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NUCLEOSYNTHESIS CONSTRAINTS ON AN EXTENDED JORDAN–BRANS–DICKE THEORY

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Abstract

In a natural extension of the Jordan–Brans–Dicke theory the scalar field can couple much more strongly to dark than to visible matter. New limits on the coupling constants of the model are obtained from standard big–bang nucleosynthesis.

1. Introduction

In a recent publication [1] we have considered the cosmological effects of a scalar–tensor theory of gravity in which a massless "dilaton" scalar field couples with different strengths to dark matter, which is often postulated to achieve the theoretically desirable closure density, and to ordinary visible matter. This theory arises naturally from the Jordan–Brans–Dicke theory when it is expressed in terms of a Weyl–rescaled metric so that it reduces to general relativity plus a dilaton field :

$$\begin{aligned} S = & \frac{1}{2\kappa^2}\int \left(R - g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma\right)(-g)^{\frac{1}{2}}\, d^4x \\ & + S_V\left[\psi_V, e^{2\beta_V\sigma}g_{\mu\nu}\right] + S_I\left[\psi_I, e^{2\beta_I\sigma}g_{\mu\nu}\right] \ , \end{aligned} \tag{1}$$

where R denotes the Ricci scalar of $g_{\mu\nu}$, and S_V and S_I denote the actions of visible and invisible matter, with the first argument of each denoting the matter fields and the second argument denoting the metric they are (minimally) coupled to. This theory has two free dimensionless parameters, β_V and β_I, that measure the strengths of the coupling of the scalar to, respectively, visible and invisible matter (the usual, one–parameter, JBD theory is the particular case where $\beta_I = \beta_V$). What we must consider the physical metric, however, is that metric to which the visible matter is coupled, and in which therefore the energy–momentum tensor of visible matter is conserved. This is because our meter sticks, clocks and other experimental apparatus are made of visible matter. If we define this physical metric $e^{2\beta_V\sigma}g_{\mu\nu} \equiv \tilde{g}_{\mu\nu}$ (with curvature scalar $\tilde{R}$) and redefine the scalar field $e^{-2\beta_V\sigma} \equiv 2\kappa^2\tilde{\phi}$, the action reads

$$\begin{aligned} S = & \int \left(\tilde{\phi}\tilde{R} - \frac{\omega_V}{\tilde{\phi}}\,\tilde{g}^{\mu\nu}\,\partial_\mu\tilde{\phi}\,\partial_\nu\tilde{\phi}\right)(-\tilde{g})^{\frac{1}{2}}\, d^4x \\ & + \ S_V\left[\psi_V, \tilde{g}_{\mu\nu}\right] \ + \ S_I\left[\psi_I, \left(2\kappa^2\tilde{\phi}\right)^{\frac{\beta_V-\beta_I}{\beta_V}}\tilde{g}_{\mu\nu}\right] \ , \end{aligned} \tag{2}$$

where $\tilde{\phi}$ is $1/16\pi$ times the Jordan–Brans–Dicke scalar and $\omega_V \equiv \left(4\beta_V{}^2\right)^{-1} - 3/2$ is the Jordan–Brans–Dicke coupling constant. As in the usual JBD theory the cosmological evolution of $\tilde{\phi}$ drives an adiabatic change of the locally measured gravitational constant $\tilde{G}$:

$$\tilde{G} = \frac{1 + 2\beta_V^2}{16\pi\tilde{\phi}_c} \ , \tag{3}$$

where $\tilde{\phi}_c$ is the cosmological value of $\tilde{\phi}$. The main new feature of the extended JBD theory is that the time variation of $\tilde{G}$ depends now also on the extra parameter β_I.

In Ref. [1] we have shown that solar–system gravitational experiments constrain $|\beta_V|$ to be smaller than 0.032 (95 % confidence level), and that the limits on the present age and cosmological expansion of the universe constrain $|\beta_I|$ to be smaller than 1.0 (95 % C.L.). In this paper we put new limits on the range of the coupling constants β_V and β_I (and hence on the time variation of Newton's constant) from the requirement that cosmological nucleosynthesis proceeds by the same scenario in our model as in the standard model and yields abundances compatible with observations. The plan of the paper is as follows : in section 2 we present the needed equations of motion in the context of homogeneous cosmology. In section 3 we review the literature on constraints on the cosmological expansion rate at the epoch of nucleosynthesis. In section 4 we translate this into constraints on β_V and β_I. Section 5 contains our conclusions. In the remainder of the paper we shall always work with the physical metric, but retain the tilde marking quantities in this metric for easy comparison with [1].

