KINETIC THEORY OF GASES
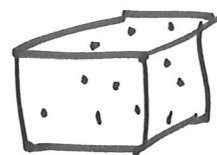
Gas are fundamentally atoms & molecules

PRINCIPLES

① A Gas is composed of a large number of molecules

② Molecules are small relative to separation

③ Molecules uniformly distributed & move randomly

④ Obey Newton's Laws of motion

⑤ No forces except in collisions with each other or the walls (hard spheres)

⑥ Collisions with walls are elastic

①, ③ & ⑥ are good approximations.

②, ⑤ break down at high density

④ molecules are really quantum objects
$$\lambda = \frac{h}{mv} \rightarrow \text{need} \quad \lambda \ll \text{separation}$$
ie at high T, high v or low density

IDEAL GASES: are described by kinetic theory
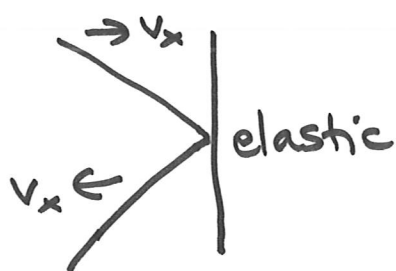
EMERGENT BULK PROPERTIES

Volume   V

Density   N molecules $\Rightarrow$ $n = N/V$ $\rho = \frac{Nm}{V}$

can vary across volume.

Pressure

$$P = \frac{Force}{area}$$ $Pa = Nm^{-2}$
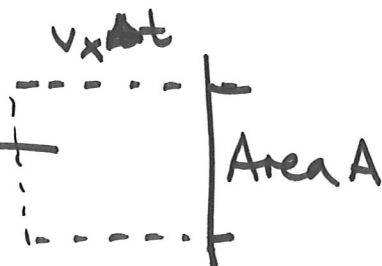
Caused by collisions (assume some average v_x)

$F = \frac{\Delta mv}{\Delta t}$ ~ $2mv_x$ per collision

In time Δt

these molecules hit the wall

except $\frac{1}{2}$ that were already travelling away

$$\text{\# collisions in } \Delta t = \frac{1}{2} n \times (v_x \Delta t \, A)$$

with $v_x \Delta t \, A$ labelled as "volume"

$$Force = \frac{2mv_x}{\Delta t} \times \frac{1}{2} n (v_x \Delta t \, A)$$

$$P = \frac{F}{A} = n M v_x^2$$

I've assumed all molecules have the same v_x.... so $v_x^2 \to \overline{v_x^2}$ the mean squared velocity

Since motion is random

$$\overline{v_x^2} = \overline{v_y^2} = \overline{v_z^2} = \tfrac{1}{3} \overline{v^2}$$

We arrive at

$$\boxed{P = \tfrac{1}{3} n m \overline{v^2}}$$

EG) At standard temp. & pressure (stp)

$$T = 273K \qquad (0°c)$$
$$P = 101\ 000\ Nm^{-2} \qquad (1 atm)$$

$$V_{\text{root mean square}} = \sqrt{\dfrac{3P}{nm}}$$

$$= \sqrt{\dfrac{3P}{\rho}}$$

eg air density is $1.2\ kg\,m^{-3}$

$$\Rightarrow V_{RMS} \approx 500\,ms^{-1}$$

So an evacuated flask will fill in

$$\sim \dfrac{0.1m}{500ms^{-1}}, \qquad \approx 0.2s\ ms$$

Internal Energy & Temperature

For an ideal gas the only energy is kinetic

$$U = \tfrac{1}{2} m \bar{v}^2 \times \text{number of molecules}$$

$$= \tfrac{1}{2} m \bar{v}^2 (nV)$$

$$m\bar{v}^2 n = \frac{2U}{V}$$

Remember $P = \tfrac{1}{3} n m \bar{v}^2 = \tfrac{2}{3} U/V$

$$\Rightarrow U = \tfrac{3}{2} PV \qquad \text{or} \qquad PV = \tfrac{2}{3} U$$

This hopefully reminds you of the empirical

EQUATION OF STATE :

$$\boxed{PV = N k T_I} \qquad \text{or} \qquad \boxed{P = n k T_I}$$

↑ Boltzmann's constant

↳ N/V

$$k = 1.38 \times 10^{-23} \, JK^{-1}$$

This gives us a link between temperature (here T_I for an ideal gas' temperature) and internal energy.

