Critical collapse of a rotating radiation fluid

Carsten Gundlach (with Thomas Baumgarte) PRL 2016

Mathematical Sciences
University of Southampton

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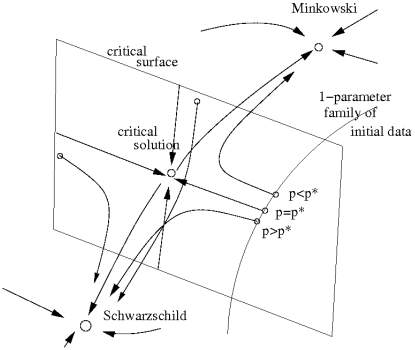
- Fine-tune initial data to threshold of collapse
- (Approximately) scale-invariant physics
  - Arbitrarily small black hole mass
  - Universality and critical exponents
- Critical solution: self-similar with exactly one growing mode



- Continuous self-similarity in adapted coordinates (τ, x^i) and variables $Z(\mathbf{x})$

$$g_{\mu\nu}(\tau,\mathbf{x}) = e^{-2\tau}\tilde{g}_{\mu\nu}(\mathbf{x}) \qquad \rho(\tau,\mathbf{x}) = e^{2\tau}\tilde{\rho}(\mathbf{x}), \qquad \mathbf{v} = \mathbf{v}(\mathbf{x})$$

- τ indicates scale

$$\text{any length} \sim e^{-\tau}, \qquad Ricc \sim e^{2\tau}, \qquad M \sim e^{-(D-3)\tau}$$

- CSS if and only if $Z(\mathbf{x},\tau) = Z(\mathbf{x})$
- but τ can also be the time coordinate

- Consider 2-parameter families of initial data with "strength" p and "rotation" $\mathbf{q}$

$$
\begin{aligned}
M(p, -\mathbf{q}) &= M(p, \mathbf{q}) \\
\mathbf{J}(p, -\mathbf{q}) &= -\mathbf{J}(p, \mathbf{q})
\end{aligned}
$$

- Restrict to axisymmetry
- Can assume $p_* = 0$ for $q = 0$

- Critical solution Z_*, unique growing perturbation Z_0, dominant $l = 1$ axial perturbation Z_1

$$Z(\mathbf{x},\tau) \simeq Z_*(\mathbf{x}) + P(p,q)\, e^{\lambda_0 \tau} Z_0(\mathbf{x}) + Q(p,q)\, e^{\lambda_1 \tau} Z_1(\mathbf{x}) + \dots$$

- From $q \to -q$ symmetry

$$\begin{aligned} P &\simeq p - Kq^2 \\ Q &\simeq q \end{aligned}$$

to leading order in p, q^2

- When AH forms/dispersion starts

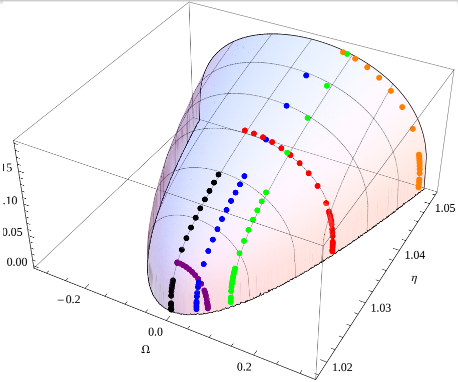
$$\begin{aligned} Z(\mathbf{x},\tau) &\simeq Z_*(\mathbf{x}) + P(p,q)\, e^{\lambda_0\tau} Z_0(\mathbf{x}) + Q(p,q)\, e^{\lambda_1\tau} Z_1(\mathbf{x}) \\ &\simeq Z_*(\mathbf{x}) \pm Z_0(\mathbf{x}) + \delta\; Z_1(\mathbf{x}) \end{aligned}$$

