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The Growth of Density Fluctuations in a Simplified Model of Extended Inflation

Nathalie Deruelle,[*+] **Carsten Gundlach,**[+] **and David Polarski**[*]

[+]Department of Applied Mathematics and Theoretical Physics
Silver Street, Cambridge CB3 9EW, England

[*]Département d'Astrophysique Relativiste et de Cosmologie
CNRS-Observatoire de Paris, 92195 Meudon, France

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Abstract

We reduce the cosmological model of extended inflation to a spatially flat Robertson-Walker geometry driven by a self-interacting scalar field with a potential of an exponential type. We quantize the gauge invariant scalar perturbations of the metric and the field, and compute the spectrum of the energy density fluctuations.

1. The model

In the cosmological model of extended inflation[1] gravity is minimally coupled to an inflaton σ, with potential $V_i(\sigma)$, in the framework of a Jordan-Brans-Dicke theory.[2] The action of the system is [see e.g. Ref. 3]:

$$S = \int d^4x\sqrt{-\bar{g}}\left[\tfrac{1}{2}\bar{R}\psi - \tfrac{1}{2}\omega\bar{g}^{\alpha\beta}\frac{\partial_\alpha\psi\partial_\beta\psi}{\psi} - V_{BD}(\psi) - \tfrac{1}{2}\bar{g}^{\alpha\beta}\partial_\alpha\sigma\partial_\beta\sigma - V_i(\sigma)\right]. \tag{1.1}$$

We use Planck units where $8\pi G = c = \hbar = 1$; $\bar{g}_{\alpha\beta}$ is the metric tensor in the "Jordan frame",[19,26] $\bar{g}$ its determinant, $\bar{R}$ the curvature scalar. The Brans-Dicke field ψ couples to the curvature so as to yield the possibility of a variable gravitational "constant" and ω is a free parameter. In the standard Brans-Dicke theory the potential $V_{BD}(\psi)$ is zero, and Einstein's general relativity is recovered when $\omega \to \infty$; current time-delay experiments[4] then constrain ω to be > 500.

As shown in Ref. 5, the field redefinition and the conformal transformation:

$$g_{\alpha\beta} \equiv \psi\,\bar{g}_{\alpha\beta} \qquad \text{and} \qquad \varphi \equiv \varphi_0 \ln\psi \quad \text{with} \quad \varphi_0 \equiv \sqrt{\omega + \tfrac{3}{2}} \tag{1.2}$$

bring the action (1.1) into a standard Einstein-Hilbert form:

$$\begin{aligned} S = \int d^4x\sqrt{-g}\Big[\tfrac{1}{2}R - \tfrac{1}{2}g^{\alpha\beta}\partial_\alpha\varphi\partial_\beta\varphi - e^{-2\varphi/\varphi_0}V_{BD}(e^{\varphi/\varphi_0}) \\ - \tfrac{1}{2}e^{-\varphi/\varphi_0}g^{\alpha\beta}\partial_\alpha\sigma\partial_\beta\sigma - e^{-2\varphi/\varphi_0}V_i(\sigma)\Big]. \end{aligned} \tag{1.3}$$

(Note however that in this "Einstein frame"[19,26] the matter field σ is non-minimally coupled to gravity.)

If, in an early phase of the evolution of the universe, the inflaton σ was trapped into a metastable "false vacuum" state $\sigma = 0$, then, in the model (1.1) (1.3) of that phase, the kinetic term for σ can be ignored and $V_i(\sigma)$ taken to be a constant: $V_i(0) \equiv V_f$. In the original version of extended inflation,[1] where the potential V_{BD} is set equal to zero, the action (1.1) (1.3) then reduces to that of the Brans-Dicke theory with a cosmological constant equal to V_f, or, equivalently, to that of Einstein's gravity, minimally coupled to a scalar field φ with the exponential potential $V_f\, e^{-2\varphi/\varphi_0}$. In Ref. 6-7 and 14 it was shown that if $\varphi_0 > \sqrt{2}$, that is if $\omega > \frac{1}{2}$, most Robertson-Walker solutions of the corresponding field equations have a spatial curvature which vanishes asymptotically, and that the scale factor of the spatially flat attractors grows as a power of cosmic time. As shown in Ref. 1 and 8 this power-law inflation[9] ensures that the phase transition of the inflaton to its stable "true vacuum" state $\sigma = \sigma_t$ (with $V_i(\sigma_t) \equiv V_t$) can be completed through bubble nucleation, in contrast to old inflation[10] where an exponential growth of the scale factor, sufficient to solve e.g. the horizon problem, prevents the bubbles from percolating.[10,11]

This model with $V_{BD} = 0$ proved however to be insatisfactory:[13,14] the bubbles formed during the phase transition do not collide and thermalize fast enough, and hence spoil the observed isotropy of the microwave background radiation, just as in old inflation,[12] unless

ω is < 25 ($\Leftrightarrow \varphi_0 < 5$), a value which is incompatible with the experimental constraint on ω ($\omega > 500$) when $V_{BD} = 0$. An (ad hoc) way to cure the problem is to assume that V_{BD} is not zero and has a stable minimum, $V_{BD} \equiv V_{GR}$, at $\psi = 1 (\Leftrightarrow \varphi = 0)$, and that ψ settled there sometime in the past.[13,14] In this way the constraint on ω is relaxed, since this modified Brans-Dicke theory would, today, be equivalent to general relativity.

If the Brans-Dicke field settles to its general relativity value $\psi = 1$ before the inflaton has tunnelled to its true vacuum state, a phase of exponential expansion occurs *before* the transition, since the theory is then equivalent to pure general relativity with a cosmological constant (equal to $V_f + V_{GR}$), and the bubbles, as in old inflation, will not percolate.[1,14] The Brans-Dicke field must therefore settle *after* the phase transition (or, more unlikely, concomitantly), yielding the following scenario: in an initial phase of power-law inflation, the inflaton is trapped in the false vacuum $V_i(0) \equiv V_f$ and $V_{BD} \approx 0$; the phase transition then occurs: $\sigma \to \sigma_t$ and $V_i(\sigma) \to V_t$, and the bubbles of true vacuum percolate and thermalize; when the transition is completed, the Brans-Dicke field settles to its general relativity value $\psi = 1$. A "graceful exit" to the standard hot Big-Bang imposes that no effective cosmological constant be present, that is $V_t + V_{GR} = 0$.