2. The equations for homogeneous cosmology

The equations obtained by varying the action (2) with respect to $\tilde{g}_{\mu\nu}$ and $\tilde{\phi}$ are as follows :

$$\begin{aligned} \tilde{G}_{\mu\nu} = & \frac{1}{2\tilde{\phi}} \left(\tilde{T}^V_{\mu\nu} + \tilde{T}^I_{\mu\nu} \right) + \frac{\omega_V}{\tilde{\phi}^2} \left[\tilde{\phi}_{,\mu} \tilde{\phi}_{,\nu} - \frac{1}{2} \tilde{g}_{\mu\nu} \left(\tilde{\nabla} \tilde{\phi} \right)^2 \right] \\ & + \frac{1}{\tilde{\phi}} \left(\tilde{\nabla}_\mu \tilde{\nabla}_\nu \tilde{\phi} - \tilde{g}_{\mu\nu} \tilde{\Box} \tilde{\phi} \right) , \end{aligned} \tag{4}$$

$$\tilde{\Box} \tilde{\phi} = \beta_V^2 \left(\tilde{T}^V + \frac{\beta_I}{\beta_V} \tilde{T}^I \right) , \tag{5}$$

while the matter conservation laws read

$$\tilde{\nabla}_\lambda \tilde{T}_V^{\lambda\mu} = 0 , \tag{6a}$$

$$\tilde{\nabla}_\lambda T_I^{\lambda\mu} = \frac{1}{2} \frac{\beta_V - \beta_I}{\beta_V} \tilde{T}_I \tilde{\nabla}^\mu \ln \tilde{\phi} . \tag{6b}$$

Here $\tilde{T}_i^{\mu\nu} \equiv 2(-\tilde{g})^{-\frac{1}{2}} \delta S_i / \delta \tilde{g}_{\mu\nu}$ for $i = V$ and I. Assuming the metric to be $d\tilde{s}^2 = -d\tilde{t}^{\,2} + \tilde{a}^2(\tilde{t}) \left(\frac{dr^2}{1-Kr^2} + r^2 d\Omega^2 \right)$ and each kind of matter to be a perfect fluid with density $\tilde{\rho}_i$ and pressure $\tilde{p}_i$, we obtain the Friedmann equation

$$\begin{aligned} 3\tilde{H}^2 + 3\frac{K}{\tilde{a}^2} = & \frac{1}{2\tilde{\phi}} \left(\tilde{\rho}_V + \tilde{\rho}_I \right) \\ & + \frac{\omega_V}{2} \tilde{Y}^2 - 3\tilde{H}\tilde{Y} , \end{aligned} \tag{7}$$

where $\tilde{H} = \dot{\tilde{a}}/\tilde{a}$ and $\tilde{Y} = \dot{\tilde{\phi}}/\tilde{\phi}$, the dot denoting the $\tilde{t}$–derivative ($\tilde{Y} = -2\beta_V \tilde{y}$ in the notation of Ref. [1]). In the following we will set $K = 0$, consistent with a dark matter cosmology. The conservation equation (6b) becomes

$$\dot{\tilde{\rho}}_I + 3\tilde{H} \left(\tilde{\rho}_I + \tilde{p}_I \right) = \left(3\tilde{p}_I - \tilde{\rho}_I \right) \frac{\beta_I - \beta_V}{2\beta_V} \tilde{Y} . \tag{8}$$

If we can neglect $\tilde{\rho}_V$ and $\tilde{p}_V$ compared to $\tilde{\rho}_I$, and if $\tilde{p}_I = (\gamma - 1)\tilde{\rho}_I$ holds for a constant γ, eqs. (7) and (8) yield an autonomous first–order evolution system for $\tilde{H}$ and $\tilde{Y}$. The right–hand sides of this system are homogeneous (of degree 2) in the variables (see Ref. [1]), which allows one to reduce the integration of this system to quadratures when using the variables $\tilde{H}$ and $\eta \equiv \tilde{Y}/\tilde{H}$. However we shall not need the explicit form of the general solution. There is an attractor in the $\tilde{H} - \tilde{Y}$ phase space given by

$$\tilde{Y} = \frac{-2\beta_I\beta_V}{\beta_I\beta_V - \frac{1-\gamma/2}{4-3\gamma}}\,\tilde{H}\ . \tag{9}$$

