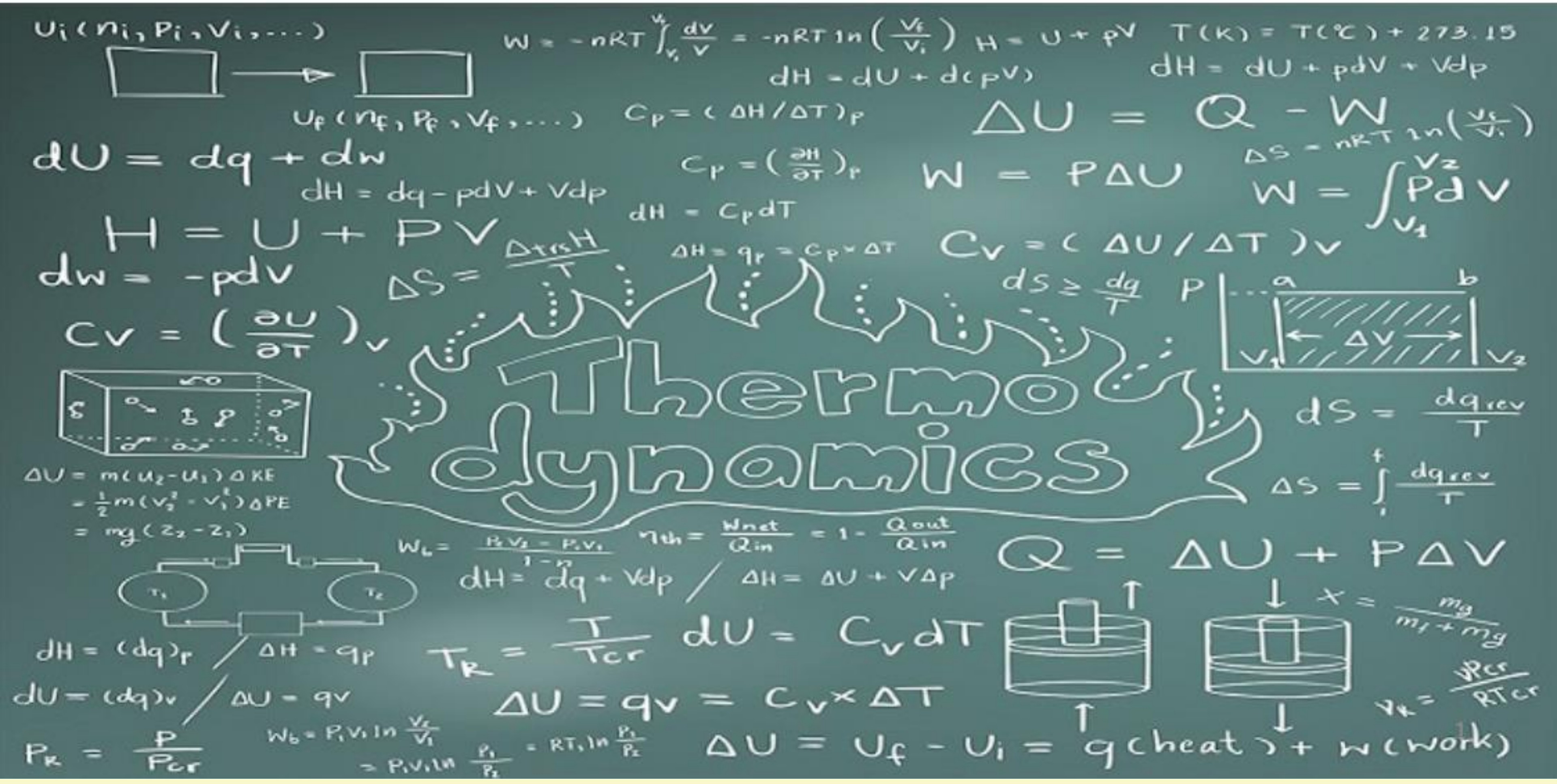
PHYS1013
Energy and Matter



THERMODYNAMICS

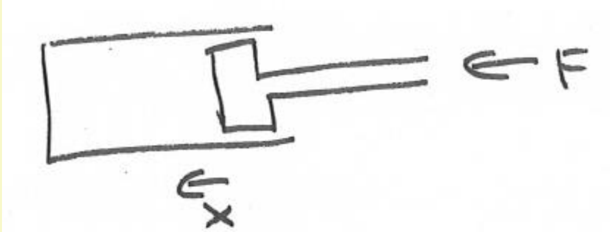
FIRST LAW:

$$\boxed{\Delta U = Q + W}$$

Reversible Change ?: best is tiny changes that can be reversed by tiny reverse changes in surroundings

? inginitessimal

Compressing a gas



$$\text{đ}W_R = PA\,\delta x = -P\,\delta V$$

reversible

volume decreases so $-(-\delta V)$

$$\boxed{W_R = -\int P\,dV}$$

C_p

$$đQ_R = dU - đW_R$$
$$= dU + PdV$$

$$PV = RT_I$$

$$C_p = \frac{đQ}{dT_I} = \frac{C_v\, dT_I + R\, dT_I}{dT_I}$$

$$\gamma = C_p / C_v$$

$$C_p = C_v + R$$

- **ISOTHERMAL** $\quad T$ constant $\underset{\text{ideal gas}}{\Longrightarrow}$ U constant

$$\Rightarrow \quad Q + W = 0$$

$$W_R = -\int_{V_1}^{V_2} P\, dV \underset{\text{ideal gas}}{=} -\int_{V_1}^{V_2} \frac{n_m R T_I}{V}\, dV$$
$$= -n_m R T_I \int_{V_1}^{V_2} \frac{dV}{V}$$
$$= n_m R T_I \ln(V_1/V_2)$$

ADIABATIC $\quad Q = 0 \quad$ no heat in or out.

