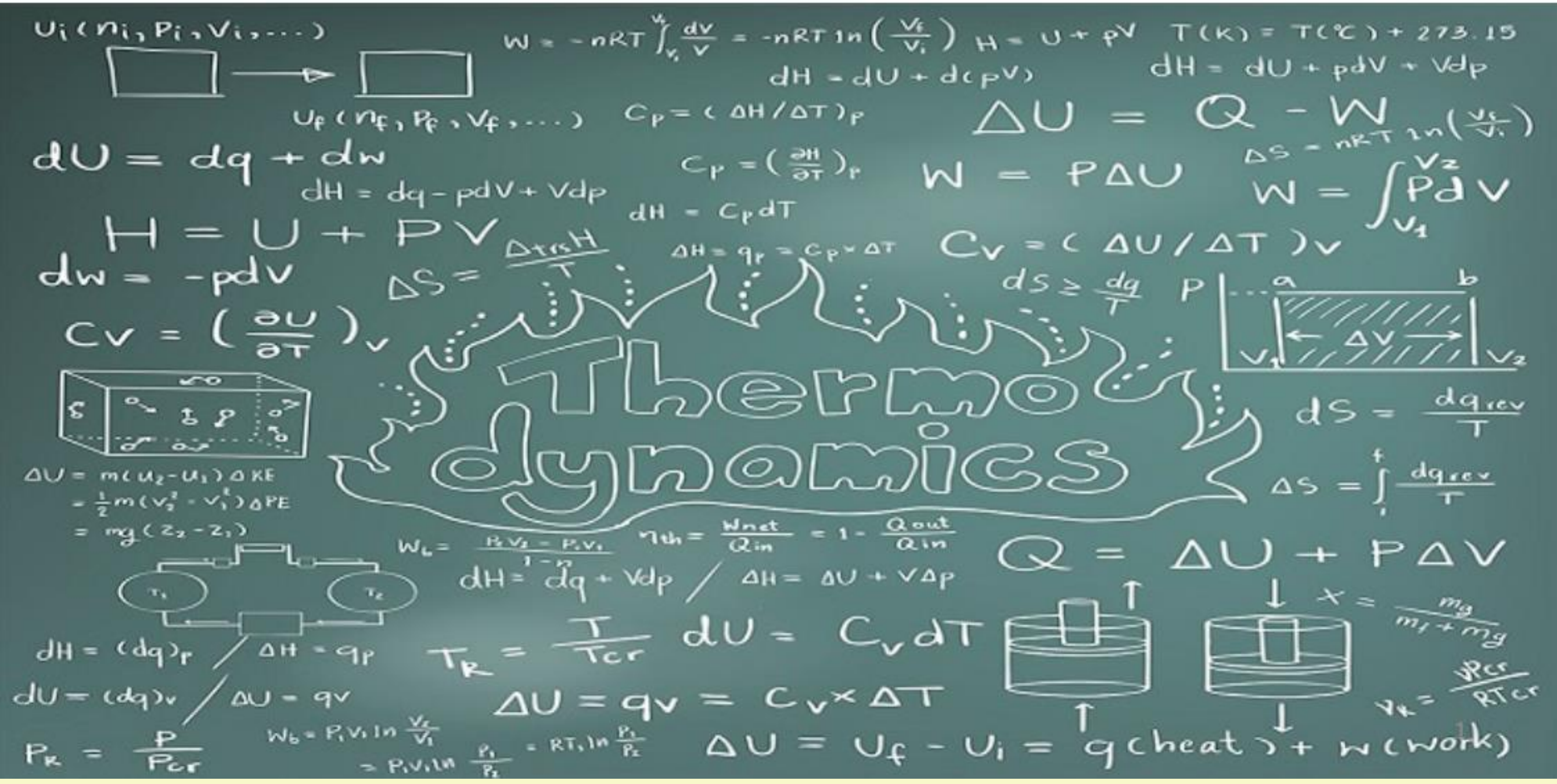
PHYS1013
Energy and Matter

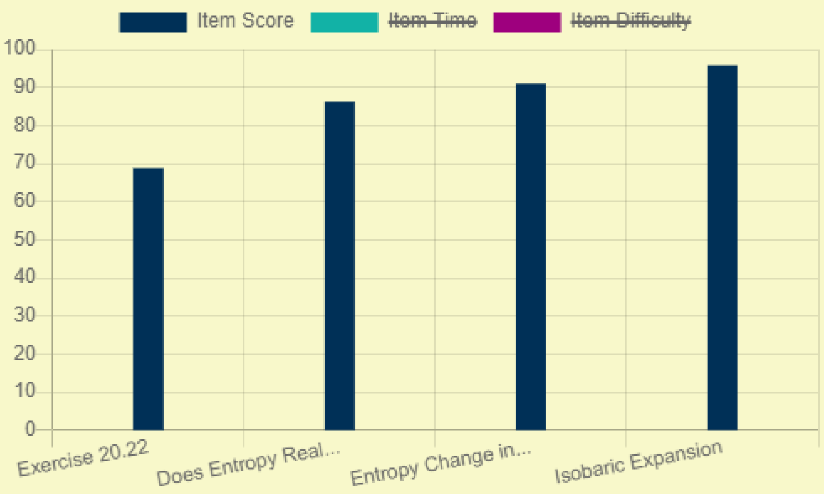


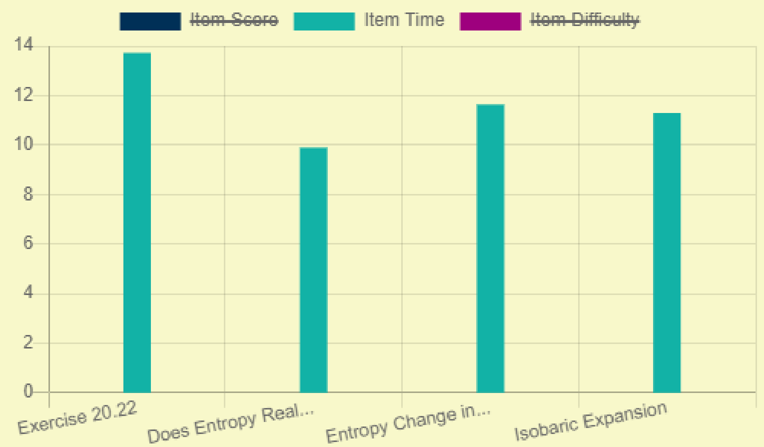
MP Week 7 – average score 67.9%

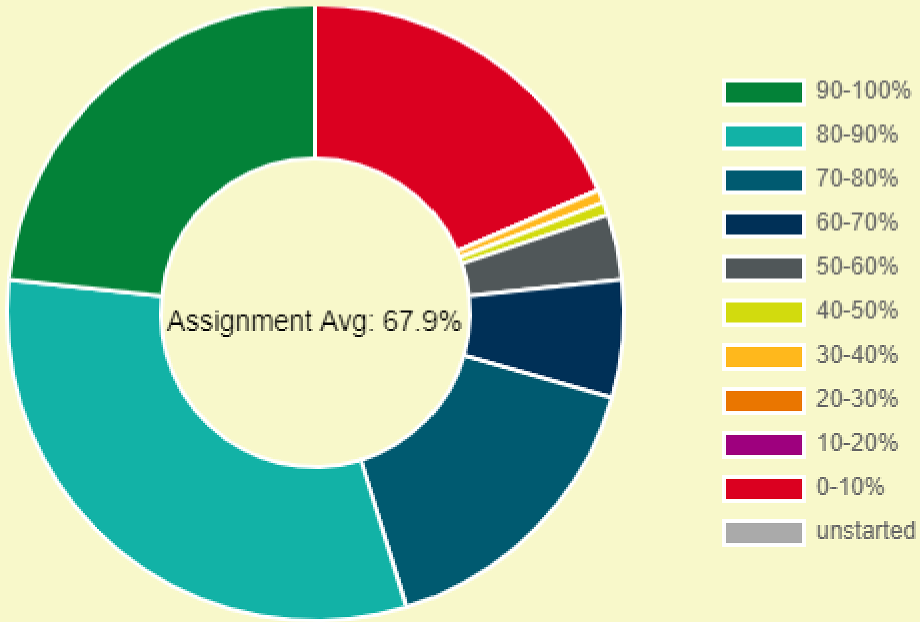
(last year 68.3%)

Average time 47 min

(last year 42 min)







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Come and see us at our Student Ambassador Pop Up in front of Hartley Library to find out more!

