

MATH3083/MATH6163

Advanced Partial Differential Equations, 2026/27

Module information and lecture plan

Content

If you have taken one of the prerequisite modules for this one (MATH2038, MATH2048, MATH2047, MATH2015, ELEC1310+20, or BIOM1005+6), you have seen a particular method for solving linear PDEs: separation of variables. Here we go further in three directions.

The material in Secs. 1-3 of the lecture notes (12 lectures) covers the crucial concept of well-posedness, and the classification of PDEs into types such as elliptic, parabolic and hyperbolic, based on their principal symbol.

Sec. 4 of the lecture notes (6 lectures) covers a type of hyperbolic PDE that is central in engineering, namely conservation laws. Here, the nonlinearity of the PDE takes centre stage, leading to discontinuous solutions. A key concept is the Riemann problem (piecewise constant initial data) and techniques for its solution.

Secs. 5-10 of the lecture notes (12 lectures) cover linear PDEs in typically three space dimensions, or three space dimensions and time. A key tool are Green's functions. If you think of the matrix equation $\mathbf{A}\mathbf{y} = \mathbf{b}$, with solution $\mathbf{y} = \mathbf{A}^{-1}\mathbf{b}$, as a toy model for the Poisson equation $\Delta\phi(\vec{x}) = f(\vec{x})$, then the Green's function is, in a certain sense Δ^{-1} . Before we introduce Green's functions for PDEs, we first introduce the δ -function and friends, and warm up with Green's functions for ODEs.

Lecturer

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Getting help

Please ask your questions in the tutorials and problem classes! If there is something you do not understand, it is very likely you are not the only one, so it makes sense to ask in class. You can also talk to me after class, or make an appointment by email to see me in my office, or just drop in to my office.

Teaching materials

The module will be delivered through **30 pre-recorded lectures** (pen capture and voiceover) in the first 10 teaching weeks before Christmas. You will find those lectures under the "Teaching materials" tab on Blackboard, under "Panopto". Recordings of the in-person tutorials and problem classes will also appear there. (To see the lectures in the right order, sort the content of Panopto by title, as they are not recorded in order. To see the latest tutorial and problem class recording, sort by date instead).

There are **full lecture notes** under the "Teaching materials" tab on Blackboard. I suggest you print them out and put them in a ringbinder: it is much easier to read mathematics on paper than on a screen.

The **lecture plan** on at the end of this document tells you which parts of the printed notes are covered in each lecture.

There is a number of **exercises** at the end of each section of the lecture notes, and numbered continuously across all sections. You will find full solutions to both the homework and the other problems under the "Assessments" tab Blackboard, in the form of a single set of solutions to which I will add new solutions each week.

There are **three revision exercises that you should attempt before the course starts** to test your understanding of the prerequisite material. These are at end of the Sec. 1 of the lecture notes, and the solutions are already in the solutions document I have just mentioned.

Some exercises in each section are additionally labelled as "**Homework n** ". Each of these is designed to reinforce what you have just learned in Lecture n , and so you should attempt it right after that lecture. The homework problems will require both time and original thought, and are an integral part of the teaching.

Teaching methods

Our teaching method is sometimes called the “flipped classroom”. You engage with the material on your own through the pre-recorded lectures, lecture notes, and homework problem. The interactive, in-person sessions are reserved for asking questions, while you work on your own. You get as much staff time as in the traditional three lectures plus one problem class model, but it is used differently.

To go with each pre-recorded lecture, there is an **in-person tutorial** each Monday, Tuesday and Thursday, and there is **in-person problem class** each Friday, as shown on your timetable.

Please **watch each pre-recorded lecture before the corresponding tutorial**. A good plan, if you can fit it in, is to watch it on the same day, so it will be fresh in your mind for the tutorial. If taking notes in in-person lectures has helped you in the past, you should try doing this also in recorded lectures. You will probably need to stop or rewind the recording to take notes, think something through, or check a calculation.

The plan for the tutorials is that you **work on the homework problem** from the preceding lecture there and then, asking for help if and when you need. So T01 supports L01 and H01, and so on. At the beginning of each tutorial, you can also ask me to go through any part or all of previous lectures, or any of the homework and other problems. This initial part of the tutorial will be recorded.

The plan for the **additional problems that are not homework** problems is that you use them for additional study or for revision. You can ask for help with any problem in any tutorial.

Self-marking

To encourage you to engage with the course, and in particular with the homework problems, day by day, we will use self-marking. **The plan for the problem classes** is that we go over the three homework problems in that week, and that **you mark your own work during the session**. You should normally be able to self-mark your work during the problem class. If you had serious problems or errors, it may be more instructive to work that part of the problem again in your own time after the tutorial, and add this to your original solution. The problem classes will be recorded but, as always, recording quality may vary depending on the room.

You will get 0, 1 or 2 marks per week. These are not for the correctness of your work, but for engaging with the material: **You will get the first mark for submitting a reasonable effort at a solution, and the second mark for accurate self-marking**. This means finding where you went wrong, and explaining to yourself what you should have done.

