The Pomeron in QCD

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2014

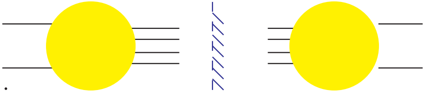
Topics to be Covered

- Introduction to Regge Theory- the “classical” Pomeron
- Building a Reggeon in Quantum Field Theory
- The “reggeized” gluon
- The QCD Pomeron - the BFKL Equation
- Some Applications
- Diffraction and the Colour Dipole Approach
- Running the Coupling
- Higher Order Corrections in BFKL
- Soft and hard Pomerons

Strong Interactions Before QCD

Extract information from unitarity and analyticity properties of S-matrix.

Unitarity - Optical Theorem



$$\Im m \mathcal{A}_{\alpha\alpha} = \sum_n \mathcal{A}_{\alpha n} \mathcal{A}^*_{n\alpha}$$

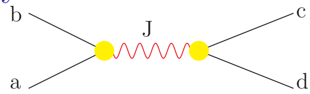
$$\Im m \mathcal{A}(s, t=0) = \frac{1}{2s} \sigma_{TOT}$$

Can be extended (Cutkosky Rules)

$$\Delta_s \mathcal{A}_{\alpha\beta} = \sum_{\text{cuts}} \mathcal{A}_{\alpha n} \mathcal{A}^*_{n\beta}$$

Leads to self-consistency relations for scattering amplitudes (Bootstrap)

Partial Wave Analysis



$$\mathcal{A}^{ab\to cd}(s,t) \; = \; \sum_J a_J(s) P_J(1-2t/s), \quad [\cos\theta = (1-2t/s), \; m_i \to 0]$$

Crossing:

$$\mathcal{A}^{a\bar{c}\to\bar{b}d}(s,t) \; = \; \mathcal{A}^{ab\to cd}(t,s) \; = \; \sum_J a_J(t) P(J, 1-2s/t)$$

In the limit $s \gg t$ (diffractive scattering)

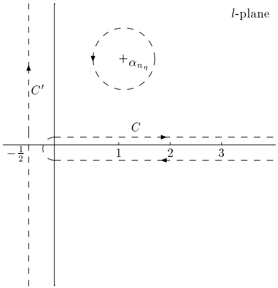
$$P(J, 1-2s/t) \sim s^J$$

so that

$$\mathcal{A}^{a\bar{c}\to\bar{b}d}(s,t) \;\stackrel{s\to\infty}{\to}\; \sum_J b_J(t) s^J$$

Sommerfeld-Watson Transformation

$$\mathcal{A}^{a\bar{c}\to\bar{b}d}(s,t) \ = \ \oint_C \sum_{\eta=\pm 1} \frac{(2J+1)}{\sin\pi J}\frac{\left(\eta+e^{i\pi J}\right)}{2} a^{\eta}(J,t)P(J,2s/t)$$



Poles at integer J

Deform contour to C' For large s, integral along C' is zero.

Pick up only contributions from poles in J-plane.

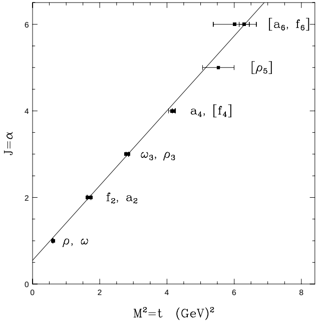
N.B. Important (and possibly incorrect) assumption is that all singularities in J-plane are poles.

$$\mathcal{A}(s,t) \ \stackrel{s\to\infty}{\to} \ \sum_i \frac{\left(\eta+e^{i\pi\alpha_i(t)}\right)}{2}\beta_i(t)s^{\alpha_i(t)}$$

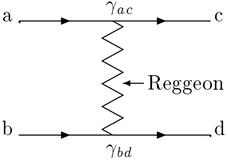
α_i are the poles in the J-plane

For $s\to\infty$ we only need the leading pole.

Chew-Frautschi Plot



Continue to negative t



$$\mathcal{A} \sim \gamma_{ac}(t)\gamma_{bd}(t)s^{\alpha_R(t)}$$

(Factorisation)

We can think of Regge exchange as the superposition of the exchange of many particles.

The Pomeron

Leading trajectory

Using Optical Theorem

$$\mathcal{A}(s,0) \stackrel{s\to\infty}{\to} \sim s^{\alpha(0)}$$

implies

$$\sigma_{TOT} \ \sim \ s^{\alpha(0)-1}$$

Okun-Pomeranchuk Theorem

If exchanged Regge trajectory does NOT have the quantum numbers of the vacuum, $\alpha(0) < 1$.

Foldy-Peierls Theorem

If $\alpha(0) \geq 1$, the Regge trajectory exchanged MUST have the quantum numbers of the vacuum.

THIS IS THE POMERON

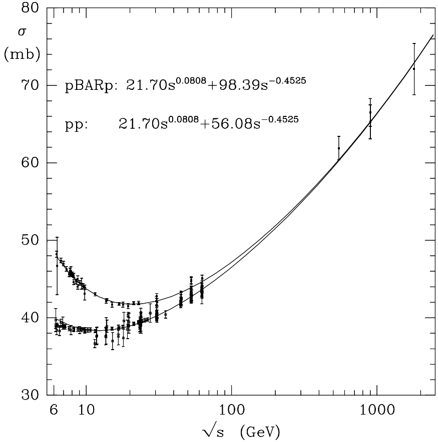
N.B. $\alpha(0) > 1$ is NOT allowed by unitarity.

Froissart-Martin bound (derived from unitarity)

$$\sigma_{TOT} \ < \ A \ln^2 s$$

Landshoff-Donnachie fit

$$\alpha_P(t) \ = \ 1.08 + 0.25\,(\mathrm{GeV}^{-2})t$$



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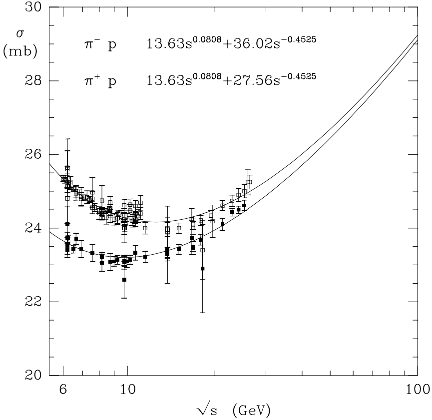

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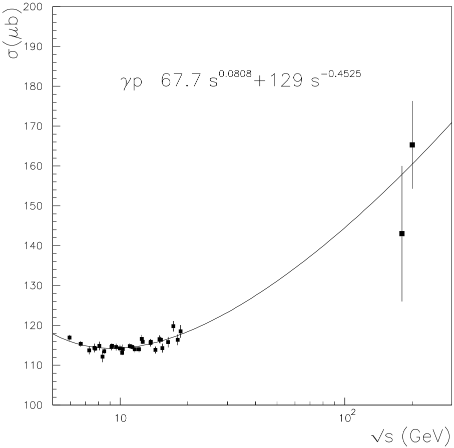


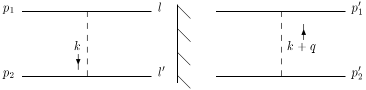
Confronting Regge Theory with QCD

- How is the Pomeron explained in QCD?
- Data fitted by Landshoff-Donnachie cannot be calculated in purely perturbative QCD - non perturbative effects must be included
- Expect a hint of the Pomeron from perturbative QCD - but we don't really find one
- Phenomenological models have to be used to explain data.
- Nevertheless perturbative QCD in the kinematic regime where Pomerons are expected to dominate give some interesting results (solutions to the BFKL equation) which can be compared with data in certain cases.

Toy Model

$$\mathcal{L}_I = \lambda\phi^3$$

Use Optical theorem to calculate $\Im m\mathcal{A}(\sqrt{s},\mathbf{q})$



Leading order
Neglecting masses.

$$k = (k^+, k^-, \mathbf{k})$$

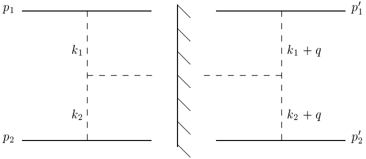
$$q = (0, 0, \mathbf{q})$$

$$dLIPS = \frac{1}{2} dk^+ dk^- d^2\mathbf{k}\,\delta(k^+(\sqrt{s} - k^-) - \mathbf{k}^2)$$

$$\begin{aligned}
\Im m\mathcal{A} &= (2\pi)\lambda^4 \int dk^+ dk^- d^2\mathbf{k} \frac{\delta(k^+(\sqrt{s}-k^-)-\mathbf{k}^2)\delta(k^-(\sqrt{s}-k^+)-\mathbf{k}^2)}{(k^+k^- - \mathbf{k}^2)(k^+k^- - (\mathbf{k}-\mathbf{q})^2)} \\
&\approx \int d^2\mathbf{k} \frac{1}{\mathbf{k}^2(\mathbf{k}-\mathbf{q})^2}
\end{aligned}$$

Higher Orders

In higher order we want to sum all terms $\sim \lambda^{2n} \ln^n(s)$.
Is this approximation valid?
Maybe not - renormalon studies suggest that leading logarithm sums are not reliable.



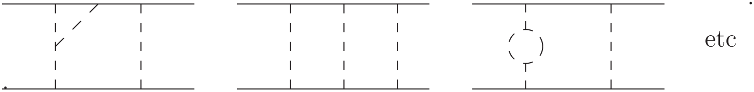
$$\frac{\lambda^6}{8\pi}\int dk_1^+ dk_1^- d^2\mathbf{k_1} dk_2^+ dk_2^- d^2\mathbf{k_2}\,\delta(k_1^+(\sqrt{s}-k_1^-)-\mathbf{k_1}^2)$$

$$\frac{\delta((k_2-k_1)^+(k_2-k_1)^- - (\mathbf{k_1}-\mathbf{k_2})^2)\delta(k_2^- - (\sqrt{s}-k_2^+)-\mathbf{k_2}^2)}{(k_1^+k_1^- - \mathbf{k_1}^2)(k_1^+k_1^- - (\mathbf{k_1}-\mathbf{q}^2)(k_2^+k_2^- - \mathbf{k_2}^2)(k_2^+k_2^- - (\mathbf{k_2}-\mathbf{q})^2)}$$

$$= \int_{k_1^+}^{\sqrt{s}} \frac{dk_2^+}{(k_2^+ - k_1^+)} \frac{d^2\mathbf{k_1} d^2\mathbf{k_2}}{\mathbf{k_1}^2(\mathbf{k_1}-\mathbf{q})^2\mathbf{k_2}^2(\mathbf{k_2}-\mathbf{q})^2}$$

Integral over k_2^+ gives $\ln(s)$
Dominated by the region $\sqrt{s} \gg k_2^+ \gg k_1^+$.

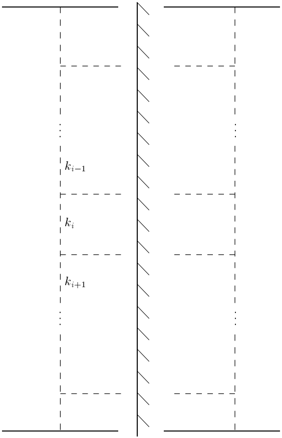
On the other hand



give an extra power of λ^2, but NO extra $ln(s)$.

Such graphs may be dropped in the leading log approximation.

Multi-rung Ladder Graph



$$\Im m \mathcal{A}^n \sim \int_{k_{n-1}^+}^{\sqrt{s}/2} \frac{dk_n^+}{(k_n^+ - k_{n-1}^+)} \cdots \int_{k_1^+}^{\sqrt{s}/2} \frac{dk_2^+}{(k_2^+ - k_2^+)}$$

$$\ln^n s$$

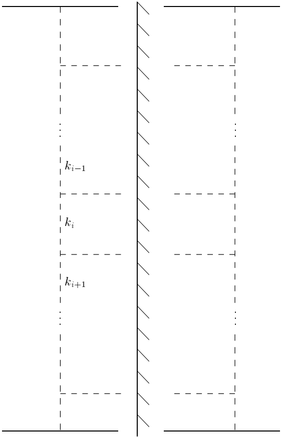
Dominant region of k^+ integration

$$\sqrt{s} \gg k_n^+ \gg k_{n-1}^+ \cdots \gg k_2^+ \gg k_1^+$$

“Multi-Regge regime”

In this model only uncrossed ladder graphs contribute to the leading log approximation.

