# Logics for Information Aggregation

Doctoral Consortium

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## ABSTRACT

With the amount of information available to us today, making sense of potentially conflicting information is a problem that we are bound to run into. We aim to study this problem from the perspective of epistemic logic. So far, we have studied logics for reasoning about information distributed across groups of agents in the epistemic logic framework, focusing on the notion of distributed belief. We have introduced the notions of *cautious* and *bold* distributed belief. These are intended as alternatives to the standard distributed belief, behaving better when agents have conflicting beliefs. With standard distributed belief, such situations lead to explosion: everything becomes distributedly believed. The idea behind the new notions is that we look at maximal consistent subgroups, allowing us to meaningfully express that a group collectively possesses information supporting a belief, also in cases where some group members disagree. Going forward, we are interested in continuing to explore the aggregation of potentially conflicting information in the setting of epistemic logic, e.g. by looking at distributed belief in richer extensions of epistemic logic and in relation to other areas that deal with similar problems.

## KEYWORDS

epistemic logic; distributed belief; inconsistent information

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## 1 INTRODUCTION

Nowadays, when studying a certain topic, it is relatively easy to find a large amount of information. The problem that arises is not to obtain information, but rather how to make sense of all the potentially conflicting information one might get. The general problem of dealing with conflicting information is studied in areas such as preference aggregation [2, 18] and judgement aggregation [14, 15], in belief revision, belief merging and belief fusion [1, 4, 11], in abstract argumentation [3, 5] and in distributed computing.

My PhD project aims at studying this problem from a logical perspective. In an initial phase, and within the epistemic logic framework, we have studied logics for reasoning about information distributed across groups of agents, focusing on the notion of distributed belief.

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## 2 THE PROPOSAL SO FAR: CAUTIOUS AND BOLD DISTRIBUTED BELIEF

### 2.1 Epistemic Logic and Distributed Belief

In *epistemic logic*, knowledge and belief are given precise formal definitions. Knowledge is defined indirectly, as lack of uncertainty. The standard way of giving semantics is through relational models [10]. The relations are interpreted as *indistinguishability*. When something is true in all possible worlds indistinguishable from the actual one, it is known. Belief in this setting can be seen as a generalization of knowledge. The intuition is the same: an agent is said to believe $\varphi$ whenever the information possessed by the agent supports $\varphi$. The difference between knowledge and belief will lie in the kind of information we take relations to represent. Making different assumptions about the relations can give us different notions of knowledge and belief. In the case of knowledge, we will be dealing with "certain" information (given by reflexive models), while for beliefs the information supporting them may be incorrect.[1]

The epistemic logic framework is a simple yet powerful tool with which to study higher-order reasoning, group notions of knowledge, and information dynamics. An important group notion is *distributed knowledge* [7, 9]. It is the "implicit" knowledge of a group: that which would be known if all the information of the individuals was somehow combined. This can be thought of as removing any uncertainty not shared by everyone in a group, and in relational semantics it has a straightforward, intuitive definition. A distributed knowledge modal operator is defined using the intersection of relations used to define individual knowledge of group members. Thus a logic with the operator will be able to capture and reason about the information contained in a group of agents.

Weaker notions of *group belief* are also studied (e.g. in [6]). However, the move from knowledge to belief seems somewhat problematic for *distributed belief* ($D$). Distributed knowledge combines the knowledge of the individuals in the group, but for distributed belief, the information on which the individual beliefs are based could be inconsistent with the information of others in the group. Disagreement in the group then leads to quantification over an empty set of worlds, and we get explosion: belief in everything. Looking for similar ways of combining the individual information of members of a group, that avoid this inconsistency, we are currently exploring two variants of the standard distributed belief. We are calling them *cautious* distributed belief ($D^{\forall}$) and *bold* distributed belief ($D^{\exists}$).

[1] What is actually being represented in the models is the support for beliefs. The beliefs generated for an agent are idealized. When considering our actual beliefs, we seem able both to believe things for which we do not have have sufficient support, and to not believe things for which we do have support. To capture that, however, a different framework would be required. Rather than belief as we may use it in an everyday context, what we have here may be better interpreted as that which should or could rationally be believed by the agent given the information they possess.

Unlike the standard notion, the fact that a group has distributed belief according to these alternative definitions will always mean that the group has some information that supports the belief.

This approach to belief aggregation focuses on operations on the relations in epistemic logic models, making it more semantic than the approach taken in the area of belief merging [11]. Two other approaches to distributed belief and inconsistent information are found in [12] and [8]. In the former, priority is given to some agents, and this is used to get a consistent set of beliefs for the group. The latter, like us, use maximal consistency to get a consistent distributed belief. They do it in a setting with belief bases rather than Kripke models generating the beliefs of agents, and look at maximal consistent sets of formulas, while we are looking at maximal consistent groups of agents in standard Kripke models.

## 2.2 Cautious and Bold Distributed Belief

The models considered are multi-agent Kripke models. For a finite non-empty set of agents $A$: $\mathcal{M} = \langle S, R, v\rangle$, where $S$ is a set of possible worlds, $R = \{R_a \subseteq S \times S \mid a \in A\}$ assigns a relation to each agent, and $v$ is a valuation function. For $a \in A$ we define the *conjecture set* relative to $s \in S$: $C_a(s) := \{s' \in S \mid sR_a s'\}$. This is generalized in the combined conjecture set of a group $G \subseteq A$: $C_G(s) := \bigcap_{a\in G} C_a(s)$. Note that standard distributed belief is defined: $\mathcal{M}, s \models D_G\,\varphi$ iff $\forall s' \in C_G(s)$: $\mathcal{M}, s' \models \varphi$.