3rd, 10th, 18th, 23rd 31st May
10:00 – 14:00

Find out more and apply!

The exam is 14:30 on the 25th May.

It will consist of

Section A – all questions to be answered

Section B – answer 2 of 3 questions

Answers to Section A and Section B must be in separate answer books

Answer **all** questions in **Section A** and **only two** questions in **Section B**.

Section A carries 1/3 of the total marks for the exam paper and you should aim to spend about 40 mins on it.

Section B carries 2/3 of the total marks for the exam paper and you should aim to spend about 80 mins on it.

SCHOOL OF PHYSICS & ASTRONOMY

PHYSICAL CONSTANTS

Candidates are advised that they should only use the number of significant figures appropriate for the problem they are attempting to solve.

GENERAL CONSTANTS:

Charge on electron	$-e$	=	$-1.60217733 \times 10^{-19}\ C$
Mass of electron	m_e	=	$9.1093897 \times 10^{-31}$ kg ($\equiv$ 0.510998902 Mev/c^2)
Mass of proton	m_p	=	$1.6726231 \times 10^{-27}$ kg ($\equiv$ 938.27200 Mev/c^2)
Mass of neutron	m_n	=	$1.6749286 \times 10^{-27}$ kg ($\equiv$ 939.56533 Mev/c^2)
Permeability of vacuum	μ_0	=	$4\pi \times 10^{-7}\ \mathrm{H\,m^{-1}}$
Permittivity of vacuum	ϵ_0	=	$8.854187817 \times 10^{-12}\ \mathrm{F\,m^{-1}}$
Fine structure constant	α	=	$1/137.035989$
Gravitation constant	G	=	$6.67259 \times 10^{-11}\ \mathrm{N\,m^2\,kg^{-2}}$
Boltzmann's constant	k_B	=	$1.3806503 \times 10^{-23}\ \mathrm{J\,K^{-1}}$
Atmospheric pressure	1 atm.	=	$1.01325 \times 10^5\ \mathrm{N\,m^{-2}}$ (Pa)
Stefan-Boltzmann constant	σ	=	$5.6704 \times 10^{-8}\ \mathrm{W\,m^{-2}\,K^{-4}}$
Avogadro's number	N	=	6.0221367×10^{23}
Velocity of light	c	=	$2.99792458 \times 10^8\ \mathrm{m\,s^{-1}}$
Bohr radius	a_0	=	$5.2917721 \times 10^{-11}$ m
Bohr magneton	μ_B	=	$9.274006 \times 10^{-24}\ \mathrm{J\,T^{-1}}$
Planck's constant	h	=	$6.62607544 \times 10^{-34}$ Js
Planck's constant/2π	$\hbar$	=	$1.05457266 \times 10^{-34}$ Js

ASTRONOMICAL CONSTANTS

Astronomical unit:	1AU	=	$1.49597871 \times 10^{11}$ m
Parsec:	1pc	=	$3.08567758 \times 10^{16}$ m
Mass of the Earth	$M_\oplus$	=	5.97×10^{24} kg
Radius of the Earth	$R_\oplus$	=	6.37814×10^6 m
Mass of the Sun	$M_\odot$	=	1.99×10^{30} kg
Radius of the Sun	$R_\odot$	=	6.96×10^8 m
Luminosity of the Sun	$L_\odot$	=	3.85×10^{26} W
Thomson cross-section	σ_T	=	$6.652459 \times 10^{-29}\ \mathrm{m^2}$

ATOMIC AND NUCLEAR PHYSICS UNITS

	1 fm	=	10^{-15} m
	1 barn	=	$10^{-28}\ \mathrm{m^2}$
Atomic mass unit	1 u.	=	$1.6605402 \times 10^{-27}$ kg
Atomic energy unit	1 a.u.	=	27.2113834 eV
Ångstrom	1 Å	=	10^{-10} m
Electron volt	1 eV	=	$1.6021765 \times 10^{-19}$ J
	$\hbar c$	=	197.32696 MeV fm

(Updated 4 June 2010)

There is not a formula sheet although you may be given some formulae in the questions (making the questions simpler).

