PHYS1022 - ELECTRICITY & MAGNETISM

NICK EVANS

LECTURE NOTES - ELECTRICITY

I ELECTRIC CHARGE

Matter is made of electrons, protons & neutrons. Electrons & protons carry opposite electric charge

$$Q_p = -Q_e = 1.602176565\,(35) \times 10^{-19}\ \mathrm{C}$$

UNIT: COULOMB

Charge is conserved - it can be transferred but not created or destroyed.

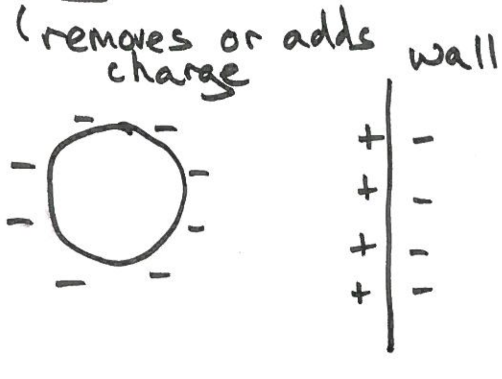
The two types of charge + & - attract

Like charges repel.

Triboelectric Effect

Touching materials can exchange charge (typically electrons move from one to other)

eg rubbing a balloon & sticking to a wall



Molecules in wall polarize

Net attraction since + closer than -.

Insulators - e^- are bound in atoms & can not move
eg wood, glass

Conductors - e^- are free to move in material
eg metals

- typically one free e^- per atom
assume 0.3 nm atomic spacing

$$\Rightarrow \text{charge density in } e^- \text{ gas} = \frac{1.6 \times 10^{-19}\,C}{(0.3 \times 10^{-9}\,m)^3}$$

$$\sim 6 \times 10^{9}\ C\,m^{-3}$$

Since Q_e is so small & electrons so numerous we often treat charge as continuous and discuss charge density

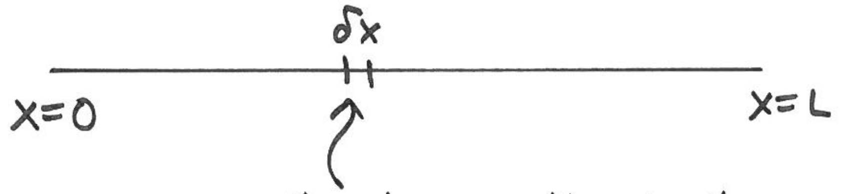
"number of charge $= \rho \times$ volume"

One needs to be careful though when ρ changes through a material - you must break the problem up into regions where ρ is constant, compute ρV, then add up for all regions.

This is most easily done using integration.

II ADDING UP WITH INTEGRATION

Consider a charged string with linear charge density $\lambda(x)$ Cm^{-1}



split into inginitessimally small lengths over which λ is constant

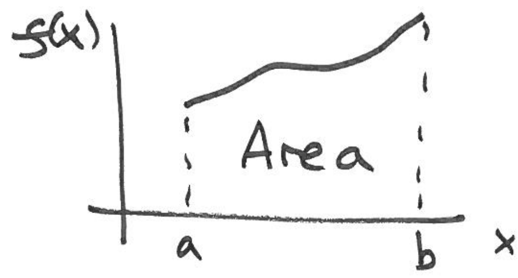
$$\text{Total charge} = \lambda(0)\,\delta x + \lambda(\delta x)\,\delta x + \lambda(2\delta x)\,\delta x + \dots \lambda(x-\delta x)\,\delta x$$

$$= \sum_{\text{bits}} \lambda(x)\,\delta x$$

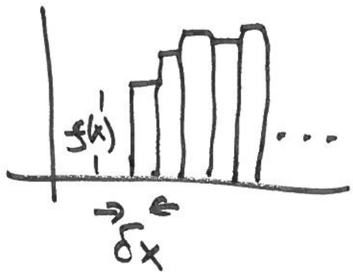
INTEGRATION

Newton taught us

$$\int_a^b f(x)\,dx = \text{Area}$$



We can also compute the area by adding areas of inginitesimal rectangles



$$\text{Area} = \sum_i f(x)\,\delta x$$

So we learn that

$$\sum_{a \text{ to } b} f(x)\,\delta x = \int_a^b f(x)\,dx$$

Now go back and look at our expression for the total charge on the wire

$$Q_{TOT} = \sum_{\text{bits}} \lambda(x)\,\delta x$$

$$= \int_0^L \lambda(x)\,dx$$

eg <u>constant density</u>

$$Q_{TOT} = \int_0^L \lambda\,dx = \lambda\,[x]_0^L = \lambda(L-0)$$

$$= \lambda L$$

ie just density × length in this case

eg $\underline{\lambda(x) = kx}$

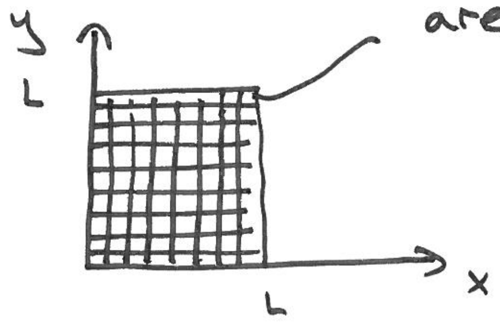
- the density of charge grows along the wire
- k has dimensions of Cm^{-2} and is a constant

