Resilient Fair Allocation of Indivisible Goods

Extended Abstract

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ABSTRACT

Fair allocation of indivisible goods has been studied extensively. However, the solutions offered to date are not resilient to subsequent changes that may occur after the allocation has been decided and executed, e.g., agents leaving the system, or additional goods are discovered. Currently, such settings require rerunning the allocation algorithm from scratch, potentially shifting most allocated goods between the agents. This can be cumbersome at best, or impossible at worst. In this paper, we study the notion of resilience, which quantifies the number of changes needed to resolve subsequent changes in the environment. We then apply it to the problem of fair allocation of indivisible goods, focusing on the EF1 and EFX solution concepts. For the EF1 solution concept, we provide constructive and efficient algorithms to restore EF1 after a simultaneous loss of goods, addition of new goods, and resignation of agents. We show that the addition of new agents cannot be resolved efficiently when the agents' valuation may be arbitrary. When agents have identical valuations, we show how to accept new agents efficiently. For the EFX solution concept, we (mostly) prove negative results, establishing that restoring EFX may be prohibitively costly, even for agents with identical valuations.

KEYWORDS

Resilience, Fairness, Multi-Agent Systems, Indivisible Goods, Allocation, Heterogeneous Preferences, EF1, EFX

ACM Reference Format:
Dolev Mutzari, Yonatan Aumann, and Sarit Kraus. 2023. Resilient Fair Allocation of Indivisible Goods: Extended Abstract. In *Proc. of the 22nd International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2023), London, United Kingdom, May 29 – June 2, 2023*, IFAAMAS, 3 pages.

1 INTRODUCTION

Fair allocation of indivisible goods (FAIG) is a central problem considered by several fields, including computer science and economics [12, 13]. It has also attracted much attention in the multiagent community [1, 10], and has been studied extensively in recent years [4, 19, 20, 27]; See [2] for an updated overview. The goal in fair allocation is to distribute a set of goods in a *fair* manner. In the indivisible setting, two of the most widely accepted and used fairness notions are EFX [15] and EF1 [14].

While the fair allocation problem has been extensively studied, a vital issue remains unaddressed. Consider, for example, a set of heirs dividing the inheritance of their late parents. They decide to use some EFX or EF1 protocol, but after the allocation has been decided and executed - each heir taking its allotted items - it turns out that one of the goods they allocated did not actually belong to their parents, and must be returned to its true owner. How many goods need to be reallocated in order to restore a fair allocation of the remaining goods? Does losing this single item necessitate an entirely new allocation, or is it possible to restore fairness with relatively few changes? What if a good is added to the inheritance? What if a new heir is discovered? What if one or more are eliminated? How many changes are necessary in order to restore fairness in each of these cases? This is the topic of this paper.

In this paper, we study the *resilience* of fair allocation procedures to subsequent small changes in the environment. Intuitively, we say that a solution is *resilient* to changes/faults if, following a small number of "faults", fairness can be restored with only a small number of changes to the allocation. In this paper, we consider four types of faults: the loss and addition of goods and agents. We believe that handling such cases can be a crucial requirement in many real-world settings.

	g_+	g_-	a_+	a_-
Hom.	0	$g_-(1-\Theta(\frac{1}{n}))$	0	0
Het.	0	$O((n-1)g_-)$	$a_+\left(\frac{m(n-a_+)}{2n(n+a_+)} - O(1)\right)$	0

Table 1: Resilient EF1 efficient constructions for various faults in FAIG instances, considering both homogeneous and heterogeneous agent preference settings.

1.1 Our Contribution

We start by providing a formal definition of the *resilience* concept, in a generic way that can be applied to a broader scope of problems (some of which were already studied in the literature). We then proceed to study resilient solutions in the scope of fair allocation of indivisible goods. We consider four types of faults: g_- goods being lost, g_+ goods being added, a_- agents leaving, and a_+ agents being added. For each such case, we are interested in how many goods must be *shifted* among the original agents in order to restore the original fairness guarantee (EF1 or EFX - as the case may be). Throughout, n denotes the number of agents, and m the number of goods. In practice, m may be much greater than n, so, first and foremost, we seek resilience that is independent of m.

Results for EF1 and EFX are summarized in Tables 1,2 below respectively. All EF1 upper bounds are constructive and efficient.

Proc. of the 22nd International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2023), A. Ricci, W. Yeoh, N. Agmon, B. An (eds.), May 29 – June 2, 2023, London, United Kingdom.

	g_+	g_-	a_+	a_-
Hom.	$\frac{m}{n} g_+$	$\Omega(\frac{m-g_-}{n})$	$\Omega(\frac{m}{n+a_+} a_+)$	$m - O(n)$
Het.	$m - O(1)$	$m - g_-$	$\Omega(\frac{m}{n+a_+} a_+)$	$m - O(n)$

Table 2: Negative results, specifying lower bounds on the number of fixing steps required to restore EFX.

