Extremal black holes are codimension-one in gravitational collapse of a spherical charged scalar field – but away from the threshold of collapse

Carsten Gundlach, Laetitia Martel (Southampton)
Satwik Mittal (LMU Munich)

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System and coordinates

Action

$$S = \int \left(R - D_a\phi (D^a\phi)^* - F_{ab}F^{ab} \right) \sqrt{-g}\, d^D x$$

The coupling constant q in

$$D_a := \nabla_a + iqA_a$$

sets a length scale q^{-1} in the equations

Restrict to spherical symmetry, single null coordinates

$$ds^2 = -2G(du\, dx - B\, du^2) + R^2 d\Omega^2$$

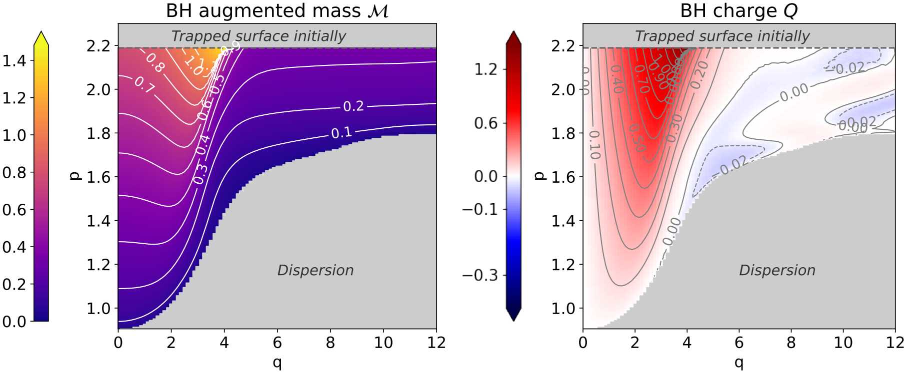
B such that $R = x = 0$ is a regular centre, $x = x_{\mathrm{max}}$ is ingoing null

A 2-dimensional solution space

Initial data ansatz

$$\phi(0,x) = p\,\mathrm{Gaussian}(x)\,e^{i\omega x}, \qquad G(0,x) = 1$$

Vary p and q [equivalent to varying all length scales in $\phi(0,x)$]:



(Figures for 3+1, Laetitia)

Aside: Type-II critical collapse

Predictions (Gundlach–Martín-García96) from complex perturbations about Choptuik solution

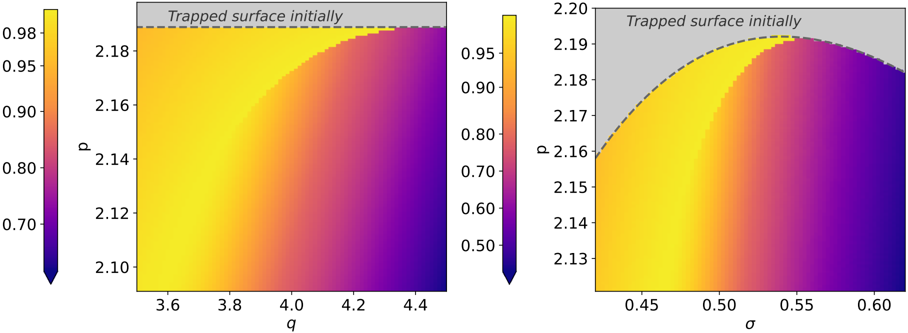
$$\begin{aligned}
\mathcal{M}_{\rm BH} &\sim (p-p_*)^{\gamma}\cdot \text{periodic}[\ln(p-p_*)] \\
\mathcal{Q}_{\rm BH} &\sim (p-p_*)^{\delta}\cdot \text{periodic}[\ln(p-p_*)] \\
\Rightarrow \left(\frac{Q}{\mathcal{M}}\right)_{\rm BH} &\sim (p-p_*)^{\delta-\gamma}\to 0
\end{aligned}$$

Verified as $p\to p_*$ for small q, in 3+1 and 4+1

- **As far as we know**, type-II with Q, $\mathcal{M}$, $Q/\mathcal{M}\to 0$ everywhere at the threshold
- For larger ω, more fine-tuning of p required for quantitative agreement with power-laws, even at small q
- periodic is universal
- periodic **is not, and changes sign** for large q

$|Q|/\mathcal{M} = 1$ at an internal threshold

Plots of $Q/\mathcal{M}$ for 2-parameter families of solutions:

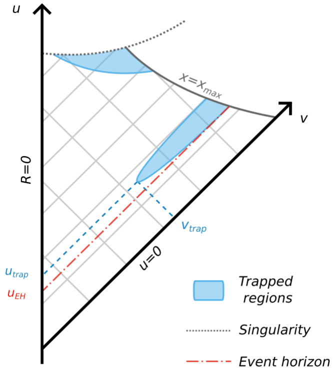


$|Q|/\mathcal{M} = 1$ **on one side of the discontinuity**, < 1 on the other

Evidence for codimension 1:

- 3+1 (Laetitia, above): 4 2-parameter families in $e^{i\omega x}$ Gaussian
- 4+1 (Satwik, original discovery): 17 1-parameter families in $e^{i\omega x}$ double Gaussian