Multi-rung Ladder Graph



$$\sqrt{s} \gg k_n^+ \gg k_{n-1}^+ \cdots \gg k_2^+ \gg k_1^+$$

Similarly

$$\sqrt{s} \ll k_n^- \ll k_{n-1}^- \cdots \gg k_2^- \gg k_1^-$$

The on-shell delta functions for cut lines

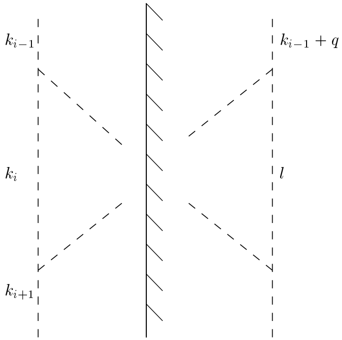
$$\delta(k_{i+1}^- k_i^+ - (\mathbf{k_i} - \mathbf{k_{i-1}})^2)$$

Propagators of the vertical lines

$$\frac{1}{k_i^+ k_i^- - \mathbf{k_i}^2} \approx -\frac{1}{\mathbf{k_i}^2} \quad (\text{since } k_i^+ \ll \mathbf{k_i}^2 / k_i^-)$$

In this model only uncrossed ladder graphs contribute to the leading log approximation.

Crossed-rung Graphs

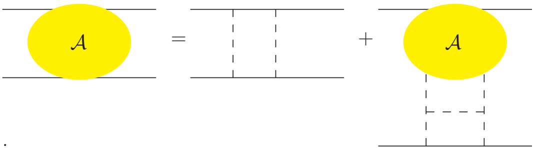


$$l^2=(k_i+k_{i+1}-k_{i-1})^2=(k_i^+ +k_{i+1}^+ -k_{i-1}^+)(k_i^- +k_{i+1}^- -k_{i-1}^-)- \sim \mathbf{k_i}^2$$

$$\approx k_{i+1}^+ k_{i-1}^- \gg k_{i+1}^+ k_i^- \gg \mathbf{k}^2$$

In the multi-Regge region crossed-ladder graphs are suppressed because denominators of propagators are larger.

Integral Equation



$$\Im m \mathcal{A}(\sqrt{s}, \mathbf{q}) \;=\; \Im m \mathcal{A}_0 + \frac{\lambda^2}{16\pi^3} \int^{\sqrt{s}/2} \frac{dk^+}{k^+} \frac{d^2\mathbf{k}}{\mathbf{k}^2 (\mathbf{k}-\mathbf{q})^2} \Im m \mathcal{A}(k^+, \mathbf{q})$$

$\uparrow$

effect of adding rung.

Mellin Transform

(Equivalent to Sommerfeld-Watson transformation)

$$\tilde{\mathcal{A}}(\omega) = \int_{s_0}^{\infty} \left(\frac{s}{s_0}\right)^{-\omega-1} \mathcal{A}(s) ds$$

If $\mathcal{A}(s) \sim s^{\omega_0}$

$$\tilde{\mathcal{A}}(\omega) \sim \frac{1}{(\omega - \omega_0)}$$

Mellin transform has a pole at $\omega = \omega_0$.
Furthermore

$$\begin{aligned}
\tilde{\mathcal{A}}(\omega)\tilde{\mathcal{B}}(\omega) &= \int_{s_0}^{\infty} \left(\frac{s}{s_0}\right)^{-\omega-1} \mathcal{A}(s) ds \int_{s_0}^{\infty} \left(\frac{s'}{s_0}\right)^{-\omega-1} \mathcal{B}(s') ds' \\
&= \int_{s_0}^{\infty} \left(\frac{s}{s_0}\right)^{-\omega-1} \int \frac{dk^+}{k^+} \mathcal{A}(k^+) \mathcal{B}\left(\frac{\sqrt{s}}{k^+}\right)
\end{aligned}$$

Integral Equation in Mellin Space

$$\Im m\tilde{\mathcal{A}}(\omega) = \Im m\tilde{\mathcal{A}}_0 + \frac{\lambda^2}{16\pi^3}\frac{1}{\omega}\int\frac{d^2\mathbf{k}}{\mathbf{k}^2(\mathbf{k}-\mathbf{q})^2}\Im m\tilde{\mathcal{A}}(\omega)$$

Solution:

$$\Im m\tilde{\mathcal{A}}(\omega) \sim \frac{1}{(\omega-\omega_0)}$$

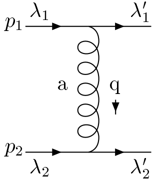
$$\Im m\mathcal{A}(\sqrt{s},\mathbf{q}) \sim s^{\omega_0}$$

$$\omega_0 = \frac{\lambda^2}{16\pi^3}\int\frac{d^2\mathbf{k}}{\mathbf{k}^2(\mathbf{k}-\mathbf{q})^2}$$

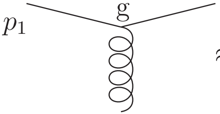
QCD

- Vertices contain momenta - cannot just consider uncrossed ladder graphs
- Need to account for tree graphs and loop corrections for cut graphs - “ladders within ladders” - bootstrap (self-consistency relations)
- Need to account for colour factors - distinguish between colour singlet and colour octet exchange

Colour Octet Exchange



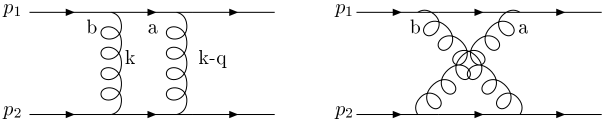
For $|\mathbf{q}|^2 \ll s$ use eikonal approximation



$\approx 2gp_1^\mu$

$$\mathcal{A}_0^{(8)} \approx g^2 \frac{2s}{\mathbf{q}^2} \tau^a \otimes \tau^a \quad (s = p_1 \cdot p_2)$$

NLO

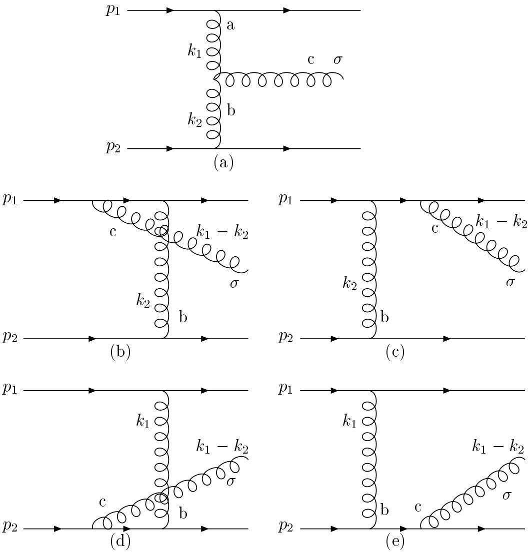


(a) (b)

$$\Im m \mathcal{A}^{(8)} = \frac{\alpha_s C_A}{4\pi^2} \int d^2\mathbf{k} \frac{-\mathbf{q}^2}{\mathbf{k}^2 (\mathbf{k}-\mathbf{q})^2} g^2 \frac{2s}{\mathbf{q}^2} \tau^a \otimes \tau^a$$

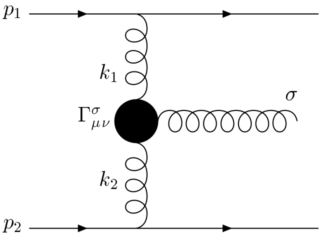
N.B. Infrared divergent - regularise by any means

NNLO



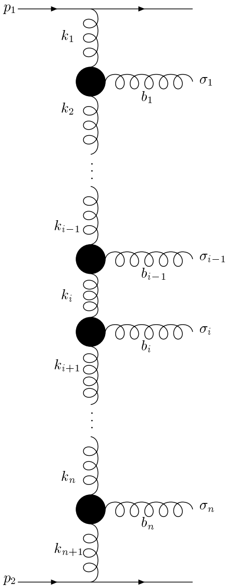
Effective vertex

$$k_1^+ \gg k_2^+, \; k_2^- \gg k_1^-$$



$$\Gamma^{\sigma}_{+-}(k_1,k_2) \;=\; 2gf^{abc}\left(k_1^+ + \frac{2\mathbf{k_1}^2}{k_2^-}, k_2^- + \frac{2\mathbf{k_2}^2}{k_1^+}, -(\mathbf{k_1}+\mathbf{k_2})\right)$$

(other components negligible)

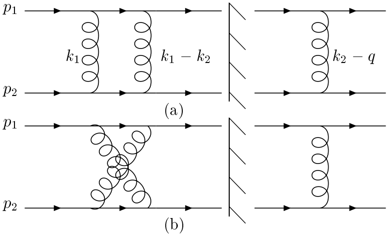


Generalises to all orders at tree-level in multi-Regge region (leading log approximation)

Each vertex replaced by $\Gamma^{\sigma_i}_{+-}(k_i, k_{i+1})$

Virtual Corrections

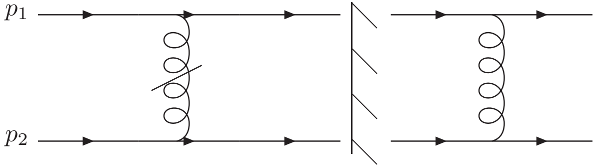
Need graphs such as



Absorb L.H. part of graph into NLO correction to gluon exchange

$$\Im m \mathcal{A}^{(8)} = \frac{\alpha_s C_A}{4\pi^2} \int d^2\mathbf{k_2} \frac{-\mathbf{q}^2 \varepsilon_G(\mathbf{k_2}^2) \ln(s)}{\mathbf{k_2}^2 (\mathbf{k_2} - \mathbf{q})^2} g^2 \frac{2s}{\mathbf{q}^2} \tau^a \otimes \tau^a$$

$$\varepsilon_G(\mathbf{k_2}^2) = -\frac{C_A \alpha_s}{4\pi^2} \int d^2\mathbf{k_1} \frac{\mathbf{k_2}^2}{\mathbf{k_1}^2 (\mathbf{k_1} - \mathbf{k_2})^2}$$

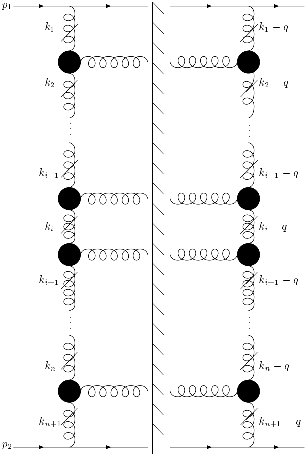


LHS part indicates that exchanged gluon has been corrected.
(also need graph with correction of RHS of cut)
Up to NNLO we get

$$\mathcal{A}^{(8)}(\sqrt{s},\mathbf{q}) \;=\; \mathcal{A}_0^{(8)}(\sqrt{s},\mathbf{q})\left(1+\varepsilon_G(\mathbf{q}^2)\ln(s)+\frac{1}{2}\varepsilon_G^2(\mathbf{q}^2)\ln^2(s)\right)$$

Bootstrap - ladders within ladders

Suggests a self-consistency ansatz for colour octet exchange in multi-Regge region



Assume solution in which vertical gluons are “reggeized”

i.e. propagator of i^{th} gluon is replaced by

$$\frac{1}{\mathbf{k_i}^2}\left(\frac{k_{i-1}^+}{k_i^+}\right)^{\varepsilon_G(\mathbf{k_i}^2)}$$

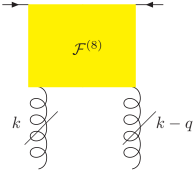
N.B. in multi-Regge region

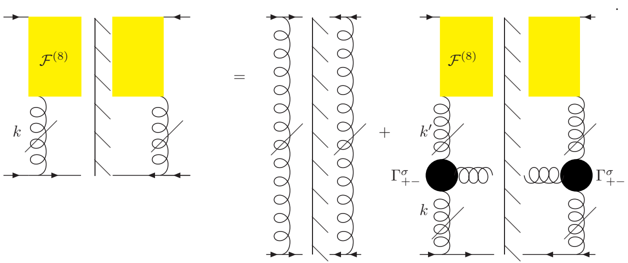
$$s_{i+1,i-1} \sim k_{i+1}^+ k_{i-1}^- \sim \mathbf{k_i}^2 \frac{k_{i-1}^+}{k_i^+}$$

