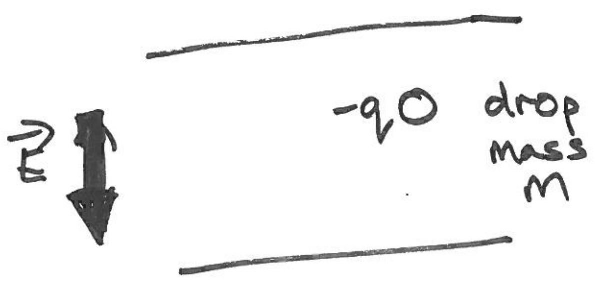
V $\vec{F} = q\vec{E}$

The definition of $\vec{E}$ ($\vec{F}/q$) tells us that a charge q experiences a force in an $\vec{E}$ field

$$\vec{F} = q\vec{E}$$

Millikan's Oil Drop Expt

Millikan measured the charge on the electron. He sprayed oil drops between two plates that generate a constant $\vec{E}$ field - spraying them makes them charged.



By tuning $\vec{E}$ can achieve force balance with gravity

static drop : $mg = qE$

He determined m_{drop} by $m = \rho V$

$$m_{drop} = \frac{4}{3}\pi r^3 (\rho_{oil} - \rho_{air})$$

uplift

To determine r_{drop} he switched off $\vec{E}$ & measured the drops terminal velocity - here gravity is balanced by the drag force

$$\frac{4}{3}\pi r^3 (\rho_{oil} - \rho_{air}) = 6\pi r \eta v_t$$

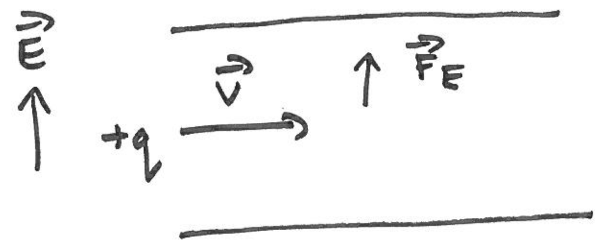
terminal velocity

viscosity

By looking at many drops he determined charge came in "electron sized" lumps.

Ink Jet Printers

Ink jet printers position ink on the page by charging the drops then using $\vec{E}$ field to position then



$$F = ma$$

$$a = \frac{qE}{m}$$

No force acts in x-direction : $\frac{dx}{dt} = v$

integrate w.r.t. t $\rightarrow$ $x = vt + x_0$

Constant acceleration in y-direction:

$$\frac{d^2y}{dt^2} = a$$

$$\frac{dy}{dt} = at + \overset{0}{\cancel{v_y^{\text{initial}}}}$$

$$y = \tfrac{1}{2}at^2 + y_0$$

VI ELECTRIC FLUX & GAUSS' LAW

Flux

Consider the bullets from an array of machine guns:

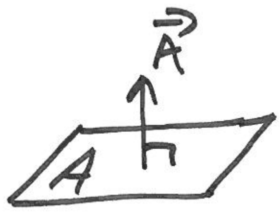
current density = number of bullets $/m^2/sec$

It is a vector (here in $\hat{x}$) called $\vec{J}$

If we put the area at an angle not so many bullets go through A

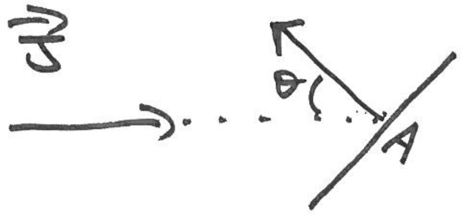
We chose to define <u>area as a vector</u> too

$\vec{A}$ = A m^2 in direction perpendicular to the area



Perp. direction is unique direction associated with the area

Now...