$$\Delta U = W$$

$$dU = -P\,dV$$

one mole: $\quad C_V\,dT_I = -P\,dV$

$$\uparrow RT_I/V$$

$$\ln T_I V^{R/C_V} = \text{const}$$

$$PV^{C_P/C_V} = \text{constant}$$

$$\boxed{PV^{\gamma} = \text{constant}}$$

$$P_1V_1^{\gamma} = P_2V_2^{\gamma}$$
$$= PV^{\gamma} \text{ in process}$$

Work in Adiabatic Compression

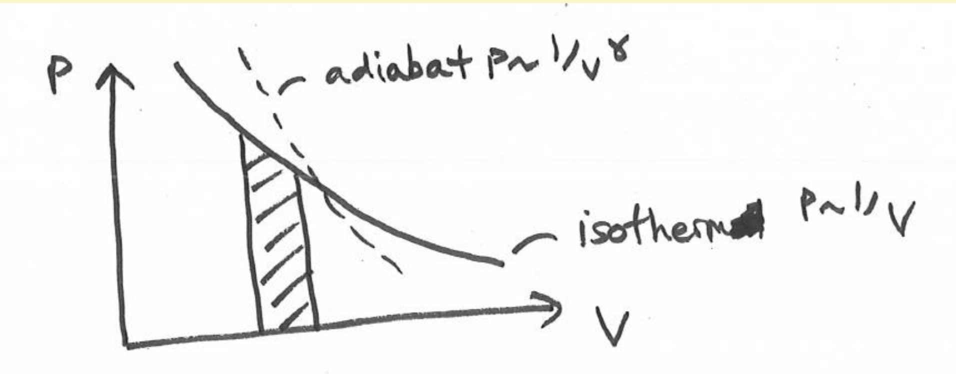
$$W_R = -\int_{V_1}^{V_2} P\,dV$$

$$= -P_1V_1^{\gamma}\int_{V_1}^{V_2}\frac{dV}{V^{\gamma}}$$

$$= -P_1V_1^{\gamma}\left[\frac{V^{1-\gamma}}{1-\gamma}\right]_{V_1}^{V_2}$$

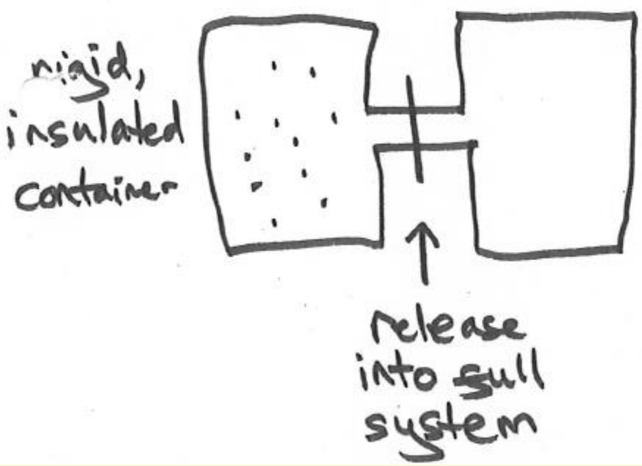
$$= \frac{P_1V_1}{\gamma-1}\left[\left(\frac{V_1}{V_2}\right)^{\gamma-1} - 1\right]$$

PV Indicator Diagrams



$dW = -PdV$ the area

- **JOULE EXPANSION**



No work is done on surroundings $W=0$

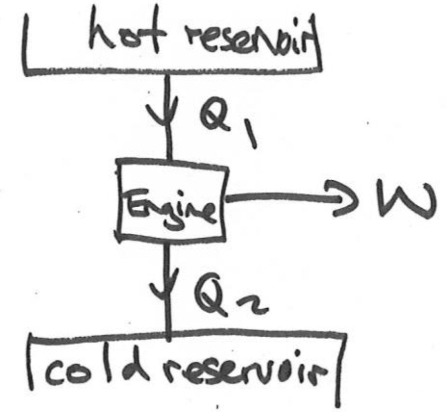
Adiabatic so $Q=0$

$\Rightarrow$ the internal energy U is unchanged

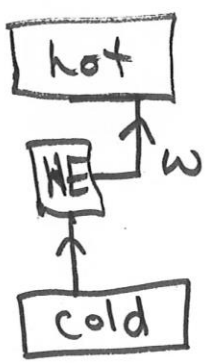
THE SECOND LAW OF THERMODYNAMICS

HEAT ENGINES

$$\eta_{\text{heat engine}} = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - Q_2/Q_1$$



Clausius: "It is impossible to construct a device that, operating in a cycle, produces no other effect than the transfer of heat from a colder to a hotter body"



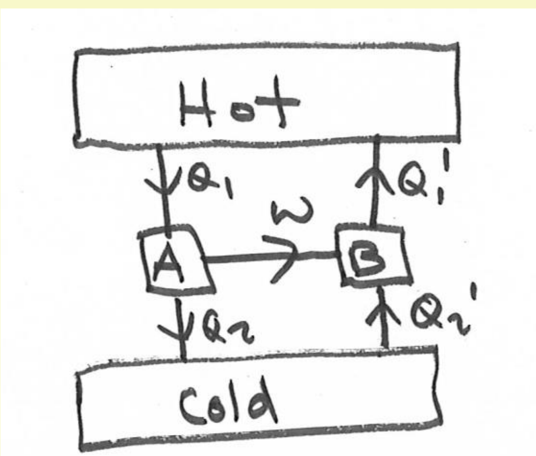
Kelvin Planck: "It is impossible to construct a device, operating in a cycle, that produces no effect other than the extraction of heat from a single body & the performance of an equal amount of work"

