ZEROTH LAW

Historically at this point people realized they needed to be more careful about temperature.

"If system A is in thermal equilibrium with B&C then B&C are in ≡ım with each other"

$\}$ ie no heat will flow between them

This law is needed if A is a thermometer & you want to believe B&C are at the same temperature.

EMPIRICAL TEMPERATURE

A classic measure of T is the length of a mercury column in a thermometer. Here one assumes

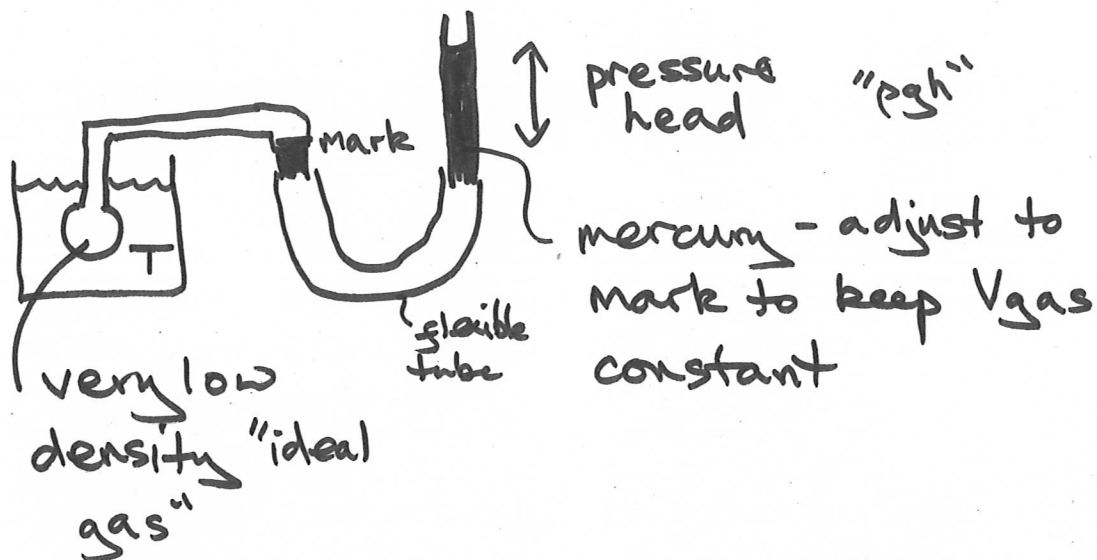$$\theta(x) = aX + b$$

One fixes a & b with measurements

eg Centigrade $\theta = 0°$ at water melting point

$\theta = 100°$ at water boiling point

$$\Rightarrow \theta = 100 \left(\frac{X - X_0}{X_{100} - X_0} \right)$$

$x = X_0 \to 0$
$x = X_{100} \to 100$

The precise melting & boiling points are hard to measure though... and there's a long extrapolation to $-270°C$.

Kelvin's Method

pressure head   "$\rho g h$"

mercury – adjust to mark to keep V_{gas} constant

very low density "ideal gas"

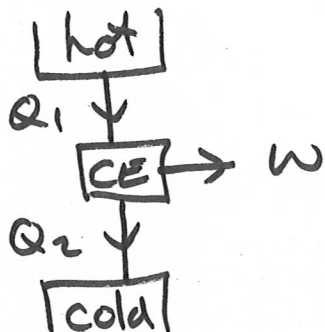$$PV = n_m R T_I \qquad \text{At const } V \quad T \propto P$$

$$\Rightarrow \quad T_I = 273.16 \left(\frac{P}{P_{TP}} \right)_{\rho \to 0}$$

At $T = 0$ $P = 0$ ✓

At triple point of water set $T = 273.16$
 → this ensures a $100°$ difference between boiling & melting points.
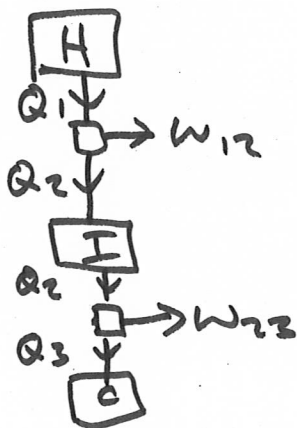
Defining Temperature with Carnot Engines