We have $P = \tfrac{1}{3} n m \bar{v}^2 = n k T_I$

$$\tfrac{3}{2} k T_I = \tfrac{1}{2} m \bar{v}^2$$

relation between KE & Temp.

& $U = \tfrac{3}{2} N k T_I \left(= N \tfrac{1}{2} m \bar{v}^2 \right)$

Avagadro's number: $N_A = 6.02 \times 10^{23}$

This number of atoms/molecules weighs the atomic mass number in grams

$$N = n_m N_A \quad \leftarrow \text{A. number}$$

$\uparrow$ number

$\uparrow$ number of moles

$$PV = n_m N_A k T_I = n_m R T$$

$\uparrow$ $R = k N_A$

gas constant

$= 8.314 \ JK^{-1}mole^{-1}$

$$\boxed{PV = n_m RT}$$

Q1) Volume of 1 mole at stp?

$$V_m = \frac{RT_I}{P} = \frac{8.314 \times 273}{101,000} = 0.0225 m^3 = 22.5 \ell$$

Q2) How many molecules in $1 m^3$ at $20C$?

$$N = \frac{PV}{RT_I} = \frac{101,000 \times 1 m^3}{1.38 \times 10^{-23} \times 295} \simeq 250 \times 10^{23}.$$

EQUIPARTITION OF ENERGY

We had for an ideal gas

$$\frac{1}{2}M\bar{v}^2 = \frac{1}{2}m(\bar{v_x^2} + \bar{v_y^2} + \bar{v_z^2}) = \frac{3}{2}kT$$

these are each "degrees of freedom" into which energy can be put

Equipartition Theorem: for a classical system in thermal equilibrium, the total energy is shared equally among all the degrees of freedom

& we can see each gets $\boxed{\frac{1}{2}kT}$

Monatomic Gases eg He

- three possible motions $E = \frac{3}{2}kT$

For a mole $U_{mol} = N_A k \frac{3}{2}T = \frac{3}{2}RT$

$\underbrace{\quad}_{R}$

Diatomic Gases   eg H_2, O_2, N_2

- 3 translations
- 2 axes of rotation

we neglect spinning on axis — I'll explain in a bit....

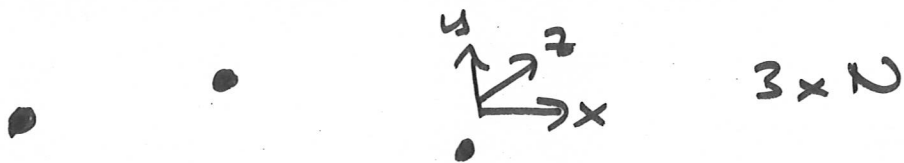- 1 vibration $\frac{1}{2}Mv^2 + \frac{kx^2}{2} = KE + PE$

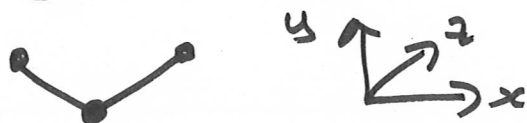so counts as 2

$$U_{mol} = \frac{7}{2}RT$$

If we ignore the bonds in a molecule there are $3N$ movements of the N atoms

$3 \times N$

When we tie them with bonds some movements are different from others:

translations: 3   the whole structure can move in x, y, z rigidly...

rotations: 3   Rotations about 3 axes - rigid
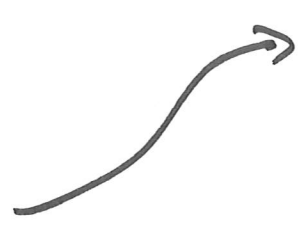
              2   rotations about this axis doesn't move the atoms...

vibrations: the rest of the $3N$ motions move one atom relative to another — this stresses the bond & needs potential as well as kinetic energy....

