DAMTP R-90/24
November 1990

NUCLEOSYNTHESIS CONSTRAINTS ON AN EXTENDED JORDAN–BRANS–DICKE THEORY

Talk delivered at the First William Fairbank Meeting on
Gravitational Experiments in Space, Rome, September 14-19, 1990

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Abstract

In a natural extension of the Jordan–Brans–Dicke theory the scalar field can couple much more strongly to dark than to visible matter. At the moment the best limits on the coupling constants of the model are obtained from standard big–bang nucleosynthesis.

This talk could serve to illustrate how cosmology places instant limits on any alternative theory of gravity, thus qualifying the need for laboratory experiments. One such limit comes from the hot big bang model of nucleosynthesis, which is very succesful in predicting the primordial abundances of He, Li and D as inferred from observations. Having to trade this scenario for one that needs fine-tuning of free parameters must be considered as unfavourable. This talk also illustrates the fact that the relation between this consideration and limits on $\dot{G}/G$ today is highly model-dependent.

1. Nucleosynthesis

The standard big–bang model of nucleosynthesis has been used in the past to put a limit on the possible change of Newton's constant with time [2–6] or on the number of neutrino species. Either parameter influences the value of the expansion rate around the time of nucleosynthesis. In the standard model [7] the $n-p$ thermal equilibrium freezes out at $T_* \sim 9.2 \times 10^9 K$. Then the neutrons decay freely until the temperature has dropped to $T_n \sim 0.9 \times 10^9 K$, when nucleosynthesis stabilizes them in light nuclei. The main effect of a change in expansion rate is to modify the freeze–out temperature T_*, and thereby the ratio $(n/p)_*$. The speed–up factor ξ at the temperature T is defined as the ratio of the actual Hubble constant $\tilde{H}$ in a nonstandard model to the value of $\tilde{H}$ in the standard model with 3 light neutrino families given by $\tilde{H}^2$ (standard) $= (8\pi/3)(43/8)G_N\tilde{\rho}_\gamma$ where $\tilde{\rho}_\gamma = aT^4$ is the energy density of (visible) photons, and where G_N denotes the usual Newtonian gravitational constant.

A recent review [8] of the determination of the number of neutrino species from nucleosynthesis data found a good fit for

$$N_\nu \ = \ 2.2 \pm 0.7 \pm 0.2 \ . \qquad (1)$$

A similar result follows from Reference [9], the main source of uncertainty coming in both cases from the determination of Y_p which is marred by systematic errors and which cannot be better than

$Y_p = 0.235 \pm 0.010$ [8,9]. The (sole) effect of $N_\nu \neq 3$ is to change the number of relativistic degrees of freedom in the heat bath, and thereby to produce a constant speed–up factor with respect to the standard model with 3 light neutrino families equal to

$$\xi^2 \;=\; 1 + \frac{7}{43}\,(N_\nu - 3)\ . \tag{2}$$

Adding the errors in (1) in quadrature and taking 2σ–limits leads to a (probably conservative) "2σ" range for ξ^2 of

$$0.63 \;\leq\; \xi^2 \;\leq\; 1.11. \tag{3}$$

Another recent publication, using new measurements of the neutron half-life, gives a limit, for $N_\nu = 3$, of

$$0.75 \;\leq\; \xi^2 \;\leq\; 1.20\ , \tag{4}$$

The range (3,4) applies, strictly speaking, to a constant speed–up factor. This will be the case in our model if the dominating matter at the time of nucleosynthesis is relativistic. Even if it was not, the change of ξ between the beginning and the end of nucleosynthesis could be neglected within the accuracy of the present calculation observational constraints. For the same reason T_*, which is ξ-dependent, can be taken at its standard model value [14]. It follows that the calculation of the nucleosynthesis constraint on $\dot{G}$ splits into two independent parts: i) Running a numerical nucleosynthesis code with ξ as a free parameter, comparing with the observational constraints on primordial He, Li and D abundances and thus placing limits on ξ. ii) Computing ξ, at T_* of the standard model, for the alternative theory of gravity, thus putting limits on its free parameters and hence on $\dot{G}$.