The area presented to $\vec{J}$ is $|\vec{A}| \cos\theta$

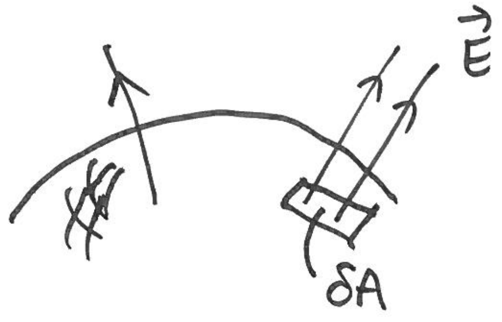
$$\text{Number of pts/sec} = \vec{J} \cdot \vec{A}$$
$$= |\vec{J}||\vec{A}| \cos\theta$$
$$\equiv \text{FLUX}$$

Electric Field Flux

We can define $\vec{E} \cdot \vec{A}$ as a flux.

Physically it is the number of field lines going through $\vec{A}$ ... (note not per second so the word "flux" is a bit loose).

What do we do if $\vec{A}$ is not a flat surface? Or if $\vec{E}$ varies across $\vec{A}$? The answer is that we break the area into many flat $\vec{dA}$ pieces over which $\vec{E}$ is constant

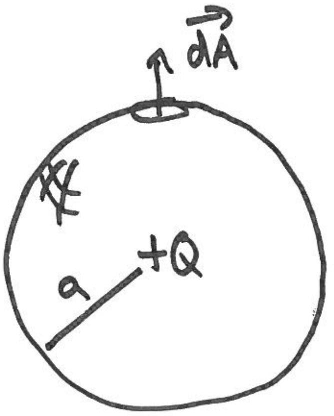


We compute $\vec{E} \cdot \vec{\delta A}$ for each bit... and add up the answers for the whole surface

$$\text{ie} \quad \text{Electric Flux} = \int_{\text{surface}} \vec{E} \cdot d\vec{A}$$

eg point charge

Let's compute the flux through a spherical surface with a charge +Q at the centre



$|\vec{E}|$ is the same over the whole surface : $Q/4\pi\epsilon_0 a^2$

$\vec{E}$ points radially so is perpendicular to the surface everywhere

We must split the surface into flat $\vec{\delta A}$ pieces. They are all of the form $\delta A\,\hat{r}$ (perp. to surface)

$$\text{Flux} = \int_{\text{surface}} \vec{E}\cdot d\vec{A} = \sum_{\text{bits}} \vec{E}\cdot\vec{\delta A}$$

Now $\vec{E}\cdot\vec{\delta A} = |\vec{E}|\,|\vec{\delta A}|\,\underbrace{\hat{r}\cdot\hat{r}}_{1}$

$$\text{Flux} = \sum_{\text{bits}} \frac{Q}{4\pi\epsilon_0 a^2}\,\delta A$$

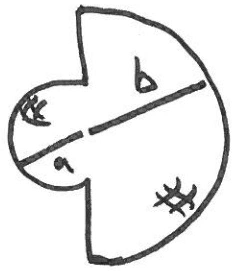
$$= \frac{Q}{4\pi\epsilon_0 a^2}\sum_{\text{bits}}\delta A$$ stuff out front just constants over all δA

$$= \frac{Q}{4\pi\epsilon_0 a^2}\cdot 4\pi a^2$$ $\Sigma\delta A$ = surface area of sphere

$$= Q/\epsilon_0$$

Maths Games & Gauss' Law

Now compute the flux for a surface made of two different sized hemi-sphere's stuck together



The curved sides are as before... the flat connecting sides are perpendicular to $\hat{r}$ - here $\vec{E} \cdot d\vec{A} = 0$

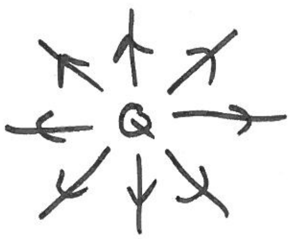
So

$$\int_{surface} \vec{E} \cdot d\vec{A} = \frac{Q}{4\pi\epsilon_0 a^2} \cdot 2\pi a^2 + \frac{Q}{4\pi\epsilon_0 b^2} 2\pi b^2$$

$\frac{1}{2}$ surface area of sphere

$$= Q/\epsilon_0$$

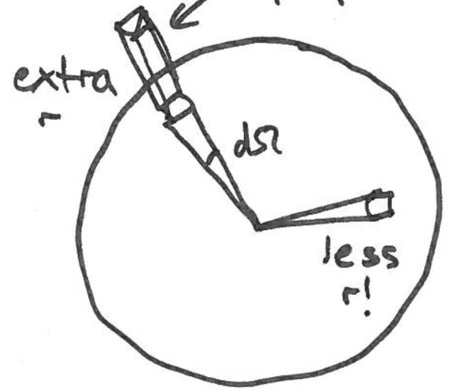
The same as before! Why? It's a measure of the number of flux lines leaving through the surface



The number of lines is the same however we encase the charge!