Define $\mathcal{F}^{(8)}(s,\mathbf{k},\mathbf{q})$

$$\mathcal{A}^{(8)}(s,\mathbf{q}) = \int d^2\mathbf{k}\,\mathcal{F}^{(8)}(s,\mathbf{k},\mathbf{q})$$

$\mathcal{F}^{(8)}(s,\mathbf{k},\mathbf{q})$ is the amplitude for a particle to emit two reggeized gluons with (transverse) momenta $\mathbf{k}$ and $(\mathbf{k}-\mathbf{q})$ in a colour octet state.





$$\Im m\mathcal{F}^{(8)}(\sqrt{s},\mathbf{k},\mathbf{q}) = \Im m\mathcal{F}_0^{(8)}(\sqrt{s},\mathbf{k},\mathbf{q}) -$$

$$\frac{\alpha_s C_A}{4\pi^2}\int^{\sqrt{s}}\frac{dk^{+\prime}}{k^{+\prime}}\Gamma^{\sigma}_{+-}(k',k)\Gamma^{\sigma}_{+-}(k'-q,k-q)\frac{\Im m\mathcal{F}_0^{(8)}(k^{+\prime},\mathbf{k}',\mathbf{q})}{\mathbf{k}'^2(\mathbf{k}'-\mathbf{q})^2}$$

$$\times\left(\frac{k^{+\prime}}{k^{+}}\right)^{\varepsilon(\mathbf{k}^2)+\varepsilon((\mathbf{k}^-\mathbf{q})^2)}$$

Take Mellin transform

$$\tilde{\mathcal{F}}^{(8)}(\omega,\mathbf{k},\mathbf{q}) \;=\; \int_{s_0}^{\infty} s^{-\omega-1}\mathcal{F}^{(8)}(\sqrt{s},\mathbf{k},\mathbf{q})ds$$

Expanding the expression for Γ gives the integral equation in Mellin space

$$\Im m\tilde{\mathcal{F}}^{(8)}(\omega,\mathbf{k},\mathbf{q}) = \Im m\tilde{\mathcal{F}}_0^{(8)}(\omega,\mathbf{k},\mathbf{q}) - \frac{\alpha_s C_A}{4\pi^2}\int d^2\mathbf{k}'\frac{\Im m\tilde{\mathcal{F}}^{(8)}(\omega,\mathbf{k}',\mathbf{q})}{\omega-\varepsilon_G(\mathbf{k}^2)-\varepsilon_G((\mathbf{k}-\mathbf{q})^2)}$$

$$\times\frac{1}{\mathbf{k}'^2(\mathbf{k}'-\mathbf{q})^2}\left(\mathbf{q}^2-\frac{\mathbf{k}^2(\mathbf{k}'-\mathbf{q})^2+\mathbf{k}'^2(\mathbf{k}-\mathbf{q})^2}{(\mathbf{k}-\mathbf{k}')^2}\right)$$

Integrate both sides over $\mathbf{k}$ to get

$$\Im m\tilde{\mathcal{A}}^{(8)}(\omega,\mathbf{q}) = \Im m\tilde{\mathcal{A}}_0^{(8)}(\omega,\mathbf{q}) - \frac{\alpha_s C_A}{4\pi^2}\int d^2\mathbf{k}'d^2\mathbf{k}\frac{\Im m\tilde{\mathcal{F}}^{(8)}(\omega,\mathbf{k}',\mathbf{q})}{\omega-\varepsilon_G(\mathbf{k}^2)-\varepsilon_G((\mathbf{k}-\mathbf{q})^2)}$$

$$\times\frac{1}{\mathbf{k}'^2(\mathbf{k}'-\mathbf{q})^2}\left(\mathbf{q}^2-\frac{\mathbf{k}^2(\mathbf{k}'-\mathbf{q})^2+\mathbf{k}'^2(\mathbf{k}-\mathbf{q})^2}{(\mathbf{k}-\mathbf{k}')^2}\right)$$

$$\Im m\tilde{\mathcal{A}}^{(8)}(\omega,\mathbf{q}) = \Im m\tilde{\mathcal{A}}_0^{(8)}(\omega,\mathbf{q}) - \frac{\alpha_s C_A}{4\pi^2}\int d^2\mathbf{k}' d^2\mathbf{k}\frac{\Im m\tilde{\mathcal{F}}^{(8)}(\omega,\mathbf{k}',\mathbf{q})}{\omega-\varepsilon_G(\mathbf{k}^2)-\varepsilon_G((\mathbf{k}-\mathbf{q})^2)}$$

$$\times\frac{1}{\mathbf{k}'^2(\mathbf{k}'-\mathbf{q})^2}\left(\mathbf{q}^2-\frac{\mathbf{k}^2(\mathbf{k}'-\mathbf{q})^2+\mathbf{k}'^2(\mathbf{k}-\mathbf{q})^2}{(\mathbf{k}-\mathbf{k}')^2}\right)$$

BUT

$$-\frac{\alpha_s C_A}{4\pi^2}\int d^2\mathbf{k}'\frac{\mathbf{q}^2}{\mathbf{k}'^2(\mathbf{k}-\mathbf{q})^2} = \varepsilon_G(\mathbf{q}^2)$$

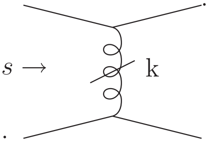
$$-\frac{\alpha_s C_A}{4\pi^2}\int d^2\mathbf{k}'\frac{\mathbf{k}^2}{\mathbf{k}'^2(\mathbf{k}'-\mathbf{k})^2} = \varepsilon_G(\mathbf{k}^2)$$

$$-\frac{\alpha_s C_A}{4\pi^2}\int d^2\mathbf{k}'\frac{(\mathbf{k}-\mathbf{q})^2}{(\mathbf{k}'-\mathbf{q}^2(\mathbf{k}'-\mathbf{k})^2} = \varepsilon_G((\mathbf{k}-\mathbf{q})^2)$$

Solved by

$$\Im m\tilde{\mathcal{A}}^{(8)}(\omega,\mathbf{q}) \sim \frac{1}{\omega-\varepsilon_G(\mathbf{q}^2)}$$

This justifies the ansatz and shows that the reggeized gluon is given (to all orders in perturbation - in the leading log approximation)
Exchange of a colour octet in the Regge region $s \gg t$



$$\frac{1}{\mathbf{k}^2}\left(\frac{s}{\mathbf{k}^2}\right)^{\varepsilon_G(\mathbf{k}^2)}$$

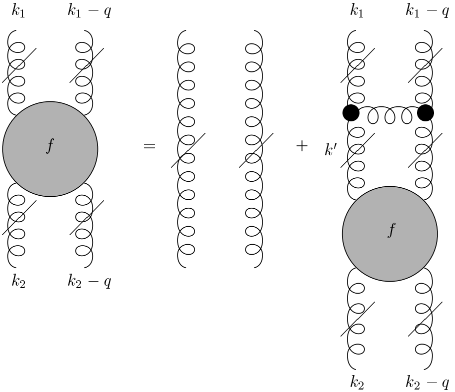
where

$$\varepsilon_G(\mathbf{q}^2) \;=\; -\frac{\alpha_s C_A}{4\pi^2}\int d^2\mathbf{k}\,\frac{\mathbf{q}^2}{\mathbf{k}^2(\mathbf{k}-\mathbf{q})^2}$$

This is IR divergent, but since it is not a colour singlet process it is unphysical.

The Pomeron in (perturbative) QCD

- Pomeron must be a colour singlet
- At leading order this is achieved by exchanging two gluons in a colour singlet state.
- As in the case of reggeized gluon, this can be generalised in leading log. approximation by
  - replacing gluons exchanged in t-channel by reggeized gluons
  - building ladders using the effective vertices Γ^{σ}_{+-}



For $\mathbf{q} = 0$

$$f(\sqrt{s}, \mathbf{k_1}, \mathbf{k_2}) = \delta^2(\mathbf{k_1} - \mathbf{k_2}) - \frac{\alpha C_A}{2\pi^2} \int d^2\mathbf{k}' \int^{\sqrt{s}} dk^+ f(k^+, \mathbf{k}', \mathbf{k_2})$$

$$\times \left(\frac{k_1^+}{k^{+\prime}} \right)^{2\varepsilon_G(\mathbf{k}'^2)} \frac{\Gamma^\sigma_{+-}(k_1, k')\Gamma^\sigma_{+-}(k_1, k')}{\mathbf{k}'^4}$$

BFKL Equation at zero momentum transfer

After a HUGE amount of algebra we get (in Mellin space)

$$\omega\tilde{f}(\omega,\mathbf{k_1},\mathbf{k_2}) = \delta^2(\mathbf{k_1}-\mathbf{k_2}) + \frac{\alpha C_A}{\pi^2}\int\frac{d^2\mathbf{k}'}{(\mathbf{k_1}-\mathbf{k}')^2}\left[\tilde{f}(\omega,\mathbf{k}',\mathbf{k_2}) - \frac{\mathbf{k_1}^2}{\mathbf{k}'^2+(\mathbf{k}'-\mathbf{k_1})^2}\tilde{f}(\omega,\mathbf{k_1},\mathbf{k_2})\right]$$

N.B. Integrand is finite as $\mathbf{k_1}\to\mathbf{k}'$
Integral is IR finite as expected for a colour singlet amplitude.

Solving the Equation

Write

$$\frac{\alpha C_A}{\pi^2}\int \frac{d^2\mathbf{k}'}{(\mathbf{k_1}-\mathbf{k}')^2}\left[\tilde{f}(\omega,\mathbf{k}',\mathbf{k_2}) - \frac{\mathbf{k_1}^2}{\mathbf{k}'^2+(\mathbf{k}'-\mathbf{k_1})^2}\tilde{f}(\omega,\mathbf{k_1},\mathbf{k_2})\right]$$

as

$$\mathcal{K}_0\cdot\tilde{f}$$

Solution may be written as

$$\tilde{f}\omega,\mathbf{k_1},\mathbf{k_2}) \;=\; \sum_i \frac{\phi_i(\mathbf{k_1})\phi_i^*((\mathbf{k_2})}{(\omega-\lambda_i)}$$

where

$$\mathcal{K}_0\cdot\phi_i \;=\; \lambda_i\phi_i$$

Eigenfunctions and Eigenvalues of BFKL Kernel

$$\text{Eigenfunctions:}, \ \phi_{n,\nu}(\mathbf{k}) = (k^2)^{-1/2+i\nu} e^{in\theta}$$

$$\text{Eigenvalues: } \frac{\alpha_s C_A}{\pi}\chi_n(\nu)$$

$$\chi(\nu) \ = \ 2\Psi(1) - \Psi\left(\frac{(n+1)}{2} + i\nu\right) - \Psi\left(\frac{(n+1)}{2} - i\nu\right)$$

General solution $(\mathbf{k} = (k, \theta))$

$$\tilde{f}(\omega, \mathbf{k_1}, \mathbf{k_2}) \ = \ \sum_{n=0}^{\infty} \int_{-\infty}^{\infty} \frac{d\nu}{2\pi^2 \mathbf{k_1}\mathbf{k_2}} \left(\frac{\mathbf{k_1}^2}{\mathbf{k_2}^2}\right)^{i\nu} \frac{e^{in(\theta_1-\theta_2)}}{\omega - \overline{\alpha_s}\chi_n(\nu)}$$

where

$$\overline{\alpha_s} \equiv \frac{\alpha_s C_A}{\pi}$$

At large s we need only the right-most singularity of the Mellin transform.
Set $n=0$
Invert Mellin transform to get

$$f(\sqrt{s},\mathbf{k_1},\mathbf{k_2}) = \int \frac{d\nu}{2\pi \mathbf{k_1}\mathbf{k_2}} \left(\frac{\mathbf{k_1}}{\mathbf{k_2}}\right)^{i\nu} s^{\overline{\alpha_s}\chi_0(\nu)}$$

$$\chi_0(\nu) \ = \ 2\Psi(1) - \Psi\left(\frac{1}{2}+i\nu\right) - \Psi\left(\frac{1}{2}-i\nu\right) \ = \ 4\ln(2) - 14\zeta(3)\nu^2 + \cdots$$

