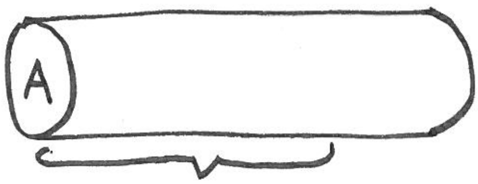
IX ELECTRIC CURRENTS

Charges in an $\vec{E}$ field experience a force & accelerate.

In a wire e^- accelerate, collide with an ion & slow, accelerate again etc. In a constant $\vec{E}$ (due to constant potential difference across wire's ends) a steady state can be reached with <u>constant mean drift velocity</u> $\vec{v}_D$

Current, I, is charge passing /second

how much passes through A in 1sec?



v_D ~ all charges within v_D m. get to A....

$$I = \frac{\Delta Q}{\Delta t} = n\, q\, v_D\, A$$

number density of charges

charge

The current can change... so define instantaneous current as

$$I = \frac{dq}{dt} \qquad \text{(ie } \Delta t \to 0 \text{ limit)}$$

I can also vary in space so define a <u>current density</u>

$$\vec{J} = nq\vec{v}_D \qquad I = \int \vec{J}\cdot d\vec{A}$$

$I = JdA$

$I = 0$

Ohm's Law

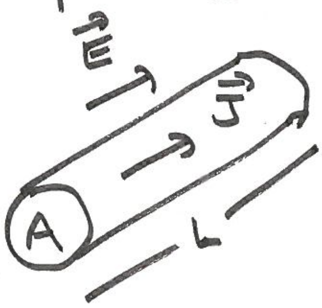
In normal materials $\quad \vec{J} \propto \vec{E}$

$$\vec{J} = \frac{\vec{E}}{\rho}$$

ρ is resistivity

$\sigma = \frac{1}{\rho}$ conductivity

This is a microscopic version of Ohm's Law. We can compute from it for a ~~wire~~ resistor



$$\text{p.d.} = V = |\vec{E}|\,L$$

$(\Delta\phi = -\int \vec{E}\cdot d\vec{l})$

$$I = JA$$

$$|\vec{J}| = \frac{|\vec{E}|}{\rho} \Rightarrow \frac{I}{A} = \frac{1}{\rho}\left(\frac{V}{L}\right)$$

$$\Rightarrow V = IR \quad , \quad R = \frac{L\rho}{A}$$

local property of material

property of macroscopic object

Power

If a charge q falls through a potential difference V it acquires energy qV. In steady state all this goes to heating

$$P = \frac{\text{energy}}{\text{sec}} = \frac{dq}{dt}V = IV$$

Circuit Components

Wires

We assume wires are perfect conductors and all points along a wire have the same potential.

Resistors

Resistors have finite ρ and hence $V = IR$

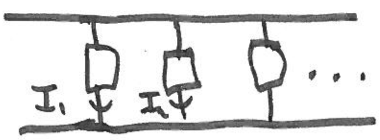
In Series

I is the same through all so we have sequential drops in V, $V_i = I R_i$

$$V_{TOT} = \sum_i V_i = I \sum_i R_i$$

$$\Rightarrow \quad R_{TOT} = R_1 + R_2 + \dots$$

In Parallel



V across each is the same

I_i add

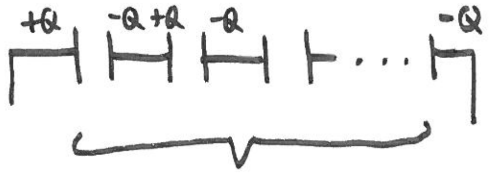
$$I_{TOT} = \frac{V}{R_{TOT}} = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \dots\right)$$

$$\Rightarrow \quad \frac{1}{R_{TOT}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots$$

Capacitors

We have seen $C = Q/V$

In Series



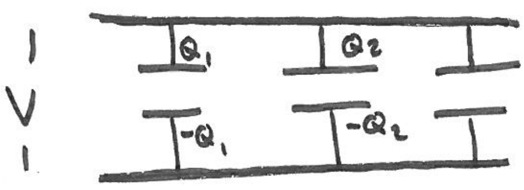
$V = V_1 + V_2 + \dots$

charge conservation places same Q on each plate

$$\frac{1}{C_{TOT}} = \frac{V_{TOT}}{Q} = \frac{V_1 + V_2 + \dots}{Q} = \sum_i \frac{V_i}{Q}$$

$$\Rightarrow \frac{1}{C_{TOT}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots$$