2. An alternative theory of gravity

The Jordan-Brans-Dicke theory [15,16] was first formulated as

$$S = \int \left(\tilde{\phi}\tilde{R} - \frac{\omega_V}{\tilde{\phi}}\, \tilde{g}^{\mu\nu}\, \partial_\mu \tilde{\phi}\, \partial_\nu \tilde{\phi} \right) (-\tilde{g})^{\frac{1}{2}}\, d^4x \\ + \; S_V\left[\psi_V, \tilde{g}_{\mu\nu}\right] \tag{5}$$

(ψ_V denotes any matter field and $\tilde{g}_{\mu\nu}$ the metric it minimally couples to. V stands for "visible" matter, as we shall later consider additional "invisible" matter.) It is clear from this action that in a cosmological context $\tilde{\phi}^{-1}$ acts as an effective gravitational constant $\tilde{G}$:

$$\tilde{G} = \frac{1 + 2\beta_V^2}{16\pi\tilde{\phi}_c} \; . \tag{6}$$

$\tilde{\phi}_c$ is the cosmological value of $\tilde{\phi}$, which depends on the matter distribution in a field-theoretic way. Brans and Dicke felt that this was a better implementation of Mach's principle (inertia derives from the distant stars) than general relativity. It was soon discovered [17] that the theory can be recast in more familiar form by redefining the metric as $\tilde{g}_{\mu\nu} \equiv e^{2\beta_V \sigma} g_{\mu\nu}$, the scalar field as $2\kappa^2\tilde{\phi} \equiv e^{-2\beta_V \sigma}$ and the coupling constant as $\omega_V \equiv \left(4\beta_V{}^2\right)^{-1} - 3/2$:

$$S = \frac{1}{2\kappa^2} \int \left(R - g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma\right)(-g)^{\frac{1}{2}} d^4x \\ + S_V\left[\psi_V, e^{2\beta_V \sigma} g_{\mu\nu}\right] \tag{7}$$

In this form it is clear that the theory has a well-posed initial value problem, as it is just general relativity plus a scalar field, that $\omega_V > -3/2$, and that $\beta_V \to 0\,(\omega_V \to +\infty)$ is the general relativity limit. Because of this limit the JBD theory cannot be ruled out experimentally but only be constrained to be uninterestingly close to general relativity. The best experimental limit at present is $\beta_V \leq 0.032$ (95% confidence level), from radar time delay experiments in the solar system[18]. The theory obeys the weak equivalence principle, as the only coupling of matter to gravitation is the minimal coupling to the metric $\tilde{g}_{\mu\nu}$ (and not to σ separately). It breaks the strong equivalence principle, as gravitational self-energy couples to the metric $g_{\mu\nu}$. This "Einstein" form of the action also shows one reason why the theory still has a certain theoretical interest: String theory seems to predict the existence of a, perhaps even massless, "dilaton" field σ with the exponential coupling to matter shown here, and with $\beta = 1/\sqrt{2}$ $(\omega = -1)$ [19], with similar results from Kaluza-Klein theory. If one considers not $\tilde{g}_{\mu\nu}$ but $g_{\mu\nu}$ as the metric in S_V, the conformal factor $\exp 2\beta_V\sigma$ cancels out of terms such as $F_{\mu\nu}F^{\mu\nu}\sqrt{g}$ or $\bar{\Psi}\gamma^\mu D_\mu \Psi\sqrt{g}$, while making particle masses σ-dependent, as in the example of a single point particle:

$$S_V = \int d\tau\, m_0 \left(\dot{x}^\mu \dot{x}^\nu g_{\mu\nu} e^{2\beta_V\sigma}\right)^{1/2} \equiv \int d\tau\, m(\sigma)\left(\dot{x}^\mu \dot{x}^\nu g_{\mu\nu}\right)^{1/2} \qquad (8)$$

Any theory with a massless or ultralight scalar σ which couples to fermions as $\bar{\Psi}\Psi m_0 \beta\sigma$ mimics the JBD theory to first order, which can therefore serve as a phenomenological model for such ultralight scalars (compare Gary Gibbons' talk). Our system of units is based on defining $\hbar, c$ and the electron mass as constant. Therefore $\tilde{g}_{\mu\nu}$ must be considered as the physical metric and $\tilde{T}_i^{\mu\nu} \equiv 2(-\tilde{g})^{-\frac{1}{2}}\,\delta S_i/\delta\tilde{g}_{\mu\nu}$ as the physical energy momentum tensor.