CO_2

$$O = C = O$$

- 3 translations
- 2 rotations
- 4 vibrations

$O = C = O$ 1

$O = C = O$ 2

$\{O = C = O\}$ & $\{O = C = O\}$
each has KE & PE

$$\Rightarrow U_{mol} = \frac{3 + 2 + 2 \times 4}{2} RT$$

$$= \frac{13}{2} RT$$

SOLIDS

KE & PE for each atom
in each direction

$$U_{mol} = 6 \times \frac{1}{2} RT = 3 RT$$

HEAT CAPACITIES : we can test the predictions

$$\Delta Q = C \, \Delta T$$

heat in — heat capacity — change in temp

or

$$\Delta Q = n_m C \, \Delta T$$

moles

specific heat capacity

$$C = \frac{dQ}{dT}$$

mono-atomic gas : $C = 3R/2 = 12.5 \ JK^{-1} mole^{-1}$

diatomic gas : $C = 7/2 R = 29 \ JK^{-1} mole^{-1}$
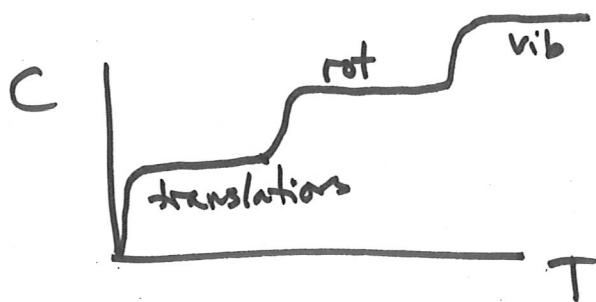
We find $C_{measured} \leq C_{predicted}$

Why Equipartition Fails: this is one of the first signs of quantum mechanics

- classically — you can put any amount of energy into a degree of freedom

- QM — energy comes in lumps. If your kT is too small you can't excite the motion or vibration at all.

Typically $\Delta E_{trans} \ll \Delta E_{rot} \ll \Delta E_{vib}$

So the modes switch on only as T goes through steps

So for example at room temp N_2 $C = \frac{5}{2}R$

$= 20.8 \ JK^{-1}mol^{-1}$

Incidentally this is why we ignore spin on axis $KE = h^2/2I$ & $I \sim mr^2$ & $r \Rightarrow 0$

so the energy to excite is very, very large.

For ~~metals~~ solids C is typically lower than predicted because harder to excite vibrations eg diamond ¼ !

Metals at room temp — prediction pretty good... but at higher ~~temps~~ T electron sea contributes $3k/2$ per electron.

Boltzman Factor : We've been pretending all particles at some T move at same v.... of course they have a spread... to understand the spread better consider gas in the atmosphere:

Hydrostatic Equilibrium
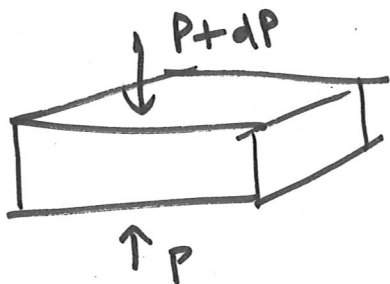
A lump of gas' weight is supported by a pressure gradient

$$\text{Weight} = mg = \rho (A \, dh) g$$

volume $= A \, dh$
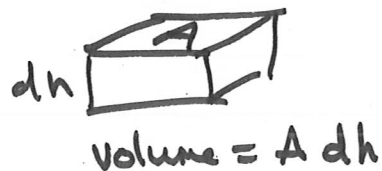
$$\text{Upward force} = PA - (P+dP)A = -A \, dP$$

(minus sign is because pressure gas down as go up)

