Distortion in Attribute Approval Committee Elections

Extended Abstract

Dorothea Baumeister
Heinrich-Heine-Universität
Düsseldorf, Germany
d.baumeister@uni-duesseldorf.de

Linus Boes
Heinrich-Heine-Universität
Düsseldorf, Germany
linus.boes@uni-duesseldorf.de

ABSTRACT

In attribute approval elections, the task is to select sets of winning candidates, while each candidate satisfies a variety of attributes in different categories (e.g., academic degree, work experience, location). Every voter specifies, which attributes in each category are desirable for a candidate, whereas each candidate might satisfy only some of the attributes. In this paper, we study questions of distortion in attribute approval committee elections. We introduce different methods to derive approval ballots, ordinal preferences, or cardinal preferences from a given attribute approval ballot. Then for a given voting method, assuming only a derived preference is provided, we compute the ratio of the voters' satisfaction for the worst possible committee, with the satisfaction of the actual winning committee, given the attribute approval ballots.

KEYWORDS

Distortion; Multiwinner Voting; Preference Aggregation

ACM Reference Format:

Dorothea Baumeister and Linus Boes. 2023. Distortion in Attribute Approval Committee Elections: Extended Abstract. In *Proc. of the 22nd International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2023), London, United Kingdom, May 29 – June 2, 2023*, IFAAMAS, 3 pages.

1 INTRODUCTION

Many different situations require the selection of a committee. For example a committee of people, but also a selection of movies on a plane, or the selection of dishes in a menu. See Faliszewski et al. [7] for a detailed discussion on different types of committee elections. Common ways to represent preferences in such elections are approval votes or rankings over the candidates, see Zwicker [13]. These preferences focus on single candidates, hence in committee elections the voters are not able to represent their opinion about possible outcomes, i.e. committees, of the election. Only if the number of candidates is small, it may be feasible to elicit preferences over all different committees. There are approaches to compactly represent complex preferences like CP-nets studied by Boutilier et al. [4], however they require expert knowledge. Another aspect is, that in the composition of the committee, the attributes of a candidate may be more important than the person (or object) itself. For an expert committee, the voter may want to ensure knowledge in some specific field, rather than a specific person to be present. Thus we study committee elections by focusing on the attributes of the candidates. Different vote representations and corresponding aggregation methods have received little attention so far (see [3], [11], [8] and [2]). We adopt the approach of Kagita et al. [9] and assume that there are different categories and each candidate has an attribute for each of them. The voters do not vote directly on the candidates, but approve a set of attributes for each category. The decision on single attributes may be easier for voters than to decide between a high number of candidates or committees. For winner determination we follow the approach by Kagita et al. [9], and aggregate the votes on the attributes to derive a decision on the candidates. In particular, we generalize voting rules for committee elections with candidate approvals to the attribute approval setting. We study the question, whether preferences on attributes of the candidates may provide better results than other commonly used types of preferences. We consider fixed-size committee elections where the candidates are associated with attributes from different categories and the votes are sets of approved attributes for each category. We investigate for six different rules on how these ballots may be aggregated to elect a committee. Then we define a notion of distortion, which measures how much the loss of information — from casting approval ballots, rankings, or cardinal preferences instead of attribute approval preferences — may affect the voters' satisfaction with the outcome negatively. This concept was formally introduced by Procaccia and Rosenschein [12] for underlying cardinal preferences, where distortion was measured with respect to a social choice function that aggregates derived ordinal preferences. In contrast, we consider underlying attribute approval preferences and measure the distortion between elections that have identically derived (e.g., ordinal) preferences. Closely related to our work is the study of so-called diversity constraints (see [6] and [10]), where the candidates have attributes, and the final committee has to fulfill certain requirements regarding the attributes.

2 PRELIMINARIES

For an integer i let $[i] = \{1, 2, \dots, i\}$ and for a set C let $\mathcal{P}(C)$ denote the power set of C and $\mathcal{P}_k(C) = \{W \subseteq C \colon |W| = k\}$ the set of all k-committees with respect to C. We follow Kagita et al. [9], who initiated a study on selecting committees using attribute approvals.