You will not get a mark for ignoring your own solution and copying down mine. Mark your own work as correct where it is correct, and mark your errors and correct them, either in place or at the end. The idea is that you think about where you have gone wrong and learn from that, and that at the end you have a complete and correct solution that you actually understand.

A typed model answer will then be released, under the “Assessments” tab, after the self-marking deadline. This is give you examples of what a complete but concise solution looks like, and to help you with revision.

Hence there are two deadlines for the first 10 teaching weeks before Christmas: **Friday 15.00 for your answers** to the three homework problems of that week, and **Monday 9.00 for your self-marking**. I cannot accept late submissions for either. The intention is that you mark your homework during the problem class, and submit it then, not that you watch the recording and self-mark from that on the weekend. Handin is under the “Assessments” tab.

A warning about the pace of the module

The teaching and assessment of this module is designed to make you work on the content regularly each week, and to attend all sessions. We start with new material on day one. You should try the three revision problems before we start to check if you know the prerequisite materials.

Weekly workplan

In Week 0: Attempt the three revision problems, familiarise yourself with the structure of the course and the lecture notes, make contact with your group.

In Week 1:

Monday	Watch Lecture 1 (L01), read corresponding sections in the notes Attempt Homework 1 (H01), attend Tutorial 1 (T01)
Tuesday	Watch L02, read corresponding sections in the notes Attempt H02, optionally attend T02
Thursday	Watch L03, read corresponding sections in the notes Attempt H03, attend T03
Thursday/Friday	Consolidate and submit your homework answers before the problem class
Friday	Attend Problem Class 1 (PC01), self-mark your homework, submit again

Weeks 2-10 continue in the same way. **Weeks 11-12** are for revision: you can ask any questions, and ask me to go over additional problems from the lecture notes or old exam questions.

Submission of weekly homework and your self-marking

In Week 1 at least, you will be asked to submit a scan of your handwritten solutions to H01-H03 on the “Assignments” tab on Blackboard, mark them on paper **using a red pen**, and submit a scan of the marked homework again.

You must submit a scan of your work as a single PDF file, black and white (not colour), containing all pages the right way up and in the right order. You must mark in red pen, or green if you prefer, but not blue or black.

You can use an electronic writing tablet, but you must save your work as a single PDF file consisting of individual A4 pages, not one huge page.

If you do not stick to this format, which is meant to help me read your work easily on a screen, I will remind you, and eventually subtract marks.

Assessment structure

If you have already taken MATH3083, you cannot take MATH6163. Both modules have the same syllabus and homework problems, but the MATH6163 exam will have some harder questions.

20% of the final mark is from self-marking 30 homework questions, submitted and self-marked in 10 weekly chunks. These marks are for engaging with the material, so by copying you are only cheating yourself.

80% of the final mark is from the final exam. All material in the printed notes is examinable except where announced in the lecture plan. On Blackboard you will find all previous actual exams and two practice exams that from the first year the course ran, all with model answers.

Books

PDEs is such a vast subject that there is no standard choice of material for a one-semester course. Therefore, there is no single book that covers the entire syllabus of this course, and that is why you have full lecture notes. On the other hand, there are lots of books on the topic, each with its own choice of material and emphasis. Have a browse in section QA374 of the library.

However, we would recommend the following two books for background reading. They have very different styles. Have a look at both in the library. Both go much wider and deeper than the lecture notes, of course.

M. Renardy and R. C. Rogers, *An Introduction to Partial Differential Equations*, Springer, 2nd edition 2004. (1st edition 1993 is also fine.) QA374 REN.

J. Ockendon, S. Howison, A. Lacey and A. Movchan, *Applied Partial Differential Equations*, Oxford University Press, revised edition 2003. (1st edition 1999 is also fine.) QA377 OCK.

Chapter 1 of Renardy and Rogers is the best introduction to PDEs at the level of this course that I know. It starts from the solutions by separation of variables that you already know, and points out some important general ideas in these examples. I am trying to do the same in Sec. 3.1 of my lecture notes.

The rest of Sec. 3 of the lecture notes (classification and the symbol) loosely follows parts of Ch. 2 of Renardy and Rogers, also taking in ideas from Ch. 1-3 of Ockendon *et al.* The main difference is that for lack of time I am not introducing characteristics.

The best background reading for Sec. 4 of the lecture notes (conservation laws) would be R. J. Leveque, *Numerical Methods for Conservation Laws*, Birkhäuser 1992. QA377 LEV. Ch. 3 of Renardy and Rogers will also work.

Chs. 4, 5 and 6 of Ockendon *et al* can be used as background reading for Secs. 7-10 of the lecture notes (Green's function methods for linear elliptic, parabolic and hyperbolic equations).

Another two books are more elementary, and do not cover the whole syllabus, but are also useful.

Y. Pinchover and J. Rubinstein, *An Introduction to Partial Differential Equations*, Cambridge University Press 2005. QA374 PIN.