The relaxation time to this attractor is of order $\tilde{H}^{-1}$ at any given epoch, as is easily seen to follow from the homogeneity properties of the general solution (or from the fact that there is no other scale in the theory, κ^2 being an arbitrary scale). Therefore, when γ remains effectively constant at $\gamma = 1,\ 4/3$ or 0 (corresponding to cold, hot and vacuum matter), the transition is short compared to the duration of the next era. Figure 1 shows, schematically, a typical phase space trajectory. Note that a constant shift in $\ln\tilde{\phi}$ corresponds only to a change in the arbitrary scale κ^2. The origin of the diagram corresponds to the point of largest expansion for a closed universe, or to the infinite future for an open universe. The Planck boundary is defined by the requirement that the Hubble time of a homogeneous universe be greater than the Compton wavelength of the mass contained within a Hubble horizon, i.e. that the Hubble time be greater than the Planck time. The other boundary is given by $\tilde{\rho}_V + \tilde{\rho}_I \geq 0$ in (7), for $K = 0$.

3. Constraints on the expansion rate at nucleosynthesis

The standard big–bang model of nucleosynthesis has been used in the past to put a limit on the possible change of Newton's constant with time [2–6] or on the number of neutrino species. Either parameter influences the value of the expansion rate around the time of nucleosynthesis. In the standard model [7] the $n-p$ thermal equilibrium freezes out at $T_* \sim 9.2 \times 10^9 K$. Then the neutrons decay freely until the temperature has dropped to $T_n \sim 0.9 \times 10^9 K$, when nucleosynthesis stabilizes them in light nuclei. The main effect of a change in expansion rate is to modify the freeze–out temperature T_*, and thereby the ratio $(n/p)_*$. The speed–up factor ξ at the temperature T is defined as the ratio of the actual $\tilde{H}$ in a nonstandard model to the value of $\tilde{H}$ in the standard model with 3 light neutrino families given by $\tilde{H}^2$ (standard) $= (8\pi/3)(43/8)G_N\tilde{\rho}_\gamma$ where $\tilde{\rho}_\gamma = aT^4$ is the energy density of (visible) photons, and where G_N denotes the usual Newtonian gravitational constant.

A recent review [8] of the determination of the number of neutrino species from nucleosynthesis data found a good fit for

$$N_\nu \;=\; 2.2 \pm 0.7 \pm 0.2 \; . \tag{10}$$

A similar result follows from Reference [9], the main source of uncertainty coming in both cases from the determination of Y_p which is marred by systematic errors and which cannot be better than $Y_p = 0.235 \pm 0.010$ [8,9]. The (sole) effect of $N_\nu \neq 3$ is to change the number of relativistic degrees of freedom in the heat bath, and thereby to produce a constant speed–up factor with respect to the standard model with 3 light neutrino families equal to

$$\xi^2 \;=\; 1 + \frac{7}{43}\,(N_\nu - 3) \;. \tag{11}$$

Adding the errors in (10) in quadrature and taking 2σ–limits leads to a (probably conservative) "2σ" range for ξ^2 of

$$0.63 \;\leq\; \xi^2 \;\leq\; 1.11 \; . \tag{12}$$

The range (12) applies, strictly speaking, to a constant speed–up factor. This will be the case in our model if the dominating matter at the time of nucleosynthesis is relativistic (see the discussion in Section 4 below).

4. The nucleosynthesis constraint in the present theory

The present theory has three simultaneous effects on the value of ξ :

1) $\tilde{G}$ could have been greater or less at nucleosynthesis than now.

2) As in all $K = 0$ cosmologies, dark matter adds to the total density.

3) If the dominating matter during nucleosynthesis is not relativistic ($\gamma \neq 4/3$) the dilaton field energy also adds its contribution.