A first issue in this scenario, which, as far as we know, still begs for an answer, is: when does reheating occur? As in old inflation, the collision of the bubbles should generate a hot fluid of radiation.[44] This radiation will dominate the subsequent evolution of the universe if, and as long as, its energy density is larger than the energy density stored in the true vacuum state of the inflaton (V_t) and the Brans-Dicke field. If the settling of the Brans-Dicke field occurs in such a radiation dominated phase, it need not be described in detail, and the standard hot Big-Bang scenario resumes soon after the phase transition. If, on the other hand, the radiation produced by the bubble collisions is negligible or outgrown once more by the potential energy (now equal to $V_t + V_{BD}$), a second period of power-law inflation occurs, after the phase transition. In that case reheating really happens when the Brans-Dicke field oscillates before settling, as in new or chaotic inflation (cf e.g. Ref. 15-16 and references therein).

Another issue is: when and how are the seeds for galaxy formation produced? In the period of power-law inflation when the inflaton is in its false vacuum state and $V_{BD} \approx 0$, density fluctuations grow from the quantum fluctuations of the inflaton, as in new or chaotic inflation,[17,18] and also, a priori, from quantum fluctuations of the Brans-Dicke field. They will continue to grow if a second period of power-law inflation occurs after the transition, when the inflaton is trapped in its true vacuum and the Brans-Dicke field "rolls" towards its minimum. The percolation, collision and thermalization of the bubbles produce inhomogeneities as well, but the problem of computing the resulting spectrum of the energy density fluctuations is, as far as we are aware, unsolved. Finally the reheating caused by the oscillations of the Brans-Dicke field around its minimum should create inhomogeneities of again another type, which have not yet been studied.

In the analyses of the growth of perturbations in extended inflation made so far,[19,20,45] the inhomogeneities of the last type are ignored and it is argued[19] that if the bubbles are constrained to spare the isotropy of the microwave background (i.e. if $\omega < 25$), they cannot by the same token be of cosmological interest, unless some fine-tuning is invoked: hence density fluctuations must mainly arise from the quantum fluctuations of the fields.

Moreover, since the inflaton σ is confined to a passive rôle, merely transiting from one state to another, its dynamics are presumably irrelevant; the argument here[19] is that the quantum fluctuations of σ are highly suppressed because its effective mass ($\approx V_f^{1/4} \ll 1$) is much larger than the Hawking-Gibbons temperature; (this however may not be true at the beginning of inflation, as the time dependence of the effective gravitational constant, $G_{eff} \approx e^{-\varphi/\varphi_0}$, causes the Hawking-Gibbons temperature, $H \approx (G_{eff}V_f)^{1/2}$, to be bigger than its general relativity value $V_f^{1/2}$; for an analysis of this effect see Ref. 46).

In this paper we shall simplify the problem even further and ignore the phase transition altogether: not only shall we assume that the transition leaves the universe homogeneous and isotropic on cosmic scales, as in Ref. 19, 29, 45, but also that it does not perturb its evolution, i.e. that $V_f \approx V_t$ (recall that in the context considered here, V_t need not be zero, only $V_t + V_{GR}$ has to); finally we assume that reheating is due to the late stage oscillations of the Brans-Dicke field. (All these assumptions are in fact implicit in Ref. 19, 20 and 45.) In this model, extended inflation therefore boils down to chaotic inflation[21] where the action for the system is:

$$S = \int d^4x\sqrt{-g}\left[\tfrac{1}{2}R - \tfrac{1}{2}g^{\alpha\beta}\partial_\alpha\varphi\partial_\beta\varphi - V(\varphi)\right] \tag{1.4}$$

with an effective potential $V(\varphi) \equiv (V_i + V_{BD})e^{-2\varphi/\varphi_0}, V_i = Const.$, which grows exponentially when $\varphi \to -\infty$, and has a zero minimum at $\varphi = 0$. In Ref. 19, 20 and 45, $V_i \equiv V_f$, and all that happens after the phase transition is ignored; here we ignore the phase transition explicitly and $V_i \equiv V_t \approx V_f$. Moreover, whenever we have to be specific, we shall choose:

$$V(\varphi) \equiv V_i\left[e^{-2\varphi/\varphi_0} - \frac{\beta}{\beta-\alpha}e^{-2(1-\alpha)\varphi/\varphi_0} + \frac{\alpha}{\beta-\alpha}e^{2(\beta-1)\varphi/\varphi_0}\right], \tag{1.5}$$

with $5 > \varphi_0 > \sqrt{2}$ and where α and β are constants, $0 < \alpha < 1$ and $\beta > 1$.

When $\varphi \to -\infty$ this potential approaches the exponential potential of extended inflation, $V \simeq V_i e^{-2\varphi/\varphi_0}$, considered in Ref. 19, 20 and 45. When $\varphi \to +\infty$ it grows exponentially, $V \propto e^{2(\beta-1)\varphi/\varphi_0}$, or reduces to an effective cosmological constant, $V \simeq \alpha V_i/(1-\alpha)$, if $\beta = 1$. Near $\varphi = 0$ we have:

$$V(\varphi) \simeq \tfrac{1}{2}m^2\varphi^2 \qquad \text{with} \qquad m^2 = \frac{4\alpha\beta V_i}{\varphi_0^2}, \tag{1.6}$$

as in the simplest model of chaotic inflation. Hence this potential can yield cosmological solutions exhibiting a period of power-law inflation (when φ starts large and negative), ending in a phase (when $\varphi \approx 0$) when the scale factor behaves as in a dust dominated era.[22] The special case $\beta = 2$, $\alpha = 1$, where $V = 4V_i \sinh^2 \varphi/\varphi_0$ was considered in Ref. 23.