Writing things down (blind) is supposed to be the best revision technique.

Do think about setting questions as you go…

Do use the problem sheets as example questions – solutions are all online (Blackboard)

Past exam papers are linked from the blackboard site…

The summaries below are intended to get you started with the key elements of the course… but all material taught in lectures can be examined.

KINETIC THEORY OF GASES

PRINCIPLES

1. A Gas is composed of a large number of molecules
2. Molecules are small relative to separation
3. Molecules uniformly distributed & move randomly
4. Obey Newton's Laws of motion
5. No forces except in collisions with each other or the walls (hard spheres)
6. Collisions with walls are elastic

$n = N/V$ $\rho = \frac{Nm}{V}$

$$\boxed{P = \tfrac{1}{3} n m \overline{v^2}}$$

$U = \tfrac{1}{2} m \overline{v^2} \times$ number of molecules

$= \tfrac{1}{2} m \overline{v^2} \, (n V)$

$$PV = \frac{2}{3} U$$

$$\boxed{PV = N k T_I}$$

$$\boxed{PV = n_m R T}$$

$R = k N_A$

Equipartition Theorem: for a classical system in thermal equilibrium, the total energy is shared equally among all the degrees of freedom

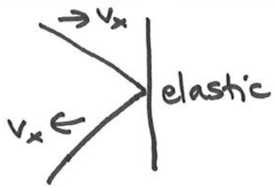
& we can see each gets $\boxed{\tfrac{1}{2} kT}$

$$U = \frac{3}{2} N k T_I$$

Pressure

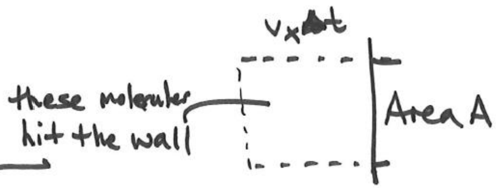
$$P = \frac{\text{Force}}{\text{area}} \qquad Pa = Nm^{-2}$$

Caused by collisions



$$F = \frac{\Delta mv}{\Delta t}$$ — Δmv: $2mv_x$ per collis[ion]

In time Δt



except 1/2 that were already travelling away

$$\#\text{ collisions in } \Delta t = \frac{1}{2} n \times (v_x \Delta t A)$$ ← volume

$$\text{Force} = \frac{2mv_x}{\Delta t} \times \frac{1}{2} n (v_x \Delta t A)$$

$$P = \frac{F}{A} = n m v_x^2$$

I've assumed all molecules have the same v_x.... so $v_x^2 \rightarrow \overline{v_x^2}$ the <u>mean squared velocity</u>

Since motion is random

$$\overline{v_x^2} = \overline{v_y^2} = \overline{v_z^2} = \frac{1}{3}\overline{v^2}$$

We arrive at

$$\boxed{P = \frac{1}{3} n m \overline{v^2}}$$

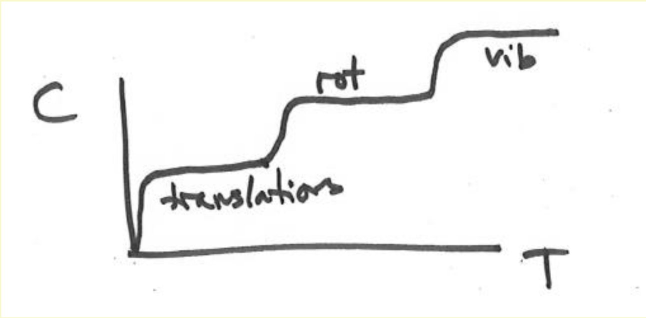
HEAT CAPACITIES

$$\Delta Q = C \, \Delta T$$

N unlinked molecules have 3N motions

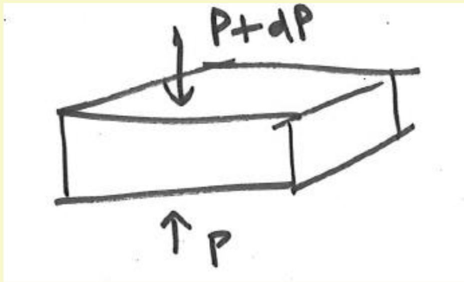
-> 3 translations (KE)
-> 2 or 3 rotations (KE)
-> rest vibrations (KE + PE)