Balancing forces
$$\rho A g \, dh = -A \, dP$$

$$\frac{dP}{dh} = -\rho g$$

Eq'n of hydrostatic equilibrium

If $\rho = \text{const}$ (eg in water) we can integrate

$$\int dP = - \int \rho g \, dh \qquad "P = \rho g h"$$

For an ideal gas $\quad P = nkT_I \quad$ & $\quad \rho = nm \leftarrow$ mass of it

at const T_I $\qquad dP = kT_I \, dn$ $\qquad\qquad \uparrow$ number of pts/V

$$\frac{dn}{dh} = -\frac{P}{kT_I} g = -\frac{mg}{kT_I} \, n$$

$$\int_{n_0}^{n} \frac{dn}{n} = -\frac{mg}{kT_I} \int_0^h dh$$

$$\ln \frac{n}{n_0} = -\frac{mgh}{kT_I}$$

$$\frac{n}{n_0} = e^{-mgh/kT_I} \qquad \left(= \frac{P}{P_0} \right)$$

Pressure & number density fall off exponentially

eg on Earth $\quad m = 30u \quad g = 9.81 \, ms^{-2} \quad T \sim 300K$

At $\quad h = 5km \quad P/P_0 = 0.55$

Interpretation: we have $\qquad \dfrac{n}{n_0} = e^{-E_{GPE}/kT_I}$

Recast this as fraction or probability that one atom is at height h with $E = mgh$

$$P(E) \propto e^{-E/kT}$$

BOLTZMAN FACTOR

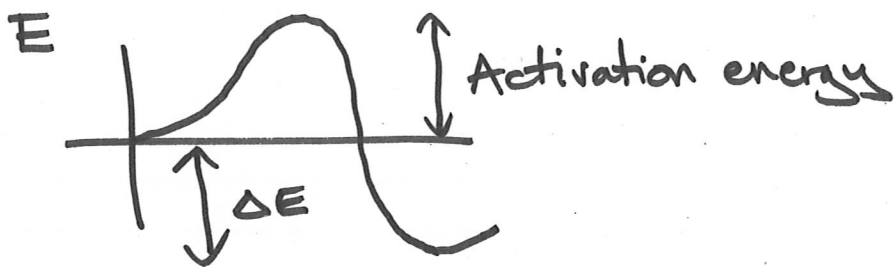We won't prove this further here... or show it leads to $\langle E \rangle = \frac{1}{2}kT$ (next year!)

<u>EXAMPLES</u>

- prob $\begin{pmatrix} \text{pt in kinetic theory} \\ \text{gas has speed } v \end{pmatrix} \propto e^{-\frac{1}{2}mv^2/k}$

- <u>Activation Energies</u>

$$2H_2 + O_2 \longrightarrow 2H_2O$$

but you have to light a match!

The Boltzman factor controls whether any atoms can get over the activation energy....
... and it can be very T dependent

EG $E_A = 100kT$ $T = 300K$

How does e^{-kT} change by $\Delta T = 20K$?

$$P(300K) \propto e^{-100}$$

$$P(320K) \propto e^{-100(300/320)}$$

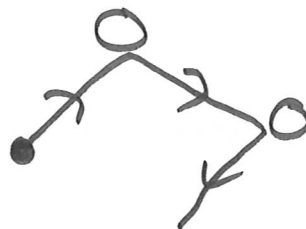$$\text{ratio} \propto e^{100 - 93.75} = e^{6.25} = 518$$

Using a bunsen burner in Chemistry can be a dangerous business!

TRANSPORT IN KINETIC THEORY

Let's think about how transport (as energy etc) happens in a gas where n depends on position. At base atoms/molecules move:

Mean Free Path

particles richochet
about in the gas:

What is the "average distance between
collisions" ?

A particle, diameter d passes through a cylinder as it moves

$$\downarrow$$
$$d \bigcirc\!\!=\!\!=\!\!=\!\!=\!\!=$$
$$\uparrow \quad -v\Delta t-$$
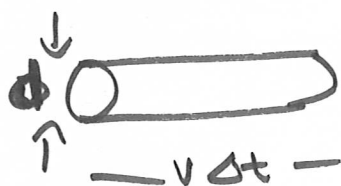
It will hit another particle if it comes within $d/2$ of the cylinder

So the number of collisions
depends on how many pts
are in a tube of $\underline{radius = d}$
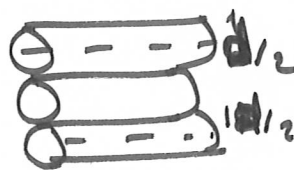
$$\text{collisions/sec} = \underbrace{\pi d^2 v}_{\text{volume}} \quad \underset{\underset{N/V}{\big\downarrow}}{n}$$

I've assumed only one pt is moving here.