We shall consider here only a conservative scenario, in which the total energy density and the effective gravitational constant at nucleosynthesis are separately close to their values in the standard model. The present theory would allow, when $\beta_V \beta_I < 0$, a much larger (invisible–matter–dominated) total density to be balanced by a much smaller effective gravitational constant. We feel, however, that this nonconservative scenario is both too unnatural, requiring fine–tuning, and too difficult to examine without a knowledge of the invisible matter and its possible nongravitational interactions with visible matter. We shall therefore assume that visible matter dominates at the epoch of nucleosynthesis. As, going backwards in time from the epoch of nucleosynthesis, $\tilde{\rho}_V$ grows as the fourth power of the redshift $\tilde{Z} \equiv 1 + \tilde{z} \equiv \tilde{a}_0/\tilde{a}$, and $\tilde{\rho}_I$ cannot grow faster than that, the universe will then have been dominated by ultrarelativistic matter from the end of inflation until well after the end of nucleosynthesis. During this span of time $\tilde{\Box}\, \tilde{\phi} = 0$, from eq. (5) with $\tilde{T}_i = -(\tilde{\rho}_i - 3\tilde{p}_i)$, and there is sufficient time for $\tilde{Y} = \dot{\tilde{\phi}}/\tilde{\phi}$ to relax to the radiation attractor $\tilde{Y} = 0$. In consequence, the effective gravitational constant is constant until after nucleosynthesis, although at a different value from today's. When computing the overall cosmological evolution of $\tilde{a}$ and $\tilde{G}$, we consider, in each era, only the effect of the dominating matter. However, when studying the influence of our nonstandard model on nucleosynthesis, we shall also take into account the effect of the small ratio $\tilde{\rho}_I/\tilde{\rho}_V$ on the expansion rate during freeze–out.

With the assumptions just described, we obtain that our model differs from the standard one by an approximately constant speed–up factor given by

$$\xi^2 = \frac{\tilde{G}_*}{\tilde{G}_0}\left(1 + \frac{\tilde{\rho}_{I*}}{\tilde{\rho}_{V*}}\right) , \tag{13}$$

where the index * refers to the freeze–out time.

In our calculation, we need to designate the following times (see Figure 2) : t_* is the freeze–out time, essentially fixed by an approximate equality between the weak interaction time scale and the expansion time scale. In our conservative scenario the corresponding redshift differs little from its standard value, and it will be sufficient to use in eq. (13) the value $\tilde{Z}_* = T_*^{\mathrm{standard}}/T_0 = 9.2 \times 10^9 K/2.7\,K = 3.4 \times 10^9$. Moreover, let t_3 denote the time when the invisible pressure changes from $\tilde{\rho}_I/3$ to 0. As is commonly done in this kind of calculation, we will assume this changeover to be instantaneous. If $t_3 > t_*$, we need to replace $\tilde{Z}_*$ by $\tilde{Z}_3$ in eq. (17) below. Let t_2 designate the switchover from visible to invisible matter domination, also approximated as instantaneous and as accompanied by instantaneous relaxation to the new attractor. Finally, t_1 denotes the time when the visible matter becomes pressure–free, given by $\tilde{Z}_1 = \tilde{\rho}_{d0}/1.68\tilde{\rho}_{\gamma 0}$, where $\tilde{\rho}_{d0}$ and $\tilde{\rho}_{\gamma 0}$ denote the present densities of visible dust and photons, and where the factor 1.68 takes into account the energy density of three neutrino species at a present temperature $T_{\nu 0} = (4/11)^{1/3}T_{\gamma 0}$ [7].

We need to know the present density of invisible matter. By the assumption of flat cosmology, $\tilde{\rho}_{V0} + \tilde{\rho}_{I0} = \tilde{\rho}_{c0}$, where the latter "critical" density can be obtained from eq. (7) using the invisible–dust–dominated attractor $\tilde{Y} \simeq 4\beta_V\beta_I\tilde{H}$ (eq. (9) with $\gamma = 1$, and $\beta_I\beta_V \ll 1$), as

$$
\begin{aligned}
\tilde{\rho}_{c0} &\simeq \frac{3\tilde{H}_0^2}{8\pi\tilde{G}_0}\,(1+2\beta_V^2)\left(1-\frac{2}{3}\,\beta_I^2+4\beta_V^2+4\beta_V\beta_I\right) \\
&\simeq \frac{3\tilde{H}_0^2}{8\pi\tilde{G}_0}\left(1-\frac{2}{3}\,\beta_I^2\right)\ . \qquad (14)
\end{aligned}
$$