A potential such as (1.5) would arise if the self-interaction for the Brans-Dicke field was described by the double well:[47]

$$V_{BD}(\psi) = \frac{V_i}{\beta-\alpha}\left(-\beta\psi^{2\alpha} + \alpha\psi^{2\beta}\right). \tag{1.7}$$

The fact that V_{BD} depends on V_i, a parameter which characterizes the a priori independent field σ, reflects the need for a fine-tuning to ensure that in the late stage ($\varphi \approx 0$), the effective cosmological constant be zero.

We recall for completeness that the potential (1.5) with $\beta = 1$ can also appear in the context of Kaluza-Klein theories, but does not yield power-law inflation, as φ_0 is then less than $\sqrt{2}$.[6,26] Indeed the action of general relativity with a cosmological constant, in a $(n+4)$ dimensional space-time compactified onto a n-sphere, is equivalent to (1.4) [see e.g. Ref. 24], with a potential of the form (1.5), if some ad hoc fine-tuning is allowed to ensure a zero minimum (see e.g. Ref. 25 for an example of such a prescription). The parameters ω (or φ_0) and α are then related to the dimension n of the internal space by: $\omega = \frac{n-6}{2(n+2)}$ and $\alpha = \frac{2}{n+2}$. Now since $n > 0$, ω must be less than $\frac{1}{2}$ ($\Rightarrow \varphi_0 < \sqrt{2}$), and the spatial curvature of the Robertson-Walker solutions with φ large and negative does not vanish asymptotically. The spatially flat solutions which inflate according to a power law are therefore not attractors. In these Kaluza-Klein models then, the evolution must start with large *positive* φ, where the potential is almost constant, yielding exponential inflation. Our analysis, where φ will start large and negative, will therefore not apply to that case.

It will not apply either when the potential (1.5) arises from the higher derivative theory of gravity where the Lagrangian R is replaced by $R + R^2/8V_i$. The action for such a theory is indeed equivalent to (1.4), with a potential given by (1.5) where $\beta = 1$, $\omega = 0$ and $\alpha = \frac{1}{2}$.[27,28] However, here again, only the spatially flat Robertson-Walker solutions inflating exponentially are attractors of the non flat solutions. Moreover, since $\omega = 0$, this model cannot be interpreted as a Brans-Dicke theory.

Notwithstanding all these restrictions, we shall briefly analyze in section 2 the inflationary background solution of the model (1.4-5) and study, in section 3, the classical evolution of the scalar gauge invariant perturbations. In section 4 we shall quantize them and compute the spectrum of the energy density fluctuations, following closely Ref. 29-32. As we shall see, the result does not depend crucially on the precise form assumed by $V(\varphi)$, as long as it behaves as $\exp(-2\varphi/\varphi_0)$ for large negative φ, and as $\frac{1}{2}m^2\varphi^2$ when $\varphi \to 0$. This will allow a comparison with the more heuristic analyses of Ref. 19, 20 and 45.

2. The background solution

We consider the system with action (1.4-5) when the geometry is of the Friedmann Robertson Walker type with flat spatial sections:

$$ds^2 = -dt^2 + a^2(t)d\vec{x}^2 = a^2(\eta)(-d\eta^2 + d\vec{x}^2) \tag{2.1}$$

where t is the cosmic time, η the conformal time, a the scale factor, $d\vec{x}^2 \equiv \delta_{ij}dx^i dx^j$, $i, j = 1, 2, 3$, and where the field φ depends only on time. The evolution of the system is determined by the Klein-Gordon equation together with the (00) component of Einstein's equations:

$$\ddot{\varphi} + 3H\dot{\varphi} + \frac{dV}{d\varphi} = 0 \qquad ; \qquad 3H^2 = \tfrac{1}{2}\dot{\varphi}^2 + V(\varphi). \tag{2.2}$$

A dot denotes differentiation with respect to cosmic time, a prime will denote differentiation with respect to conformal time, and $H \equiv \dot{a}/a$ is the Hubble "constant". A useful relation follows from (2.2):

$$\dot{H} = -\tfrac{1}{2}\dot{\varphi}^2. \tag{2.3}$$

For φ large and negative, $V(\varphi) \to V_i e^{-2\varphi/\varphi_0}$, and a large class of solutions of (2.2) starts evolving, for small t, as:[6,7,14]

$$\varphi \simeq \varphi_0 \ln\left(\sqrt{\frac{4V_i}{\varphi_0^2(3\varphi_0^2-2)}}\; t\right) \qquad ; \qquad a \simeq t^{\varphi_0^2/2} \tag{2.4}$$

(where we chose $a(t=1)=1$) as shows[6] an analysis of the trajectories in the phase plane $(\varphi, \tilde{\varphi})$ with $\tilde{\varphi} \equiv \sqrt{3/V_i}\dot{\varphi}e^{\varphi/\varphi_0}$. We shall also use:

$$H \simeq \frac{\varphi_0^2}{2t} \qquad ; \qquad -\eta \simeq \frac{t^{-(\varphi_0^2/2-1)}}{\varphi_0^2/2-1} \simeq \left(\frac{\varphi_0^2}{\varphi_0^2-2}\right)\frac{1}{aH}. \tag{2.5}$$

Note that (2.4) and (2.5) would be exact solutions for an exactly exponential potential. Since $\varphi_0 > \sqrt{2}$ the growth of the scale factor is characteristic of power-law inflation.[9] After this initial phase, the trajectories spiral towards the origin of the phase plane, where $V(\varphi)$ is given by (1.6), and the solutions, for large t, tend to:[22]

$$\varphi = 2\sqrt{\frac{2}{3}}\frac{\sin(mt+\phi)}{mt} + \mathcal{O}\left(\frac{1}{m^3t^3}\right) \quad ; \quad H = \frac{2}{3t} + \mathcal{O}\left(\frac{1}{t^2}\right) \tag{2.6}$$

where ϕ is an integration constant. The scale factor grows as $t^{2/3}$ as in a dust dominated universe. The order of magnitude of the time t_e when the transition between the two regimes (2.4) (2.6) takes place is given by $\varphi \approx \varphi_0$ in (2.4), that is (dropping a factor e):

$$mt_e \approx \sqrt{\alpha\beta}\sqrt{3\varphi_0^2-2} \qquad \text{with} \qquad m^2 = \frac{4\alpha\beta V_i}{\varphi_0^2} \tag{2.7}$$

which scales like φ_0 for generic values of α and β.