Next we can have more fun (!) - imagine taking every solid angle "square" on the surface of the sphere & putting it at a different r...

any exposed sides are perpendicular to $\vec{E}$

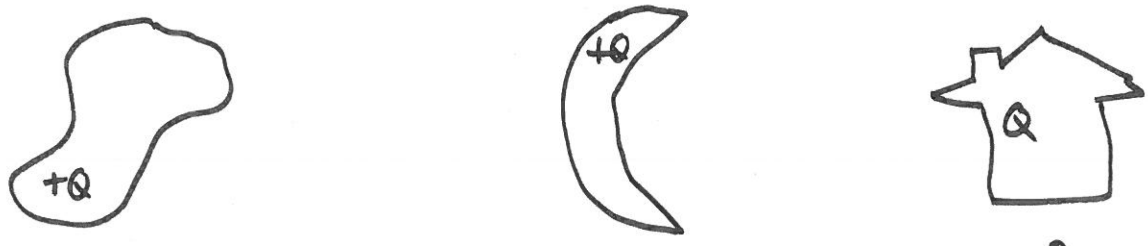


$$\int_{surf} \vec{E}\cdot d\vec{A} = \sum \frac{Q}{4\pi\epsilon_0 r^2} \underbrace{r^2 \sin\theta\, d\theta\, d\phi}_{|d\vec{A}|}$$

$$= \frac{Q}{4\pi\epsilon_0} \underbrace{\sum \sin\theta\, d\theta\, d\phi}_{4\pi}$$

$$= \frac{Q}{\epsilon_0}$$

Now we realize we can make <u>any</u> closed shape we like (use infinitessimal shifts between neighbouring dΩs)

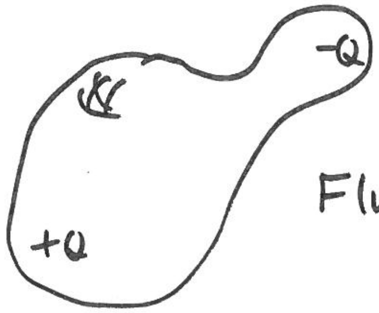


Flux = Q/ϵ_0 $\qquad$ Q/ϵ_0 $\qquad$ Q/ϵ_0

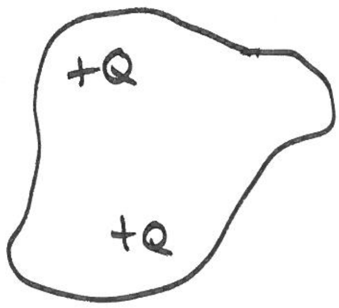
Electric fields from more than one charge just vectorially add so

$$\int_{\substack{\text{closed}\\\text{surface}}} \vec{E}\cdot d\vec{A} = \int_{cs} (\vec{E}_1 + \vec{E}_2)\cdot d\vec{A} = \int_{cs} \vec{E}_1\cdot d\vec{A} + \int_{cs} \vec{E}_2\cdot d\vec{A}$$

$$= NQ/\epsilon_0$$

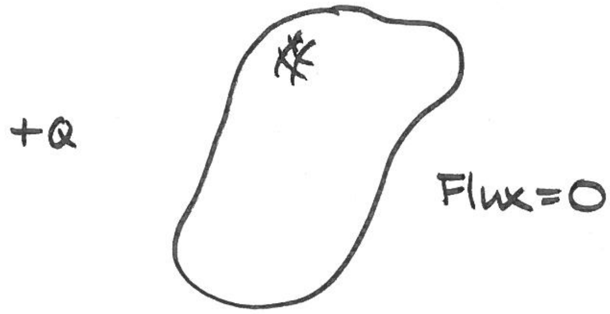


Flux = 0



Flux = $\frac{2Q}{\epsilon_0}$

Finally, charges outside the volume do not contribute to the flux through the surface.