Integral can be performed in saddle-point approximation (truncate χ_0 at $O(\nu^2)$)

$$f(\sqrt{s},\mathbf{k_1},\mathbf{k_2}) \sim \frac{1}{\mathbf{k_1}\mathbf{k_2}} s^{4\overline{\alpha_s}\ln(2)} \frac{1}{\sqrt{\ln(s)}} \exp\left\{ \frac{-\ln^2(\mathbf{k_1}/\mathbf{k_2})}{14\zeta(3)\overline{\alpha_s}\ln(s)} \right\}$$

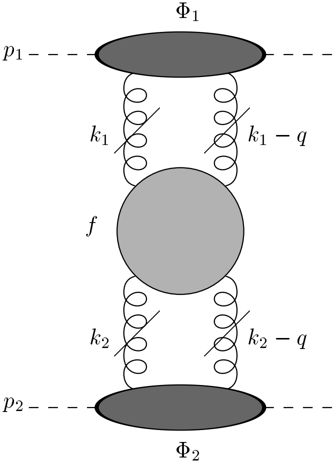
THE QCD POMERON

Properties of BFKL Pomeron

$$f(\sqrt{s},\mathbf{k_1},\mathbf{k_2}) \sim \frac{1}{\mathbf{k_1 k_2}} s^{4\overline{\alpha_s}\ln(2)} \frac{1}{\sqrt{\ln(s)}} \exp\left\{\frac{-\ln^2(\mathbf{k_1}/\mathbf{k_2})}{14\zeta(3)\overline{\alpha_s}\ln(s)}\right\}$$

- Exchange of colour singlet object in leading log. approximation
- Terms in $\ln(s)$ indicate that Pomeron has a cut with a branch-point at
$$4\ln(2)\overline{\alpha_s}$$
- The amplitude grows very fast as $s\to\infty$ - violates unitarity (needs correcting)
- Does NOT resemble the Landshoff-Donnachie pomeron ($\sim s^{.08}$) in any way !!

Impact factors



In order to calculate the amplitude for a physical process, the gluons at the top and bottom of the BFKL ladder must be attached to physical initial and final states. The amplitude for this process is

$$\mathcal{A} = \int \frac{d^2\mathbf{k_1}}{\mathbf{k_1}^2}\frac{d^2\mathbf{k_2}}{\mathbf{k_2}^2}\Phi_1(\mathbf{k_1})\Phi_2(\mathbf{k_2}) \times f(s,\mathbf{k_1},\mathbf{k_2},\mathbf{q})$$

Φ_1 and Φ_2 are process dependent impact factors, but they can sometimes be calculated in perturbation theory.

For the perturbation expansion to be totally reliable, the impact factors should only have support for $\mathbf{k} \gg \Lambda_{QCD}$
This is not usually possible - but it is often possible that one of the impact factors is in the perturbative regime.

Hamiltonian Approach to BFKL Equation

For the forward case ($q = 0$) the BFKL equation may be written

$$\frac{\overline{\alpha_s}}{\pi}\left[\int \frac{d^2k'k^2}{k'^2+(k-k')^2}f(k) - \int \frac{d^2k'}{(k-k')^2}f(k')\right] = \omega f(k)$$

The first term gives

$$\overline{\alpha_s}\ln\left(\frac{k^2}{\lambda^2}\right)$$

and the second is the convolution with the (2-dimensional) Fourier transform of

$$-\ln\left(\rho^2\lambda^2\right)$$

where the impact parameter ρ is conjugate to the transverse momentum k.
Consider the wavefunction

$$\tilde{\phi}(\rho) = \int d^2k e^{ik\cdot\rho}\phi(k)$$

and write the 2-d vector ρ as a complex number $\rho = \rho_x + i\rho_y$, The BFKL equation may be written as

$$\hat{H}\tilde{\phi}(\rho,\rho^*) = \omega\tilde{\phi}(\rho,\rho^*)$$

where the Hamiltonian operator is

$$\hat{H} = \overline{\alpha_s}\left[\ln(\hat{k}) + \ln(\rho) + h.c.\right]$$

with the operator

$$\hat{k} = -i\frac{\partial}{\partial\rho}$$

Note that the Hamiltonian is "holomorphically separable" i.e. it can be written as a sum of terms depending only on ρ and only on ρ^*' This means that the eigenfunctions are of the form

$$\phi(\rho,\rho^*) = \phi_1(\rho)\phi_2(\rho^*)$$

The holomorphic Hamiltonian $\hat{H} = \overline{\alpha_s} \left[\ln(\hat{k}) + \ln(\rho) \right]$

is invariant under rescaling

$$\rho \to \Lambda \rho, \quad k \to \frac{k}{\Lambda},$$

i.e. under the Weyl group whose generator is

$$\hat{M} = \rho \frac{\partial}{\partial \rho}$$

.
The eigenfunctions are therefore representations of this group, which are

$$\phi_m(\rho) = \rho^m$$

so that the general eigenfunction of the full Hamiltonian is

$$\phi_{m,\overline{m}}(\rho, \rho^*) = \rho^m (\rho^*)^{\overline{m}}$$

$$\rho^m (\rho^*)^{\overline{m}}$$

It can be shown that

$$\left[\ln(\hat{k}) + \ln(\rho)\right] \rho^m = \Psi(1) - \frac{1}{2}\left(\Psi(m) - \Psi(1-m)\right)$$

m and $\overline{m}$ do not have to be related, but by performing a trivial phase rotation on ρ the imaginary parts can be taken to be equal.
In order for the eigenfunctions to be normalizable we require

$$\Re e\{m + \overline{m}\} = -1,$$

so that we have

$$m = \frac{1}{2} + n + i\nu, \quad \overline{m} = \frac{1}{2} - n + i\nu, \quad (n \text{ integer, },\nu \text{ real })$$

$$\phi_{m,\overline{m}}(\rho,\rho^*) = \frac{1}{|\rho|}\left(\frac{\rho}{\rho^*}\right)^n |\rho|^{2i\nu}$$

n is called the "conformal weight", if it is zero then there is no dependence of the eigenfunction on the azimuthal angle of the 2-d vector ϕ.

BFKL Equation for $t \neq 0$

$$
\begin{aligned}
\frac{\partial}{\partial \ln(s)} f(s,\mathbf{k_1},\mathbf{k_2},\mathbf{q}) \quad &= \quad \delta^2(\mathbf{k_1}-\mathbf{k_2}) \\
&+ \frac{\bar{\alpha}_s}{2\pi}\int d^2\mathbf{k}' \left[\frac{-\mathbf{q}^2}{(\mathbf{k}'-\mathbf{q})^2\mathbf{k_1}^2} f(s,\mathbf{k}',\mathbf{k_2},\mathbf{q}) \right. \\
&+ \frac{1}{(\mathbf{k}'-\mathbf{k_1})^2}\left(f(s,\mathbf{k}',\mathbf{k_2},\mathbf{q}) - \frac{\mathbf{k_1}^2 f(s,\mathbf{k_1},\mathbf{k_2},\mathbf{q})}{\mathbf{k}'^2+(\mathbf{k_1}-\mathbf{k}')^2}\right). \\
&+ \frac{1}{(\mathbf{k}'-\mathbf{k_1})^2}\left(\frac{(\mathbf{k_1}-\mathbf{q})^2\mathbf{k}'^2 f(s,\mathbf{k}',\mathbf{k_2},\mathbf{q})}{(\mathbf{k}'-\mathbf{q})^2\mathbf{k_1}^2} \right. \\
&\qquad \left.\left. - \frac{(\mathbf{k_1}-\mathbf{q})^2 f(s,\mathbf{k_1},\mathbf{k_2},\mathbf{q})}{(\mathbf{k}'-\mathbf{q})^2+(\mathbf{k_1}-\mathbf{k}')^2}\right)\right].
\end{aligned}
$$

Solutions for $t \neq 0$

Solution for $t = -\mathbf{q}^2$, is still of the form

$$f(s, \mathbf{k_1}, \mathbf{k_2}, \mathbf{q}) = \int d\nu \, s^{\overline{\alpha_s}\chi_0(\nu)} f_\nu(\mathbf{k_1}, \mathbf{k_2}, \mathbf{q})$$

$$\begin{aligned} f_\nu(\mathbf{k_1}, \mathbf{k_2}, \mathbf{q}) \;\; = \;\; & \mathbf{k_2}^2 (\mathbf{k_1} - \mathbf{q})^2 \int d^2\rho_1 d^2\rho_1' d^2\rho_2 d^2\rho_2' e^{\mathbf{k_2}\cdot(\rho_2 - \rho_2') - \mathbf{k_1}\cdot(\rho_1 - \rho_1') + \mathbf{q}\cdot(\rho_1 - \rho_2)} \\ & \times \delta^2(\rho_1 + \rho_1' - \rho_2 - \rho_2') \tilde{f}^\nu(\rho_1, \rho_1', \rho_2, \rho_2') \end{aligned}$$

$$\tilde{f}^\nu(\rho_1, \rho_1', \rho_2, \rho_2') = \int d^2\rho_0 \left(\frac{(\rho_1 - \rho_1')^2 (\rho_2 - \rho_0)^2 (\rho_2' - \rho_0)^2}{(\rho_2 - \rho_2')^2 (\rho_1 - \rho_0)^2 (\rho_1' - \rho_0)^2} \right)^{i\nu}$$

Cut with (t-independent) branch-point $4\overline{\alpha_s}\ln(2)$.
t- dependence only in the eigenfunctions.

Non-Forward Pomeron

For non-zero momentum transfer the wavefunction depends on two impact parameters ρ_1, ρ_2 where $\rho_1 - \rho_2$ is conjugate to the pomeron momentum transfer q.
The Hamiltonian is again separable into holomorphic and anti-holomorphic parts

$$\tilde{H}_{12} = H_{12} + \overline{H}_{12}$$

$$\hat{H}_{12} = \overline{\alpha}_s \sum_{i=1}^{2} \left[\ln(\hat{k}_i) + \frac{1}{\hat{k}_i} \ln(\rho_{12}) \hat{k}_i - \Psi(1) \right], \quad (\rho_{12} \equiv (\rho_1 - \rho_2)$$

This Hamiltonian is invariant under 2-D conformal (Möbius transformations)

$$\rho_i \to \frac{a\rho_i + b}{c\rho_i + d}$$

$$\hat{H}_{12} = \overline{\alpha}_s \sum_{i=1}^{2} \left[\ln(\hat{k}_i) + \frac{1}{\hat{k}_i} \ln(\rho_{12}) \hat{k}_i - \Psi(1) \right]$$

The Möbius transformations are generated by

$$\hat{M}_+ = \sum_{i=1}^{2} \frac{\partial}{\partial \rho_i}, \ \hat{M}_- = \sum_{i=1}^{2} \rho_i^2 \frac{\partial}{\partial \rho_i}, \ \hat{M}_3 = \sum_{i=1}^{2} \rho_i \frac{\partial}{\partial \rho_i},$$

$$\hat{M}^2 \equiv \hat{M}_+ \hat{M}_- + \hat{M}_- \hat{M}_+ + \hat{M}_3^3 = -\rho_{12}^2 \frac{\partial}{\partial \rho_1} \frac{\partial}{\partial \rho_2}$$

$$[\hat{M}_i, \hat{H}_{12}] = 0$$

The eigenfunctions of H_{12} are simultaneously eigenfunctions of M^2 and M_3

$$\phi_m(\rho_1,\rho_2) \equiv \left(\frac{\rho_{12}}{\rho_1\rho_2}\right)^m$$

$$\hat{M}^2\phi_m(\rho_i) = m(m-1)\phi_m(\rho_i), \quad \hat{M}_3\phi_m(\rho_i) = m\phi_m(\rho_i)$$

$$\hat{H}_{12}\phi_m(\rho_i) = \frac{1}{2}\overline{\alpha}_s\left[2\Psi(1)-\Psi(m)-\Psi(1-m)\right]\phi_m(\rho_i$$

The complete wavefunction is

$$\phi_{m,\overline{m}}(\rho_1,\rho_1^*,\rho_2,\rho_2^*) = \left(\frac{\rho_{12}}{\rho_1\rho_2}\right)^m \left(\frac{\rho_{12}^*}{\rho_1^*\rho_2^*}\right)^{\overline{m}}$$

with eigenvalue

$$\omega = \overline{\alpha}_s\left[2\Psi(1)-\Psi(m)-\Psi(\overline{m})-\Psi(1-m)-\Psi(1-\overline{m})\right)$$

There is a further degeneracy generated by the translation invariance. The origin ρ_0 is arbitrary, so strictly we should write

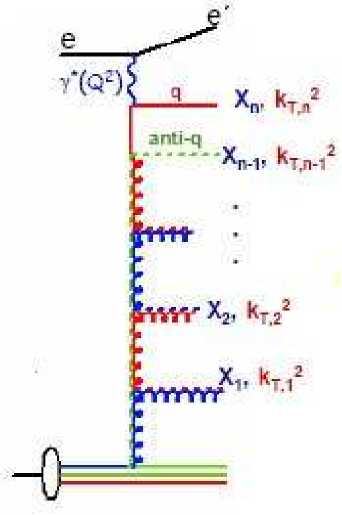
$$\phi_{m\overline{m}}(\rho_1,\rho_2,\rho_0) = \left(\frac{\rho_{12}}{\rho_{10}\rho_{20}}\right)^m \left(\frac{\rho_{12}^*}{\rho_{10}^*\rho_{20}^*}\right)^{\overline{m}}$$

By making a phase rotation on all ρ we can require that m and $\overline{m}$ have the same imaginary part, ν.

