# Non-abelian dyons in anti-de Sitter space

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# Outline

1. A brief history of Einstein-Yang-Mills
2. $\mathfrak{su}(N)$ EYM with $\Lambda < 0$
3. Dyonic solutions
4. Conclusions and outlook

# A brief history of Einstein-Yang-Mills

Asymptotically flat EYM studied for over 20 years

- Purely magnetic $\mathfrak{su}(2)$ solitons and black holes found numerically 1989-90
- Have no magnetic charge

## Non-abelian baldness of asymptotically flat $\mathfrak{su}(2)$ EYM

If $\Lambda = 0$, the only solution of the $\mathfrak{su}(2)$ EYM equations with a non-zero (electric or magnetic) charge is Abelian Reissner-Nordström

Rules out dyonic $\mathfrak{su}(2)$ asymptotically flat solutions

[ Ershov and Gal'tsov, PLA **138** 160 (1989), **150** 159 (1990) ]

- Ershov/Gal'tsov result no longer holds for larger gauge group in asymptotically flat space
- What about asymptotically anti-de Sitter space?

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# The model for $\mathfrak{su}(N)$ EYM

## Einstein-Yang-Mills theory with $\mathfrak{su}(N)$ gauge group

$$S = \frac{1}{2} \int d^4x \sqrt{-g} \left[ R - 2\Lambda - \mathrm{Tr}\, F_{\mu\nu} F^{\mu\nu} \right]$$

## Field equations

$$\begin{aligned} R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} &= T_{\mu\nu} \\ D_\mu F^\mu_\nu = \nabla_\mu F^\mu_\nu + [A_\mu, F^\mu_\nu] &= 0 \end{aligned}$$

## Stress-energy tensor

$$T_{\mu\nu} = \mathrm{Tr}\, F_{\mu\lambda} F^\lambda_\nu - \frac{1}{4} g_{\mu\nu} \mathrm{Tr}\, F_{\lambda\sigma} F^{\lambda\sigma}$$

# Static, spherically symmetric, configurations

## Metric

$$ds^2 = -\mu(r)\sigma(r)^2\, dt^2 + [\mu(r)]^{-1}\, dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)$$

$$\mu(r) = 1 - \frac{2m(r)}{r} - \frac{\Lambda r^2}{3}$$

## $\mathfrak{su}(N)$ gauge potential [ Kunzle *Class. Quant. Grav.* **8** 2283 (1991) ]

Static, dyonic, gauge potential

$$A_\mu\, dx^\mu = \mathcal{A}\, dt + \frac{1}{2}\left(C - C^H\right) d\theta - \frac{i}{2}\left[\left(C + C^H\right)\sin\theta + D\cos\theta\right] d\phi$$

$N - 1$ electric gauge field functions $h_j(r)$ in matrix $\mathcal{A}$

$N - 1$ magnetic gauge field functions $\omega_j(r)$ in matrix $C$

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# Field equations

## Yang-Mills equations

$$
\begin{aligned}
h_k'' &= h_k' \left( \frac{\sigma'}{\sigma} - \frac{2}{r} \right) + \sqrt{\frac{2\,(k+1)}{k}} \frac{\omega_k^2}{\mu r^2} \left( \sqrt{\frac{k+1}{2k}} h_k - \sqrt{\frac{k-1}{2k}} h_{k-1} \right) \\
&\quad + \sqrt{\frac{2k}{k+1}} \frac{\omega_{k+1}^2}{\mu r^2} \left( \sqrt{\frac{k}{2\,(k+1)}} h_k - \sqrt{\frac{k+2}{2\,(k+1)}} h_{k+1} \right) \\
0 &= \omega_k'' + \omega_k' \left( \frac{\sigma'}{\sigma} + \frac{\mu'}{\mu} \right) + \frac{\omega_k}{\sigma^2 \mu^2} \left( \sqrt{\frac{k+1}{2k}} h_k - \sqrt{\frac{k-1}{2k}} h_{k-1} \right)^2 \\
&\quad + \frac{\omega_k}{\mu r^2} \left( 1 - \omega_k^2 + \frac{1}{2} \left[ \omega_{k-1}^2 + \omega_{k+1}^2 \right] \right)
\end{aligned}
$$

## Einstein equations

Give $\mu'(r)$ and $\sigma'(r)$ in terms of $\omega_k$, $h_k$ and their derivatives

# Solving the field equations

Numerically solve the field equations for $\mathfrak{su}(2)$ and $\mathfrak{su}(3)$, with different values of $\Lambda < 0$

## Soliton solutions

- Regular at the origin $r = 0$
- Solutions parameterized by $\omega_k''(0)$ and $h_k'(0)$