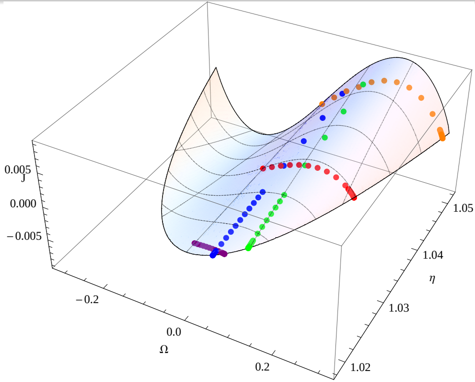
- This happens at $\tau_*(p,q)$ defined by

$$|P|\; e^{\lambda_0\tau_*} = 1 \qquad \Rightarrow \qquad \delta := Q|P|^{-\frac{\lambda_1}{\lambda_0}}$$

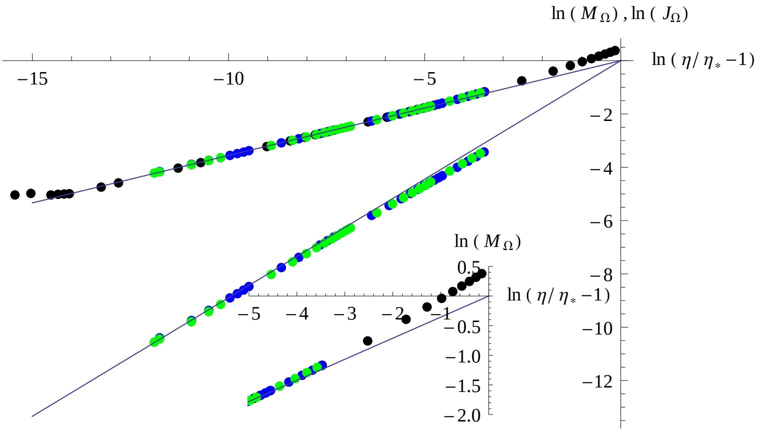
- Black hole forms for $P > 0$, with

$$\begin{aligned} M &\simeq e^{-\tau_*}\; F_M(\delta) \simeq P^{\frac{1}{\lambda_0}}\; 1 \simeq (p - Kq^2)^{\frac{1}{\lambda_0}} \\ J &\simeq e^{-2\tau_*}\; F_J(\delta) \simeq P^{\frac{2}{\lambda_0}}\; \delta \simeq (p - Kq^2)^{\frac{2-\lambda_1}{\lambda_0}}\; q \end{aligned}$$

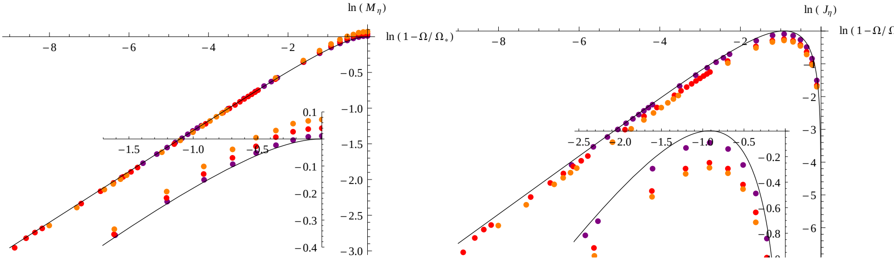




- Black hole mass M (left) and angular momentum J (right), against η (strength) and Ω (rotation)
- fluid density $\rho \sim \eta e^{-r^2}$
- angular velocity $\sim \Omega/(1 + r^2)$
- Same colour-coding of data points on next two slides



- $\ln M$ (upper curves) and $\ln J$ (lower curves), against $\ln(\eta/\eta_* - 1)$



- $\ln M$ (left) and $\ln J$ (right), against $\ln(1 - \Omega/\Omega_*)$

- For the fluid with $P = \kappa\rho$ with $0 < \kappa < 1/9$, the dominant roation mode Z_1 is unstable (ballerina effect)
- δ now **increases** as $P \to 0$
- A black hole now forms for

$$P(p,q) > 0 \quad \textbf{and} \quad |\delta(p,q)| < \text{some } \delta_{\rm crit}$$

- What is the universal value of J/M^2 at $\delta = \delta_{\rm crit}$?
- What is the universal maximum of J/M^2?