HEAT PUMPS

$$\eta_{HP} = \frac{Q_1}{W}$$

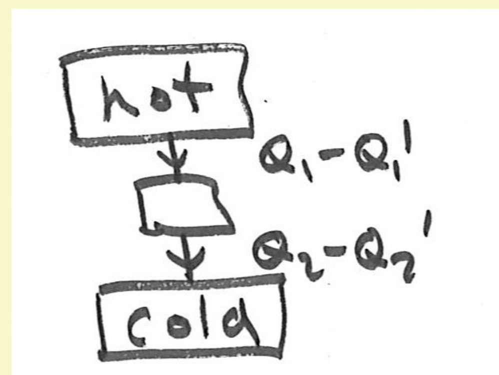
$$\eta_{HP} = \frac{Q_2 + W}{W} > 1$$

Reversible Heat Engines Are Most Efficient



$$\eta_A = \frac{W}{Q_1}$$

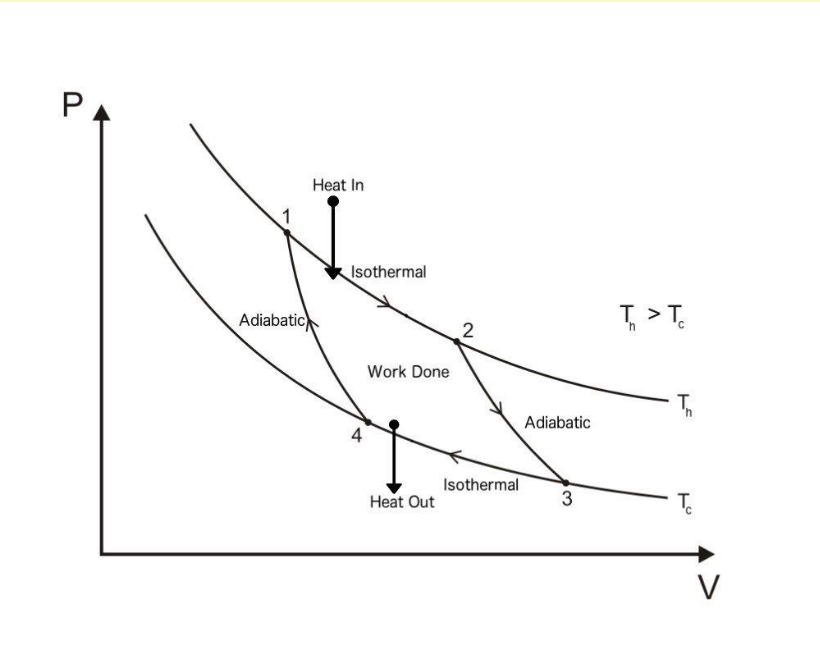
$$\eta_B = \frac{W}{Q_1'}$$



$$Q_1 \geq Q_1'$$

$$\eta_A \leq \eta_B$$

Ideal gas realization of a Carnot engine



$$\eta_{max} = 1 - T_2/T_1$$

$$Q = kT$$

In step ① - isothermal expansion $T_I^{(1)} = \text{const}$

$$Q_1 = n_m R T_I^{1} \ln \frac{V_2}{V_1}$$

Step ③ - isothermal compression at $T_I^{(2)} = \text{const}$

$$Q_2 = n_m R T_I^{(2)} \ln V_3/V_4$$

$$\frac{Q_1}{Q_2} \equiv \frac{T_1}{T_2} = \frac{T_I^{(1)}}{T_I^{(2)}} \frac{\ln V_2/V_1}{\ln V_3/V_4}$$

