Critical phenomena in gravitational collapse

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Cosmic censorship

- Cosmic censorship: is it possible to form a naked singularity, visible to distant observers, starting from smooth initial conditions in a self-gravitating system which is regular without gravity?
- Yes (1993-1998+):
  - Codimension 1 in the space of all smooth initial data
  - Approach to the singularity via a self-similar spacetime
  - Universality and scaling (as in a critical phase transition)
  - A new type of discrete symmetry

Choptuik 1993: numerical setup

- Spherical symmetry in Schwarzschild-like coordinates

$$ds^2 = -\alpha^2(t,r)dt^2 + a^2(t,r)dr^2 + r^2(d\theta^2 + \sin^2\theta\, dvarphi^2)$$

- Minimally coupled massless scalar field $\phi(r,t)$:
  $\nabla^a\nabla_a\phi = 0$ and $R_{ab} = 8\pi G\nabla_a\phi\nabla_b\phi$
- One-parameter (p) families of smooth, asymptotically flat initial data, such that
  - Small p leads to no BH formation (small finite data)
  - Large p produces a BH (large data)
  - For example, width or amplitude of a Gaussian
- Now bisect in p to find threshold value p_*
- Different families map out the collapse threshold in phase space

Choptuik 1993: results

- Single well-defined p_* for each family: the threshold appears to be a smooth hypersurface in phase space.
- It is possible to form arbitrarily small black holes as $p \to p_*$.
- *Scaling*: $M_{BH}(p) \propto (p - p_*)^\gamma$ for $p \gtrsim p_*$.
  $R_{max}(p) \propto (p_* - p)^{-2\gamma}$ for $p \lesssim p_*$.
- A spacetime region of *discrete self-similarity*:
  $\phi(t, r) \simeq \phi_*(t, r) = \phi_*(t/e^\Delta, r/e^\Delta)$
- *Universality*: $\gamma \simeq 0.371$, $\Delta \simeq 3.44$, and same $\phi_*(t, r)$, for all families of initial data.

Is this generic?

- Many different matter systems in spherical symmetry (Yang-Mills, Dirac, perfect fluid, scalar electrodynamics)
- Higher and lower spacetime dimensions
- Good theoretical understanding (this talk)
- Axisymmetric vacuum (Brill gravitational waves)
- Black hole charge and angular momentum also scale

Idea 1: Continuous and discrete self-similarity

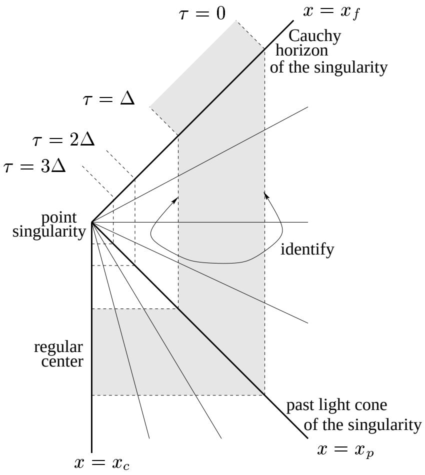
Definition of CSS:

$$\mathcal{L}_\xi\, g_{ab} = -2\, g_{ab}$$

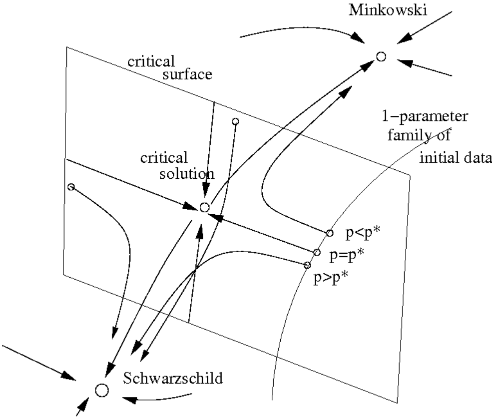
In adapted coordinates $x^\mu \equiv (\tau, x^i)$:

$$\xi = \frac{\partial}{\partial\tau}, \quad g_{\mu\nu}(\tau, x^i) = e^{-2\tau}\bar{g}_{\mu\nu}(x^i)$$

In DSS, $\bar{g}_{\mu\nu}$ is periodic in τ with period Δ instead.
Length and time scales $\propto e^{-\tau}$
Spacetime curvature $R_{\mu\nu} \propto e^{2\tau}$



Idea 2: Phase space picture



Critical solution has one unstable perturbation mode. Explains universality and scaling.