Damour, Gibbons and Gundlach have proposed an extension of the JBD theory [20]. In the Einstein frame it is natural to consider a different value of β for the exotic dark or "invisible" matter that is often postulated to achieve $\Omega = 1$, that is, to add a carbon copy of the matter action S_V:

$$\begin{aligned} S = &\ \frac{1}{2\kappa^2}\int \left(R - g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma\right)(-g)^{\frac{1}{2}}\, d^4x \\ &+ S_V\left[\psi_V, e^{2\beta_V\sigma} g_{\mu\nu}\right] + S_I\left[\psi_I, e^{2\beta_I\sigma} g_{\mu\nu}\right] . \end{aligned} \qquad (9)$$

This proposal is not based on an underlying fundamental theory, but on our lack of knowledge about the low energy limit of such a theory. It is motivated by thinking of the invisible matter as the shadow matter of naive string theory, or by the requirement $\omega < 20(\beta > 0.11)$ which seems to be needed for bubble percolation in extended (old) inflation [13]. As the invisible matter couples minimally to a new metric $e^{2\beta_I\sigma}g_{\mu\nu} \equiv \hat{g}_{\mu\nu}$, the weak equivalence principle is now broken. The physical energy-momentum tensor of the invisible matter is not conserved with respect to the physical metric. (This can be interpreted as the result of spacetime-dependent particle masses with respect to our choice of physical units.) As our only evidence for the invisible matter is $\Omega = 1$, all this is not in contradiction with experiment. In contrast to the original JBD theory, $\dot{G}/G > 0$ if $\beta_V \beta_I < 0$. The best limit on β_I before the present work was $|\beta_I| < 1$ from the requirement that $H_0 t_0 > 0.4$ [20].

3. FRW cosmology

Details of the calculation of ξ from β_V and β_I can be found in [1]. Because of our motivation for the presence of invisible matter we assume a flat FRW metric

$$ds^2 = dt^2 + a^2 d\mathbf{x}^2 \tag{10a}$$

$$\tilde{ds}^2 = e^{2\beta_V \sigma} ds^2 = d\tilde{t}^2 + \tilde{a}^2 d\mathbf{x}^2 \tag{10b}$$

$$\hat{ds}^2 = e^{2\beta_I \sigma} ds^2 = d\hat{t}^2 + \hat{a}^2 d\mathbf{x}^2 \tag{10c}$$

and two homogeneous perfect fluids that interact only gravitationally, with pressure $\tilde{\rho}_V = \tilde{T}^0_0$, $\tilde{p} = \tilde{T}^1_1$ and correspondingly for $\hat{\rho}_I, \tilde{\rho}_I$ etc. The equations of motion can be most simply stated in a mixture of metrics:

$$\tilde{\rho}_V \, \tilde{a}^3(4) = const. \quad \text{for dust (radiation)} \tag{11a}$$

$$\hat{\rho}_I \, \hat{a}^3(4) = const. \quad \text{for dust (radiation)} \tag{11b}$$

$$3H^2 = 8\pi G(\rho_V + \rho_I) + \frac{1}{2}\dot{\sigma}^2 \tag{11c}$$

$$\ddot{\sigma} + 3H\sigma = 8\pi G \left[\beta_V(3p_V - \rho_V) + \beta_I(3p_I - \rho_I)\right]. \tag{11d}$$

Here $\dot{} \equiv \frac{d}{dt}$, and $G = \kappa^2/8\pi$ is an arbitrary scale which coincides with Newton's constant if we define $\sigma_0 = 0$. $\rho = e^{4\beta_V \sigma}\tilde{\rho} = e^{4\beta_I \sigma}\hat{\rho}$ follows from the definitions of $T_{\mu\nu}, \tilde{T}_{\mu\nu}$ and $\hat{T}_{\mu\nu}$. σ introduces an additional degree of freedom compared to general relativity, but for $p = 0, \rho/3$ or $-\rho$ there are attractors in the $\dot{\sigma} - H$ phase space, with a relaxation time which is of the order of the Hubble time. We have neglected this relaxation time, as well as the transition times from $p = \rho/3$ to $p = 0$, and from $\rho_V \ll \rho_I$ to $\rho_V \gg \rho_I$, which are of the same order, as these transitions are separated by many e-foldings of the scale factor. The calculation reduces then to powers of the scale factor. The present-day variation of the effective gravitational constant for visible matter is given by [20]

$$\dot{\tilde{G}}_0/(\tilde{G}_0\tilde{H}_0) = -\frac{4\beta_V\beta_I}{1-2\beta_V\beta_I} \simeq -4\beta_V\beta_I. \; , \tag{12}$$

The extended JBD theory has three simultaneous effects on the value of ξ :

1) $\tilde{G}$ could have been greater or less at nucleosynthesis than now.

