Simulating gravitational collapse on null cones

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"Bondi-like" coordinates

$$ds^2 = -2G\,du\,dx - H\,du^2 + R^2\gamma_{ij}(d\theta^i + \beta^i\,du)(d\theta^j + \beta^j\,du)$$

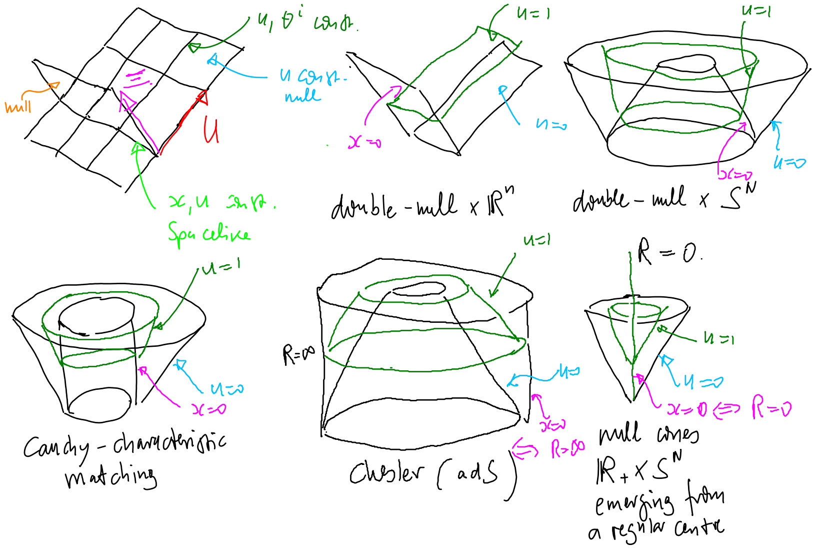
- time slices of constant u are null $\Leftrightarrow g^{uu} = 0$
- Their null generators are curves of constant $(u, \theta^i) \Leftrightarrow g_{xi} = 0$
- Radial coordinate x still free
- Area radius R defined by $\det\gamma_{ij} = \det\gamma_{ij}(\text{flat})$
- Affinely parameterised generators of u-slices $U = G^{-1}\partial_x$
- Ingoing null vector normal to 2-surfaces of constant (u, x)

$$\Xi := \partial_u - \frac{H}{2G}\partial_x - \beta^i\partial_i$$

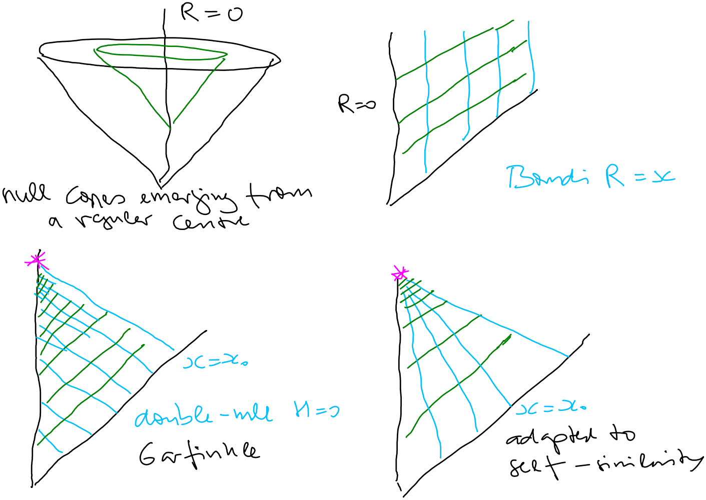
Some examples of previous uses

- Bondi $R = x$ in spher symm for scalar (Goldwirth-Piran) and fluid (Ori-Piran) collapse, scal crit collapse (G-Price-Pullin)
- Double null $H = 0$ in spher symm for scal crit coll (Garfinkle)
- Bondi for CCM in 3D (Winicour+, Moxon+)
- Affine $G = 1$ for adS problems (Chesler-Yaffe)
- Double null for BH interiors (Dafermos-Luk) and exteriors
- Shifted double-null for spher crit coll (G-Baumgarte-Hilditch)
- Shifted Bondi for scalar field crit coll in axisymm (G-B-H): **first simulation of gravitational collapse beyond spherical symmetry on null cones**

Local and global geometry



Choice of coordinate x for critical collapse



Hierarchical structure of the vacuum Einstein eqns 1

- Assume $R(x,\theta^i)$, $\gamma_{ij}(x,\theta^i)$ given on $u=0$

$$\left(\ln\frac{G}{R_{,x}}\right)_{,x} = S_G[R,\gamma_{ij},\psi] \quad (1)$$
$$(R^4\gamma_{ij}\beta^j_{,x})_{,x} = S_b[G,...] \quad (2)$$
$$(R\Xi R)_{,x} = S_R[\beta^i,...] \quad (3)$$
$$(R\Xi\gamma_{ij})_{,x} = S_\gamma[\Xi R,...] \quad (4)$$

- ∂_u and H appear only in $\Xi := \partial_u - \frac{H}{2G}\partial_x - \beta^i\partial_i$
- Solve by integration for G, β^i, ΞR, $\Xi\gamma_{ij}$
- Example double null gauge: Set $H=0$, get $R_{,u}$, $\gamma_{ij,u}$
- Example Bondi gauge: Set $R=x \Rightarrow R_{,u}=0$, get H, $\gamma_{ij,u}$
- This works only for $R_{,x}>0$ (null generators expand)

Hierarchical structure of the vacuum Einstein eqns 2

- Now instead assume $G(x,\theta^i)$, $\gamma_{ij}(x,\theta^i)$ given on $u=0$

$$R_{,xx} + (\ln G)_{,x} R_{,x} + S_G[\gamma_{ij},\psi]\, R = 0 \qquad (1')$$

$$(R^4\gamma_{ij}\beta^j_{,x})_{,x} = S_b[G,R,...] \qquad (2)$$

$$(R\Xi R)_{,x} = S_R[\beta^i,...] \qquad (3)$$

$$(R\Xi\gamma_{ij})_{,x} = S_\gamma[\Xi R,...] \qquad (4)$$

$$\left(\frac{H_{,x}}{G}\right)_{,x} = S_H[...] + 2(\ln G)_{,{\color{red}u}x} \qquad \text{from } (1',3)$$

- $R_{,x} > 0$ not required: can go inside black hole
- Solve ODE for R, solve for β^i, ΞR, $\Xi\gamma_{ij}$, H by integration
- Example affine gauge: Set $G=1 \;\Rightarrow\; U=\partial_x,\; H_{,xx}=S_H$
- **General "Bondi-like" gauge**: set $G(u,x,\theta^i)$ freely

Key ingredients of my code

- Regular origin $R = x = 0$ sets trivial BC there
- Make $x = x_{\max}$ future spacelike by making $H < 0$ there: domain of dependence, no outer BC needed (or could compactify to scri)
- Make $x = x_0$ approximately the past light cone of the singularity by making $H \simeq 0$ there: **critical collapse** resolved without mesh refinement
- Numerical stability (CFT) requires $\Delta u < \Delta x (\Delta\theta)^2$
  But setting spherical harmonics (ℓ, m) to zero for $x < \ell \Delta x$ gives $\Delta u < \Delta x$ instead
- There can be no MOTS on a regular lightcone
  But we can look for 2-surfaces (u, x) with Hawking compactness $C > 1$, or for generators with expansion $R_{,x} < 0$