String theory and galactic rotation

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String theory explanation of galactic rotation

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The unique spherically symmetric metric which has vanishing weyl tensor, is asymptotically desitter, and can model constant galactic rotation curves is presented. Two types of field equations are shown to have this metric as an exact solution. The first is palatini varied scalar-tensor theory. The second is the low energy limit of string theory modified by inclusion of a contrived potential.

Introduction

Galactic rotation curves often exhibit speeds which are a constant independent of distance from the center of the galaxy. This is less than would be expected from solid body rotation and more than would be expected from free orbit rotation. From a newtonian perspective the modification which works is the replacement of the reciprocal potential by a logarithmic potential, the spherically symmetric relativistic generalization of this has one free function which is fixed by requiring that the weyl tensor vanishes. This leaves the problem of finding which field equations the einstein tensor obeys and both palatini varied scalar-tensor theory and a low energy limit of string theory are found to work. The usual method of approaching problems by starting with a lagrangian, deriving field equations, finding solutions, and then comparing with observations is thus reversed: in the present case it is observation, then metric, then field equations and finally lagrangian.

The metric

Consider the line element

$$ds^2 = -(X + \lambda_t r^2)dt^2 + \frac{(X - v^2)^2}{X(X + \lambda_r r^2)}dr^2 + r^2 d\Sigma_2^2, \qquad (1)$$

where

$$X \equiv (1 + 2v^2 \ln(r)), \qquad (2)$$

and v is the constant speed of galactic rotation. $g_{\theta\theta}$ and g_{tt} are fixed upto a radial coordinate transformation by requiring constant rotation curves, g_{rr} is fixed by requiring that the weyl tensor vanishes. When $\lambda_t(\lambda_t - \lambda_r) = 0$ the weyl tensor vanishes, from now on only consider the solution $\lambda = \lambda_t = \lambda_r$.
In conformally flat form the metric requires lambert W functions.

Low energy string theory with a potential

The lagrangian for low energy string theory with a potential is

$$\mathcal{L} = \exp\left(-2\phi\right)\left\{R + \frac{\alpha}{2}\Box\phi - \beta\left(\nabla\phi\right)^2 - \frac{\gamma}{12}H^2 - V\left(\phi\right)\right\}, \qquad (3)$$

where $H^2 \equiv H_{abc}H^{abc}$, performing metric variation and then expressing the field equations in terms of the ricci tensor

$$8\pi\kappa^2 \exp\left(2\phi\right) S_{ab} = R_{ab} - \alpha\phi_{;ab} + 2\beta\phi_a\phi_b + \frac{\gamma}{2}H_{acd}H_{b..}^{\ \ cd} \qquad (4)$$

$$+g_{ab}\left(-\frac{\gamma}{6}H^2 + V\right), \qquad (5)$$

the solution is

$$\phi = -\frac{1}{\alpha}\ln(X), \quad H^{abc} = \pm\sqrt{(\alpha^2 - 4\beta)/2\gamma}\,\epsilon^{abcd}\phi_d, \qquad (6)$$

$$V = \frac{v^2}{r^2(X - v^2)^3}\left(X(X - 4v^2) + v^4\right). \qquad (7)$$

Metric properties M1

(1) has constant rotation curves when λ_r is negligible, the easiest way to see this is that the circular vector

$$P^a = \left(\frac{v}{r}\delta^a{}_\theta + \delta^a{}_t\right) f(r), \tag{8}$$

where f is an arbitrary function of r has acceleration

$$\dot{P}_a = \lambda_r f^2 \delta^r_a, \tag{9}$$

so that when λ_r vanishes the vector is acceleration free or geodesic. For a metric conformal to the schwarzschild solution with the same vector (8)

$$\dot{P}_a = m\sqrt{X}(X - 3v^2)\frac{f^2}{r^2}\delta^r_a, \tag{10}$$

so that when m vanishes the vector (8) is again geodesic.