The flaws of the standard hot Big-Bang model (horizon, flatness problems etc) are cured if the scale factor increases by a factor $\ln Z > 65$ during the inflationary stage.[10] Given that inflation ends at a time given by (2.7) and that the scale factor grows according to (2.4) it follows that inflation must start at:

$$t_{in} \approx t_e\, Z^{-2/\varphi_0^2} \quad \text{with} \quad t_e \approx \varphi_0\sqrt{(3\varphi_0^2-2)/4V_i}, \tag{2.8}$$

which corresponds to the initial values: $\varphi_{in} \approx -\frac{2}{\varphi_0}\ln Z$ (which is indeed $<< -1$ for $\sqrt{2} < \varphi_0 < 5$) and $V_{in} \approx V_i Z^{4/\varphi_0^2}$. If V_i corresponds to the grand unification scale, $V_i \approx 10^{-16}$ (in Planck units), then V_{in} remains smaller than 1 (the Planck energy density) if $\varphi_0 > 2.7$. Hence enough inflation is achieved for generic values of the parameters.[48] In the full-fledged model of extended inflation, a large enough Z is also obtained for generic values of the parameters,[1] although the condition is different, being that the phase transition be slow enough, which implies that the tunnelling probability be small.[1,14,44]

In the model considered here reheating is supposed to happen during the phase (2.6), when, with the input of additional physics, the Brans-Dicke field converts into ordinary matter, and forces the universe into a radiation era, $H \simeq 1/2t$ (see e.g. Ref. 15-16). Since after recombination the universe will again evolve as if dominated by dust we shall ignore, for simplicity, this transitory radiation era and assume that the behaviour (2.6) holds ever after the transition (2.7).

Finally we note that in the limit $\varphi_0 \to \infty$, $V(\varphi) \to V_i$ for all finite φ. The solution of the Friedmann equations (2.2) is then the de Sitter universe: $\varphi = Const., H = \sqrt{V_i/3}$. This limit is obtained formally from the general solution (2.4-5) by "freezing" t at the value $t_{GR} = (\varphi_0^2/2)\sqrt{V_i/3}$. This is consistent with the fact that the times (2.8) t_{in} and t_e of the beginning and end of inflation coalesce to t_{GR} when $\varphi_0 \to \infty$; (as measured in cosmic time, the duration of the inflationary era then shrinks to zero).

3. Classical evolution of the perturbations

a. The evolution equation for the scalar perturbations

We outline here the analysis of Ref. 33 and 29, as applied by Ref. 18, 30-32 to a spatially flat Robertson-Walker background driven by a scalar field. It can be decomposed into the following steps:

Perturb the metric and the scalar field. Classify the metric perturbations into scalar, vectorial and tensorial types and consider the scalar perturbations only, as only these couple to the perturbation of the scalar field. See how they transform under coordinate changes. Look for linear combinations which are invariant under these changes. Find three such independent gauge invariant perturbations, denoted Φ, Ψ, χ. Write the linearized Einstein equations in terms of Φ, Ψ and χ. Obtain[18] two constraint equations:

$$\Psi = \Phi \qquad ; \qquad \tfrac{1}{2}\dot{\varphi}\chi = \dot{\Phi} + H\Phi \tag{3.1}$$

and one evolution equation:

$$\ddot{\Phi} + \dot{\Phi}\left(H - 2\frac{\ddot{\varphi}}{\dot{\varphi}}\right) - \Phi\left(2H\frac{\ddot{\varphi}}{\dot{\varphi}} + \dot{\varphi}^2\right) - \frac{\triangle\Phi}{a^2} = 0 \tag{3.2}$$

where $\triangle \equiv \delta^{ij}\partial_i\partial_j$. The set of scalar perturbations therefore possesses only one true dynamical degree of freedom. The physical interpretation of Φ, Ψ, χ is that in the "longitudinal" gauge, χ is the perturbation of φ and $ds^2 = -(1+2\Phi)dt^2 + a^2(1-2\Psi)d\vec{x}^2$. In that gauge then Φ is the relativistic equivalent of the perturbation of the Newtonian gravitational potential.

An alternative form of (3.2) is:[30]

$$Y'' - \triangle Y - \frac{w''}{w} Y = 0 \qquad \text{with} \qquad Y \equiv 2\frac{\Phi}{\dot\varphi} \quad \text{and} \quad w \equiv \frac{H}{\varphi'} \,. \tag{3.3}$$

When $\triangle\Phi = 0$, that is when $\Phi \equiv \Phi_0$ is spatially homogeneous, equation (3.2) can be rewritten as:

$$\dot\Phi_0 = -\dot H \Gamma_0 - H\Phi_0 \qquad , \qquad \dot\Gamma_0 = \Phi_0, \tag{3.4}$$

Γ_0 being defined by the first equation. The system (3.4) has the useful first integral:[18,34,35]

$$\zeta \equiv \Phi_0 + H\Gamma_0 = \Phi_0 \left(\frac{2H^2}{\dot\varphi^2} + 1 \right) + \frac{2H}{\dot\varphi^2} \dot\Phi_0 = Const. \tag{3.5}$$

Finally we will consider the gauge invariant density perturbation δ defined by the "Poisson" equation:

$$\delta \equiv \frac{2}{3} \frac{\triangle\Phi}{(Ha)^2} \,. \tag{3.6}$$

In the "comoving" gauge where the matter world-lines are perpendicular to the $t = Const$ surfaces (so that the perturbation of φ vanishes), δ reduces to the density constrast $\delta\rho/\rho$, where $\rho = 3H^2$ is the background energy density.[33,18]

b. The inflationary stage

During the inflationary stage $t_{in} < t < t_e$ with t_{in} and t_e given by (2.7-8), H, φ and a are approximated by (2.4) (2.5), so that $w \simeq \varphi_0/(2a)$ and equations (3.2) (3.3) reduce to:

$$\Phi_k \simeq \tfrac{1}{2}\varphi_0 \frac{Y_k}{t} \qquad ; \qquad Y_k'' + Y_k \left(k^2 - \frac{2\varphi_0^2}{(\varphi_0^2 - 2)^2} \frac{1}{\eta^2} \right) \simeq 0 \tag{3.7}$$

where we have written Φ (and Y) as a sum over modes:

$$\Phi = \int \frac{d^3k}{(2\pi)^{\frac{3}{2}}} \left(\Phi_k(\eta) e^{\mathrm{i}\vec k\cdot\vec x} + \Phi_k^*(\eta) e^{-\mathrm{i}\vec k\cdot\vec x} \right), \tag{3.8}$$

$k \equiv \sqrt{\vec k^2}$ being the comoving wave-number.

