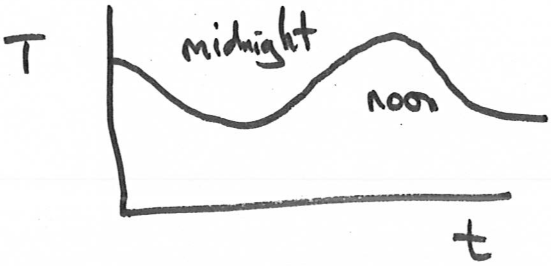
PARTIAL DIFFERENTIATION

Derivatives where some quantity is held constant.

EG Consider the temperature across some lake



If I want to know how T changes over time at a point "$\frac{dT}{dt}$" but we mean at a fixed point so we write

$$\frac{\partial T}{\partial t} \quad \text{or} \quad \left(\frac{\partial T}{\partial t}\right)_x$$

This is what you measure if you stand still.

eg $T = \sin kx \sin \omega t$

eg $\left(\frac{\partial T}{\partial t}\right)_x = \omega \sin kx \cos \omega t$

Equally we might want to know

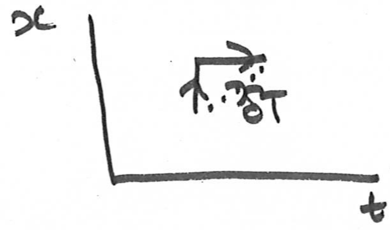
$$\left(\frac{\partial T}{\partial x}\right)_t$$

How the temperature changes with x at one time.

eg $\left(\frac{\partial T}{\partial x}\right)_t = k \cos kx \sin \omega t$

If you change t & x between a first & second measurement

$$\delta T = \left(\frac{\partial T}{\partial x}\right)_t \delta x + \left(\frac{\partial T}{\partial t}\right)_x \delta t$$



So $$\frac{dT}{dt} = \left(\frac{\partial T}{\partial t}\right)_x + \left(\frac{\partial T}{\partial x}\right)_t \frac{\partial x}{dt}$$

partial since at fixed y

This is the change in T with time including you swimming along the x-line eg from hot to cold section - it depends on your speed $\partial x/\partial t$.

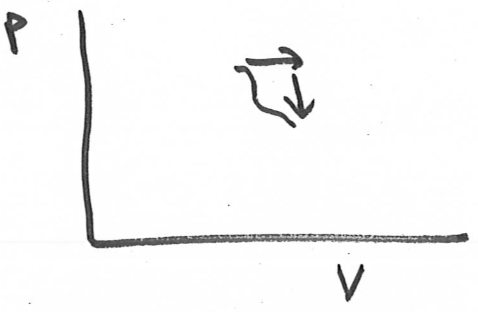
So partial derivatives give the gradient when you have $\phi(x, y \ldots)$ & you keep ~~some~~ all variables but one fixed.

THERMODYNAMICS A gas is described by P, V, T

for a fixed n_m but $PV = n_m RT$

So if we know P, V we know T
P, T " " V
T, V " " P

Can pick two & work in that space



(Indicator diagram – more to come)

Consider something like internal energy

$$U(P, V)$$

Then $$dU = \left(\frac{\partial U}{\partial V}\right)_P \delta V + \left(\frac{\partial U}{\partial P}\right)_V \delta P$$

We might be interested in how U changes with T along some path in the PV plane...

eg

$$C_p = \left(\frac{\partial U}{\partial T}\right)_P \qquad C_v = \left(\frac{\partial U}{\partial T}\right)_V$$

here $T = \frac{\cancel{V}\, P_{fixed}}{n_m R}$ $\qquad$ $T = \frac{\cancel{P}\, V_{fixed}}{n_m R}$