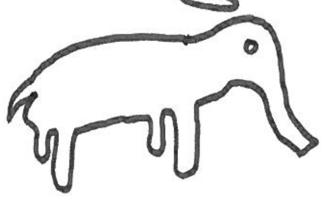


We're counting the number of field lines cutting the surface. Since they can't end in the volume (no charges) any that enter must leave.

GAUSS' LAW

$$\int_{\substack{\text{closed} \\ \text{Gaussian} \\ \text{surface}}} \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\epsilon_0}$$

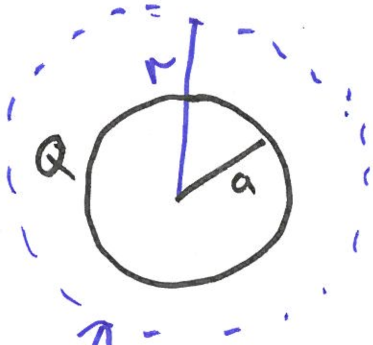
NB A Gaussian surface is <u>any</u> closed surface you care to imagine



Gauss' Law Examples

Whilst one can choose any Gaussian surface, in simple problems one can use symmetry arguments

eg Spherical Shell of Charge



Symmetry: the $\vec{E}$ field lines must be radial - if tilted they would pick out one side of the ball...

$|\vec{E}|$ can only depend on r (not θ, ϕ) since all θ, ϕ look the same

$$\text{ie} \quad \vec{E} = E(r)\,\hat{\underline{r}}$$

A sensible Gaussian surface to pick is a sphere of radius r - it respects the rotational symmetry.

Any $d\vec{A}$ piece on the sphere is in $\hat{\underline{r}}$ direction.
$|\vec{E}|$ is constant over whole sphere.

$$r > a \qquad \int_{CS} \vec{E} \cdot d\vec{A} = \sum_{dA} |\vec{E}|\,|d\vec{A}| \underbrace{\hat{\underline{r}} \cdot \hat{\underline{r}}}_{1}$$

$$= E(r) \sum_{dA} |d\vec{A}|$$

$$= 4\pi r^2 E(r)$$

$$\text{Gauss' Law} \qquad 4\pi r^2 E(r) = Q/\epsilon_0$$

$$\vec{E} = \frac{Q}{4\pi\epsilon_0 r^2}\,\hat{\underline{r}}$$

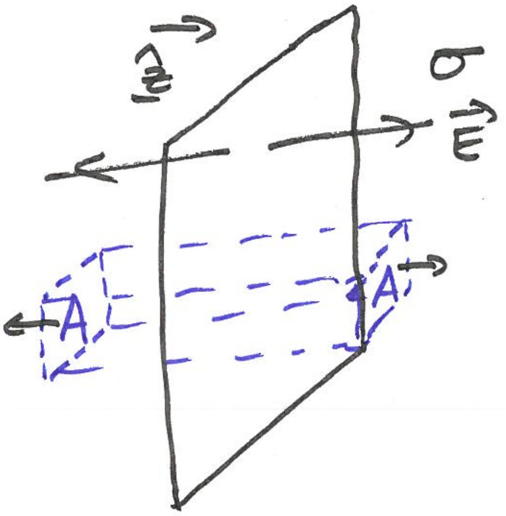
Note the result is the same as for a point charge.

$r<a$ If the Gaussian surface lies inside the charged shell then $Q_{enclosed} = 0$

$$\Rightarrow \quad E(r) = 0$$

$$\Rightarrow \quad \vec{E} = 0$$

eg <u>Infinite Plane Sheet of Charge</u>



σ Cm^{-2}

Symmetry: $\vec{E}$ must be perpendicular to the plane or we would distinguish a direction on it's surface.