When $k \neq 0$ the solution of (3.7) is $\sqrt{k|\eta|}$ times a linear combination of Hankel functions of the first and second kind with argument $k|\eta|$ and index $\nu = \frac{1}{2}\frac{\varphi_0^2+2}{\varphi_0^2-2}$ (see e.g. Ref. 36). Qualitatively, it behaves as follows:

As η increases towards zero the k^2-term in (3.7) first dominates and the mode Y_k oscillates sinusoidally in conformal time; this reads, using equation (2.5):

$$Y_k(\eta) \simeq \frac{\alpha_k}{\sqrt{2k^3}} \exp\left(-\mathrm{i} \frac{\varphi_0^2}{\varphi_0^2 - 2} \frac{k}{Ha} \right) + \frac{\beta_k}{\sqrt{2k^3}} \exp\left(\mathrm{i} \frac{\varphi_0^2}{\varphi_0^2 - 2} \frac{k}{Ha} \right) \tag{3.9}$$

where α_k and β_k are (complex) integration constants. (The factor $\sqrt{2k^3}$ is added for later convenience.) "Hubble radius crossing" occurs when the physical wavelength a/k of the mode becomes larger than the Hubble length H^{-1}, that is for $\eta = \eta_{hc}$ or $t = t_{hc}$ such that:

$$-k\eta_{hc} \simeq \frac{\varphi_0^2}{\varphi_0^2 - 2} \qquad ; \qquad t_{hc} \simeq \left(\frac{2}{\varphi_0^2}k\right)^{\frac{2}{\varphi_0^2-2}} . \tag{3.10}$$

After that time the exponentials in (3.9) soon tend to 1 (unless $\varphi_0 \approx \sqrt{2}$) and the mode Y_k is "frozen". As for the mode Φ_k it decreases in $1/t$ (see (3.7)).

When now the k^2-term in (3.7) becomes negligible, that is after the transition time $\eta = \eta_{trans}$ or $t = t_{trans}$ such that:

$$-k\eta_{trans} \simeq \sqrt{2}\,\frac{\varphi_0}{\varphi_0^2 - 2} \qquad ; \qquad t_{trans} \simeq \left(\frac{\sqrt{2}}{\varphi_0}k\right)^{\frac{2}{\varphi_0^2-2}} , \tag{3.11}$$

the solution of (3.7) behaves like the homogeneous ($k = 0$) mode, and Φ_k is "frozen" at a constant value:

$$\Phi_k \;\rightarrow\; \tilde{\alpha} \times k^{-\frac{3\varphi_0^2-2}{2(\varphi_0^2-2)}} \tag{3.12}$$

(the other solution decays as $t^{-(\varphi_0^2/2+1)}$), where $\tilde{\alpha}$ is related to the initial amplitudes α_k and β_k by:[36]

$$\tilde{\alpha} \simeq \mathrm{i}\frac{\Gamma(\nu)}{\sqrt{\pi}}2^{-\frac{\varphi_0^2-4}{\varphi_0^2-2}}\frac{\varphi_0}{\sqrt{2}}\left(\frac{\varphi_0^2}{2}-1\right)^{\frac{2}{\varphi_0^2-2}}\left[\alpha_k \exp\left(-\mathrm{i}\frac{\pi}{2}\frac{\varphi_0^2}{\varphi_0^2-2}\right) - \beta_k \exp\left(\mathrm{i}\frac{\pi}{2}\frac{\varphi_0^2}{\varphi_0^2-2}\right)\right]. \tag{3.13}$$

For φ_0 significantly larger than $\sqrt{2}$ (e.g. $\varphi_0 \approx 5$), this simplifies to:

$$\tilde{\alpha} \approx \frac{\varphi_0}{2\sqrt{2}}(\alpha_k + \beta_k). \tag{3.14}$$

In our derivation of (3.12-14) we have assumed that during inflation equation (3.7) is both first dominated by the k^2 term (so that we can specify the initial conditions in terms of α_k and β_k) and later by the $1/\eta^2$ term (so that the decaying solution can be neglected with respect to the constant one). This will be true for a range of wavelengths such that $t_{in} < t_{trans} < t_e$. In order to see that all the modes of cosmological interest belong to that range, we introduce the physical wavelength $\lambda_{phys} \equiv a(t_{now})\lambda$, with $\lambda \equiv 1/k$, which is the size the mode would have today. To compute $a(t_{now})$ we use the fact that the temperature T of the radiation present in the universe is such that $aT = Const$ together with the observation that $T_{now} \simeq 3°$K and the hypothesis that the universe was reheated to a temperature $T_e \simeq 10^{28}$°K at the end of inflation. Given that inflation ended at a time given by (2.8) and that $a(t)$ grows according to (2.4), the condition $t_{in} < t_{trans} < t_e$ therefore reads:

$$\frac{\lambda_{phys}}{\lambda_{hc}} Z^{-\frac{\varphi_0^2-2}{\varphi_0^2}} < \frac{\varphi_0}{\sqrt{2}} < \frac{\lambda_{phys}}{\lambda_{hc}} \quad \text{with} \quad \lambda_{hc} \equiv \frac{T_e}{T_{now}}\sqrt{\frac{3\varphi_0^2-2}{\varphi_0^2 V_i}}. \tag{3.15}$$

Putting in numbers ($\varphi_0 \approx 5$ and $V_i \simeq 10^{-16}$), we have: $\lambda_{hc} \approx 10$ m and $Z \approx Z^{\frac{\varphi_0^2-2}{\varphi_0^2}} \approx 10^{26}$. This means that the formulae (3.12-14) are valid for all structures of cosmological relevance (from λ_{galaxy} to $\lambda_{obs.univ.}$) if φ_0 is in the range: $\varphi_0/\sqrt{2} > \lambda_{obs.univ.}/Z\lambda_{hc}$ and $\varphi_0/\sqrt{2} < \lambda_{galaxy}/\lambda_{hc}$. The first condition is satisfied as soon as φ_0 is of order 1. The second is satisfied unless φ_0 is *very* big, in which case the linear approximation is probably no longer valid; moreover we assumed $\varphi_0 < 5$.