A better job is to work out the average relative velocity of two particles

$$\vec{v}_{rel} = \vec{v}_A - \vec{v}_B$$

$$\vec{v}_{rel} \cdot \vec{v}_{rel} = v_A^2 + v_B^2 - 2\vec{v}_A \cdot \vec{v}_B$$

$$\underset{v_{rel}^2}{\underbrace{}}$$

$$\underset{v_A v_B \cos\theta_{AB}}{\underbrace{}}$$

In a randomly moving gas every θ_{AB} is equally likely $\langle \cos\theta_{AB} \rangle = 0$

So $\quad v \rightarrow \langle v_{rel} \rangle \simeq \sqrt{2} \, v$

The mean free path is then $\dfrac{\text{speed}}{\text{collision rate}}$

which divides distance travelled by number of collisions

$$\Rightarrow \boxed{\lambda = \frac{1}{\sqrt{2} \, \pi d^2 \, n}}$$

EG air at STP $\qquad P = n k T$

$$n = \frac{P}{kT} = \frac{1.01 \times 10^5 \, Pa}{1.38 \times 10^{-23} \, JK^{-1} \times 273K} = 2.7 \times 10^{25} \, m^{-3}$$

$$\lambda = \frac{1}{\sqrt{2}} \times 3.142 \times (0.3nm)^2 \times 2.7 \times 10^{25} \, m^{-3}$$

$$\underset{\simeq 10^{-9}}{\underbrace{}}$$

$$= 10^{-7} \, M$$

Distribution of Path Lengths

P (collision in δr travel) = time of travel $\times$ $\begin{array}{c}\text{collision}\\\text{rate}\end{array}$

$$= \frac{\delta r}{v} \times \underset{2v/\lambda}{\text{collision rate}}$$

$$= \frac{\delta r}{\lambda}$$

This assumes an equal chance of collision through travelling λ rising to 1 when $\delta r = \lambda$ ... good at small δr...

Now

$$P (\text{travel } r + \delta r) = P(\text{travel } r) \, P(\text{travel } \delta r)$$

$$= P(\text{travel } r) \, (1 - P_{\text{collision}}(\delta r))$$

$$= P(\text{travel } r) \, (1 - \delta r/\lambda)$$

$$\Rightarrow \quad \frac{dP}{dr} = \frac{P(r + \delta r) - P(r)}{\delta r} = -\frac{P(r)}{\lambda}$$

$$\frac{dP}{P} = -\frac{dr}{\lambda}$$

integrate

$$\int \frac{dP}{P} = -\frac{1}{\lambda} \int dr$$

$$\ln P = -r/\lambda + \text{const}$$

$\uparrow$ zero since
$P(r=0) = 1$

$$P = e^{-r/\lambda}$$

$P(\text{travel } \lambda) = e^{-1} = \frac{1}{2.7}$ $P(\text{travel } 10\lambda) = e^{-10}$ ie $\frac{5 \text{ in}}{100,000}$

Distance Travelled On A Random Walk

The particle is constantly
scattered randomly

$$\vec{r} = \vec{s}_1 + \vec{s}_2 + \dots$$

$$r^2 = \vec{r} \cdot \vec{r} = s_1^2 + s_2^2 + \dots + \vec{s}_1 \cdot \vec{s}_2 + \vec{s}_2 \cdot \vec{s}_1 + \dots$$

As we saw before in a random environment

$$\langle \cos \theta_{12} \rangle = 0 \qquad \vec{s}_1 \cdot \vec{s}_2 = 0$$

$$\Rightarrow \quad r^2 = \sum_{i=1}^{N} s_i^2 \simeq N \lambda^2$$

↑ assume go λ each step.

$$\boxed{r = \sqrt{N} \; \lambda}$$

This is much less than $N\lambda$ – the total
distance travelled.

