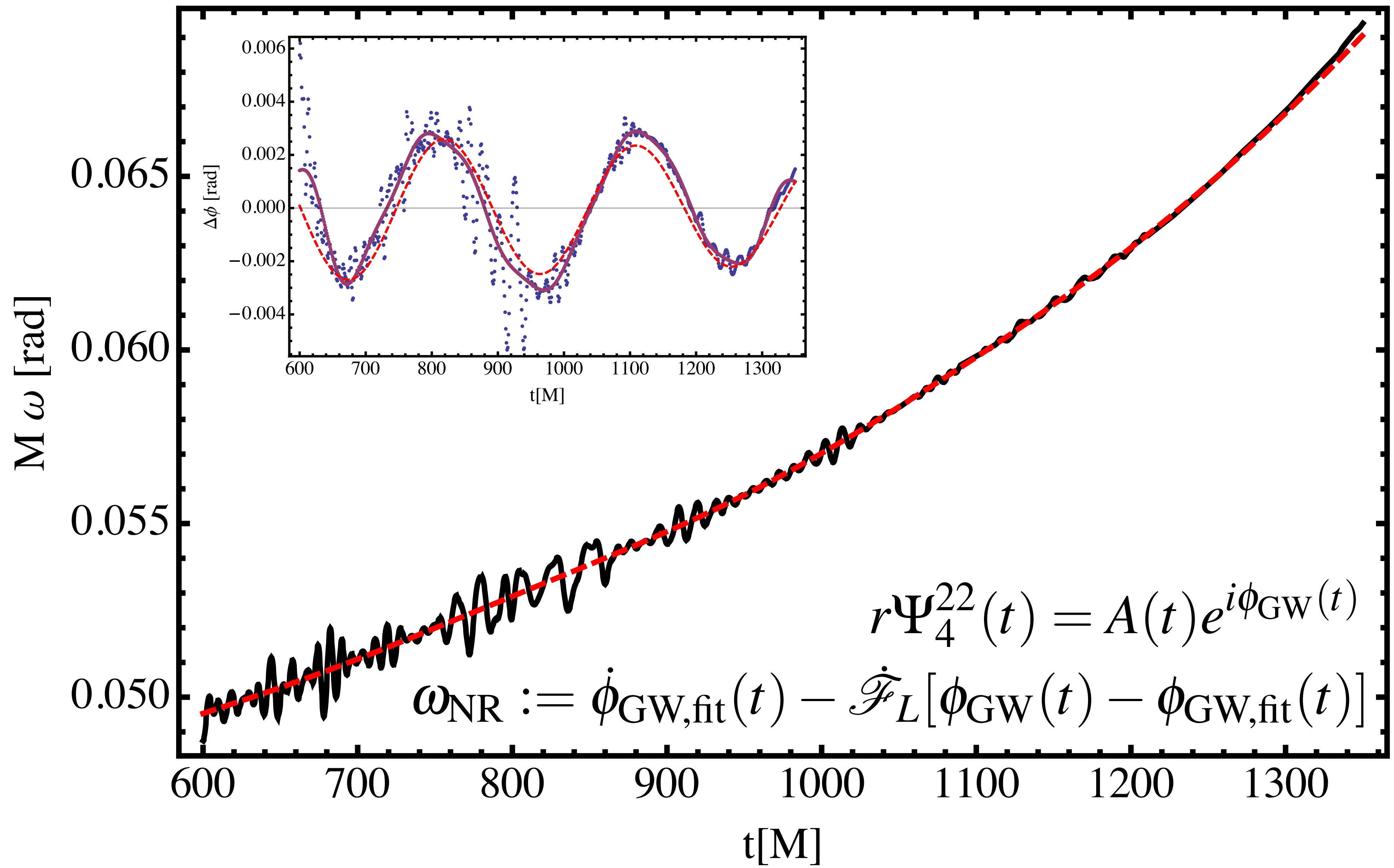


Iteration	p_r	p_t	$e_{\phi,\mathrm{GW}}$	e_Ω	λ_r	λ_t
0	0.000758	0.11710	0.006	0.0045	1	1.0028
1	0.000758	0.11677	0.003	0.0029	1.15	0.999
2	0.000660	0.11689	0.0003	0.0013		



- At each step the NR residual $\tilde{\mathscr{R}}^i$ is calculated from the filtered GW signal (red).
- The best-match scale factors lead to the residuals $\tilde{\mathscr{R}}^\lambda_\mathrm{M}$ (green, dashed).
- GW phase eccentricities show our progress $e_{\phi,\mathrm{GW}}(t) := [\phi_\mathrm{GW}(t) - \phi_{\mathrm{GW,fit}}(t)]/4$ while orbital frequency eccentricities are gauge-limited $e_\Omega(t) := (\Omega(t) - \Omega_\mathrm{fit}(t))/(2\Omega_\mathrm{fit}(t))$.

Filtering the numerical GW signal



A combination of fitting and filtering methods is used to produce a cleaned GW frequency while preserving the delicate eccentricity oscillations.

How low should residual eccentricity be in practice?