Let us nevertheless consider the case $\varphi_0/\sqrt{2} > \lambda_{phys}/\lambda_{hc}$, that is the limit $\varphi_0 \to \infty$, to see how the general relativity limit is recovered in this model. In that case $t_{trans} > t_e$ and Y_k oscillates according to (3.9) during the *whole* inflationary era. However we still have that $t_{in} < t_{hc} < t_e$. Indeed this condition reads: $\lambda_{hc} < \lambda_{phys} < \lambda_{hc}Z$ which is satisfied for all scales of cosmological relevance. Therefore, although Y_k is now given by (3.9) $\forall t < t_e$, we have that it is frozen after t_{hc} at the value $(\alpha_k + \beta_k)/\sqrt{2k^3}$. As for Φ_k (see (3.7)), it is also frozen after t_{hc}, because $t_{in} \approx t_{hc} \approx t_e \approx t_{trans}$ when φ_0 is large. (That $t_{in} \approx t_e$ can be seen from (2.8); that $t_{hc} \approx t_e$ follows from the inequality $t_{in} < t_{hc} < t_e$, and also from rewriting (3.10) as: $t_{hc} \simeq [2T_e/(\varphi_0^2 T_{now}\lambda_{phys})]^{2/(\varphi_0^2-2)}\, t_e^{\varphi_0^2/(\varphi_0^2-2)} \to t_e$ when $\varphi_0 \to \infty$ with λ_{phys} fixed; rewriting t_{trans} (eq (3.11)) similarly, we see that it also tends to t_e (from above, as we are in the case $t_{trans} > t_e$). The fact that in the limit $\varphi_0 \to \infty$ all these times coalesce to $t_{GR} \approx (\varphi_0^2/2)\sqrt{3/V_i}$ was already mentionned at the end of &2.) Hence we have, $\forall t \in [t_{hc}, t_e]$:

$$\Phi_k \simeq \sqrt{\frac{V_i}{3}}\frac{1}{\varphi_0}\frac{\alpha_k + \beta_k}{\sqrt{2k^3}} \qquad \text{when} \qquad \frac{\varphi_0}{\sqrt{2}} > \frac{\lambda_{phys}}{\lambda_{hc}}. \tag{3.12bis}$$

We therefore see that a structure of size λ_{phys} measured today (from the size of a galaxy to that of the observable universe) has evolved from a fluctuation in the gravitational potential which, at the end of inflation, had the amplitude: 1) given by (3.12-14) if $\varphi_0/\sqrt{2} < \lambda_{phys}/\lambda_{hc}$, with $\lambda_{hc} \approx 10$ m (a formula which therefore applies for all structures as long as $\varphi_0/\sqrt{2} < \lambda_{galaxy}/\lambda_{hc}$, which is certainly the case if $\varphi_0 < 5$); 2) given by (3.12 bis) otherwise. Hence, for a given λ_{phys}, the fluctuation in the gravitational potential Φ_k grows linearly in φ_0 as long as $\varphi_0/\sqrt{2} < \lambda_{phys}/\lambda_{hc}$, and then decreases as $1/\varphi_0$ and tends to zero, in agreement with the fact that in a strictly de Sitter background the perturbations of the scalar field trapped in a false vacuum do not produce fluctuations in the metric; (although this cannot be seen from (3.2) which is not valid when $\dot\varphi = 0$, it follows straightforwadly from the evolution equations given in e.g. Ref. 29).

c. The late time stage

In the late time stage the background evolves according to (2.6) and $V(\varphi)$ is given by (1.6). Since $\dot\varphi$ regularly goes through zero, we first rewrite the evolution equation (3.2) for that phase in the well-behaved form:

$$\begin{aligned}\dot\Phi_k &\simeq -\dot H\Gamma_k - H\Phi_k(1+\gamma^2\dot\varphi)\\ \dot\Gamma_k &\simeq H\gamma^2\dot\varphi\Gamma_k + \Phi_k(1+2\gamma^2\dot\varphi - 2\gamma^4H^2)\end{aligned} \qquad \text{with} \quad \gamma^2 = \frac{1}{6}\left(\frac{k}{Ha}\right)^2\frac{\varphi}{H} \tag{3.16}$$

where Γ_k is defined by the first equation. In order to compare the values of the various coefficients we note that:

$$< \gamma^2 \dot{\varphi} > \simeq -\frac{1}{3} \left(\frac{k}{Ha} \right)^2 \frac{1}{m^2 t^2} \quad ; \quad < \gamma^4 H^2 > \simeq \frac{1}{27} \left(\frac{k}{Ha} \right)^4 \frac{1}{m^2 t^2} \tag{3.17}$$

where $< ... >$ denotes the average over the period $2\pi/m$ of the oscillations of φ. In the late stage mt is typically larger than 1 (see (2.7)), and k/Ha is less than 1 as long as the physical wavelength a/k of the mode remains larger than the Hubble length H^{-1}. Therefore, until at least the mode reenters the Hubble radius, the terms proportional to γ in the coefficients of Φ_k in (3.16) can be neglected. Taking then the average of (3.16), differentiating the first equation and eliminating Γ_k (using again (3.17)), we obtain:

$$\ddot{\Phi}_k + \frac{8}{3t} \dot{\Phi}_k \simeq 0\,, \tag{3.18}$$

the solution of which tends to a constant:

$$\Phi_k(t) \longrightarrow a_k. \tag{3.19}$$

(The other solution decays as $t^{-5/3}$.) This result is in keeping with the fact that, in a spatially flat universe dominated by dust, the perturbations of the Newtonian potential do not grow (see e.g. Ref. 37-38).