EG   Air molecules: previously we've computed
$$v = 500 \; ms^{-1} \quad \& \quad \lambda = 10^{-7} m$$

In 1s it travels 500 m...

But collides $N = \dfrac{500}{10^{-7}} \sim 5 \times 10^{9}$ times

Total net distance is then $\sqrt{N} \lambda = 7mm$

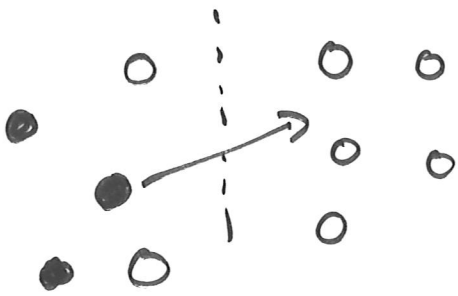Note that since $N \propto t$ (more collisions in more time)

$$r \propto t^{1/2}$$

If one were to simulate a finite step walk particle by particle then $\langle \cos \theta_{12} \rangle \neq 0$ precisely... one gets a distribution of distances travelled

As $N \Rightarrow \infty$ the Gaussian like peak narrows....

Modelling Transport

black & white mix over time

movement of:

identity $\longrightarrow$ diffusion

energy $\longrightarrow$ thermal conductivity

momentum $\longrightarrow$ viscosity

We can do simple models of all these... assume particles move in $\pm x, \pm y, \pm z$ only.

$1/6^{th}$ do each motion.

Ignore collisions - work in small Δt where pts cross line unimpeded

Diffusion

this defines Diffusion Coefficient $m^2 s^{-1}$

Fick's Law

$$\frac{dN}{dt} = -DA \frac{dn}{dx}$$

number flowing across dotted line /sec

Area flowing over

spatial density gradient

Let's use kinetic theory to study the same process.

Number flowing
$L \Rightarrow R$
$$= \frac{1}{6} n(x-\lambda) A \bar{v} \Delta t$$

1 in 6 go right way

density to left by λ

volume that can cross in Δt sec.

Number flowing
$R \Rightarrow L$
$$= \frac{1}{6} n(x+\lambda) A \bar{v} \Delta t$$

so there's a net flow if the density is bigger on one side.

Net flow $\frac{dN}{dt}$
$L \Rightarrow R$ Δt
$$= \frac{1}{6} \left[n(x-\lambda) - n(x+\lambda) \right] A \bar{v}$$

$n(x) - \frac{dn}{dx} \lambda$

$n(x) + \frac{dn}{dx} \lambda$

$$= \frac{1}{3} \lambda \bar{v} A \frac{dn}{dx}$$

$$\Rightarrow \boxed{D = \frac{1}{3} \lambda \bar{v}}$$

EG Evaporation from a test tube

$$\text{Flow rate} = -\tfrac{1}{3} \lambda \bar{v} A \frac{dn}{dx}$$

$$? \, dN/dt$$

Assume saturation density n_s just above surface; and zero at top

$$\frac{dn}{dx} = \frac{n_s}{h}$$

Ideal gas: $P_s = n_s k T_I$ $= \dfrac{P_s}{h k T_I}$

$$|\text{Flow rate}| = \tfrac{1}{3} \frac{\lambda \bar{v} A P_s}{h k T_I}$$

$$\text{Mass loss} = \frac{M_{mole}}{N_A} \frac{\lambda \bar{v} A P_s}{3 h k T_I}$$

$$\nwarrow M_{mole} = M_{atom} N_A$$

$$= \frac{M_{mole}}{R} \frac{\lambda \bar{v} A P_s}{3 h T_I}$$

$\div$ density $\text{Volume loss} = \dfrac{M_{mole}}{R} \dfrac{\lambda \bar{v} A P_s}{3 h T_I \rho}$
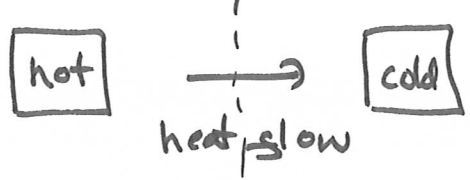
$\div$ area $\text{Height loss} = \dfrac{M_{mole}}{R} \dfrac{\lambda \bar{v} P_s}{3 h T_I \rho}$