adiabatic

② $T_I^{(1)} V_2^{\gamma-1} = T_I^{(2)} V_3^{\gamma-1}$

④ $T_I^{(1)} V_1^{\gamma-1} = T_I^{(2)} V_4^{\gamma-1}$

$$\Rightarrow \frac{V_2}{V_1} = \frac{V_3}{V_4}$$

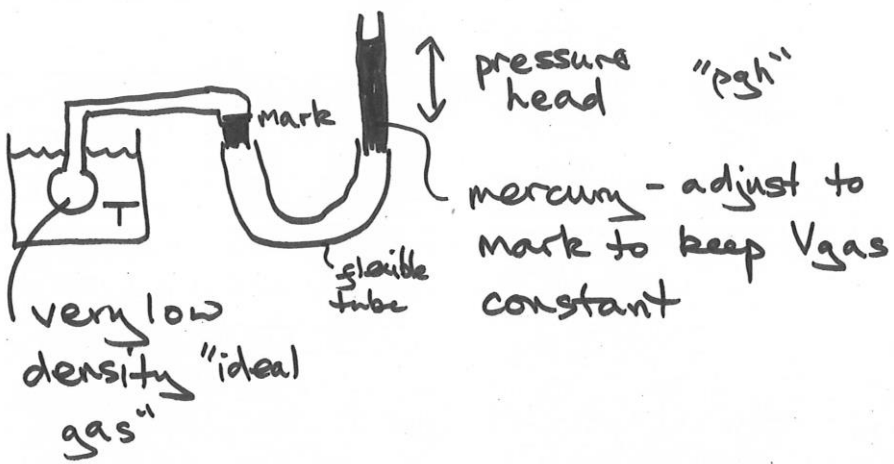
$$\frac{Q_1}{Q_2} \equiv \frac{T_1}{T_2}$$

ZEROTH LAW

"If system A is in thermal equilibrium with B & C then B & C are in Eℓm with each other"

mercury column

$$\theta = 100\left(\frac{X - X_0}{X_{100} - X_0}\right)$$



$$T_I = 273.16\left(\frac{P}{P_{TP}}\right)_{\rho \to 0}$$

A3. A reversible heat engine is used as a refrigerator to run a domestic freezer. It consumes 50 W of electrical power and maintains the cold reservoir at $-18\,^\circ\mathrm{C}$ in a room at $20\,^\circ\mathrm{C}$. Draw a heat engine diagram depicting the flow of heat and work, find the coefficient of performance of this refrigerator, and calculate the rate at which it extracts heat from the cold reservoir. [5]

A4. A reversible heat engine performs 10^6 J of work and dumps 10^6 J of waste heat to a cold reservoir at $15\,^\circ\mathrm{C}$. Calculate the temperature of the reservoir from which the engine draws heat. [4]

B3. An ideal gas initially at volume $V_1 = 20\,\mathrm{L}$, temperature $T_1 = 20\,^\circ\mathrm{C}$, and pressure $P_1 = 10^5$ Pa is isothermally compressed to $P_2 = 20 \times 10^5$ Pa.

The adiabatic index of the gas is $7/5$. Note: you may find useful to know that $\ln(20) = 3$ and $20^{5/7} = 8.5$ are precise enough approximations for this question.

(a) What is the volume V_2 and temperature T_2 after this compression? [1]

(b) Compute W_{12}, the work done on the gas during the compression. [2]

(c) Compute Q_{12}, the amount of heat into the gas during the compression. [1]

The same gas is then brought back adiabatically to a pressure $P_3 = 10^5$ Pa.

(d) What is the volume V_3 and temperature T_3 after this expansion? [2]

(e) Compute Q_{23}, the amount of heat into the gas during the expansion. [1]

(f) Compute W_{23}, the work done on the gas during the expansion. *Hint: Show that $c_V = R/(\gamma - 1)$ using Mayer's relation and compute the change in internal energy.* [3]

The gas is brought back to the initial conditions through an isobaric expansion.

(g) Compute W_{31}, the work done on the gas during this expansion. [1]

(h) Compute Q_{31}, the amount of heat into the gas during this expansion. *Hint: as in (f), you can show that $c_P = \gamma R/(\gamma - 1)$.* [3]

Now consider the complete cycle $1 \to 2 \to 3 \to 1$.

(i) Sketch this cycle on the $P - V$ plane. [2]

(j) What is the net work done on the gas during the complete cycle? [1]

(k) During which part of the cycle is heat flowing into the gas? [1]

(l) Is this cycle a heat engine or a refrigerator? Compute its efficiency (or coefficient of performance). [2]