$$U = N_A \tfrac{1}{2} kT \times \text{dof}$$



Quantum thresholds

Hydrostatic Equilibrium



$$\frac{dP}{dh} = -\rho g$$

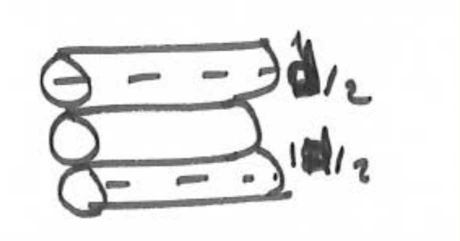
$$P(E) \propto e^{-E/kT}$$

BOLTZMAN FACTOR

TRANSPORT

Mean Free Path

$$\text{collisions/sec} = \underbrace{\pi d^2 v}_{\text{volume}} \; n \quad (n = N/V)$$



$$\vec{v}_{rel} \cdot \vec{v}_{rel} = v_A^2 + v_B^2 - 2\vec{v}_A \cdot \vec{v}_B$$

$$v \rightarrow \langle v_{rel} \rangle \simeq \sqrt{2}\, v$$

$$\boxed{\lambda = \frac{1}{\sqrt{2}\,\pi d^2 n}}$$

$$\frac{\text{speed}}{\text{collision rate}}$$

$$P(\text{collision in } \delta r \text{ travel}) = \frac{\delta r}{\lambda}$$

$$\begin{aligned} P(\text{travel } r+\delta r) &= P(\text{travel } r)\, P(\text{travel } \delta r) \\ &= P(\text{travel } r)\,(1 - P_{\text{collision}}(\delta r)) \\ &= P(\text{travel } r)\,(1 - \delta r/\lambda) \end{aligned}$$

$$\frac{dP}{dr} = \frac{P(r+\delta r) - P(r)}{\delta r} = -\frac{P(r)}{\lambda}$$

$$P = e^{-r/\lambda}$$

Distance Travelled On A Random Walk

$$r^2 = \vec{r}\cdot\vec{r} = s_1^2 + s_2^2 + \dots + \vec{s}_1\cdot\vec{s}_2 + \vec{s}_2\cdot\vec{s}_1 + \dots$$

$$\boxed{r = \sqrt{N}\,\lambda}$$

Diffusion

Fick's Law

$$\frac{dN}{dt} = -DA\frac{dn}{dx}$$

- $\frac{dN}{dt}$: number flowing across dotted line /sec
- D: this defines Diffusion Coefficient $m^2 s^{-1}$
- A: Area flowing over
- $\frac{dn}{dx}$: spatial density gradient

Net flow L ⇒ R $= \frac{1}{6}\left[n(x-\lambda) - n(x+\lambda)\right] A\,\bar{v}$

where $n(x-\lambda) \approx n(x) - \frac{dn}{dx}\lambda$ and $n(x+\lambda) \approx n(x) + \frac{dn}{dx}\lambda$

$$= \frac{1}{3}\lambda\bar{v}A\frac{dn}{dx}$$

$$\Rightarrow \boxed{D = \tfrac{1}{3}\lambda\bar{v}}$$

Fourier's Law:

$$\frac{dQ}{dt} = -KA\frac{dT}{dx}$$

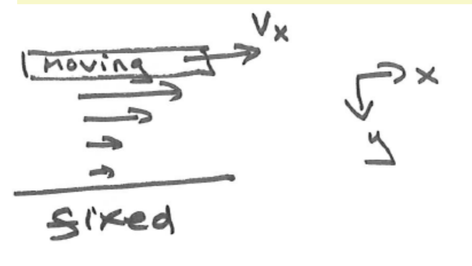
- $\frac{dQ}{dt}$: energy flow/s
- K: thermal conductivity
- A: Area
- $\frac{dT}{dx}$: temperature gradient