In Parallel



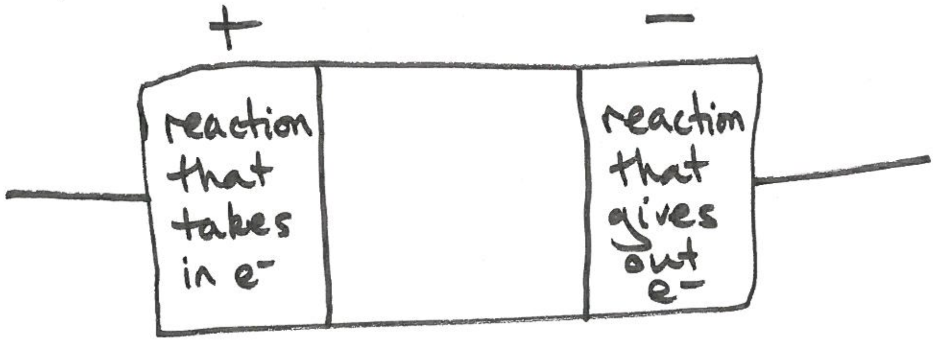
All the plates make one effective capacitor storing total charge $\sum_i Q_i$

$$C_{TOT} = \frac{Q_{TOT}}{V} = \frac{Q_1 + Q_2 + \dots}{V} = \sum_i C_i$$

$$C_{TOT} = C_1 + C_2 + \dots$$

Batteries

The idea is to separate two chemical reactions that would occur together - one needs e^- from the other.

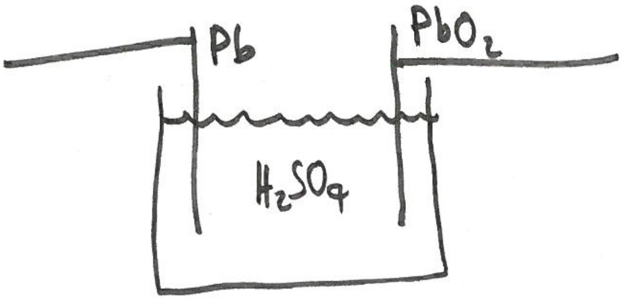


If the two ends are connected e^- run between the two ends allowing both reactions to go. Without the energy is chemically stored.

eg Car Battery

−VE TERMINAL $\quad Pb + SO_4^{2-} \rightarrow PbSO_4 + 2e^-$

+VE TERMINAL $\quad PbO_2 + SO_4^{2-} + 4H^+ + 2e^- \longrightarrow PbSO_4 + 2H_2O$



Kirchoff's Laws

(i) Charge is conserved - at any point in a circuit

$$\sum_i I_{\text{ingoing}} = 0$$

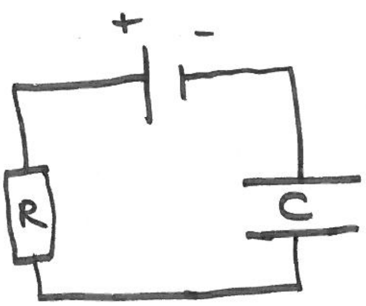
If this weren't true then charge would accumulate at the point.

(ii) Potential is single valued so if you go round a loop

$$\oint_{\substack{\text{round}\\\text{loop}}} \vec{E}\cdot d\vec{l} = 0$$

$\Rightarrow$ in a circuit $\quad \displaystyle\sum_{\substack{\text{any}\\\text{loop}}} V_i = 0$

eg <u>RC Circuit</u>

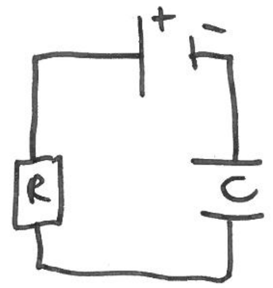


$$V_R + V_C = V_S$$

$$IR + Q/C - V_S = 0$$

$$R\frac{dQ}{dt} + Q/C - V_S = 0$$

eg <u>charging a capacitor</u>



$$IR + Q/C = V_s$$

diff w.r.t t :

$$\frac{dI}{dt} R + \frac{I}{C} = 0 \qquad \frac{dV_s}{dt} = 0$$

Over any short period $\quad \dfrac{dI}{I} = -\dfrac{dt}{RC}$

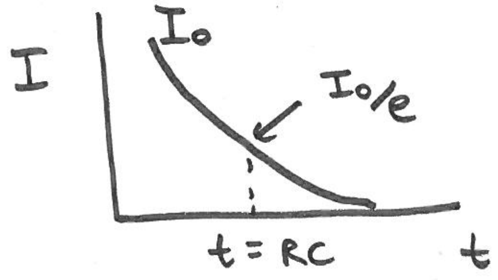
We can sum both sides over some period as long as we make the limits match

$$\int_0^t -\frac{dt}{RC} = \int_{I(t=0)=I_0}^{I(t=t)=I} \frac{dI}{I}$$

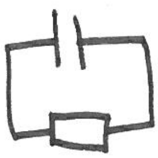
$$\frac{-t}{RC} = \ln I/I_0$$

$$\Rightarrow \quad I = I_0 e^{-t/RC}$$

The current falls exponentially as charge grows on the plates resisting the current.



eg <u>Discharging a capacitor</u>



$$IR + \frac{Q}{C} = 0$$

$$\frac{dQ}{dt} R + \frac{Q}{C} = 0$$

$$\int_{Q_0}^{Q} \frac{dQ}{Q} = - \int_0^t \frac{dt}{RC}$$

$$\ln Q/Q_0 = - t/RC$$

$$Q = Q_0 \, e^{-t/RC}$$

The charge on the plates falls exponentially with characteristic time RC again.