Week 3/4 Key Concepts

- λ is the mean distance travelled by an atom before a collision occurs. In kinetic theory $\lambda = \frac{1}{\sqrt{2}\pi d^2 n}$
- The probability that a molecule travels a distance r without collision is $p(r) = e^{\frac{-r}{\lambda}}$
- For a random walk of N steps, the distance travelled $r = \sqrt{N}\lambda$.
- The kinetic theory predictions for transport coefficients are
  Diffusion coefficient $D \sim \lambda \bar{v}$
  Thermal conductivity $K \sim C\lambda \bar{v}$
  Viscosity coefficient $\eta \sim \rho\lambda \bar{v}$
- Equipartition implies $\bar{v} \sim T^{1/2}$; $\lambda \sim 1/n$ and $n = P/kT$ (but λ can never be smaller than the container size); ρ and C grow as n.

Tutorial problems

1. If we use peas of diameter 6 mm to represent the molecules of a gas, their number density would be about 2500 m^{-3} at s.t.p. Estimate the average separation between neighbouring peas, and compare this with the mean free path of a pea.

2. The mean free path of photons inside the sun, before they are absorbed and re-emitted, in a random direction, is about 0.01m. How many steps will photons take to travel from the centre of the sun to it's surface, a distance of 7×10^8m? How long will this take, if the time between absorption and re-emission can be neglected?

3. Ants tend to head off looking for food. If they encounter another ant then they change direction at random to avoid duplicating effort.

   A biologist separates 1000 red ants and 1000 black ants on a 1m^2 table top, then removes the barrier between them. What is the diffusion coefficient for ants of each colour to penetrate the other ants territory?

Problem Class Questions

1. The diameter of a hydrogen molecule is 2.7×10^{-10}m. Calculate the mean free path for the hydrogen molecule in hydrogen gas at s.t.p., when one mole occupies 22.4 litres.

   Recompute λ for interstellar hydrogen, where the number density is about 1 molecule per cm^3.

2. The mean free path in gaseous CO_2 at s.t.p. is 4.2×10^{-8}m. How far will one molecule actually travel along its path to reach a distance of 1m from it's starting point? If the mean velocity of a CO_2 molecule is 250 ms^{-1} (which incidentally is a typical airline speed), how long will this take?

3. A test tube of length 16cm and diameter 1.2 cm has a little liquid ether at the bottom. Just above the liquid the density of ether molecules will correspond to the saturated vapour pressure of ether, which is 0.59 atm at 20C. Given that one mole of gas at atmospheric pressure and 20C occupies a volume of 24 litres, what is the density of the ether molecules just above the liquid?

   At the top of the test tube the density of ether molecules is zero, as they are rapidly carried away by air currents. The diffusion coefficient of ether molecules in air is 8.9×10^{-6} m^{2}s^{-1}. At what rate do ether molecules diffuse up the test tube?

   How long will it take for 0.01 moles of liquid ether (about 1cm^3) to evaporate?

4. In a chaotically run city, buses arrive at bus stops entirely at random, once every τ hours. Show that the chance that no bus arrives during a time period of t hours is given by

   $$P = e^{\frac{-t}{\tau}}$$

   What is the maximum value of τ that will ensure that the chance of having to wait more than 1 hour for a bus is less than 1%?