$$Q_{TOT} = \int_0^L kx\,dx = k\left[\frac{x^2}{2}\right]_0^L$$

$$= kL^2/2$$

AREA INTEGRALS

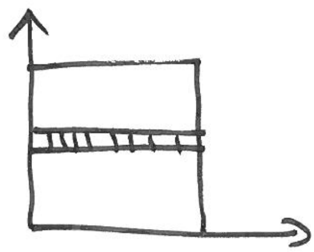
The simplest example of a 2d integral is to compute the area of a square (!). We split it up into lots of tiny squares



area of tiny square $= \delta x \, \delta y$

$$\text{TOTAL AREA} = \sum_{\substack{\text{all} \\ \text{squares}}} \delta x \, \delta y$$

First consider a row of squares at some fixed y



Each has same δy & y

$$\text{Area of strip} = \delta y \sum \delta x$$

$$= \delta y \int_0^L dx$$

$$= \delta y \, L$$

Now we can add all the strips across all y

$$\text{TOTAL AREA} = L \int_0^L dy$$

$$= L^2$$

eg <u>Charge on a plane</u>

Assume the square has charge density

$$\sigma = k\,x\,y \qquad k \text{ - constant } \mathrm{C\,m^{-4}}$$

Since σ varies we must compute the charge inginitessimal square at a time

$$\delta Q = \underbrace{\sigma(x,y)}_{\text{density}} \times \underbrace{\delta x\, \delta y}_{\text{area}}$$

& then sum

$$Q_{TOT} = \int_0^L \int_0^L \overset{kxy}{\sigma(x,y)}\, dx\, dy$$

we can do the sums in either order

$$Q_{TOT} = \underbrace{\int_0^L dy\, k\, y}_{\text{all constant at fixed } y} \underbrace{\int_0^L x\, dx}_{\text{sum strip in } x}$$

$$= \int_0^L dy\, k\, y\, L^2/2$$

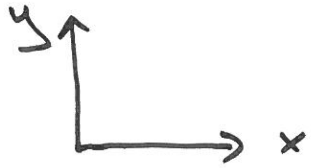
$$= \frac{kL^2}{2} \cdot L^2/2$$

$$= kL^4/4$$

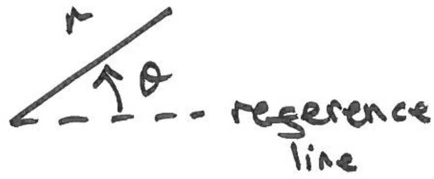
2d Polar Coordinates

In problems with rotational symmetry it's best to

$$x, y \longrightarrow r, \theta$$



$\longrightarrow$



$0 < r < \infty$

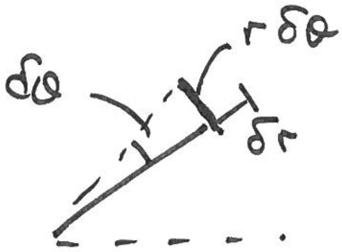
$0 < \theta < 2\pi$

area element $\delta y \, \square^{\delta x}$?

We make a tiny square by shifting

$r \Rightarrow r + \delta r$ draws a side of length δr

$\theta \Rightarrow \theta + \delta\theta$ draws an arc of length $r\delta\theta$

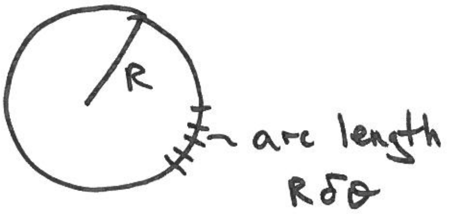


We have now drawn a square of area $\underline{r \delta r \delta\theta}$

Note the size of the square depends on r!

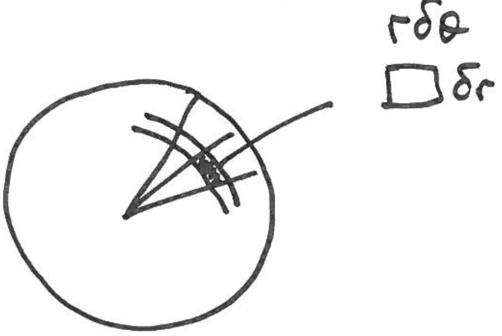
Examples of Adding with Rotational Sym

eg line integral



$$\text{Circumference} = \sum R\,\delta\theta = \int_0^{2\pi} R\,d\theta = 2\pi R$$