## Black hole solutions

- Regular event horizon at $r = r_h = 1$
- Solutions parameterized by $\omega_k(r_h)$ and $h_k'(r_h)$

Electric functions $h_k$ are monotonically increasing

Colour-code solution space by number of zeros of magnetic gauge functions $\omega_k$

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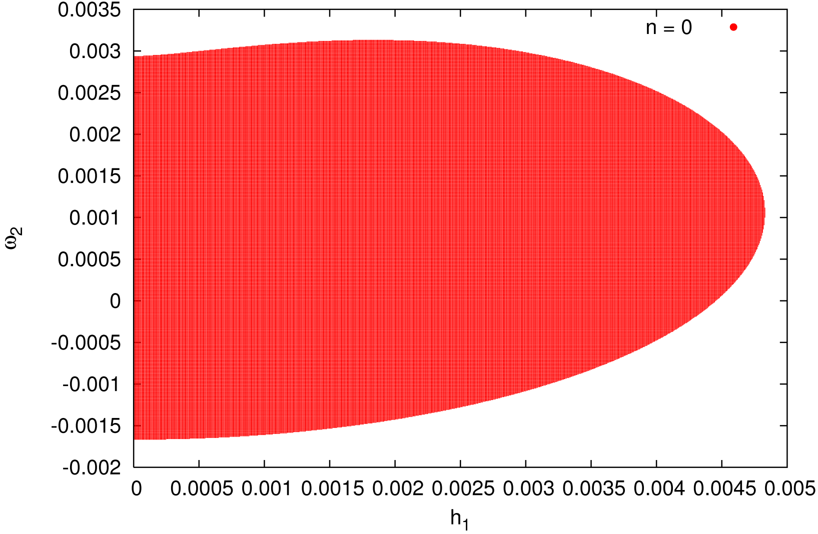
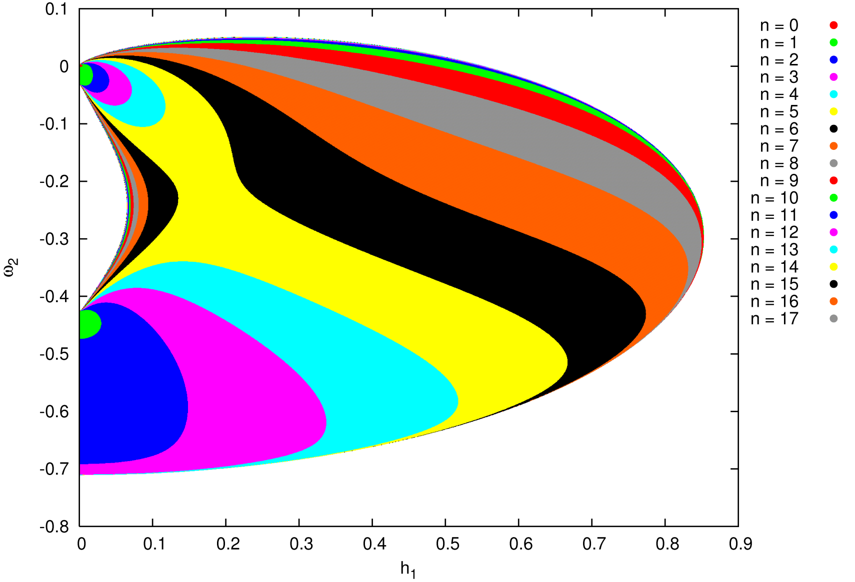
# $\mathfrak{su}(2)$ solitons, $\Lambda = -0.01$

[ Bjoraker and Hosotani, PRL **84** 1853 (2000), PRD **62** 043513 (2000) ]

