In week 7 your tutors should return your mid-term test solutions to work through.

Tutorial problems

1. Two gas filled rubber balloons, initially of equal volume, are at the bottom of a lake. One balloon rises rapidly and expands adiabatically; the other rises slowly and expands isothermally. Which balloon is larger when it reaches the surface?

2. Use the equipartition theorem and the relation between C_P and C_V for ideal gases to calculate the ratio of specific heats γ, for (a) helium, and (b) nitrogen at room temperature.

Problem Class Questions

1. Consider an ideal gas inside a cylinder fitted with a piston. The system goes from the initial conditions T and V_i to the final conditions T and V_f. Calculate $Q, W, \Delta U$ when the change of state occurs through:

   a - a free expansion with no external pressure

   b - a reversible, isothermal expansion

   c - an irreversible, isothermal expansion with constant non-zero external pressure.

2. The Sterling cycle (used in a heat engine that does not emit exhaust, instead reusing the gas over and over) comprises four steps, starting with an ideal gas at temperature T_1, volume V_1 and pressure P_1:

   (1) the gas is expanded isothermally to volume V_2

   (2) the gas is cooled at constant volume to temperature T_1

   (3) the gas is compressed isothermally to volume V_1.

   (4) the gas is heated at constant volume to temperature T_2

   Sketch this cycle on a P-V indicator diagram. Indicate the direction in which steps occurs, and the region of the diagram corresponding to the work performed by the engine.

   Which stages involve work? Which stages involve heat exchange?

   Calculate the work performed during the cycle in terms of T_1, T_2, V_1, V_2 and n_m, the number of moles of gas.

3. The *bathysphere* was an early deep-sea submersible constructed from a spherical steel shell 25mm thick, with inner radius 0.68m. It was lowered to depths of 800m in the sea near Bermuda. One worry was how cold the occupants (William Beebe and Otis Barton) would get (they were submerged for several hours on occasion). In fact the sea temperature at that depth is about 4°C so wrapping up warm probably sufficed.

To understand how quickly the submersible would cool down we can calculate how much power would be needed to maintain the submersible interior at 20°C. The thermal conductivity of the steel is 25 W m^{-1} K^{-1}. Imagine that a steady power input P was used to heat the submersible to keep it's temperature T_i, above the sea water temperatujre T_w. Show that the temperature gradient in the shell of radius r from the centre of the submersible is given by

$$\frac{dT}{dr} = -\frac{P}{4\pi K r^2}$$

Thus show that the power required would be

$$P = 4\pi K R_1 R_2 \left(\frac{T_i - T_w}{R_2 - R_1}\right)$$

where R_1 and R_2 are the inner and outer radii of the shell.

What power would be required to keep the 20°C to 4°C temperature difference. Look up the power output of the heat from a human and from an electric heater to compare.