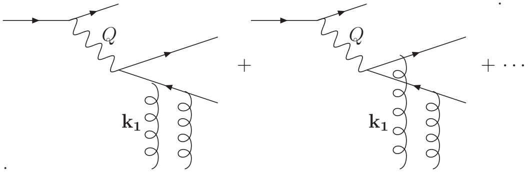
For normalizable wavefunctions we require $\Re e\{m+\overline{m}\} = 1$ so that the wavefunction may be written

$$\begin{aligned}\phi_{n,\nu}(\rho_1,\rho_2,\rho_0) &= \left(\frac{\rho_{12}}{\rho_{10}\rho_{20}}\right)^{(1/2+n+i\nu)} \left(\frac{\rho_{12}^*}{\rho_{10}^*\rho_{20}^*}\right)^{(1/2-n+i\nu)} \\ &= \left[\frac{\rho_{12}\rho_{10}^*\rho_{20}^*}{\rho_{12}^*\rho_{10}\rho_{20}}\right]^n \left|\frac{\rho_{12}}{\rho_{10}\rho_{20}}\right|^{(1+2i\nu)}\end{aligned}$$

Deep Inelastic Scattering at Low-x



$$s = Q^2 \frac{(1-x)}{x}$$



$$\Phi_2^{\gamma^*} = 4\pi\alpha\alpha_s \sum_f Q_f^2 \int_0^1 d\rho d\tau \frac{1-2\rho(1-\rho)-2\tau(1-\tau)+12\rho(1-\rho)\tau(1-\tau)}{Q^2\rho(1-\rho)+\mathbf{k_1}^2\tau(1-\tau)}$$

Define probability to find gluon inside proton with fractional momentum x and transverse momentum $\mathbf{k_1}$

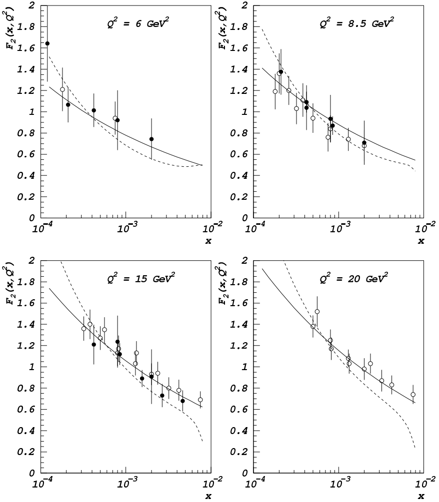
$$\mathcal{F}(x,\mathbf{k_1}^2) \equiv \frac{1}{2\pi^3}\int \frac{d^2 k_2}{\mathbf{k_2}^2} \Phi_p(\mathbf{k_2}) f(Q^2/x,\mathbf{k_1},\mathbf{k_2})$$

Φ_p is unknown impact factor of proton (must be modelled)

$$F_2(x,Q^2) \;=\; \frac{Q^2}{4\pi\alpha}\int \frac{d^2 k_1}{\mathbf{k_1}^4}\mathcal{F}(x,\mathbf{k_1}^2)\Phi_2^{\gamma^*}(Q^2,\mathbf{k_1})$$

Insert expression for $f(Q^2/x, \mathbf{k_1}, \mathbf{k_2})$

$$F_2(x,Q^2) \sim \left(\frac{1}{x}\right)^{4\overline{\alpha_s}\ln(2)} \frac{1}{\sqrt{-\ln(x)}} \exp\left\{\frac{\ln^2(Q^2/\mu^2)}{56\zeta(3)\overline{\alpha_s}\ln(x)}\right\}$$



dashed line $\alpha_s = 0.1$
solid line $\alpha_s = 0.2$

Relationship with DGLAP

Moments of structure functions:

$$F_N(Q^2) = \int_0^1 x^{N-1} F_2(x, Q^2) dx = \frac{Q^2}{4\pi\alpha} \int \frac{d^2\mathbf{k_1}}{\mathbf{k_1}^2} \mathcal{F}_N(\mathbf{k_1}) \Phi_2^{\gamma^*}(Q^2, \mathbf{k_1})$$

$$\mathcal{F}_N(\mathbf{k_1}) \sim \int d^2\mathbf{k_2} \Phi_p(\mathbf{k_2}) \int_{-\frac{1}{2}-i\infty}^{-\frac{1}{2}+i\infty} d\gamma \left(\frac{\mathbf{k_1}^2}{\mathbf{k_2}^2}\right)^{\gamma} \frac{1}{N - \overline{\alpha_s}\chi(\gamma)}$$

$$\gamma = \frac{1}{2} + i\nu$$

$$\chi(\gamma) = 2\Psi(1) - \Psi(\gamma) - \Psi(1-\gamma) = \frac{1}{\gamma} + 2\sum_r \zeta(2r+1)\gamma^{2r}$$

Integral over γ picks up a pole at

$$\gamma = \overline{\gamma} = \chi^{-1}\left(\frac{N}{\overline{\alpha_s}}\right)$$

$$\mathcal{F}_N(\mathbf{k_1}) \sim \int d^2\mathbf{k_2}\Phi_p(\mathbf{k_2}) \int_{-\frac{1}{2}-i\infty}^{-\frac{1}{2}+i\infty} d\gamma \left(\frac{\mathbf{k_1}^2}{\mathbf{k_2}^2}\right)^\gamma \frac{1}{N-\overline{\alpha_s}\chi(\gamma)} \quad \sim \left(\mathbf{k_1}^2\right)^{\overline{\gamma}}$$

$$F_N(Q^2) = \frac{Q^2}{4\pi\alpha}\int \frac{d^2\mathbf{k_1}}{\mathbf{k_1}^2}\mathcal{F}_N(\mathbf{k_1})\Phi_2^{\gamma^*}(Q^2,\mathbf{k_1}) \ \sim (Q^2)^{\overline{\gamma}}$$

[$\Phi_2^{\gamma^*}(Q^2,\mathbf{k_1})$ peaks at $Q^2 \sim \mathbf{k_1}^2$]
DGLAP equation:

$$\frac{\partial}{\partial \ln(Q^2)} F_N(Q^2) = \overline{\gamma} F_N(Q^2)$$

Invert

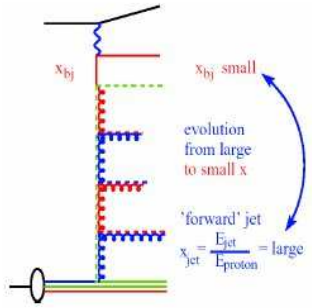
$$\chi(\gamma) = 2\Psi(1) - \Psi(\gamma) - \Psi(1-\gamma) = \ \frac{1}{\gamma} + 2\sum_r \zeta(2r+1)\gamma^{2r}$$

to get DGLAP gluon splitting function near N=0 to all orders in α_s

$$\lim_{N\to 0}\gamma_N = \overline{\gamma} = \left(\frac{\overline{\alpha_s}}{N}\right) + 2\zeta(3)\left(\frac{\overline{\alpha_s}}{N}\right)^4 + \ \cdots$$

Mono-jets

Is it possible to isolate a region of phase-space which has a "clean" BFKL pomeron - no non-perturbative model.



Mono-jet: Parton with transverse momentum $\mathbf{k_j}$ and fraction x_j of proton momentum

If $\mathbf{k_j} \gg \Lambda_{QCD}$ proton impact factor becomes

$$\Phi_p = 8\pi^2 \alpha_s f_i^P(x_j, \mathbf{k_j})$$

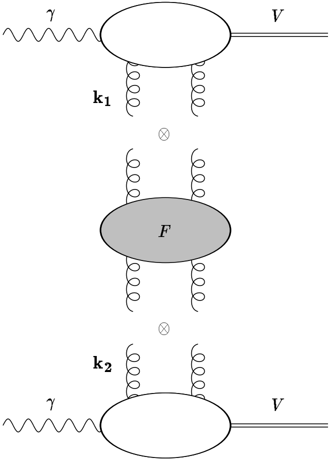
$x_j \sim 1$ so PDF $f_i^P(x_j, \mathbf{k_j})$ is well-measured.

Experimentally difficult to observe - jet in forward direction

$$\frac{\partial^2 F_2(x, Q^2, x_j, \mathbf{x_j})}{\partial \ln(x) \partial \mathbf{k_j}^2} \sim Q^2 \int \frac{d^2\mathbf{k_1}}{\mathbf{k_1}^2} \phi_2^{\gamma^*}(\mathbf{k_1}) f(x/x_j, \mathbf{k_1}, \mathbf{k_j}) f_i^P(x_j, \mathbf{k_j})$$

Vector Meson Production

Another possible way of guaranteeing that the top and bottom of BFKL ladder has $\mathbf{k} \gg \Lambda_{QCD}$ is in vector meson production

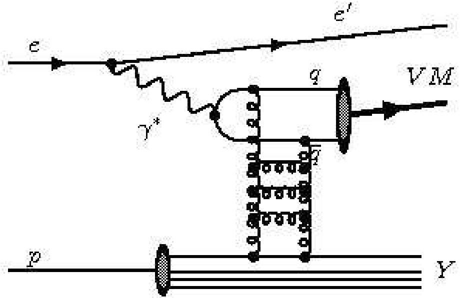


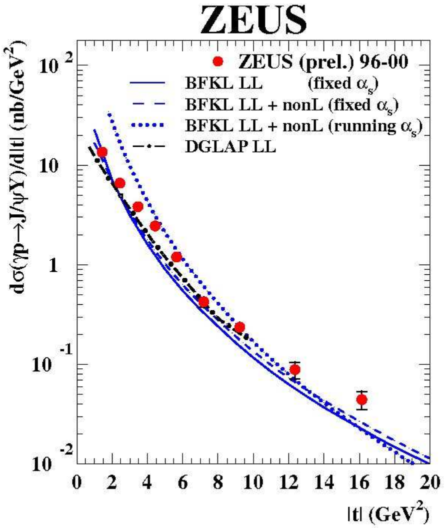
Either:
V is a heavy meson (e.g. J/Ψ)

$$\mathbf{k_1}^2 \sim \mathbf{k_2}^2 \sim m^2_{J/\Psi}$$

Or:
V is emitted with transverse momentum, $-t >> \Lambda^2_{QCD}$
but $-t \ll s$

More realistically only one vector-meson is produced

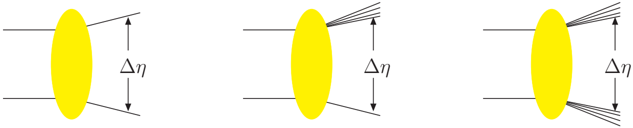




ZEUS
ZEUS (prel.) 96-00
BFKL LL (fixed α_s)
BFKL LL + nonL (fixed α_s)
BFKL LL + nonL (running α_s)
DGLAP LL
$d\sigma(\gamma p \to J/\psi Y)/d|t|$ (nb/GeV^2)
10^2
10
1
10^{-1}
10^{-2}
0
2
4
6
8
10
12
14
16
18
20
|t| (GeV^2)