Here and in the following we neglect whenever we can all fractional corrections of order β_V or β_V/β_I, purely to keep expressions simple. In this approximation the evolution equation (8) for $\tilde{\rho}_I$ yields, when $\tilde{Z} < \tilde{Z}_2$, i.e. on the invisible–dust–dominated attractor (9),

$$
\tilde{\rho}_I \propto \tilde{a}^{-(3+2\beta_I^2)} \propto \tilde{Z}^{3+2\beta_I^2}\ ,
$$

from which one deduces already that β_I^2 must be strictly smaller than $\frac{1}{2}$ for the switchover from visible to invisible matter domination to exist. On the other hand, when $\tilde{Z} > \tilde{Z}_2$, i.e. on the visible–radiation–dominated attractor ($\tilde{Y} = 0$), the evolution of $\tilde{\rho}_I$ follows the usual scaling laws : $\propto \tilde{Z}^3$ for dust, and $\propto \tilde{Z}^4$ for radiation (see Figure 2). The visible/invisible domination switchover is therefore determined by $\tilde{Z}_2^{1-2\beta_I^2} = \tilde{\rho}_{I0}/1.68\tilde{\rho}_{\gamma 0}$. Using eq. (14), we obtain

$$
\tilde{Z}_2 = \left[\left(1-\frac{2}{3}\,\beta_I^2\right)R\right]^{\frac{1}{1-2\beta_I^2}}\ , \qquad (15a)
$$

where we have introduced the notation

$$
R = \frac{3}{8\pi\tilde{G}_0}\,\frac{\tilde{H}_0^2}{1.68\tilde{\rho}_{\gamma 0}} \ \simeq\ 1.5\times 10^4\left(\frac{\tilde{H}_0}{75\,\mathrm{km\ s^{-1}Mpc^{-1}}}\right)^2\ . \qquad (15b)
$$

We find further

$$
\frac{\tilde{G}_*}{\tilde{G}_0} = \tilde{Z}_2^{4\beta_V\beta_I}\ , \qquad (16)
$$

$$
1+\frac{\rho_{I*}}{\rho_{V*}} = 1+0.31\,\tilde{Z}_2\,\tilde{Z}_*^{-1}\ , \qquad (17)
$$

where the factor 0.31=1.68/(43/8) takes into account the change of the effective number of radiation degrees of freedom before and after e^+e^- annihilation.

In the case $\beta_V \beta_I > 0$, both right–hand sides of eqs. (16) and (17), i.e. both factors in the right–hand side of eq. (13), are greater than one, and as we are demanding (eq. (12)) $\xi^2 \leq 1.11$, it follows that necessarily $\rho_{I*} < 0.11\ \rho_{V*}$, which is within our conservative scenario assumption. The resulting limit on β_V and β_I, following from $\xi^2 \leq 1.11$, is plotted in Figure 3. In the case $\beta_V \beta_I < 0$, the factor $\tilde{G}_* / \tilde{G}_0$ in eq. (13) is smaller than one, while the other factor, $1 + \tilde{\rho}_{I*}/\tilde{\rho}_{V*}$, is greater than one. Therefore, in that case, part of the nucleosynthesis limit on β_V and β_I derives from the lower limit in eq. (12), $\xi^2 \geq 0.63$, and part from the upper limit $\xi^2 \leq 1.11$. These two lines (see Figure 3) never intersect, but are cut off by the line $\tilde{\rho}_{I*} = \tilde{\rho}_{V*}$ which marks the boundary of our conservative scenario assumption. If we were to allow a strong departure from the standard scenario, we would expect these two lines to approach each other and to form a narrow "corridor" centered around the fine–tuned line defined by equating to one the product of the right–hand sides of eqs (16) and (17). This corridor is schematically indicated in Figure 3.