The receding cigar mechanism



- The tip of the cigar is extremal

$$RR_{,uv} + R_{,u}R_{,v} = -\frac{G}{2}\left(1 - \frac{Q^2}{R^2}\right)$$

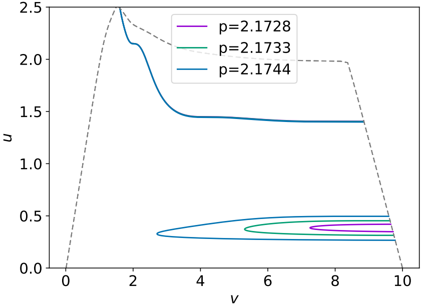
$$R_{,v} = R_{,uv} = 0 \quad \Rightarrow \quad Q = \mathcal{M} = R$$

(in 3+1, similar in 4+1)

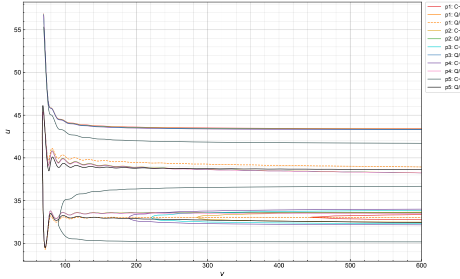
- As the tip recedes to I^+, it leaves behind an extremal EH
- For $q = 0$, Q is constant, $(Q/R)_{,u} < 0$, and the cigar is the only trapped region

- Murata-Reall-Tanahashi13: real scalar, $q = 0$, Q constant, so no regular centre possible, null rectangle domain
- Angelopoulos-Kehle-Unger26: proof of MRT13, extremality at horizon formation threshold (codimension 1), scaling laws
- Gelles-Pretorius26: charged scalar, null rectangle, scaling laws

Cigars in numerical charged collapse simulations



3+1 (Laetitia): contours $R_{,v} = 0$



4+1 (Satwik): contours $R_{,v} = 0$ and $Q/R = 1$, for different p

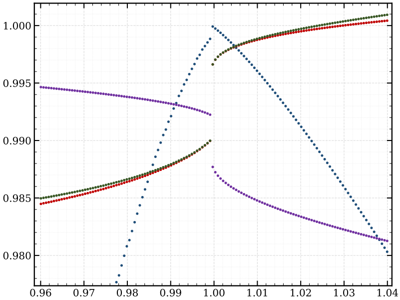
Scalings in solution space near the threshold

AKU26: near event horizon formation threshold, event horizon properties scale as

$$\frac{|Q|}{\mathcal{M}} \simeq 1 - C(p - p_*)$$
$$r \simeq a + b(p - p_*)^{\frac{1}{2}}$$
$$u \simeq c + d(p - p_*)^{\frac{1}{2}}$$

GP26: also, near threshold, tip of cigar recedes to I^+ as

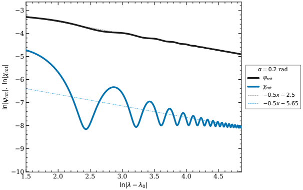
$$V_{\text{trap}} \sim (p - p_*)^{-\frac{1}{2}}$$



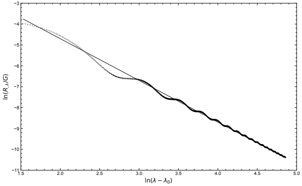
4+1 (Satwik): Q (red), $\mathcal{M}$ (green), u_{FMOTS} (purple) (all scaled), $Q/\mathcal{M}$ (blue), against p

We see these scalings on the extremal side of the discontinuity

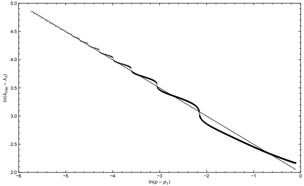
- Assume $\phi \sim \lambda^{-\sigma}$ and mostly real near EH
- On EH, with $p = p_*$, Raychaudhuri $R_{,\lambda\lambda} + |\phi_{,\lambda}|^2 R = 0$ then gives $R \simeq R_{\mathrm{EH}} - c\lambda^{-2\sigma}$, and hence $R_{,\lambda} \sim \lambda^{-(2\sigma+1)}$
- Assume further $R_{,\lambda}[p, \lambda, u_{\mathrm{EH}}(p)] \simeq a(p - p_*) + b\lambda^{-(2\sigma+1)}$
  Then $R_{,\lambda} = 0$ at $\lambda = \lambda_{\mathrm{trap}}$ gives $\lambda_{\mathrm{trap}} \sim (p - p_*)^{-1/(2\sigma+1)}$
- In 4+1 (Satwik, below) $\sigma = 1/2$ fits all three (but not in 3+1)



$\mathrm{Re}, \mathrm{Im}\phi \sim (\lambda - \lambda_0)^{-1/2}$



$R_{,\lambda} \sim (\lambda - \lambda_0)^{-2}$



$\lambda_{\mathrm{trap}} \sim (p - p_*)^{-1/2}$

at $p = p_*$, $u = u_{EH}$