Diffraction

Processes for which $-t \ll s$



Elastic diffraction   Diffractive Dissociation   Double Diffractive Dissociation

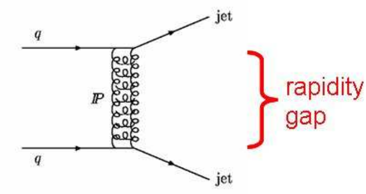
In each case the "jets" have large positive or negative rapidity

$$\eta = \frac{1}{2}\ln\left(\frac{E+p_z}{E-p_z}\right)$$

Large rapidity gap between the two jets

$$\Delta\eta \sim \ln\left(\frac{s}{-t}\right)$$

Such rapidity gaps are possible is object exchanged between scattering particles is a colour singlet (a pomeron)

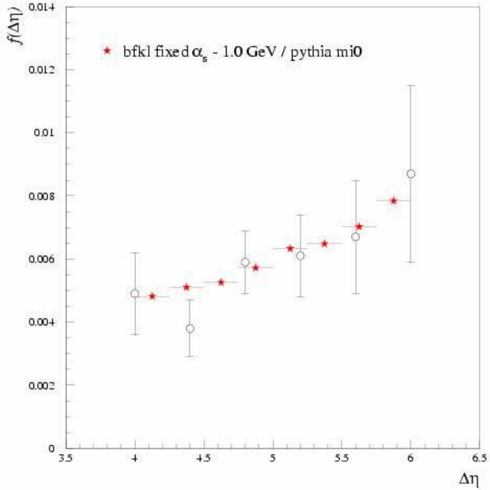


BFKL amplitude grows as

$$e^{4\overline{\alpha_s}\ln(2)\Delta\eta}$$

Expect the number of events with rapidity gap $\Delta\eta$ to grow with $\Delta\eta$

Gap fraction $f(\Delta\eta)$

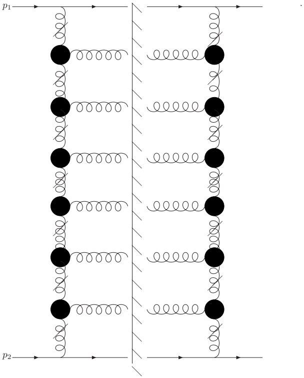


Rapidity Gap Survival

The theoretical gap fraction calculated using BFKL should be larger than that observed

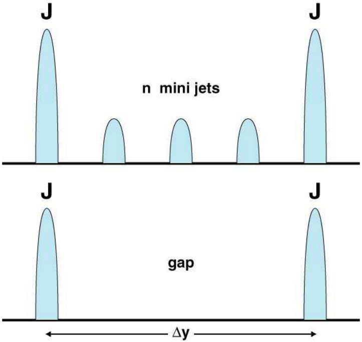
The spectator partons which do NOT partake in the BFKL evolution can nevertheless radiate gluons which can populate the rapidity gap between the primary jets.

Mini-jets



Optical Theorem

Imaginary part of BFKL amplitude is probability to find two primary jets and any number of low-energy "mini-jets"



Write the BFKL amplitude as

$$f(s,\mathbf{k_1},\mathbf{k_2}) = \sum_n P_n \ln^n(s)$$

P_n is the probability that the ladder has n rungs (equal to the probability that there are n mini-jets)
This gives an expression for the average number of mini-jets

$$< n > = \ln(s) \frac{\partial}{\partial \ln(s)} f(s,\mathbf{k_1},\mathbf{k_2})$$

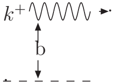
Using

$$f(s,\mathbf{k_1},\mathbf{k_2}) \sim s^{4\ln(2)\overline{\alpha_s}}, \quad (s \sim e^{\Delta\eta})$$

$$< n > \approx 4\ln(2)\overline{\alpha_s}\Delta\eta$$

Can get $< n > \sim 3-4$ at Tevatron - MORE AT LHC!!!

Diffraction Quantum Optics



Describe a photon in terms of longitudinal momentum k^+ and impact parameter $\mathbf{b}$, $|k^+,\mathbf{b}\rangle$
("energy" eigenstate in light-cone quantisation)

$$|in\rangle = \int dk^+ d^2\mathbf{b}\, \phi_{in}(k^+,\mathbf{b})|k^+,\mathbf{b}\rangle$$

$|k^+,\mathbf{b}\rangle$ are eigenstate of diffraction operator T

$$T|k^+,\mathbf{b}\rangle = t(k^+,\mathbf{b})|k^+,\mathbf{b}\rangle$$

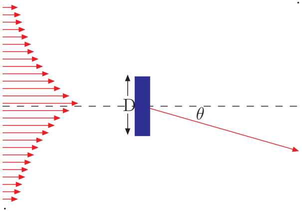
$$|out\rangle = \int dk^+ d^2\mathbf{b}\, \phi_{out}(k^+,\mathbf{b})|k^+,\mathbf{b}\rangle$$

e.g amplitude to scatter into state with transverse momentum $\mathbf{k}$

$$\mathcal{A} = \int dk^+ d^2\mathbf{b}\, \phi_{in}(k^+, \mathbf{b})\, t(k^+, \mathbf{b}) \langle k^{+\prime}, \mathbf{k} | k^+, \mathbf{b} \rangle = \int d^2\mathbf{b}\, \phi_{in}(k^+, \mathbf{b})\, t(k^+, \mathbf{b})\, e^{i\mathbf{k}\cdot\mathbf{b}}$$

e.g. Diffraction due to opaque slab

$$\phi_{in} = e^{-\alpha b} \quad (k^+ \gg \alpha)$$



$$\mathcal{A} = \int d^2\mathbf{b} e^{-\alpha b} \theta(b - D/2) e^{ik^+ b \sin\theta}$$

$$= 4 \frac{e^{-\alpha D/2}}{k^+ \sin\theta} \cos(\frac{1}{2} k^+ D \sin\theta)$$

Diffraction pattern

Diffraction in Particle Scattering

In particle physics it is also the case that for diffractive processes an incoming particle (helicity λ) described in terms of k^+, $\mathbf{b}$, $\boldsymbol{\lambda}$ is an eigenstate of the diffractive scattering operator.

$$T|k^+, \mathbf{b}, \lambda\rangle = t(k^+, \mathbf{b}\lambda)|k^+, \mathbf{b}, \lambda\rangle$$

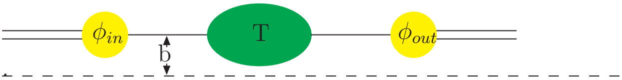
Incoming state with given k^+ may be written

$$|in\rangle = \sum_\lambda \int d^2\mathbf{b}\phi_{in}(\mathbf{b})|k^+, \mathbf{b}, \lambda\rangle$$

and out-going state with given k^+ may be written

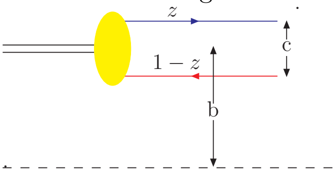
$$|out\rangle = \sum_\lambda \int d^2\mathbf{b}\phi_{out}(\mathbf{b})|k^+, \mathbf{b}, \lambda\rangle$$

The diffractive scattering process is



Colour Dipole Approach

In QCD the diffraction eigenstates are colour dipoles of transverse size $\mathbf{c}$ with (average) impact parameter $\mathbf{b}$, with the coloured particles carrying fractions z and $1-z$ of the longitudinal momentum k^+.



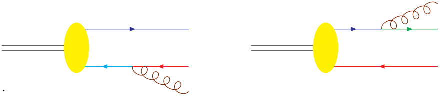
The amplitude for an incoming particle to split into such a dipole depends on z and $\mathbf{c}$ and likewise the amplitude for the dipole to form a final state also depends on these variables so the diffraction amplitude is given by

$$\int d^2\mathbf{c}\, dz\, \phi_{in}(z,\mathbf{c}) t(k^+,\mathbf{b},\mathbf{c}) \phi_{out}(z,\mathbf{c})$$

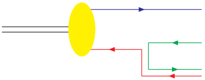
$\phi_{in}(z,\mathbf{c})\phi_{out}(z,\mathbf{c})$ plays then role of the impact factor (Fourier transformed) and $t(k^+,\mathbf{b},\mathbf{c})$ plays the part of the (process independent) BFKL evolution.

Dipole Evolution

As an incoming colour dipole propagates it can emit gluons,



thereby splitting into two colour dipoles.



Probability to emit dipole into rapidity interval $d\eta$ and impact parameter interval $d^2\mathbf{w}$ is

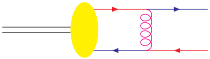
$$dP = \frac{\overline{\alpha_s}}{2\pi} \frac{(\mathbf{x}-\mathbf{y})^2}{(\mathbf{x}-\mathbf{w})^2(\mathbf{w}-\mathbf{y})^2} d^2\mathbf{w} d\eta$$

This increases the effective incoming dipole flux, thereby increasing the cross-section.

Rapidity dependence of dipole scattering cross-section

$$\frac{\partial\sigma(\mathbf{x},\mathbf{y},\eta)}{\partial\eta} = \frac{\overline{\alpha_s}}{2\pi}\int d^2\mathbf{w}\frac{(\mathbf{x}-\mathbf{y})^2}{(\mathbf{x}-\mathbf{w})^2(\mathbf{w}-\mathbf{y})^2} \times\left(\sigma(\mathbf{x},\mathbf{w},\eta)+\sigma(\mathbf{y},\mathbf{w},\eta)-\sigma(\mathbf{x},\mathbf{y},\eta)\right)$$

The first two terms represent the scattering of either of the two dipoles from the target, the third represents the case in which the



original dipole is destroyed (virtual corrections) .

This is the BFKL equation in impact parameter space.

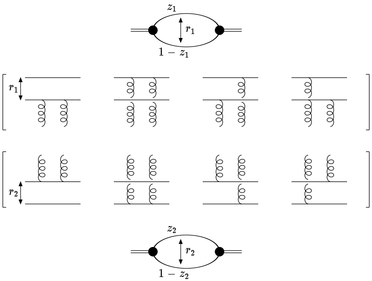
From the Optical Theorem

$$\sigma(\mathbf{x},\mathbf{y},\eta) = \frac{1}{s}\Im m(t(\sqrt{s},\mathbf{b},\mathbf{c})$$

$$\mathbf{b} = \frac{\mathbf{x}+\mathbf{y}}{2}, \quad \mathbf{c} = \mathbf{x}-\mathbf{y}, \quad \eta \sim \ln(s)$$

Rapidity dependence of $\boldsymbol{\sigma}$ gives rapidity dependence of diffractive scattering eigenvalue.

In the standard BFKL approach, impact factor is probability for dipole production/decay - the rapidity dependence is encoded in the gluon ladder.



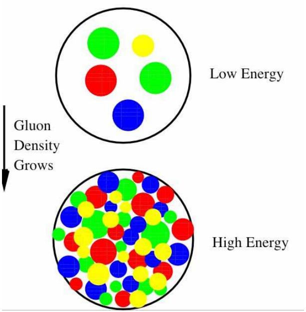
In dipole approach the gluon ladder is replaced by 2 gluons (LO colour singlet exchange) - rapidity dependence is absorbed into the impact factor leading to an incoming dipole density that grows with rapidity.

Saturation

Dipole density grows as

$$e^{4\overline{\alpha_s}\ln(2)\eta}$$

Growth cannot continue for ever - unitarity violated.



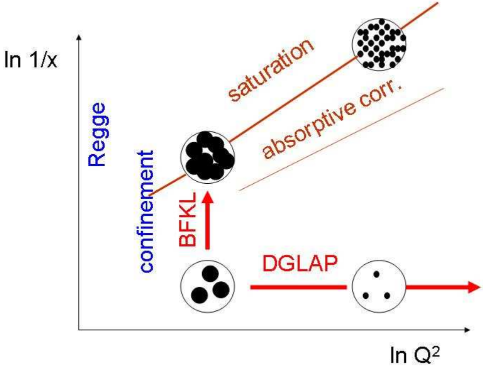
As rapidity grows dipole density increases so that dilute dipole approximation (single dipole scattered by target) breaks down.