d. The amplification factor

In order to relate the final amplitude (3.19) of the modes to the amplitudes α_k and β_k of their oscillations during the inflationary phase before they "freeze", we recall that after the transition (3.11) all the modes, until the end of the inflationary era, behave like the $k = 0$ mode. They still do afterwards, until they have reentered the Hubble radius at least, as their evolution is governed by the k-independent equation (3.18). Now the evolution equation for the homogeneous mode is (3.4) which has the first integral (3.5). To obtain the amplification factor, we therefore evaluate ζ, first in the inflationary era, when Φ_k tends to the constant (3.12):

$$\zeta \simeq \tfrac{1}{2} \tilde{\alpha} \, (\varphi_0^2 + 2) \, k^{-\frac{3\varphi_0^2 - 2}{2(\varphi_0^2 - 2)}}. \tag{3.20}$$

We then evaluate ζ in the late stage, when Φ_k is given by (3.19):

$$\zeta \simeq \frac{5}{3} a_k \tag{3.21}$$

where we used (2.6), (3.19) and averaged over the oscillations of φ. Finally we equate the two values of ζ and obtain:

$$a_k \simeq \frac{3}{10} \, \tilde{\alpha} \, (\varphi_0^2 + 2) \, k^{-\frac{3\varphi_0^2 - 2}{2(\varphi_0^2 - 2)}} \tag{3.22}$$

where $\tilde{\alpha}$ is given by (3.13).

In short then, the modes Φ_k of the perturbations of the gravitational potential (in the longitudinal gauge) first oscillate in the inflationary era according to (3.7) (3.9), stop oscillating and decrease as $1/t$ when they cross the Hubble radius (at a time given by (3.10)), tend, after the transition time (3.11), to the constant (3.12), are amplified at the end of inflation and then remain constant (equation (3.19)) until they reenter the Hubble radius; their final amplitude is related to the amplitude of their oscillations during the inflationary stage by (3.22).

As for the evolution of the modes δ_k of the density contrast (in the comoving gauge), it follows from (3.6). Thus, in the late stage, when Φ_k is given by (3.22), we have:

$$\delta_k \simeq -\frac{1}{5}\frac{1}{(Ha)^2}\,\tilde{\alpha}\,(\varphi_0^2+2)\,k^{\frac{\varphi_0^2-6}{2(\varphi_0^2-2)}}. \tag{3.23}$$

For $\varphi_0 >> \sqrt{2}$, $\tilde{\alpha}$ is given by (3.14) and this reduces to:

$$\delta_k \approx -\frac{1}{5}\frac{k^{\frac{1}{2}(1-4/\varphi_0^2)}}{(Ha)^2}\left(\frac{\varphi_0}{\sqrt{2}}\right)^3(\alpha_k+\beta_k). \tag{3.24}$$

4. Quantization of the perturbations

The quickest route to the classical equations of motion (3.1) (3.2) for the perturbations Φ, Ψ and χ is to linearize Einstein's equations around the background (2.1) (2.2) (Ref. 18). Alternatively these can be derived from the expansion of the action (1.4) to quadratic order in Φ, Ψ and χ (Ref. 30). This alternative route to the equations of motion is compulsory when the system is to be quantized. Indeed quantization cannot be based on the equations of motion (3.1) (3.2) alone: an action is needed, to define the conjugate momenta. In the case at hand, using the constraints (3.1) to eliminate two of the three variables, the action for the only true dynamical degree of freedom can be written as:[30]

$$\delta S = -\tfrac{1}{2}\int d^4x\left[v'^2-(\vec{\nabla}v)^2+\frac{z''}{z}v^2\right] \tag{4.1}$$

where $z \equiv \varphi'/H = 1/w$ and where v is defined as:

$$v \equiv a\left(\chi+\frac{\dot{\varphi}}{H}\Psi\right) = Y' + \frac{z'}{z}Y \qquad \text{or} \qquad \triangle Y = z\left(\frac{v}{z}\right)'. \tag{4.2}$$

Because its action can be cast into the familiar form (4.1), the gauge invariant perturbation v can be interpreted as a scalar field with a time dependent mass ($M^2 \equiv -z''/z$) propagating on Minkowski space-time.

The equation of motion for v follows readily from (4.1):

$$v'' - \triangle v - \frac{z''}{z} v = 0 \tag{4.3}$$

and is equivalent to the evolution equation (3.3) for Y (see Ref. 30-32). During the inflationary stage $z \simeq 2a/\varphi_0$, and the equation for v reduces to:[45]

$$v_k'' + v_k \left(k^2 - \frac{2\varphi_0^2(\varphi_0^2 - 1)}{(\varphi_0^2 - 2)^2} \frac{1}{\eta^2} \right) \simeq 0 \tag{4.4}$$

where v has been decomposed into modes according to the template (3.8). For $k \neq 0$ the solution of (4.4) is $\sqrt{k|\eta|}$ times a linear combination of Hankel functions of the first and second kind with argument $k|\eta|$ and index $\nu = \frac{3\varphi_0^2 - 2}{2(\varphi_0^2 - 2)}$. As long as $k|\eta| > \sqrt{2}\,\varphi_0 \sqrt{\varphi_0^2 - 1}/(\varphi_0^2 - 2)$ (which corresponds to a time anterior to the Hubble crossing time defined by (3.10) and falls in the inflationary era for all scales of cosmological interest), v_k oscillates in conformal time as:

$$v_k \simeq c_k e^{\mathrm{i}k\eta} + d_k e^{-\mathrm{i}k\eta} \tag{4.5}$$

where c_k and d_k are related to the amplitudes α_k and β_k of the oscillations of Y_k through (4.2) and (3.9) by:

$$c_k = \frac{\mathrm{i}}{\sqrt{2k}}\,\alpha_k \qquad \text{and} \qquad d_k = -\frac{\mathrm{i}}{\sqrt{2k}}\,\beta_k. \tag{4.6}$$