2) As in all $K = 0$ cosmologies, dark matter adds to the total density.

3) If the dominating matter during nucleosynthesis is not relativistic ($p \neq \rho/3$) the dilaton field energy also adds its contribution.

We have considered only a conservative scenario, in which the total energy density and the effective gravitational constant at nucleosynthesis are separately close to their values in the standard model. The present theory would allow, when $\beta_V\beta_I < 0$, a much larger (invisible–matter–dominated) total density to be balanced by a much smaller effective gravitational constant. We feel, however, that this nonconservative scenario is both too unnatural, requiring fine–tuning, and too difficult to examine without a knowledge of the invisible matter and its possible nongravitational interactions with visible matter. We shall therefore assume that visible matter dominates at the epoch of nucleosynthesis. As, going backwards in time from the epoch of nucleosynthesis, $\tilde{\rho}_V$ grows as the fourth power of the redshift $\tilde{Z} \equiv 1 + \tilde{z} \equiv \tilde{a}_0/\tilde{a}$, and $\tilde{\rho}_I$ cannot grow faster than that, the universe will then have been dominated by ultrarelativistic matter from the end of inflation until well after the end of nucleosynthesis. During this span of time $\Box\sigma = 0$, from equation (11d), and there is sufficient time to relax to the radiation attractor $\dot{\sigma} = 0$. In consequence, the effective gravitational constant is constant until after nucleosynthesis, although at a different value from today's. When computing the overall cosmological evolution of $\tilde{a}$ and $\tilde{G}$, we consider, in each era, only the effect of the dominating matter. However, when studying the influence of our nonstandard model on nucleosynthesis, we also take into account the effect of the small ratio

$\tilde{\rho}_I/\tilde{\rho}_V$ on the expansion rate during freeze–out. We have assumed that the invisible matter has been pressure-free since before nucleosynthesis: It must have been pressure-free most of the time or it would have been dominant at nucleosynthesis. On the other hand, only a proposal for the invisible matter could fix when it became pressure-free.

4.Results

With the premises and assumptions just described, the limit (3) on ξ^2 give the limits on $\beta_V\beta_I$-space plotted in figure 1. The nucleosynthesis constraint puts a tighter constraint on β_I than the age of the universe. $\omega_I = -1$ is definitely ruled out, while $\omega_I < 20$ is still possible for all values of ω_V within the time delay constraint. The range of $\dot{G}_0$ allowed by the constraints on β_V, β_I is

$$-0.011 \leq \dot{\tilde{G}}_0/\tilde{G}_0\tilde{H}_0 \leq +0.039\ . \tag{13}$$

It is larger than the range derived by using the original JBD theory ($\beta_V \equiv \beta_I$) or a phenomenological ansatz $G \propto t^\alpha$, but, even in the favourable case where $\tilde{H}_0 = 100$ km/s/Mpc $= 1.02 \times 10^{-10}$ yr^{-1} the range (13) becomes $-1.1 \leq \dot{\tilde{G}}_0/\tilde{G}_0 \leq 4.0$ in units of 10^{-12} yr^{-1}, which seems too small to be measurable by existing observational techniques [10–12] because of "asteroid" [11] or "galactic" [12] noise. There remains, however, the interesting case $\beta_V = 0,\ \beta_I \neq 0$, in which $\tilde{G}$ is strictly constant, but in which dark matter has different gravitational couplings than visible matter, resulting in spacetime–dependent particle masses. This particular one–parameter theory allows strong couplings ($\omega_I > -\frac{1}{2}$) to a massless field, which could be interesting for the viability of bubble percolation in extended inflation [13].

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Figure 1:
a) $Ht > 0.4$, b) $\omega_V > 250$, c) $\xi^2 > 0.63$, d) $\xi^2 < 1.11$. In ddrawing c) and d) we have taken $H_0 = 75$ km s^{-1} Mpc^{-1}. The shaded region is still allowed. The fine–tuned corridor e) is drawn only schematically.