In a nutshell, we exhibit a trade-off between resilience and fairness. For the strict fairness notion of EFX, we prove m-dependent lower bounds, whereas for EF1, efficient constructions resilient to merely all types of faults are presented.

1.2 Related Work

We are not aware of any previous work that directly considers our setting. We review works related to resilience in other settings.

For the divisible goods setting (aka *cake cutting*) [25] prove that there always exists a division of the cake into n pieces so that no matter which player leaves, there is an envy-free assignment of the pieces to the remaining $n-1$ agents. The paper does not discuss what happens with more agents leaving, other types of faults, or the cost of restoration.

Resilience to changes in agents' valuations is related to the line of research on ordinal valuations. [3] analyze the trade-off between the number of ordinal queries of the valuation of the agents, and the distortion obtained for a solution thereof. The context of that paper is partial information, but the work can also allow (approximate) fair solutions resilient to ordinal-preserving changes in the valuation of the defenders. There has also been a lot of work on ordinal-maximin share, e.g., [22]. This robust approximation notion depends only on the agents' ordinal rankings of bundles, therefore, can be resilient to small changes in the agents' valuations, even if the ordinal preference of individual goods that had a close valuation is flipped. Finally, [7] provided a solution in the divisible domain, robust against ordinal-preserving changes in the valuations of the agents.

For the kidney exchange setting, several works (e.g. [16, 18, 24]) consider the problem of minimizing the cost of re-matching following failures. Their results, however, which mostly utilize integer and mixed integer programming, do not provide the bounds we seek for our fair division setting, nor provably efficient restoration procedures for this setting.

In Hedonic games, [23] study *robust* solutions, which are defined as those that can tolerate agent faults *without any change of the coalition structure*. This *robustness* notion is different from our *resilience*, which allows for restoration steps following the faults. Similarly, in security games, [17] study SSG in a setting where defenders face unanticipated disruptions of their schedules, and aim for *robust* solutions, that withstand disruptions without any restoration steps.

[28] consider a cloud-based computing system servicing multiple heterogeneous clients in a real-time environment, where the cloud resources may fail, and such failures must be handled without affecting most of the already allocated resources and running clients. [8] consider the problem of allocating Virtual Network Functions (VNFs) on top of the physical network infrastructure, and were the first to consider the possibility of failures in this infrastructure.

In combinatorial auctions, a winner may regret or fail to provide her bid. In [21, 29], mechanisms to deal with shill bids were offered and in [26], collusion-resilient combinatorial auctions were studied. However, in these works, there is no need to change the allocation after detecting the shill bids or colluded parties.

2 TECHNIQUES

Next, we briefly go over the main techniques developed to obtain the results above. All positive resilience results for EF1 are related to the Round-Robin (RR) solution where agents greedily pick goods in a RR manner. The question is how to restore EF1 starting with such allocations.

Adding Goods. Assume a set of goods $\mathcal{M}_+$ is added forming a new instance. Then we can form a new EF1 solution by adding the goods in $\mathcal{M}_+$ in a reversed RR allocation. Once adding goods is established, removing agents is obtained by first removing all of their allocated goods and then adding them back as new goods. We can also handle the removal of goods by removing the entire rounds where some of the goods were lost, and add them as new goods (hence the n-factor in g_-). The latter two arguments work since removing entire allocation rounds, and/or agents along with their allocated goods, maintains the RR allocation structure.

Homogeneous Scenario. For homogeneous instances, where all agents agree on a common preference order over the goods, we utilize recursively-balanced (RB) allocations to handle g_- and a_+ faults optimally. Essentially, RB is a generalization of RR, where in each round each agent greedily picks a single good, but the order of turns may change across rounds. We show that such a structure can be restored when adding agents or losing goods. This is sufficient since RB allocations are known to be EF1 (see [5, 6, 9, 11]).

Negative Results. We present three main negative results. (i) Adding agents in general may require $O(m)$ fixing steps even to restore EF1; (ii) There are instances where RR is not the most resilient fair solution; (iii) Restoring EFX (and different relaxations of it) essentially requires to rerun the allocation from scratch. For example of (i), consider an instance with n identical agents and m identical goods. Clearly, any EF1 will evenly distribute the goods. Now consider adding a_+ agents, where agent i' only values the goods in agent i's bundle. Then to restore EF1, agent i cannot keep more than half of her original bundle due to agent i', but she should still have as many goods as any other agent. This results with the conclusion that $\Omega(\frac{m}{n} a_+)$ fixing steps are necessary. For (iii), consider an instance with two homogeneous agents, $m-1$ goods with $v(o_i) = 1$ and an additional special good with $v(o^*) = m$. Clearly, in EFX, the agent that gets o^* cannot get any additional good. However, if o^* is lost, even restoring EF1 requires to evenly distribute the left goods, which requires shifting $\frac{m-1}{2}$ goods. For the heterogeneous scenario, we may need $m - O(1)$ shifts.

ACKNOWLEDGMENTS

This research has been partly supported by the Israel Science Foundation under grant 1958/20 and the EU Project TAILOR under grant 952215.

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