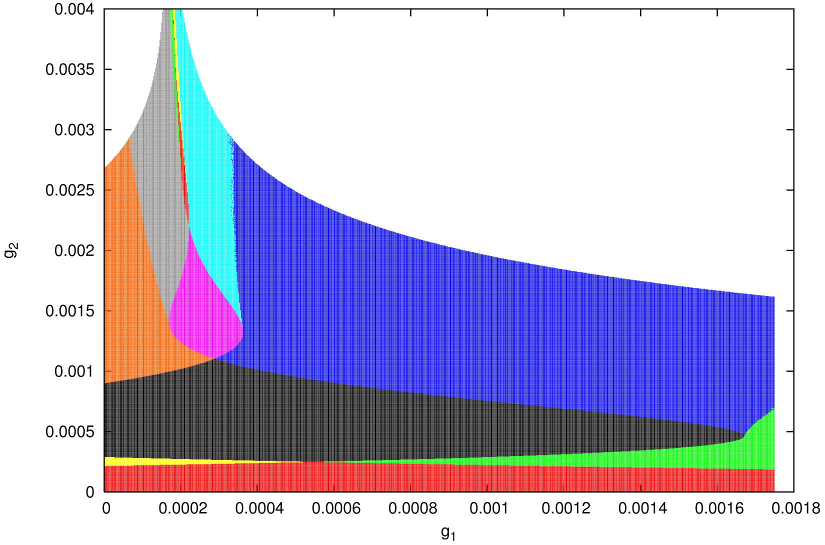


# $\mathfrak{su}(2)$ solitons, $\Lambda = -0.01$ [ Shepherd and EW ]

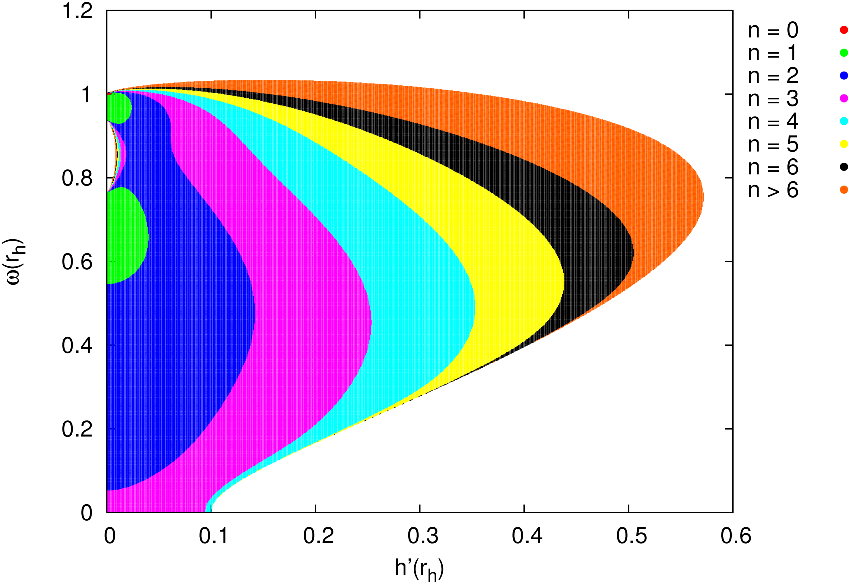


# $\mathfrak{su}(3)$ solitons - part of the solution space for $\Lambda = -0.01$

[ Shepherd and EW ]

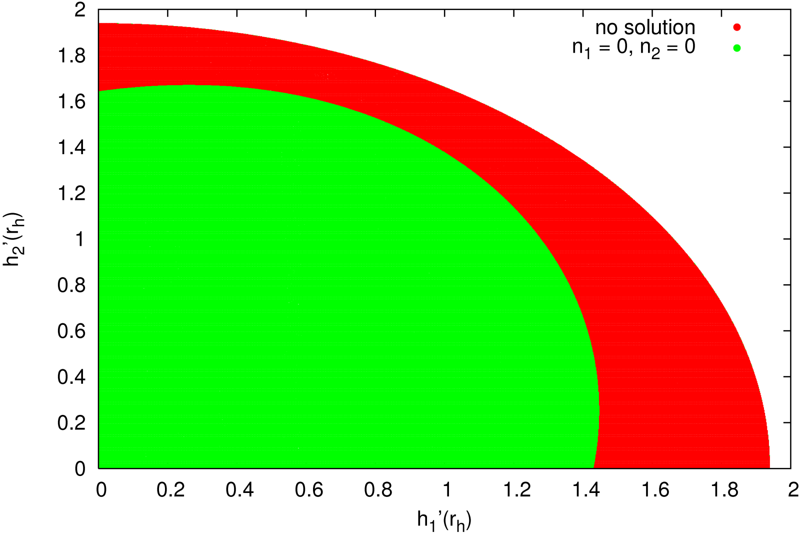


# $\mathfrak{su}(2)$ black holes, $\Lambda = -0.01$, $r_h = 1$ [ Shepherd and EW ]



# $\mathfrak{su}(3)$ black holes $\Lambda = -3$, $r_h = 1$, $\omega_1(r_h) = 1.3$, $\omega_2(r_h) = 1.2$

[ Shepherd and EW ]



# Conclusions and outlook

## Dyons in $\mathfrak{su}(N)$ EYM in adS

Black hole and soliton solutions with both electric and magnetic gauge fields:

- $\mathfrak{su}(2)$ dyonic solutions found previously by Bjoraker and Hosotani
- Examination of larger part of solution space shows a very rich menagerie of solutions
- New $\mathfrak{su}(3)$ dyonic solutions
- Solutions in which magnetic functions $\omega_k$ have no zeros for larger $|\Lambda|$

## Open questions

For sufficiently large $|\Lambda|$

- Prove analytically the existence of dyonic solutions for all $N$
- Stability when $\omega_k$ have no zeros?

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