Metric properties M2, M3, M4

M2 the metric (1) is asymptotically desitter as λr^2 increases much faster than $\ln(r)$. For $\lambda = 0$ the line element is not asymptotically flat as the *ln* term in g_{tt} diverges, this is a problem with many models of galactic rotation which have no natural long radial distance cut off, having an asymptotically desitter spacetime provides such a cut off. Whether this can be thought of as evidence of a non-vanishing cosmological constant or just an indication of the effect of distant matter does not have to be chosen. *M3* the short distance cut off for the metric is good, for short distances a metric conformal to the schwartzschild solution shows that the galactic metric can approach schwarzschild spacetime. *M4* at $r = 1$, $\ln(r)$ changes sign and it is not immediate where this is in meters, however note that the solution is still a solution with $r \rightarrow r/r_o$ so that the length scale r_o is arbitrary and has to be fixed by other means.

Metric properties M5,M6

M5 it is not clear what, if anything, corresponds to the vanishing of the three metric functions in (1), $(X - v^2)$ occurs in the denominator of curvature invariants so as it approaches zero they diverge.

M6 the line elements (1) and a metric conformal to the schwarzschild solution were taken to be spherically symmetric rather than axi-symmetric, but rotation of galactic spacetime would be expected, spacetime rotation would need a more elaborate model, one could simply choose $d\hat{s}^2 = ds^2_{kerr}$ however kerr rotation is fundamentally short range whereas galactic rotation is long range, the simpler case of the newtonian model uses just the log potential $v^2 \ln(r)$ so that the newtonian model is spherically symmetric and this suggests that the simplest relativistic models are also spherically symmetric.

Metric properties M7

M7 why choose a line element with vanishing weyl tensor in the first place: from the perspective of the jordan formulation of einstein's equations one might expect that at large distances the weyl scalar to be larger than the ricci scalar; however from the perspective of the schwarzschild solution which has vanishing ricci tensor and robertson-walker spacetime which has vanishing weyl tensor one might expect that vanishing weyl tensor characterizes large distances, then the question arises as to at what range the weyl tensor becomes non-negligible, presumably this depends on the distance r_o introduced above, so far there the conformal schwarzshild metric suggests that it is a universal length rather than a length dependent on the mass of the galaxy under consideration.

Properties of the string solution

S1 the values of the constants in the string solution lead to a real three-form H, they could have led to a complex one; although there are three constants after absorbtion there is only one free parameter. *S2* the potential in the string solution is contrived, perhaps with the inclusion of additional fields it will no longer be necessary. *S3* the lagrangian (3) includes a $\Box\phi$ term, as for palatini-scalar-tensor theory this term is necessary in order for the field equation involving differences in components in the ricci tensor to vanish, however the term can be removed from the lagrangian by integration by parts.

General properties

G1 the introduction of the absorbable constant α is done because in the better known case of spherically symmetric static minimal scalar fields the value of the constant in the equation analogous to that of the scalar field depends on schwarzschild mass, but the value of the constant in the palatini field equations does not, so it is anticipated that something similar could happen here. *G2* the palatini field equations are assumed to be consistent as they can be lagrangian based, however conservation equations and the initial value problem are not looked at: conservation and euler equations are not immediate in palatini scalar-tensor theory, also the initial value problems is not straightforward when there are $\phi_{;ab}$ terms involved; these consistency problems depend on the variables and connection in which they are formulated. *G3* it is not immediate how the result impacts other areas of physics, for example globular star clusters also exhibit unusual dynamics, but usually have no overall v as in the present case, so far the post-newtonian approximation of (1) has not been studied.

Conclusion

A relativistic model of galactic rotation curves was produced which has the properties discussed in the last section. A problem with the metric is how many field equations and lagrangians can it be a solution to. The most important problem with the scalar-tensor theory is the physical origin of the scalar field and the most likely explanation is that the scalar field comes from dimensional reduction. The most important problem with the string theory solution is the contrived potential which hopefully will at some time in the future be replaced by fields.