The canonical quantization of v is standard (see e.g. Ref. 40 and 32). As for the quantization of the other gauge invariant scalar perturbations, e.g. Y or δ, it follows from that of v through the constraints which relate them to v (e.g. equation (4.2) or (3.6)). One thus constructs the field operator:

$$\hat{v} = \int d^3k \left(\tilde{v}_{\vec{k}} \hat{a}_{\vec{k}} + \tilde{v}_{\vec{k}}^* \hat{a}_{\vec{k}}^+ \right) \tag{4.7}$$

where the eigenmodes

$$\tilde{v}_{\vec{k}} \equiv \frac{1}{(2\pi)^{\frac{3}{2}}} v_k(\eta) \exp(\mathrm{i}\vec{k} \cdot \vec{x}) \tag{4.8}$$

form a complete set of orthonormal solutions of (4.3) with respect to the Klein-Gordon norm. This implies a condition on the Wronskian: $v_k \partial_\eta v_k^* - v_k^* \partial_\eta v_k = \mathrm{i}$ and hence:

$$|d_k|^2 - |c_k|^2 = \frac{1}{2k} \qquad \text{or} \qquad |\beta_k|^2 - |\alpha_k|^2 = 1. \tag{4.9}$$

The action being (4.1), the conjugate momentum of the field $\hat{v}$ is $\partial_\eta \hat{v}$, and quantization is implemented by imposing on the operators $\hat{a}_{\vec{k}}$ and $\hat{a}_{\vec{k}}^+$ the canonical commutation relations:

$$\left[\hat{a}_{\vec{k}}, \hat{a}_{\vec{k}}^+ \right] = \delta(\vec{k} - \vec{k}'). \tag{4.10}$$

Finally we choose the state of the system to be the vacuum state $|\,0>$ defined by $\hat{a}_{\vec{k}}\,|\,0>= 0\;\forall\,\vec{k}$ together with the requirement:

$$c_k = 0 \quad\Rightarrow\quad |d_k|^2 = \frac{1}{2k} \qquad \text{or} \qquad \alpha_k = 0 \quad\Rightarrow\quad |\beta_k|^2 = 1. \tag{4.11}$$

This state is the analogue of the so-called Bunch-Davies vacuum (see e.g. Ref. 41) which is usually chosen in the context of quasi-de Sitter inflation (see e.g. Ref. [17]). We see on (4.11) how the quantization of v determines the normalization of the modes Y_k, and hence their amplitude before transition in the inflationary stage.

The quantity of interest is the two-point function of the field $\hat{\delta}$:

$$<0\,|\hat{\delta}(\eta,\vec{x})\hat{\delta}(\eta,\vec{x}')\,|\,0>= \frac{1}{(2\pi)^3}\int d^3k|\delta_k(\eta)|^2 e^{\mathrm{i}\vec{k}\cdot(\vec{x}-\vec{x}')} \tag{4.12}$$

which is identified with its equal time correlation function, as defined in classical statistics (see e.g. Ref. 38-39, and Ref. 42-43 for a discussion of the transition from the quantum to the classical regime). The power spectrum $P(k)$, or Fourier transform of the correlation function of δ, is hence given by:

$$P(k) = |\delta_k(\eta)|^2. \tag{4.13}$$

¿From the classical analysis of the evolution of the perturbations we know on one hand that in the late stage δ_k is given by (3.23). On the other hand quantization has determined the initial amplitudes of the modes α_k and β_k, which are given by (4.11). Therefore the power spectrum (4.13) is known. The average value δ_λ of the fluctuation on a given comoving scale $\lambda = 1/k$ is then defined as:[34,38]

$$\delta_\lambda^2 = k^3\, P(k). \tag{4.14}$$

At Hubble radius crossing ($a\lambda = H^{-1}$) it reads:

$$\delta_\lambda|_{hc} \simeq \frac{1}{5}\,|\tilde{\alpha}|\,(\varphi_0^2+2)\,\lambda^{\frac{2}{\varphi_0^2-2}}. \tag{4.15}$$

If we introduce the physical scale $\lambda_{phys} \equiv a(t_{now})\lambda$ and compute $a(t_{now})$ using the constancy of aT, T being the temperature of the background radiation, we obtain, given that inflation ends at a time given by (2.7) and that $a(t)$ grows according to (2.4):

$$\delta_\lambda|_{hc} \simeq \frac{|\Gamma(\nu)|}{10\sqrt{2\pi}}\,(\varphi_0^2+2)\left(\frac{3\varphi_0^2-2}{4V_i}\right)^{-\frac{\varphi_0^2}{2(\varphi_0^2-2)}}\left(\frac{\varphi_0^2-2}{\varphi_0}\lambda_{phys}\frac{T_{now}}{T_e}\right)^{\frac{2}{\varphi_0^2-2}}, \tag{4.16}$$

where, in our units, $T_{now} \approx 3°\mathrm{K} \approx 10^{-32}$ and $V_i \approx 10^{-16}$, $T_e \approx 10^{-28}$ if they correspond to the grand unification energy and temperature. For large φ_0^2 the spectrum (4.16) reduces to:

$$\delta_\lambda|_{hc} \approx \frac{1}{5\sqrt{6}}\,(\omega V_i)^{\frac{1}{2}}\left(\varphi_0\lambda_{phys}\frac{T_{now}}{T_e}\right)^{\frac{2}{\varphi_0^2}}. \tag{4.17}$$

The spectrum (4.16) (4.17) is our main result and agrees in order of magnitude with the more heuristic analyses of Ref. 19-20, as well as with Ref. 45. In Ref. 20 for example, the numerical coefficient in the amplitude of the fluctuations at Hubble radius crossing was found to be $\approx \sqrt{\frac{2\pi}{3}}(V_i\omega)^{1/2}$ (for large ω, and taking into account that in their units $G=1$: see their equation (4.10)), instead of $\frac{1}{5\sqrt{6}}(V_i\omega)^{1/2}$. This coefficient can be as small as 10^{-8} if $\omega = 25$ and $V_i \approx 10^{-16}$, the typical energy scale of grand unified theories. The spectrum scales as $\lambda^{\frac{2}{\varphi_0^2-2}} = \lambda^{\frac{4}{2\omega-1}}$. For $\omega = 25$, the largest allowed value according to Ref. 13-14, $\delta_\lambda|_{hc} \propto \lambda^{0.08}$. Whether or not this slight increase of power in the large range could account for the very large structures observed in the universe is beyond the scope of this work.

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