In eq. (17) (and in Fig. 3) we have assumed $\tilde{Z}_3 > \tilde{Z}_* (> \tilde{Z}_2)$ because, in absence of a theory of the invisible matter, it is simpler to assume that invisible matter is already pressure–free at nucleosynthesis. Note, however, that if $\tilde{Z}_* > \tilde{Z}_3 > \tilde{Z}_2$ one must replace $\tilde{Z}_*^{-1}$ by $\tilde{Z}_3^{-1}$ in eq. (17) and that this leads to tighter limits on β_I^2. One would have to specify a theory of the invisible matter to discuss more precisely this case.

Finally, recalling from Ref. [1] that eq. (3) above predicts a present time variation of $\tilde{G}$ given by

$$\dot{\tilde{G}}_0 / (\tilde{G}_0 \tilde{H}_0) = -\frac{4\beta_V \beta_I}{1 - 2\beta_V \beta_I} \simeq -4\beta_V \beta_I \ , \tag{18}$$

we find from Fig. 3 that the nucleosynthesis constraints tighten the limits on $\dot{\tilde{G}}_0$ obtained in Ref. [1] down to the range

$$-0.011 \leq \dot{\tilde{G}}_0 / \tilde{G}_0 \tilde{H}_0 \leq +0.039 \ . \tag{19}$$

Even in the favourable case where $\tilde{H}_0 = 100$ km/s/Mpc $= 1.02 \times 10^{-10}$

yr^{-1} the range (19) becomes $-1.1 \leq \dot{\tilde{G}}_0/\tilde{G}_0 \leq 4.0$ in units of 10^{-12} yr^{-1}, which seems too small to be measurable by existing observational techniques [10–12] because of "asteroid" [11] or "galactic" [12] noise.

5. Conclusions

The present theory [1] was conceived as a consistent field theoretic model for a time–dependent G which would allow a larger value of $\dot{G}$ at a given small Jordan–Brans–Dicke coupling (large ω) than the original JBD theory. The motivation was that ω (ω_V in the present theory) had been more tightly measured than $\dot{G}$. A second motivation was that the new coupling ω_I could be given the value -1 (or $\beta_I^2 = \frac{1}{2}$) predicted by naive string theory. This last possibility is now ruled out. If we assume that the invisible matter becomes dominant only after nucleosynthesis, the upper limit $\beta_I^2 < \frac{1}{4}$ or $\omega_I > -\frac{1}{2}$ follows (see Figure 3), which is still very weak. The nucleosynthesis calculation puts weaker limits on the present time variation of G in our theory than it does in a purely phenomenological approach to $\dot{G}$ [2–6], where one assumes a power–law variation of $G(t)$ from nucleosynthesis until now. The allowed range for $\dot{G}$ in the present theory is also larger than in standard JBD theory ($\beta_I \equiv \beta_V \equiv \beta$), but smaller than present experimental limits, and possibly outside their reach. This result diminishes the interest of the extended JBD theory as a model for the time variation of the gravitational attraction between bodies made of visible matter. There remains, however, the interesting case $\beta_V = 0$, $\beta_I \neq 0$, in which $\tilde{G}$ is strictly constant, but in which dark matter has different gravitational couplings than visible matter, resulting in spacetime–dependent particle masses. This particular one–parameter theory allows strong couplings ($\omega_I > -\frac{1}{2}$) to a massless field, which could be interesting for the viability of bubble percolation in extended inflation [13], which requires $\omega < 20$.

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Figure captions.

Figure 1

The phase space for $K = 0$ cosmology (schematic).

Figure 2

Evolution of the various contributions to the matter density (the numbers on the figure indicating the corresponding slopes), and definition of the various transition times t_1, t_2 and t_3 (schematic). The figure assumes $t_3 < t_* < t_2$. The kink in the visible radiation contribution occurs at e^+e^- annihilation, i.e. at $\tilde{Z} \simeq 5 \times 10^9$ K $/2.7$ K $\simeq 0.54\ \tilde{Z}_*$.

Figure 3

Constraints on β_V and β_I for $t_3 < t_* < t_2 < t_1$.

a) $Ht\ > 0.4$, b) $\omega_V > 250$, c) $\xi^2 > 0.63$, d) $\xi^2 < 1.11$.

In drawing c) and d) one has taken $H_0 = 75$ km s^{-1} Mpc^{-1}, i.e. $R = 1.5 \times 10^4$. The shaded region is still allowed. The fine–tuned corridor e) is drawn only schematically.