The most rigorous way to define temperature is using Carnot engines (the most efficient & reversible engine) - the idealized engine is independent of design.

```
┌─────┐
│ hot │
└─────┘
  Q₁ ↓
   ┌────┐
   │ CE │→ W
   └────┘
  Q₂ ↓
┌──────┐
│ cold │
└──────┘
```

The idea is to use Q extracted or dumped as a direct measure of temperature

$$Q = kT$$

A consistency check is

```
  ┌───┐
  │ H │
  └───┘
 Q₁↓
  □ → W₁₂
 Q₂↓
  ┌───┐
  │ I │
  └───┘
 Q₂↓
  □ → W₂₃
 Q₃↓
  ┌───┐
  │ C │
  └───┘
```

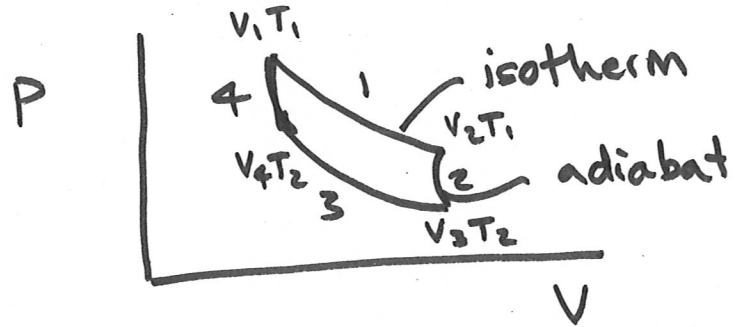$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2} \quad \& \quad \frac{Q_2}{Q_3} = \frac{T_2}{T_3}$$

which means $\dfrac{Q_1}{Q_3} = \dfrac{T_1}{T_3}$ ✓

Note $T=0$ gives no Q... and then fix k with eg triple point of water → 273.16 K again so the gap between boiling & freezing points is 100°.

$T_{CARNOT} = T_{IDEAL}$

Another good check of this T definition is that it directly works for ideal gas T_I

Carnot cycle:

- In step ① - isothermal expansion $T_I^{(1)} = const$

$$W_R = -\int_{V_1}^{V_2} P dV$$

$$= -\int_{V_1}^{V_2} n_m R T_I^{(1)} \frac{dV}{V}$$

$$= n_m R T_I^{(1)} \ln V_1/V_2$$

The Q_1 lost by the hot sink is $\quad Q_1 = n_m R T_I^1 \ln\frac{V_2}{V_1}$

- In step ③ - isothermal compression at $T_I^{(2)} = const$

$$Q_2 = n_m R T_I^{(2)} \ln V_3/V_4$$

So we would have

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2} = \frac{T_I^{(1)}}{T_I^{(2)}} \frac{\ln V_2/V_1}{\ln V_3/V_4}$$

We can link the volumes by the adiabatic phases where $PV^\gamma = \text{const}$

$$\Rightarrow \quad T_I V^{\gamma-1} = \text{const}$$

② $\quad T_I^{(1)} V_2^{\gamma-1} = T_I^{(2)} V_3^{\gamma-1}$

④ $\quad T_I^{(1)} V_1^{\gamma-1} = T_I^{(2)} V_4^{\gamma-1}$

$\div \quad \left(\dfrac{V_2}{V_1}\right)^{\gamma-1} = \left(\dfrac{V_3}{V_4}\right)^{\gamma-1}$

$$\Rightarrow \quad \frac{V_2}{V_1} = \frac{V_3}{V_4} \qquad \text{\& the logs above cancel.}$$

ie $\quad \dfrac{Q_1}{Q_2} = \dfrac{T_1}{T_2} = \dfrac{T_I^{(1)}}{T_I^{(2)}}$

The ideal gas temperature & our empirical Carnot engine temperature are the same... we can stop writing T_I!