$T = 293 K$; $P_s = 1710 \, Nm^{-2}$; $\rho = 1000 \, kg \, m^{-3}$

I.S. $h = 0.1 m$, $\lambda = 10^{-7} m$, $\bar{v} = 650 \, ms^{-1}$ (here weighs less than N_2)

$$\Rightarrow 0.24 \, mm/day$$

Thermal Conduction

$$\boxed{\text{hot}} \quad \xrightarrow{\text{heat flow}} \quad \boxed{\text{cold}}$$

Fourier's Law:

$$\frac{dQ}{dt} = -KA\frac{dT}{dx}$$

↗ Area

↑ thermal conductivity

↘ temperature gradient

$\frac{dQ}{dt}$? energy flow/s

As before assume $\frac{1}{6}nA\bar{v}$ molecules flow/sec

↑↑ assume don't change much

So   L → R $\frac{1}{6}nA\bar{v}\,E(x-\lambda)\Delta t$ energy flow

↳ energy/molecule

R → L $\frac{1}{6}nA\bar{v}\,E(x+\lambda)\Delta t$

$$\Rightarrow \quad \frac{dQ}{dt} = \text{difference} = -\frac{1}{3}nA\bar{v}\lambda\frac{dE}{dx}$$

$$= -\frac{1}{3}nA\bar{v}\lambda\frac{dE}{dT}\frac{dT}{dx}$$

Now $n\dfrac{dE}{dT} = \dfrac{dQ}{dT}$ for a unit volume
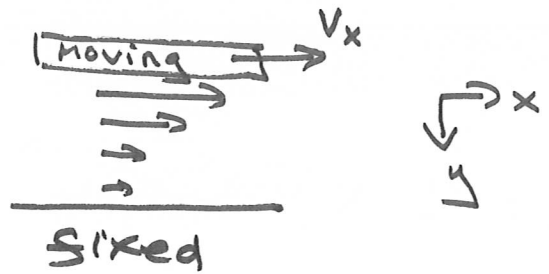
$$= C \quad \text{specific heat capacity per volume}$$

$$\Rightarrow \quad \boxed{\frac{dQ}{dt} = -\frac{1}{3}AC\bar{v}\lambda\frac{dT}{dx}}$$

& $\boxed{K = \frac{1}{3}\lambda\bar{v}C}$ which is C times the result for D.

.scosity

Molecules stick
briefly to the surface
& acquire x-momentum

This contradicts Ideal Gas rules!

ie there's a drag force in x, dependent on the
velocity gradient in y

$$F_x = -\eta A \frac{dv_x}{dy}$$

now area as
moving surface

in opposite direction to V_x

viscosity coefficient
$Nm^{-2}s^{-1}$

Consider a slice at some y & look at flow as x
momentum across it

$$\frac{\Delta p}{dt} \text{ top to bottom} = \frac{1}{6} n A \bar{v} \; m v_x(y-\lambda)$$

$$\frac{\Delta p}{dt} \text{ bottom to top} = \frac{1}{6} n A \bar{v} \; m v_x(y+\lambda)$$

net momentum flow $= -\frac{1}{3} n A \bar{v} \lambda m \frac{dv_x}{dy}$

$nm = $ mass/volume $=$ density

$\Rightarrow$ $\boxed{\eta = \frac{1}{3} \lambda \bar{v} \rho}$

~~viscos
density
dens~~ $\times$ diffusion constant

note $\frac{dv_x}{dy}$ is
usually only
large near
eg an aeroplane

Diffusion $\quad D = \frac{1}{3}\lambda \bar{v}$

Thermal cond. $\quad K = \frac{1}{3}\lambda \bar{v} C$

Viscosity $\quad \eta = \frac{1}{3}\lambda \bar{v} \rho$

$$\frac{C_{molecule} \, N/V}{M \, N/V} \sim$$

$\frac{5}{2}k$ diatomic molecules

$$\frac{K}{\eta} = \frac{C_{molek}}{M} = \frac{5k}{2m}$$

eg $O_2 \sim 650 \, m^2 s^{-2} k^{-1}$
order of
magnitude ok....