Idea 3: Mass scaling

Near-critical initial data, near the critical solution:

$$\begin{aligned}\phi(x,\tau) &\simeq \phi_*(x) + \sum_{i=0}^{\infty} C_i(p) e^{\lambda_i \tau} \phi_i(x) \\ &\simeq \phi_*(x) + (\text{some constant})(p - p_*)\ e^{\lambda_0 \tau}\ \phi_0(x) \\ &\simeq \phi_*(x) + (\text{some constant})\ \phi_0(x) \qquad \text{when AH forms}\end{aligned}$$

This happens at some $\tau = \tau_{\mathrm{AH}}(p)$ defined by

$$(p - p_*)\ e^{\lambda_0 \tau_{\mathrm{AH}}} \simeq (\text{some constant})$$

Hence

$$M(p) \propto e^{-\tau_{\mathrm{AH}}(p)} \propto |p - p_*|^{\frac{1}{\lambda_0}}$$

How well-understood is this?

Two numerical approaches:

- Nonlinear evolution and fine-tuning of initial data
- (Spherically symmetric) similarity solution plus (nonspherical) linear perturbations

Successes:

- Universality
- Scaling of BH mass and maximum curvature
- Universality classes
- Angular momentum and charge scaling in perturbation theory

Open questions:

- Far from spherical symmetry, for example axisymmetric gravitational waves
- Angular momentum
- Universal scaling functions

Angular momentum scaling

One spherical unstable mode λ_0 and
one axial $l = 1$ unstable mode λ_1 (ballerina effect)
Let p strength of the initial data, $r\mathbf{q}$ their angular momentum.

$$\bar{p} := C_0(p - p_*)$$
$$\bar{\mathbf{q}} := C_1\mathbf{q}$$
$$\delta := |\bar{p}|^{-\frac{\lambda_1}{\lambda_0}} |\bar{\mathbf{q}}|$$

Then for $\bar{p} > 0$, black hole mass and angular momentum scale as

$$M(p, \mathbf{q}) = \bar{p}^{\frac{1}{\lambda_0}} F_M(\delta) \simeq \bar{p}^{\frac{1}{\lambda_0}}$$
$$\mathbf{L}(p, \mathbf{q}) = \bar{p}^{\frac{1}{\lambda_0}} F_L(\delta)\, \bar{\mathbf{q}} \simeq \bar{p}^{\frac{1-\lambda_1}{\lambda_0}} \bar{\mathbf{q}}$$

$F_{M,L}$ are universal scaling functions. Gundlach 2002, confirmed in Newtonian fluid collapse, Aguilar, PhD 2015.

Vacuum

- Abrahams & Evans 1993-4:
  Brill waves: axisymmetry, vacuum, Killing vector hypersurface-orthogonal (one gravitational degree of freedom).
  Maximal slicing, quasi-isotropic spatial coordinates.
  Find DSS critical solution, $\Delta \simeq 3.4$, $\gamma \simeq 0.37$. Some evidence of universality.
- Several failed attempts to reproduce *or falsify* this using other (BSSN, GH) formulations of the Einstein equations.
- 3D vacuum (to get both degrees of freedom and angular momentum) even harder.
- Bizoń et al 2005-6: interesting 4+1 toy model depending only on (r, t).

Open questions

- Do critical solutions exist for any matter/symmetry?
- Are they always highly symmetric?
- Who ordered discrete self-similarity?
- Is the Cauchy horizon of the naked singularity stable?
- What happens beyond spherical symmetry?
- What role does angular momentum play?
- Einstein-Vlasov and other non-field theories?