Definition 1. *Let $\mathcal{E}$ be the set of all attribute approval elections. A single such election is given by a tuple $(D, C, V) \in \mathcal{E}$, with*

- *$D = D^1 \times \ldots \times D^d$, where $D^1, \dots, D^d$ for $d \in \mathbb{N}$ are attribute domains. We assume $|D^j| \geq 2$ and $D^j \cap D^h = \emptyset$ for all $j \neq h$.*
- *$C = \{c_1, \dots, c_m\}$ is a set of m candidates, where each candidate is associated with attributes from different categories, i.e., each candidate $c_i \in C$ satisfies exactly one attribute $c_i^j \in D^j$ for each domain $j \in [d]$. Let $a : C \to D$ be a function, which maps from a candidate c_i to her attribute vector $a(c_i) = (c_i^1, \dots, c_i^d)$.*
- *$V = \{v_1, \dots, v_n\}$ is a set of n voters, each $v_i \in V$ is associated with her ballot, represented as a vector $b_i = (B_i^1, \dots, B_i^d) \in \mathcal{D}$,*

Proc. of the 22nd International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2023), A. Ricci, W. Yeoh, N. Agmon, B. An (eds.), May 29 – June 2, 2023, London, United Kingdom.

with $\mathcal{D} = \mathcal{P}(D^1)\times\ldots\times\mathcal{P}(D^d)$, i.e., each voter specifies which subset of attributes she approves of in each attribute category.

A voting rule F maps an election $E = (D, C, V) \in \mathcal{E}$ along with a positive integer $k \in \mathbb{N}_{>0}$ to a nonempty set of winning k-committees, i.e., $F(E, k) \subseteq \mathcal{P}_k(C)$. We assume that $|C| \geq k$ always holds.

We study voting rules, where the output is a set of k-committees that maximize the overall satisfaction of the voters. An *individual scoring function f* models any single voter's individual agreement (called satisfaction or score) of her attribute approval ballot $b_i \in \mathcal{D}$ with a given committee $W \subseteq C$, such that $f(b_i, W) \in \mathbb{Q}_{\geq 0}$. Examples for individual scoring functions are *Simple Scoring* (f^{si}), *Chamberlin-Courant Scoring* (f^{cc}), and *Committee Scoring* (f^{co}):

$$f^{\mathrm{si}}(b_i, W) = \frac{1}{d} \sum\nolimits_{c\in W} \sum\nolimits_{j\in[d]} |\{c^j\} \cap B_i^j|$$

$$f^{\mathrm{cc}}(b_i, W) = \frac{1}{d} \max_{c\in W} \sum\nolimits_{j\in[d]} |\{c^j\} \cap B_i^j|$$

$$f^{\mathrm{co}}(b_i, W) = \frac{1}{d} \left|\left\{j \in [d] : \exists c \in W \text{ with } c^j \in B_i^j\right\}\right|$$

To obtain voting rules for attribute approval ballots we extend a given individual scoring function f from single ballots to *extended scoring functions* for voter profiles, considering two prominent approaches. Given a set V of voters and a k-committee W we either maximize the **utilitarian welfare** $f_\Sigma(V, W) = \sum_{v_i\in V} f(b_i, W)$ or the **egalitarian welfare** $f_{\min}(V, W) = \min_{v_i\in V} f(b_i, W)$. Finally, for each individual scoring function $f^y \in \{f^{\mathrm{si}}, f^{\mathrm{cc}}, f^{\mathrm{co}}\}$ paired with an extension $x \in \{\Sigma, \min\}$, a voting rule F_x^y maximizes the score of an extended scoring function f_x^y, outputting a set of winning k-committees, i.e., $F_x^y(E, k) = \arg\max_{W\in\mathcal{P}_k(C)} f_x^y(V, W)$. The modular setup provides six extended scoring functions and thus six voting rules[1] $F_\Sigma^{\mathrm{si}}, F_\Sigma^{\mathrm{cc}}, F_\Sigma^{\mathrm{co}}, F_{\min}^{\mathrm{si}}, F_{\min}^{\mathrm{cc}}$, and $F_{\min}^{\mathrm{co}}$.

Preference Derivation Methods. In many natural situations attribute approval ballots (in combination with a scoring function) model the underlying preferences realistically. If voters can only express their preferences in more common forms, it is reasonable to assume a voter either (i) assigns a utility to each candidate linear in the number of satisfied attributes, (ii) weakly ranks the candidates based on the number of satisfied attributes, or (iii) approves those candidates that satisfy a threshold amount of attributes.