This is the basis of the

Colour Glass Condensate

Effective theory that describes QCD in limit of high gluon density.

First applied to heavy ion collision (RHIC) but can also be applied to DIS at sufficiently low x or diffractive scattering at sufficiently large rapidities.


ln 1/x
Regge
confinement
saturation
absorptive corr.
BFKL
DGLAP
ln Q^2

Balitsky-Kovchegov Equation

In terms of the dipole evolution multiple scatterings are introduced by adding a term that account for both dipoles to scatter form the target simultaneously

$$\frac{\partial\sigma(\mathbf{x},\mathbf{y},\eta)}{\partial\eta} = \frac{\overline{\alpha_s}}{2\pi}\int d^2\mathbf{w}\frac{(\mathbf{x}-\mathbf{y})^2}{(\mathbf{x}-\mathbf{w})^2(\mathbf{w}-\mathbf{y})^2} \times\left(\sigma(\mathbf{x},\mathbf{w},\eta)+\sigma(\mathbf{y},\mathbf{w},\eta)-\sigma(\mathbf{x},\mathbf{y},\eta)-\frac{1}{2}\sigma(\mathbf{x},\mathbf{w},\eta)\sigma(\mathbf{y},\mathbf{w},\eta)\right)$$

Non-linear equation (difficult to solve)

Analogous to

- Langevin equation (stochastic processes)
- Fisher, Kolmogov, Petrovski, Pisconov equation

$$\frac{\partial\sigma(\mathbf{x},\mathbf{y},\eta)}{\partial\eta} = \frac{\overline{\alpha_s}}{2\pi}\int d^2\mathbf{w}\frac{(\mathbf{x}-\mathbf{y})^2}{(\mathbf{x}-\mathbf{w})^2(\mathbf{w}-\mathbf{y})^2}$$
$$\times\left(\sigma(\mathbf{x},\mathbf{w},\eta)+\sigma(\mathbf{y},\mathbf{w},\eta)-\sigma(\mathbf{x},\mathbf{y},\eta)-\frac{1}{2}\sigma(\mathbf{x},\mathbf{w},\eta)\sigma(\mathbf{y},\mathbf{w},\eta)\right)$$

Non-linear term moderates the growth so that $\boldsymbol{\sigma}$ saturates.
Expect a solution of the form

$$\sigma(\mathbf{x},\mathbf{y},\eta)\sim\left[1-\exp\left(-(\mathbf{x}-\mathbf{y})^2 e^{4\overline{\alpha_s}\ln(2)\eta}\right)\right]$$

Unitarity is respected in terms of dipole density.
BUT to get a physical cross-section need F.T. in $\mathbf{b}=(\mathbf{x}+\mathbf{y})/2$.
Region itself in **b**-space over which $\boldsymbol{\sigma}$ is non-zero (after saturating) grows exponentially with rapidity - thereby violating the Froissart-Martin bound)
The unitarization of the BFKL amplitude even with BK modification is still not fully resolved.

Cuts not Poles

The Mellin transform of the BFKL amplitude involves an integral over ν.

$$f(\omega, \mathbf{k_1}, \mathbf{k_2}) \sim \int d\nu \left(\frac{\mathbf{k_1}^2}{\mathbf{k_2}}\right)^{i\nu} \frac{1}{\omega - \overline{\alpha_s}\chi(\nu)}$$

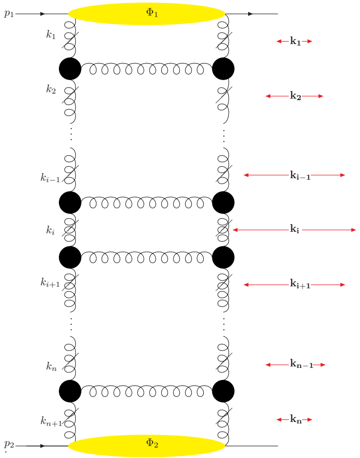
The singularity is a cut with branch-point

$$\overline{\alpha_s}\chi(0)$$

Regge theory predicts a pole !

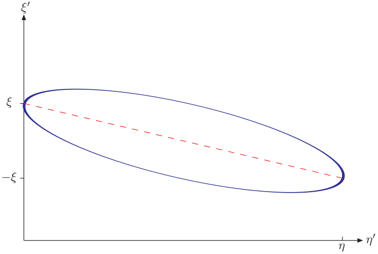
Diffusion

As we go down the BFKL ladder away from the impact factor the spread of **k** for which the integral has support, increases (until we start getting near the bottom of the ladder)



Bartels' Cigar

The BFKL equation is a diffusion equation in $\eta \sim \ln(s)$ and $\xi = \ln(\mathbf{k}^2/\Lambda^2_{QCD})$.



Eigenvalue equation

$$\mathcal{K}_0 e^{i\nu\xi} = \overline{\alpha_s}\chi(\nu) e^{i\nu\xi} = \overline{\alpha_s}\chi\left(-i\frac{\partial}{\partial\xi}\right) e^{i\nu\xi}$$

BFKL equation (as a diffusion equation)

$$\left[\frac{\partial}{\partial\eta} - \overline{\alpha_s}\chi\left(-i\frac{\partial}{\partial\xi}\right)\right] f(\eta,\xi) = \delta(\xi-\xi_0)$$

What value should be used for $\overline{\alpha_s}$?

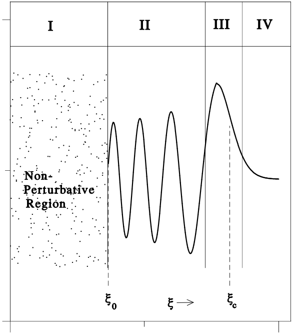
$$\overline{\alpha_s}(\xi) \sim \frac{C_A \beta_0}{\pi \xi}$$

Eigenfunction with eigenvalue ω

$$e^{i\nu\xi} \quad \left(\nu = \chi^{-1}\left(\frac{\omega\pi\xi}{\beta_0 C_A}\right)\right)$$

$$\xi_c = 4\ln(2)\frac{\beta_0 C_A}{\omega\pi}$$

[For $\xi > \xi_c$, ν is imaginary]



- Region II: $\xi \ll \xi_c$ Oscillating solution
- Region IV: $\xi \gg \xi_c$ Exponential decay
- Region III: ν is small

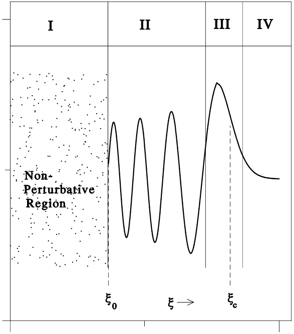
$$\omega - \chi_(\nu) \approx \frac{\omega}{\beta_0 C_A} (\xi - \xi_c)$$

Expanding χ to order ν^2

$$\left[\frac{\omega\pi}{\beta_0 C_A} (\xi - \xi_c) + \frac{\chi''(0)}{2} \frac{\partial^2}{\partial \xi^2} \right] f_\omega(\xi) = 0$$

Airy's equation.
Airy functions chosen to match functions and their derivatives in regions II and IV (fixes phase at II-III boundary).

- Region 1: $\xi < \xi_0$, too small for perturbation theory to be reliable.



Now suppose the infrared behaviour in the non-perturbative region fixes the phase at the I-II boundary.

This means that only solutions with discrete values of ω can fit between regions I and II an obey the phase fixing at both boundaries

This gives separate poles, rather than a cut for the BFKL amplitude - consistent with the predictions of Regge theory.

BFKL Kernel at Next-to Leading Logarithm Order

For the calculation in sub-leading log

- Cannot restrict the phase-space to the multi-Regge region
- Cannot neglect crossed-ladder and other graphs
- Cannot neglect section of ladder involving fermions

Took 22 years to complete the calculation !!

BFKL kernel is invariant under two-dimensional conformal transformations (SL2(C)).
In impact parameter space, kernel $\mathcal{K}(\mathbf{b}, \mathbf{b}')$ is invariant under

$$\mathbf{b} \rightarrow \frac{A\mathbf{b}+B}{C\mathbf{b}+D} \quad (AD = BC)$$

Eigenfunctions of BFKL kernel are of the form

$$\left(\mathbf{k}^2\right)^{i\nu}$$

also in higher orders (representations of conformal group) with eigenvalues

$$\omega = \overline{\alpha_s}\chi_0(\nu) + \overline{\alpha_s}^2\chi_1(\nu) + \cdots$$

Leading s behaviour in NLO

$$\omega(\nu) = \overline{\alpha_s}\chi_0(\nu) + \overline{\alpha_s}^2\chi_1(\nu) + \cdots$$

$$f(s, \mathbf{k_1}, \mathbf{k_2}) \sim \int d\nu \left(\frac{\mathbf{k_1}^2}{\mathbf{k_2}^2}\right)^{i\nu} s^{\omega(\nu)}$$

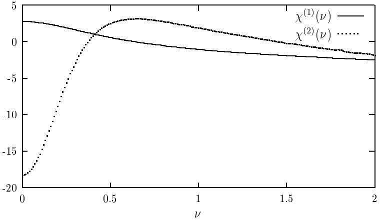
$$\chi_0(0) = 4\ln(2) = 2.8$$

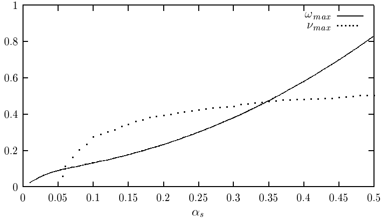
$$\chi_1(0) = -18.3$$

NLO term dominates leading s growth, (branch-point of pomeron cut) unless $\alpha_s \ll 0.1$

BUT we need to integrate over all ν - not just look at branch-point.

$\chi_1(\nu)$ varies rapidly with ν





At leading order ω decreases with ν so that $\omega_{max} = \omega(0)$ and $\nu_{max} = 0$ - but this is not true at NLO

To calculate

$$\int d\nu \left(\frac{\mathbf{k_1}^2}{\mathbf{k_2}^2}\right)^{i\nu} s^{\omega(\nu)}$$

need to go beyond saddle-point approximation and expand ω to quartic order

$$\omega(\nu) = \omega_0 + a\nu^2 - b\nu^4 + \cdots,$$

For $\alpha_s = 0.15$,

$$\omega_0 = 0.021, \quad a = 4.19, \quad b = 47.4$$

$$f(s, \mathbf{k_1}, \mathbf{k_2}) \sim \frac{1}{\sqrt{\mathbf{k}_1{}^2 \mathbf{k}_2{}^2}} s^{(\omega_0 + a^2/4b)} \frac{1}{\sqrt{a \ln s}} \exp\left(\frac{3b}{4a^2 \ln s}\right)$$
$$\cos\left(\sqrt{\frac{a}{2b}}\left(1 - \frac{3b}{4a^2 \ln s}\right) \ln\left(\frac{\mathbf{k_1}^2}{\mathbf{k_2}^2}\right)\right)$$

Forward amplitude (proportional to total cross-section by Optical Theorem) oscillates if $\mathbf{k_1}^2 \gg \mathbf{k_2}^2$ or $\mathbf{k_1}^2 \ll \mathbf{k_2}^2$

Reconciling NLO BFKL with DIS

Beyond the leading log we must write the BFKL amplitude as

$$f(s,\mathbf{k_1},\mathbf{k_2}) \sim \int_{-\frac{1}{2}-i\infty}^{-\frac{1}{2}+i\infty} d\gamma \left(\frac{\mathbf{k_1}^2}{\mathbf{k_2}^2}\right)^{\gamma} \left(\frac{s}{s_0}\right)^{\omega(\gamma)} \frac{1}{(\omega-\chi(\gamma))}$$

where

$$\omega(\gamma) = \overline{\alpha_s}\chi_0(\gamma) + \overline{\alpha_s}^2\chi_1(\gamma) \equiv \chi(\gamma)$$

Ambiguity in s_0.
Change $s_0 = \mathbf{k_1}^2$ to $s_0 = \mathbf{k_1}\mathbf{k_2}$ is equivalent to replacing γ by $\gamma + \frac{1}{2}\omega$ in $\chi(\gamma)$
In DIS the maximum log dependence in order n is

$$\alpha_s^n \ln^n(x)$$

But $\chi_1(\gamma)$ contains a triple pole

$$\sim 1/\gamma^3.$$

which implies a $\alpha_s^2 \ln^3(x)$ dependence

Replacing

$$\chi_0(\gamma) = 2\Psi(1) - \Psi(\gamma) - \Psi(1-\gamma)$$

by

$$2\Psi(1) - \Psi\left(\gamma + \frac{\omega}{2}\right) - \Psi\left(1 - \gamma + \frac{\omega}{2}\right)$$

reproduces the

$$\alpha_s^2/\gamma^3$$

term when expanded as a power series in α_s, but removes the spurious $\alpha_s^2 \ln^3(s)$ dependence.