Diffusion	$D = \frac{1}{3}\lambda\bar{v}$
Thermal cond.	$K = \frac{1}{3}\lambda\bar{v}C$
Viscosity	$\eta = \frac{1}{3}\lambda\bar{v}\rho$

Viscosity

$$F_x = -\eta A\frac{dv_x}{dy}$$

- A: now area of moving surface
- $-$: in opposite direction to v_x
- η: viscosity coefficient, $Nm^{-2}s^{-1}$



Temperature Dependence

D: $\frac{1}{2}mv^2 = \frac{3}{2}kT \rightarrow v \sim T^{1/2}$

$$\lambda = \frac{1}{\sqrt{2}\pi d^2 n} \sim \frac{1}{n} \overset{ideal}{\sim} \frac{kT_I}{P} \sim T$$

$$D \sim T^{3/2}$$

K: C is energy/volume & grows as $n \sim \frac{P}{kT_I} \sim 1/T$

$$K \sim T^{1/2}$$

η: $\rho \sim n$ too $\Rightarrow \eta \sim T^{1/2}$

They all grow with temperature

Pressure Dependence

We had $\rho, C \sim n$ $\quad \lambda \sim 1/n$ $\quad n = \frac{P}{kT_I}$

$\Rightarrow \eta, K$ are predicted to be independent of P!

λ can't be bigger than the container

A4. (a) Explain the two main effects that are assume to be negligible in the definition of an ideal gas, and describe the regimes in which these effects become non-negligible.

(b) Even for warm and dilute gases, some approximations are valid only in specific cases. What are the necessary additional assumptions for the following statements to be good approximations?

(i) The internal energy of a gas with N molecules at temperature T is $U = \frac{3}{2} N k_B T$.

(ii) The specific heat capacity of a gas of O_2 molecules is $c_V = \frac{7}{2} R$. [5]

A1. Solids which contain only one kind of atom typically follow the Dulong and Petit rule which predicts that an ideal monatomic crystal should have a molar heat capacity of $3R$. State the equipartition theorem and explain how it can be applied to an ideal monoatomic crystal. What causes the molar heat capacity to drop below $3R$ at lower temperature? [5]

A4. Three identical flasks contain three different gases at the same pressure and temperature. Flask A contains CO_2, flask B contains C_2H_4 and flask C contains He. Answer the following questions with a brief explanation. [4]

(a) Which flask contains the highest number of molecules?

(b) Which flask contains the gas with highest density?

(c) Which flask contains the fastest molecules?

(d) Which flask has the gas with lowest heat capacity at high temperature?

(Note: the correct answers may include all or none of the flasks)

B2. Fourier's law for heat conduction says that the energy flow rate across area A driven by a temperature gradient $\mathrm{d}T/\mathrm{d}x$ for a material of thermal conductivity κ is given by

$$\frac{\mathrm{d}Q}{\mathrm{d}t} = -\kappa A \frac{\mathrm{d}T}{\mathrm{d}x}.$$

A long rod has a uniform cross-sectional area of $4.0\,\mathrm{cm}^2$. It is made of two sections joined end-to-end: a 75.0 cm long section of copper, and a 25.0 cm section of steel. The rod is insulated to prevent heat loss along its sides. The copper end is placed in perfect thermal contact with boiling water at 100 °C and the steel end is placed in perfect thermal contact with an ice-water mixture at 0 °C. The thermal conductivities of copper and steel are $385.0\,\mathrm{W\,m^{-1}\,K^{-1}}$ and $50.2\,\mathrm{W\,m^{-1}\,K^{-1}}$ respectively.

(a) Draw a sketch graph to show how the temperature varies along the rod in the steady state. [4]

(b) What is the temperature of the copper-steel junction in the steady state? [6]

(c) What is the rate of heat transfer from the steam bath to the ice-water mixture in the steady state? [4]

(d) What is the rate of entropy production? [4]

(e) What would be the junction temperature, heat flow and entropy creation rate if the cross-sectional area of the rod were doubled? [2]