Maxwell Measures N_A, 1865

He noted $\quad \eta/D = \rho$

The D here is for slow of molecules — multiply by m to get slow rate of mass — this he could measure!

$$D_m = M D$$

$$\frac{\eta}{D_m} = \frac{\rho}{M} = n = \frac{N_A}{V_m}$$

So he could measure N_A via η, D_m, V_m .... he had to work out how to measure η .. but ...

Temperature Dependence

$\underline{D}$: $\quad \frac{1}{2}mv^2 = \frac{3}{2}kT \quad \rightarrow v \sim T^{1/2}$

$$\lambda = \frac{1}{\sqrt{2}\pi d^2 n} \sim \frac{1}{n} \overset{ideal}{\sim} \frac{kT_f}{P} \sim T$$

$$D \sim T^{3/2}$$

$\underline{K}$: C is energy/volume & grows as $n \sim \frac{P}{kT_f} \sim \frac{1}{T}$

$$K \sim T^{1/2}$$

η: $\rho \sim n$ too $\quad \Rightarrow \eta \sim T^{1/2}$

They all grow with temperature

Pressure Dependence

We had $\rho, C \sim n$ $\lambda \sim 1/n$ $n = \frac{P}{RT_I}$

$\Rightarrow$ η, K are predicted to be independent of P!

As the number of pts falls off... their mean free path grows...
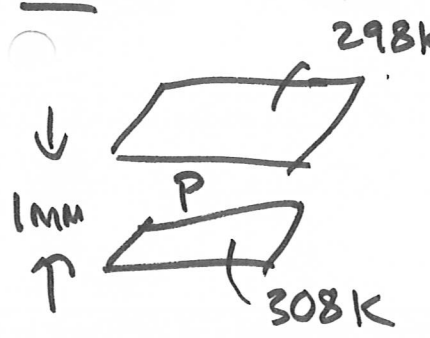
which matches data reasonably well.

BUT: surely as $P \to 0$ transport should stop?
Point is λ can't be bigger than the container size. If λ saturates $\eta, K \sim n$
$\to 0$
"Knudsen low pressure regime"

ALSO $P \to \infty$ intermolecular forces enter & mess up analysis...

EG conduction between two plates

298K

1mm

P

308K

$\sqrt{} \sim 10^{-7}m \ll d$

$K = \frac{1}{3} \lambda \bar{v} C$

$\swarrow$ 500ms^{-1}

Heat capacity /unit volume $C = \frac{5}{2} kn = \frac{5P}{2T_I}$

$P = 10^5 \, Nm^{-2} \rightarrow K = 0.014 \, Wm^{-1}K^{-1}$

$P = 0.1 \, Nm^{-2} \quad \hat{\lambda} \simeq 0.1m \gg 1mm$

K drops by $\hat{\lambda}/d = \frac{0.001}{0.001} \simeq 100$

Dependence On Gas

Conduction grows with $\lambda \sim \frac{1}{d^2}$ $\Rightarrow$ small molecules good.

& $D \sim \bar{v}$ so light molecules are good too

$$\tfrac{1}{2} M v^2 = \tfrac{3}{2} k T_I$$

Thus H_2 & He are better heat conductors than CO_2

LIMITATIONS OF KINETIC THEORY

MAINLY THAT IT ONLY DESCRIBES GASES!

No liquids or solids!

Hence no phase changes (boiling etc)!

Can't cope with interactions between molecules or quantum effects.

But we have learnt a gas is described
by
$$P, \; T \; (U), \; V, \; N \; (\text{or } n)$$

We will return at the end of the course to an improved gas model with interactions