Definition 2. *Let $E = (D, C, V) \in \mathcal{E}$ and $b_i = (B_i^1, \ldots, B_i^d)$ be the attribute approval ballot of voter $v_i \in V$. We study the following preference derivation methods for different types of preferences.*

- **Cardinal Preference Ballots:** $\mathfrak{c} : \mathcal{D}\times C \to \mathbb{Q}$, *where* $\mathfrak{c}(b_i, c) = \frac{1}{d}|\{j \in [d] : c^j \in B_i^j\}|$ *is the cardinal preference, voter v_i associates with candidate $c \in C$.*
- **Ordinal Preference Ballots:** $\mathfrak{o} : \mathcal{D} \to \mathcal{P}(C \times C)$, *such that* $\mathfrak{o}(b_i) = \succsim_{b_i}$ *is a weak ranking over C with $c \succ_{b_i} c'$ if $\mathfrak{c}(b_i, c) > \mathfrak{c}(b_i, c')$ and $c \sim_{b_i} c'$ if $\mathfrak{c}(b_i, c) = \mathfrak{c}(b_i, c')$ for all $c, c' \in C$.*
- **Candidate Approval Ballots:** $\mathfrak{a}_\tau : \mathcal{D} \to \mathcal{P}(C)$, *such that* $\mathfrak{a}_\tau(b_i) = \{c \in C : \mathfrak{c}(b_i, c) \geq \frac{\tau}{d}\}$ *is the set of preferred candidates for a given threshold $\tau \in [d]$.*

With $\Delta(E)$, for a preference derivation method $\Delta \in \{\mathfrak{c}, \mathfrak{o}, \mathfrak{a}_\tau\}$, we refer to the election (C, V'), where each voter $v_i \in V$ with attribute approval ballot b_i is substituted by $v_i' \in V'$ with derived ballot $\Delta(b_i)$.

[1] F_Σ^{si} has also been studied by Kagita et al. [9] under the name Approval Voting.

Table 1: Summary of our results on distortion for each scoring rule f_x^y paired with a derivation method Δ. Entry ∞ indicates unbounded distortion, while 1 indicates no distortion.

Δ	f_Σ^{si}	f_Σ^{cc}	f_Σ^{co}	$f_{\min}^{\mathrm{si}}$	$f_{\min}^{\mathrm{cc}}$	$f_{\min}^{\mathrm{co}}$
$\mathfrak{a}_\tau$	∞	∞	∞	∞	∞	∞
$\mathfrak{a}_1$	d	d	d	d	d	d
$\mathfrak{c}$	1	1	$\min(k, d)$	1	1	$\min(k, d)$
$\mathfrak{o}$	d	d	∞	∞	∞	∞

3 DISTORTION

In our setting distortion measures how much the voters' satisfaction can decline, if we derive the voters' preferences using a (possibly) less expressive method instead of attribute approvals. If the distortion is high, the potential upside for voting on attributes is huge. In contrast, if the distortion is low, there is no downside in voting on candidates directly. We are interested in the distortion associated with a preference derivation method and an extended scoring function. That is the maximum factor the satisfaction can be higher by considering attribute ballots instead of other common forms of ballots. In contrast to related work on distortion (see [12], [5] and [1]) we do not assume an underlying cardinal utility for each candidate, but that the attribute approvals capture the voters' opinions entirely.

Definition 3. *Let Δ be a preference derivation method, which maps from an attribute approval election (D, C, V) to an election (C, V'). Further, let f_x^y be an extended scoring function and $\sigma_\Delta : \mathcal{E} \to \mathcal{P}(\mathcal{E})$ be a function with $\sigma_\Delta(E) = \{E' \in \mathcal{E} : \Delta(E) = \Delta(E')\}$ for every $E \in \mathcal{E}$. That is, $\sigma_\Delta(E)$ is the set of attribute elections, that yield ballots equivalent to E if the votes are derived using Δ. In the following, for $\mathcal{E}' \subseteq \mathcal{E}$, let $\mathcal{W}(\mathcal{E}', k) = \bigcup_{E'\in\mathcal{E}'} F_x^y(E', k)$, be the collection of all winning k-committees for all elections in $\mathcal{E}'$. For a fixed attribute approval election $E = (D, C, V) \in \mathcal{E}$, the distortion associated with Δ, f_x^y, and E is given by*

$$\mathrm{dist}(\Delta, f_x^y, E) = \max_{W'\in\mathcal{W}(\sigma_\Delta(E),k)} \frac{\max_{W\in\mathcal{P}_k(C)} f_x^y(V, W)}{f_x^y(V, W')}$$

In case only the denominator is zero, we say the distortion is unbounded. The overall distortion for Δ and f_x^y (not depending on a specific election) is given by $\mathrm{dist}(\Delta, f_x^y) = \max_{E\in\mathcal{E}} \mathrm{dist}(\Delta, f_x^y, E)$.