$|\vec{E}|$ can only depend on z

Choose a rectangular Gaussian surface

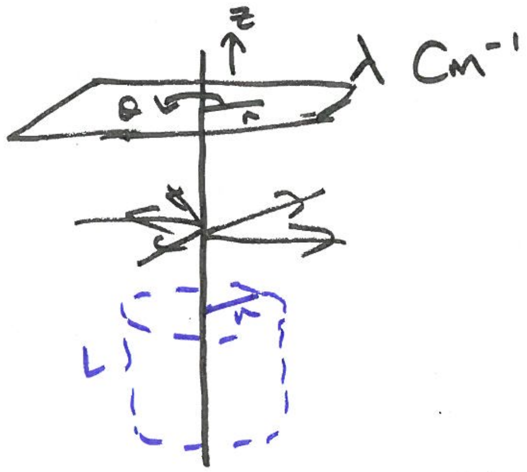
$$\int_{GS} \vec{E} \cdot d\vec{A} = \underbrace{2\,E(z)\,A}_{\text{from ends}} + 0 \quad \text{from sides at right angles to } \vec{E}$$

Gauss' Law: $$2E(z)A = \frac{\overbrace{\sigma A}^{Q_{encl}}}{\epsilon_0}$$

$$\Rightarrow \quad \vec{E} = \frac{\sigma}{2\epsilon_0}\,\hat{\underline{z}}$$

As we saw before.... note here $E(z)$ turned out to be independent of z.

eg <u>Infinite Line of Charge</u>



Symmetry: $\vec{E}$ must point radially away from wire so no z, θ distinguished

$|\vec{E}|$ can only depend on r

Choose a cylinderical Gaussian surface

$$\int_{\text{cylinder}} \vec{E} \cdot d\vec{A} = |\vec{E}| \, 2\pi r L \quad + 0 \text{ from end caps } \perp \text{ to } \vec{E}$$

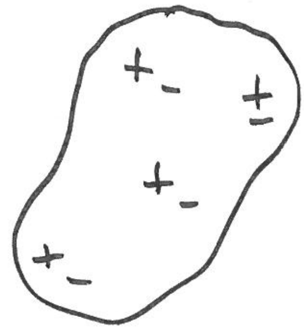
all $d\vec{A}$ pieces on curved surface point in same direction as $\vec{E} = E(r)\,\hat{\underline{r}}$

Gauss' Law: $$|\vec{E}| \, 2\pi r L = \frac{\overbrace{\lambda L}^{Q_{enc}}}{\epsilon_0}$$

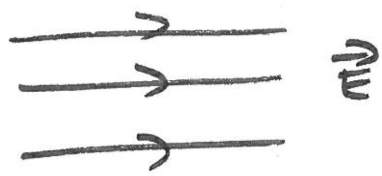
$$\Rightarrow \quad \vec{E} = \frac{\lambda}{2\pi\epsilon_0 r} \, \hat{\underline{r}}$$

Charged Conductors

In a neutral conductor there are charges that are free to move

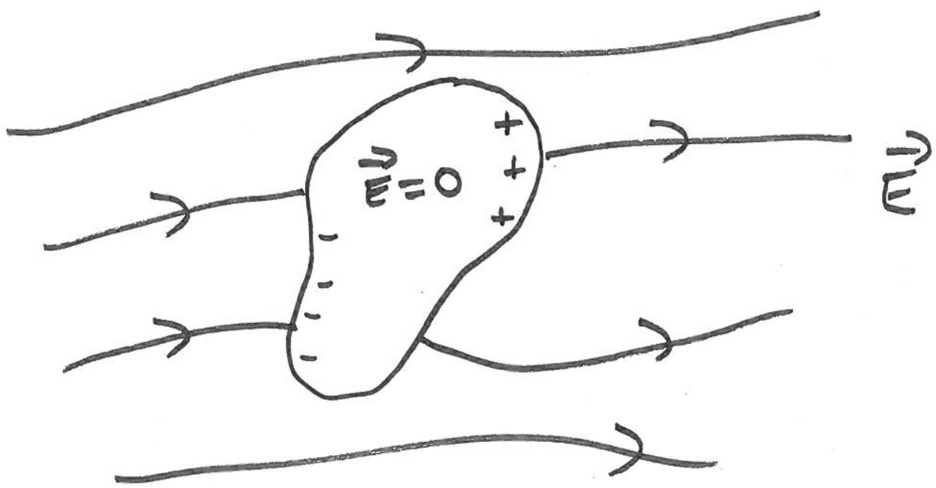


If we apply an external $\vec{E}$ field...