eg area integral



$$\text{Area} = \sum r\,\delta r\,\delta\theta = \int_0^R \int_0^{2\pi} r\,d\theta\,dr = \int_0^R 2\pi r\,dr = \pi R^2$$

eg charge on a disc $\quad \sigma = k/r^3 \ \ \mathrm{Cm}^{-2}$

$$Q_{TOT} = \sum \frac{k}{r^3}\,r\,\delta r\,\delta\theta$$

$$= k\int_0^R \int_0^{2\pi} d\theta\,\frac{dr}{r^2}$$

$$= k\,2\pi \int_0^R \frac{dr}{r^2}$$

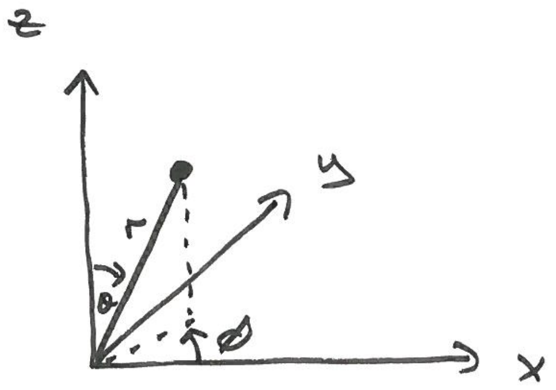
$$= 2\pi k\left[-1/r\right]_0^R \to \infty$$

There was an ∞ amount of charge at $r=0$!

VOLUME INTEGRALS

In problems with "cubic" symmetry one can use x, y, z coordinates & an inginitessimal volume element is just $\underline{\delta x\, \delta y\, \delta z}$.

For spherically symmetric problems use "spherical polar coordinates"

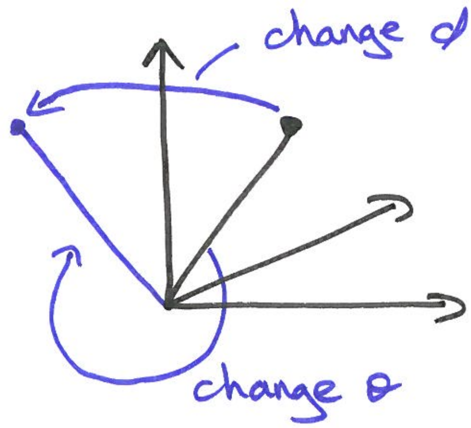


$0 < r < \infty$

$0 < \theta < \pi$

$0 < \phi < 2\pi$

Note θ doesn't extend to 2π to ensure we don't double count this point

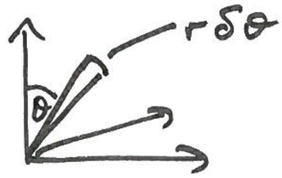


To make 3 sides of an inginitessimal cube we:

$r \to r + \delta r$ makes a side of length δr

$\theta \to \theta + \delta\theta$

makes an arc of length $r\,\delta\theta$



$\phi \to \phi + \delta\phi$

makes an arc of length $r \sin\theta\, \delta\phi$

radius is $r\sin\theta$

arc is of length $r\sin\theta\,\delta\phi$

$$\text{volume element} = r^2 \sin\theta\, \delta r\, \delta\theta\, \delta\phi$$

eg <u>volume of a sphere</u>

$$V = \int_0^a \int_0^\pi \int_0^{2\pi} r^2 \sin\theta \, d\phi\, d\theta\, dr$$

$$= \int_0^a r^2\, dr \int_0^\pi \sin\theta\, d\theta \int_0^{2\pi} d\phi$$

$$= \left[\frac{r^3}{3}\right]_0^a \left[-\cos\theta\right]_0^\pi \left[\phi\right]_0^{2\pi}$$

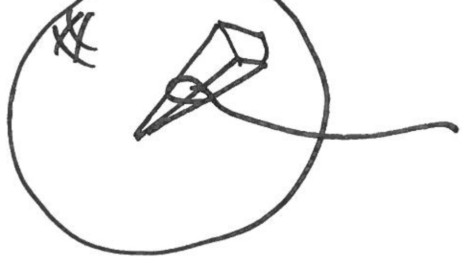
$$= \frac{a^3}{3} \cdot 2 \cdot 2\pi$$

$$= \frac{4}{3}\pi a^3$$

SOLID ANGLE

How do you cover the surface of a sphere in little squares? Change θ, ϕ as above but at fixed radius, a

$$\delta A = a^2 \sin\theta \, \delta\theta \, \delta\phi$$



the range in $\delta\theta$ & $\delta\phi$ is the "solid angle"

eg moon & sun subtend the same solid angle at the earth despite lying at different distances $\rightarrow$ solar eclipses.

eg <u>Area of sphere</u>

$$A = a^2 \int_0^{2\pi} \int_0^{\pi} \sin\theta \, d\theta \, d\phi$$

$$= a^2 . 2\pi . 2$$

$$= 4\pi a^2$$