The intuition for $\mathrm{dist}(\Delta, f_x^y, E)$ is, that E represents the undistorted voters' preferences, i.e., their attribute-based preferences. If the voters' preferences were instead cast by Δ with a loss of information (e.g., approval ballots), there is no way to determine which of the attribute approval elections in $\sigma_\Delta(E)$ coincides with the voters' actual ballots. If we pick an election $E' = (D', C, V') \in \sigma_\Delta(E)$, then a winning committee $W' \in F_x^y(E', k)$ maximizes the satisfaction for V'. Yet, the set of voters V might be dissatisfied with W', that is, $f_x^y(V, W')$ could be much lower than the score of a winning committee. We use a given extended scoring function f_x^y as a metric to evaluate the satisfaction of the voters with a committee. Our results are summarized in Table 1.

ACKNOWLEDGEMENTS

The authors would like to thank the anonymous reviewers for their helpful comments. This work was supported in part by the project "Online Participation," funded by the NRW Ministry for Innovation, Science, and Research and by the DFG grant BA-6270/1-1. Current affiliation of the first author: Federal University of Applied Administrative Science, Brühl, Germany.

REFERENCES

[1] E. Anshelevich, A. Filos-Ratsikas, N. Shah, and A. Voudouris. 2021. Distortion in Social Choice Problems: The First 15 Years and Beyond. In *Proceedings of the 30th International Joint Conference on Artificial Intelligence*. ijcai.org, 4294–4301. Survey Track.

[2] N. Barrot and J. Lang. 2016. Conditional and Sequential Approval Voting on Combinatorial Domains. In *Proceedings of the 25th International Joint Conference on Artificial Intelligence*. AAAI Press, 88–94.

[3] D. Baumeister, T. Böhnlein, L. Rey, O. Schaudt, and A. Selker. 2016. Minisum and Minimax Committee Election Rules for General Preference Types. In *Proceedings of the 22nd European Conference on Artificial Intelligence*. IOS Press, 1656–1657. Short paper.

[4] C. Boutilier, R. Brafman, C. Domshlak, H. Hoos, and D. Poole. 2004. CP-nets: A Tool for Representing and Reasoning with Conditional Ceteris Paribus Preference Statements. *Journal of Artificial Intelligence Research* 21 (2004), 135–191.

[5] Craig Boutilier, Ioannis Caragiannis, Simi Haber, Tyler Lu, Ariel D. Procaccia, and Or Sheffet. 2015. Optimal Social Choice Functions: A Utilitarian View. *Artificial Intelligence* 227 (2015), 190–213.

[6] R. Bredereck, P. Faliszewski, A. Igarashi, M. Lackner, and P. Skowron. 2018. Multiwinner Elections with Diversity Constraints. In *Proceedings of the 32nd AAAI Conference on Artificial Intelligence*. AAAI Press, 933–940.

[7] P. Faliszewski, P. Skowron, A. Slinko, and N. Talmon. 2017. Multiwinner Voting: A New Challenge for Social Choice Theory. In *Trends in Computational Social Choice*, U. Endriss (Ed.). AI Access Foundation, 27–47.

[8] P. Jain and N. Talmon. 2020. Committee Selection with Multimodal Preferences. In *Proceedings of the 24nd European Conference on Artificial Intelligence*. IOS Press, 123–130.

[9] V. Kagita, A. Pujari, V. Padmanabhan, H. Aziz, and V. Kumar. 2021. Committee Selection using Attribute Approvals. In *Proceedings of the 20th International Conference on Autonomous Agents and MultiAgent Systems*. IFAAMAS, 683–691.

[10] J. Lang and P. Skowron. 2018. Multi-Attribute Proportional Representation. *Artificial Intelligence* 263 (2018), 74–106.

[11] S. Obraztsova, M. Polukarov, Z. Rabinovich, and E. Elkind. 2017. Doodle Poll Games. In *Proceedings of the 16th International Joint Conference on Autonomous Agents and Multiagent Systems*. ACM Press, 876–884.

[12] A. Procaccia and J. Rosenschein. 2006. The Distortion of Cardinal Preferences in Voting. In *Proceedings of the 10th International Workshop on Cooperative Information Agents*. Springer-Verlag *Lecture Notes in Artificial Intelligence #4149*, 317–331.

[13] W. Zwicker. 2016. Introduction to the Theory of Voting. In *Handbook of Computational Social Choice*, F. Brandt, V. Conitzer, U. Endriss, J. Lang, and A. Procaccia (Eds.). Cambridge University Press, Chapter 2, 23–56.