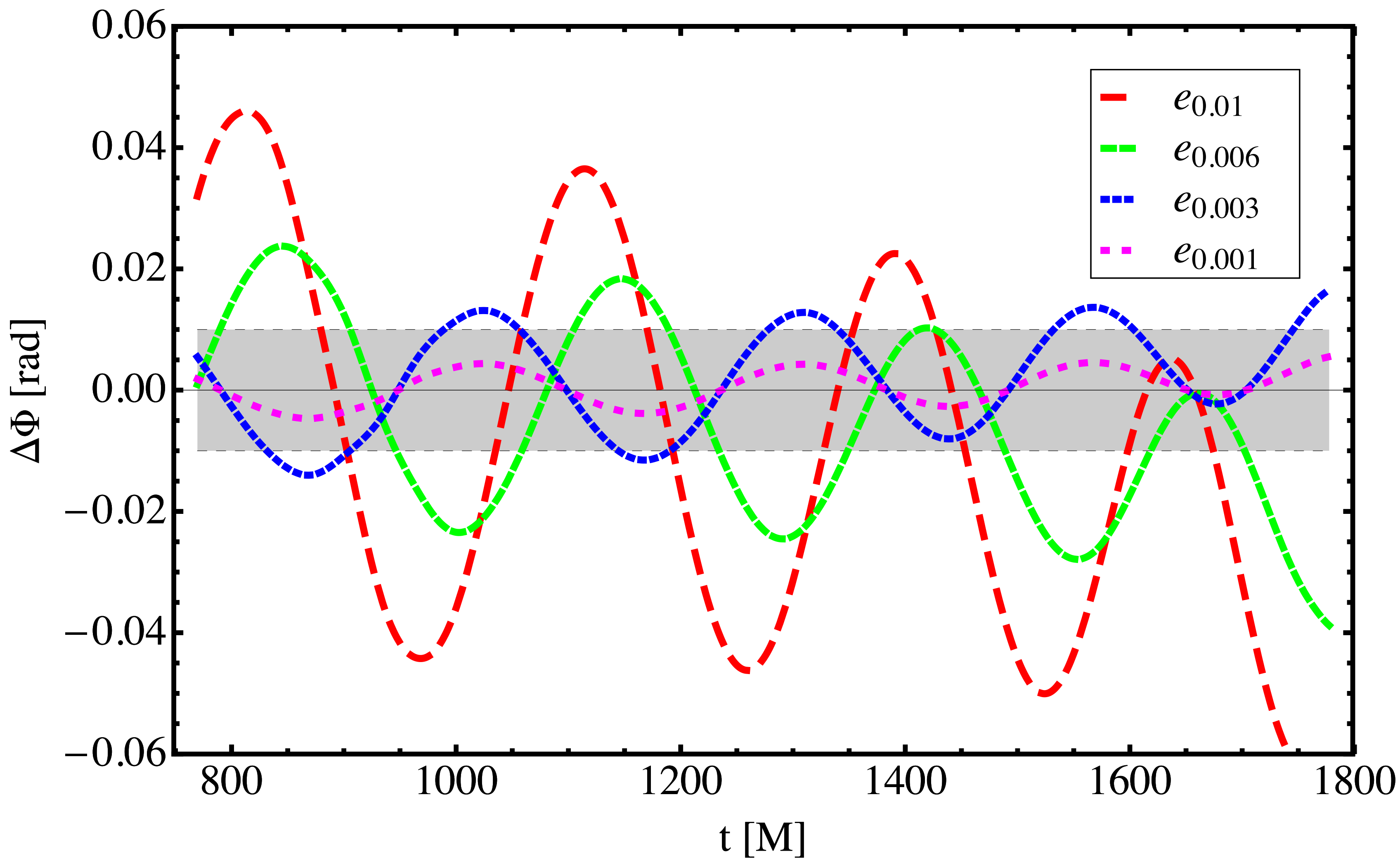
Match

- Study the match (detector-based criterion of how "close" two WFs are) between hybrid PN+NR waveforms with different residual eccentricity in the NR part.
- Have seen no evidence that eccentricities as high as 0.01 in the final ~10 orbits will have any noticeable influence on GW searches or parameter estimation in the Advanced detector era.
- Somewhat surprising, since at this level the eccentricity is visible by eye in the waveform.

Phase error

- Dominant numerical error that accrues during the inspiral of a BH binary
- Reduce eccentricity to a level where the eccentricity-induced drifts in the GW phase are well below the numerical phase errors in our simulations.
- Choose a tolerance of $e \sim 10^{-3}$.
  - Produces oscillations in the GW phase with an amplitude of 0.01 rad during inspiral.
  - Accumulated phase drift through merger and ringdown is less than 0.2 rad.
  - This is well within our NR phase errors.

Inspiral phase differences



Secular and oscillatory contributions

Alignment in time and phase offset based on QC phases

Phase differences from Ψ_4 with respect to the $e_{\phi,\mathrm{GW}} = 0.0003$ simulation. A stringent NR phase error requirement of ±0.01 rad is indicated by the shaded region.

Conclusion

- Because our method is applied to the GW signal, it can be adapted to any evolution method, and is not limited to moving-puncture simulations. It could also be adapted to other compact binary simulations, for example neutron-star (NS-NS) binaries, and black-hole--neutron-star (BH-NS) binaries.

- Our method can typically reduce eccentricity below 0.001 in one or two iteration steps using a semi-automated procedure to obtain the scale factors.

- A subset of our method was already presented in previous work [Husa ea, PRD, 77, 044037 (2008), Hannam ea, PRD, 82, 124008 (2010)].

- We will consider an extension of our method to precessing binaries in future work.