For a given value of γ the power of s is given by $\omega(\gamma)$, which is the solution of the implicit equation

$$\omega(\gamma) = \overline{\alpha_s}\left[2\Psi(1) - \Psi\left(\gamma + \frac{\omega}{2}\right) - \Psi\left(1 - \gamma + \frac{\omega}{2}\right)\right] + \overline{\alpha_s}^2 \chi_1^{sub}(\gamma)$$

where

$$\chi_1^{sub}(\gamma) = \chi_1(\gamma) - \frac{1}{2\gamma^3}$$

A further large but summable correction srises from the order ω correction to the gluon anomalous dimension

$$\gamma = \overline{\alpha}_s \left[\frac{1}{\omega} + A_1 \right], \quad \left(A_1 = -\frac{\beta_0}{2C_A} - \frac{11}{12} - \frac{n_f}{6C_A^2} \right)$$

The $1/\omega$ term matches DGLAP to (L.O) BFKL in the DLL limit. For NLO BFKL we must match the constnt term also by replacing γ by $\gamma - \overline{\alpha}_s A_1$ (for small γ).
We therefore have an expression for ω

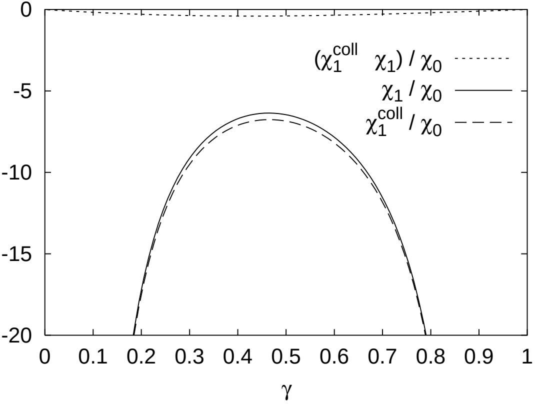
$$\omega(\gamma) = \overline{\alpha_s} \left[2\Psi(1) - \Psi\left(\gamma + \frac{\omega}{2} - \overline{\alpha}_s A_1\right) - \Psi\left(1 - \gamma + \frac{\omega}{2} - \overline{\alpha}_s A_1\right)\right] + \cdots$$

Solving by iteration up to order $\overline{\alpha}_s^2$

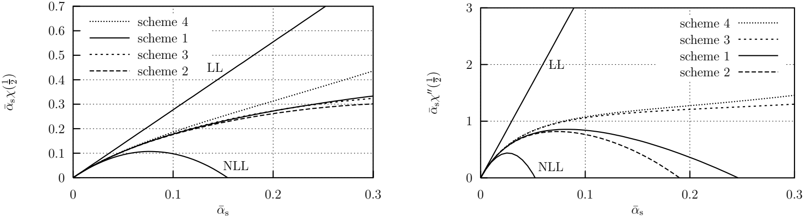
$$\omega = \overline{\alpha}_s \left[2\Psi(1) - \Psi(\gamma) - \Psi(1-\gamma)\right] + \overline{\alpha}_s^2 \left[\frac{2}{\gamma^3} + \frac{2}{(1-\gamma)^3} - \frac{A_1}{\gamma^2} - \frac{A_1}{(1-\gamma)^2} \right]$$

(plus terms which are less singular as $\gamma \to 0$ or $\gamma \to 1$)

This “collinear” correction ($\gamma \to 0(1)$ corresponds to gluons emitted parallel (anti-parallel) to emitter) accounts for mearly all the NLO correction



The summation of these “collinear singular” terms considerably moderates the effect of the higher order contribution.
Taking steps to ensure that the leading log. DGLAP dependence is correctly reproduced in all orders, the higher order BFKL amplitude suitably modified) produces a modest effect.



Hard and Soft Pomerons?

Hard Pomeron:

Solution to BFKL equation with leading energy dependence

$$s^{\omega}$$

$$\omega = 4\ln(2)\overline{\alpha_s} + O(\overline{\alpha_s}^2)$$

Rapid rise with increasing s.

Controversial evidence for BFKL behaviour in low-x DIS, diffractive processes etc.

Soft Pomeron:

Regge trajectory $\alpha_P(t)$ with vacuum quantum numbers.

For $t > 0$, glue-balls of mass $\sqrt{t}$ and spin J, where $\alpha_P(t) = \mathrm{J}$.

For $t = 0$, $\alpha_P(t)$ is close to one.

Landshoff-Donnachie:

$$\alpha_P(t) = 1.08 - 0.25(\mathrm{Gev}^{-2})t$$

Good phenomenological fit.

The "two" pomerons seem to be mutually incompatible.

[problem has existed for 36 years !!]

Landshoff-Donnachie (soft) pomeron is used to calculate total hadronic cross-sections.

These are not accessible to perturbative QCD since they always probe the infrared regime.

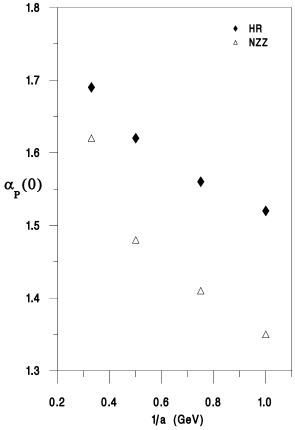
To soften the s^{ω} behaviour of the perturbative (hard) pomeron, these infrared effects must somehow exactly cancel the perturbative contributions (at least below the saturation scale)

Landshoff-Nachtmann Model

- Based on Low-Nussinov model of two gluon exchange
- At low $\mathbf{k}^2$ gluon propagators should be replaced by non-perturbative propagators
- Postulated that confined gluons have propagator which had a stronger singularity as $\mathbf{k}^2 \to 0$.

Studies of Dyson-Schwinger equation show that this does NOT but gluon acquires an effective mass at the IR scale a

$$D(\mathbf{k}^2) \sim \frac{a^2}{1+a^2\mathbf{k}^2}$$

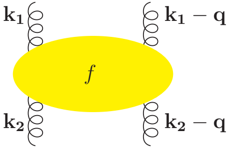


HR: insert non-perturbative propagators into BFKL equation and solve numerically.

NZZ: recalculate dipole evolution using effective mass for gluons.

Find a modest decrease in pomeron intercept $\alpha_P(0)$, but not sufficient.

Heterotic Pomeron



$$f(s,\mathbf{k_1},\mathbf{k_2},\mathbf{q}) = \int d^2\mathbf{b}\, e^{i\mathbf{q}\cdot\mathbf{b}} \tilde{f}(s,\mathbf{k_1},\mathbf{k_2},\mathbf{b}).$$

Diffusion in s, $\mathbf{k_1}$ and $\mathbf{b}$.

$$\frac{\partial \tilde{f}}{\partial s} = \int \frac{ds'}{s'} d^2\mathbf{k}'\, d^2\mathbf{b}\, \mathcal{K}\left(\frac{s}{s'},\mathbf{k}',\mathbf{k_2},(\mathbf{b}-\mathbf{b}')\right) \tilde{f}(s',\mathbf{k}',\mathbf{k_2},\mathbf{b}')$$

For $\mathbf{k_1},\mathbf{k_2} \gg \Lambda_{QCD}$ - hard pomeron

$$\mathcal{K}\left(\frac{s}{s'},\mathbf{k}',\mathbf{k_2},(\mathbf{b}-\mathbf{b}')\right) = \delta(\mathbf{b}-\mathbf{b}')\,\mathcal{K}^{BFKL}(\mathbf{k}',\mathbf{k_2})$$

For $\mathbf{k_1},\mathbf{k_2} \leq \Lambda_{QCD}$ - soft pomeron

$$\mathcal{K}\left(\frac{s}{s'},\mathbf{k}',\mathbf{k_2},(\mathbf{b}-\mathbf{b}')\right) = \delta\left(\mathbf{k}'-\mathbf{k_1}\right)\left(\frac{s}{s'}\right)^{\alpha_0-1} B(\mathbf{b}-\mathbf{b}')$$

e.g. $B(\mathbf{b}-\mathbf{b}')$ is a random walk diffusion

$$B \sim \exp\left\{-\frac{(\mathbf{b}-\mathbf{b}')^2}{4\alpha'\ln(s)}\right\}$$

This is the Fourier transform of

$$s^{\alpha' t}$$

$\alpha_0 = 1.08,\ \alpha' = .25$ matches soft pomeron

The full $\mathcal{K}\left(\frac{s}{s'}, \mathbf{k}', \mathbf{k_2}, (\mathbf{b}-\mathbf{b}')\right)$ interpolates between hard and soft extremes.

The Pomeron and Gauge-String Duality

This is the basis of work by Brower. et. al. which seeks to exploit the duality between QCD in four dimensions and string-theories of $ADS(5) \times S^5$.

The fifth dimension of the $ADS(5)$ serves as a renomalization scale so that a solution to the diffusion equation in the full 5-dimensional space gives rise to the extrapolation between the infrared and ultraviolet regimes of QCD.

For large negative t in diffractive processes, the usual perturbative BFKL behaviour is recovered.

For positive t, there are trajectories for which integer values of the power of s correspond to glue-ball masses.

Donnachie-Landshoff - Two Pomerons

Soft Pomeron: $\sim s^{1.08}$
Hard Pomeron: $\sim s^{1.4}$
Soft pomeron dominates in total cross-sections hadronic diffraction differential cross-sections, but hard pomeron dominates at sufficiently large energies, provided other momentum scales are also large.
In the dipole picture, hard pomeron dominates for small dipole sizes, r, with the dipole cross-section growing as r^2, but this saturates at some $r = R$

$$\sigma \sim a r^2 s^{0.4} \theta(R-r) + b R^2 s^{0.08}$$

Model gives good fits to proton structure function - including HERA data.
Even if low r contributions are heavily attenuated for total hadronic cross-section and low-t processes, the hard pomeron should eventually dominate at large enough s - size of R must decrease with energy.

SUMMARY

- The pomeron predicted by Regge theory is almost the one used by Donnachie-Landshoff

$$\alpha_P(t) = 1.08 - .25t$$

- In perturbative QCD the exchange of a colour singlet object (perturbative pomeron) summed to all orders in leading $\ln(s)$ leads to a pomeron cut singularity with branch-point

$$1 + 4\overline{\alpha_s} ln(2), \qquad \overline{\alpha_s} \equiv \frac{\alpha_s C_A}{\pi}$$

- The running coupling together with some "phase-fixing" from non-perturbative effects in the IR limit can convert this cut into a set of isolated poles.
- BFKL dynamics is expected to be probed at
  - DIS at low-x
  - Events with large rapidity gaps
  - Events with rapidity gaps populated with mini-jets
  - several others

SUMMARY

- "Evidence" for BFKL behaviour is still not convincing, but much data can be successfully analysed using models which incorporate BFKL dynamics.
- The BFKL evolution violates unitarity.
  At sufficiently high energies the cross-section saturates, when the density of colour dipoles is so large that the approximation of single dipole scattering off a target becomes invalid, and multiple scatterings have to be taken into account.
- The NLO BFKL kernel gives rise to large negative corrections, but most of these can be re-summed by requiring that the (low-x) DGLAP splitting function behaves like

$$\alpha_s^n \frac{\ln^n(x)}{x}$$

  in order α_s^n.

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  $$\alpha_s^n \frac{\ln^n(x)}{x}$$

  in order α_s^n.
- Over the years many attempts have been made to reconcile hard and soft pomeron behaviour (most recently based on gauge-string duality)