... it generates a force on the free charges so they will move ... eventually a steady state is reached the charges settle in a static state ... ie $\vec{F} = 0$ eventually ... $\vec{E} = 0$ inside the conductor

(If it wasn't the charges would move ....)

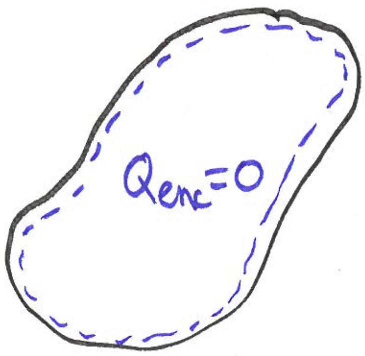


Now if we place a Gaussian surface inside the conductor

$$\int_{cs} \vec{E} \cdot d\vec{A} = 0$$

$\vec{0}$!

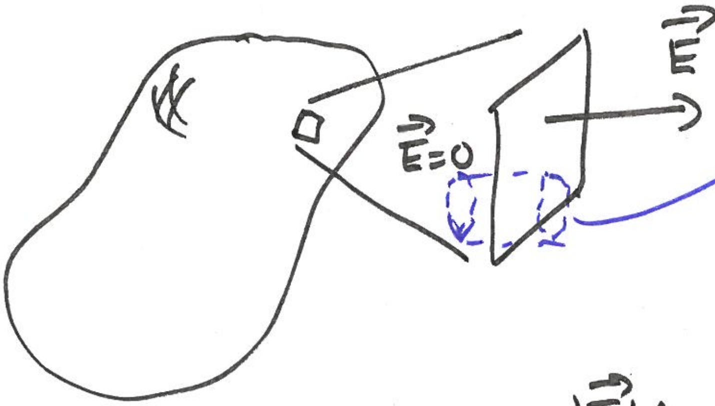
Gauss' law tells us there is no net charge inside.... we can push the Gaussian surface right to the edge...



We conclude the charges must have moved to the surface edges of the conductor.

At the surface $\vec{E}$ must be perpendicular to the surface (a component along the surface would move the free charges along the surface).

If we zoom in on a bit of the surface until it looks locally flat



Now use a cylinderical Gaussian surface

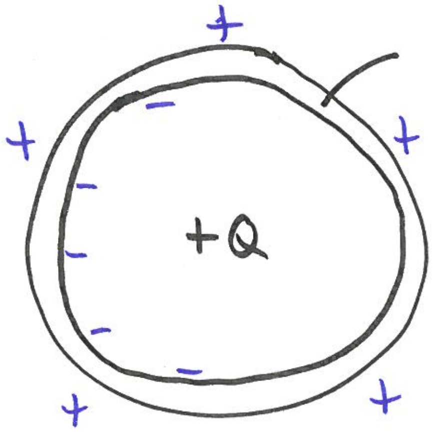
$$\int_{cylinder} \vec{E} \cdot d\vec{A} = |\vec{E}|A$$

$$|\vec{E}|A = \frac{\sigma A}{\epsilon_0}$$

$$\vec{E} = \frac{\sigma}{\epsilon_0}\hat{p}$$ ← perpendicular

Note there is a factor of 2 relative to the result for a charged sheet (no lines go into the conductor)

A final slightly surprising example: put a charge inside a neutral conducting shell...



$\vec{E} = \vec{0}$ inside shell... a Gaussian surface in the shell must enclose no net charge... there must be $-Q$ on the inside surface...

Since $\vec{E} = 0$ in the shell the interior charge distributions are totally hidden to an external observer... the $+Q$ of charge in the conductor moves to the outside of the shell where it is evenly distributed (even if the central charge is not in the middle).