W. A. Strauss, *Partial Differential Equations, An Introduction*, John Wiley 1992. QA374 STR.

Lecture plan

The following list shows you exactly which part of the lecture notes is covered in each lecture, by lecture number from 1 to 30.

Boldface and numbering denote sections in the printed notes.

(L01a) and so on refer to the parts of a pre-recorded lecture (not all lectures are in parts). These are on Blackboard. To see them listed in the right order, L01a, L01b, and so on, select “Sort by Name”.

A few sections shown in round brackets are for self-study, but everything in the printed notes is examinable.

1. **1 Overview.** (L01a) 1.1.1 Partial derivative notation. (1.1.2 systems of PDEs), 1.1.3 PDE problems, (1.1.4 Linear PDEs, 1.2 Vector calculus, 1.3 Separation of variables, 1.4 Fourier series and Fourier transforms). (L01b) 1.4.4 FT and falloff BCs. (L01c) 1.5.1 Laplace, Poisson and Helmholtz equations.
2. (L02a) 1.5.2 Heat equation. (L02b) 1.5.3 Wave equation.
3. (L03a) 1.6 Cauchy data, Cauchy-Kowalewski. (L03b) 1.7 Weak solutions of the 1D wave equation.
4. **2 Well-posedness.** (L04a) 2.1 Definition of well-posedness. 2.2 Examples of ill-posed problems: 2.2.1 Dirichlet problem for the wave equation BCs. (L04b) 2.2.2 Cauchy problem for the Laplace equation: no unique solution. (L04c) 2.2.3 Cauchy problem for the Laplace equation: no continuous dependence on the data.
5. (L05) 2.3 Continuous dependence on the data.
6. (L06a) ...continued. (L06b) 2.4.1 Energy estimate for the heat equation.
7. (L07aa) 2.4.2 Energy estimate for the wave equation. (L07b) 2.4.3 Uniqueness of the boundary value problem for the Poisson equation. (2.5 The importance of well-posedness.)
8. **3 The symbol and classification** (L08a) 3.1 Introductionm 3.2 The symbol. (L08b) 3.2.3 Plane-wave solutions.
9. (L09a) 3.3 Ellipticity. (3.4 A model parabolic PDE.) (L09b) 3.3.3 High-frequency approximation.
10. (L10a) 3.5.1 A model hyperbolic PDE. (L10b) 3.5.2 Strict hyperbolicity.
11. (L11) 3.6 Examples.
12. (L12a) 3.7.1 First-order scalar PDEs. (L12bb) 3.7.2 Second order scalar PDEs. (c) 3.8 Nonlinear PDEs.
13. **4 Conservation laws.** (L13aa) 4.1.1 1D conservation law: integral and differential form. (L13b) 4.1.2 Higher space dimensions.
14. (L14a) 4.2.1 The 1D advection equation. (L14b) 4.2.2 Method of characteristics for 1D scalar conservation laws. (L14c) 4.2.3 Propagation of small disturbances.
15. (L15) 4.3.1 Shock formation and Riemann problems.
16. (L16) 4.3.2 Propagating shock solutions.
17. (L17a) 4.3.3 Rarefaction wave solution. (L17b) 4.3.4 Lax condition. (L17c) 4.3.5 Systems and higher dimensions.
18. (L18) 4.4 Traffic flow.
19. (L19) **5 Generalized functions.**
20. (L20a) 5.5 Properties of the δ -function. (L20b) **6 Green’s functions for ODEs.** 6.1 First order.
21. (L21a) 6.2 Harmonic oscillator. (6.3 General second order.) (L21b) 6.4 Initial value problem.

22. (L22) **7 Green's functions for the Poisson and Helmholtz equations.** 7.1 Three dimensional δ -function. 7.2 Green's function for the Poisson equation on free space, derivation using the divergence theorem.
23. (L23a) 7.3 Green's function for the Helmholtz equation. (7.4 Derivation using differentiation under an integral.) (L23b) 7.5 Large distance and long wave-length approximation. (7.6 Uniqueness of the solution to the free-space problem.)
24. **8 Bounded regions.** (8.1 Green's theorem. 8.2 Reciprocal theorem.) (L24) 8.3 Kirchhoff-Helmholtz formula. 8.4 Problems on bounded regions.
25. (L25) 8.5 Method of images, two examples.
26. (L26) **9 The diffusion equation.** 9.1 Green's function for the 1D heat equation: dimensional analysis.
27. (L27) 9.2 1D heat equation: initial value problem, error function.
28. (L28) 9.3 Green's function for the 3D heat equation from 1D.
29. **10 The wave equation.** (L29a) 10.1 One space dimension: 10.1.1 Green's function. (L29b) 10.1.2 initial value problem.
30. (L30a) 10.2 Three space dimensions: 10.2.1 Green's function. (L30b) 10.2.2 retarded potentials. (10.3 Method of descent, from 3D to 1D and 2D.)