When we consider models that are not reflexive, the group can end up inconsistent ($C_G(s) = \emptyset$), even if all members are individually consistent. Consider a group of agents, who are all individually consistent, but disagree on some belief. Then take some belief that everyone in the group agrees on, e.g. that it is raining in Lisbon. The group has distributed belief that it is raining in Lisbon, but it also has distributed belief that it is not raining in Lisbon. This seems undesirable. We therefore look at ways of partially combining the information.

We do this by looking at *maximal consistent* subgroups.[2] A group $G$ is consistent at a world $s$ when $C_G(s) \neq \emptyset$. A subgroup $G' \subseteq G$ is maximally consistent relative to $G$ (in symbols, $G' \subseteq_s^{max} G$), when it is consistent and there is no consistent $G' \subset H \subseteq G$. Cautious distributed belief is defined:

$$\mathcal{M}, s \models D_G^{\forall}\varphi \qquad \text{iff} \qquad \forall G' \subseteq_s^{max} G, \forall s' \in C_{G'}(s)\colon \mathcal{M}, s' \models \varphi$$

For bold distributed belief we simply replace the universal quantification over maximal consistent groups with existential:

$$\mathcal{M}, s \models D_G^{\exists}\varphi \qquad \text{iff} \qquad \exists G' \subseteq_s^{max} G, \forall s' \in C_{G'}(s)\colon \mathcal{M}, s' \models \varphi$$

For both operators we look at the worlds in the conjecture sets of all maximal consistent subgroups. In the example above, we get both cautious and bold distributed belief that it is raining in Lisbon, but not in its negation. More interesting are of course cases where the proposition under consideration is not believed by everyone, and some combination of information is needed. And, as long as at least some of the agents are mutually consistent, we get cases where not everyone in the group, or where no one in the group believes something, but it is cautious or bold distributed belief.

We have been mapping out properties of cautious and bold distributed belief. In [13], we did this for cautious distributed belief, and we have now been focusing on bold distributed belief. Here are some ways the introduced modalities differ from each other and the standard distributed belief modality:

- While $D_G^{\forall}$ is a normal modal operator, $D_G^{\exists}$ is not. The latter avoids belief in contradictions by not being closed under conjunction introduction. It cannot be represented using a standard relational semantics.
- Cautious distributed belief is inconsistent only when all agents are individually inconsistent. Bold distributed belief is never inconsistent.
- In reflexive models all three notions of distributed belief coincide. Thus, they can all be seen as different generalizations of distributed knowledge.
- While both standard and bold distributed belief are coalition monotonic, cautious distributed belief is not (beliefs can be lost upon adding agents).

Similar to what is done for standard distributed belief in [6], for $D^{\forall}$, we investigated whether some relational properties, relevant to epistemic logic, are inherited from individual relations (giving insight into how assumptions about the individual knowledge or belief will affect the behaviour of the group notion). We have also investigated the relative expressivity of the proposition language extended with each distributed belief modality (for standard: $\mathcal{L}_D$, for cautious: $\mathcal{L}_{D^{\forall}}$ and for bold: $\mathcal{L}_{D^{\exists}}$). In [13] we showed that $\mathcal{L}_{D^{\forall}}$ is strictly less expressive than $\mathcal{L}_D$, and that adding an "inconsistency constant" for each group to the former makes them equally expressive. For $\mathcal{L}_{D^{\exists}}$, we have since found that the same relationship to $\mathcal{L}_D$ holds: the bold variant is strictly less expressive, but equally expressive with the inconsistency constant added.

# 3 CURRENT PLAN FOR THE FUTURE

We have some work still to do in our investigation of cautious and bold distributed belief. Regarding expressivity, we have yet to figure out whether $D^{\exists}$ is strictly less expressive than $D^{\forall}$. We are interested in studying the complexity profile for both new modalities and we are working on axiomatizing them. There are some challenges with the latter. For example, it seems difficult to find something to replace the coalition monotonicity axiom usually used for $\mathcal{L}_D$ in the case of $\mathcal{L}_{D^{\forall}}$. In the case of $\mathcal{L}_{D^{\exists}}$, things are complicated by the fact that it is not a normal modality.

We conclude with some ideas for where to go next. It would make sense to look at the problem of combining information from potentially inconsistent sources in richer epistemic logic frameworks. One direction that seems worth exploring, is looking at distributed belief in the setting of plausibility models [16]. In that setting, multiple kinds of information are represented, and the models contain additional structure on the information that helps deal with information change. Defining distributed belief in that setting may not be entirely trivial, but it seems worth exploring, and the way plausibility is represented (as a "preference" relation on possible worlds) seems to lend itself well to comparisons to preference aggregation from computational social choice. Another interesting direction would be to look at distributed belief in extensions or variants of epistemic logic, in which the logic deals explicitly with the information of the agents, e.g. in the form of evidence [17] or arguments [5].

[2] Looking at maximally consistency is a standard approach for merging potentially inconsistent information, and has been used, for example, in formal argumentation theory [5] and evidence logics [17].

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