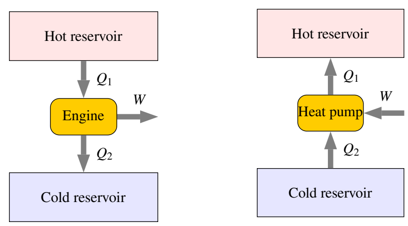
Energy and Matter: Problem Sheet 7

Week 7 Key Concepts:

- **2nd Law - Clausius**: No cyclical device can just transfer heat from a colder to a hotter body.
- **2nd Law - Kelvin-Planck**: No cyclical device can just transfer heat into work.
- A **perpetual motion machine of the second kind** can perform work with energy from just ambient thermal energy.
- **0th Law**: If A is in thermal equilibrium with B and C, then B and C are in equilibrium.
- **Carnot Engines**: in heat engine and heat pump modes



  For the heat pump $\eta=\frac{W}{Q_1}=\frac{Q_1-Q_2}{Q_1}=\frac{T_1-T_2}{T_2}$. To increase the efficiency one should maximize the temperature difference between the reservoirs.

  For a refrigerator $\eta=\frac{Q_2}{W}=\frac{Q_2}{Q_1-Q_2}=\frac{T_2}{T_1-T_2}$ which is most efficient when T_1-T_2 is small.

- **Carnot's theorem**: states that a reversible engine is the most efficient possible.
- The **Carnot cycle** is a realisation of a Carnot engine using two isothermal transitions and two adiabatic transitions on an ideal gas.
- The **entropy change** in a reversible process is given by $dS=\frac{dQ}{T}$

Tutorial problems

1. Devise an arrangement of a heat engine that shows that if the Clausius statement of the second law is violated then so must the Kelvin-Planck statement be violated.

2. A house is maintained at a temperature T_H by means of a heat pump that extracts heat from a stream at the outdoor temperature T_S. The pump works at a constant power P, and the house loses heat at a rate $A(T_H-T_S)$. Show that the maximum temperature the house can reach is given by

$$T_H=T_S+\frac{P}{2A}+\sqrt{(T_S+\frac{P}{2A})^2-T_S^2}$$

3. A large mass M of porous hot rock at temperature T is to be used to generate electrical power by injecting water and using the steam to drive a turbine.

   As the heat is extracted the temperature of the rock decreases according to $dQ = -MC_V dT$ where C_V is the temperature independent heat capacity. Since the ideal thermodynamic efficiency $\eta = 1 - T_0/T$ this means that the efficiency decreases with time as T falls.

   If the rock is initially at a temperature T_i and the plant is closed down when the temperature has dropped to T_f, calculate the maximum electric energy that can extracted.

   Estimate this energy if $M = 10^{14}$kg (about 30 km^3), $C_V = 1000$ Jkg^{-1}K^{-1}, $T_i = 600°$C, $T_f = 100°$C and $T_0 = 20°$C.

Problem Class Questions

1. A reversible heat engine is run between a metal block of unknown temperature and a reference system held at the triple point. The temperature of the block is held constant by electrical heating at a rate of 15 W while power of 4 W is extracted from the heat engine. What is the temperature of the metal block?

2. A 3kg mass in equilibrium with its surroundings at 15°C is dropped from a height of 4 m onto a flat surface where it comes to rest and returns to equilibrium with its surroundings. What is the entropy change of the Universe as a result of this process?

3. Calculate the entropy change when Argon at 25° C and 1 atm expands from 500 cm^3 to 1000 cm^3 when heated to 100° C.

4. What path does an adiabatic expansion or compression of a gas follow on a $T - S$ graph?

   What path does an isothermic expansion or compression of a gas follow on a $T - S$ graph?

   Show that when a mass m of gas is heated from T_1 to T_2 at constant volume

   $$\Delta S = mC_V \ln \frac{T_2}{T_1}$$

   Hence sketch the paths of a Carnot engine and a Stirling engine (described on problem sheet 6) on graphs of temperature versus entropy.