PHYS1022 - ELECTRICITY & MAGNETISM

LECTURE NOTES - MAGNETISM

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X MAGNETISM

Magnetic rocks were known for many years. They attract & repel like electric dipoles

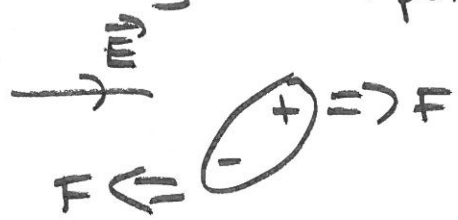
Electric Dipoles

Molecules like H_2O have uneven charge distribution

dipole moment $= q d \, \hat{r}_{-+}$

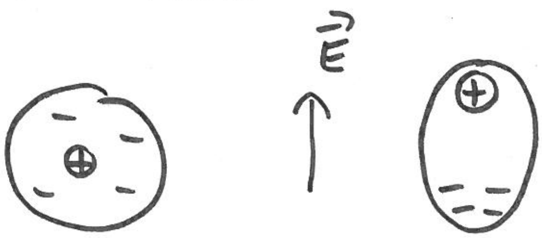
unit vector from -ve to +ve

In an $\vec{E}$ field dipoles align with field



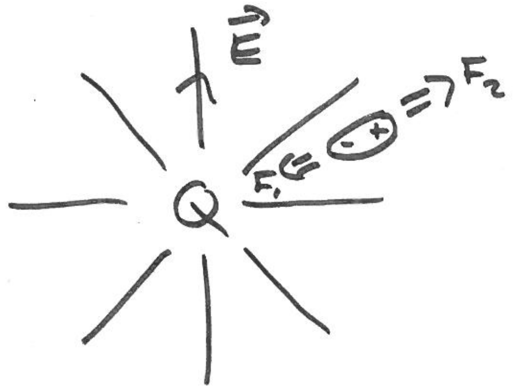
There's a torque that spins to cause alignment

$\vec{E}$ fields induce dipoles in molecules, reducing $\vec{E}$ inside the material



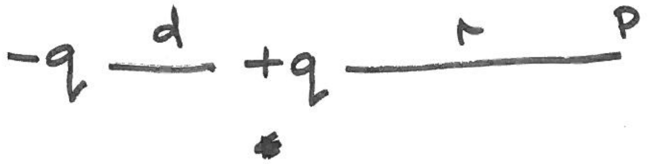
polarized

In non-uniform $\vec{E}$ fields dipoles experience a net force



$F_1 > F_2$

We can compute the $\vec{E}$ field from a dipole - let's work on axis



$$E_P = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{r^2} - \frac{1}{(r+d)^2}\right)$$

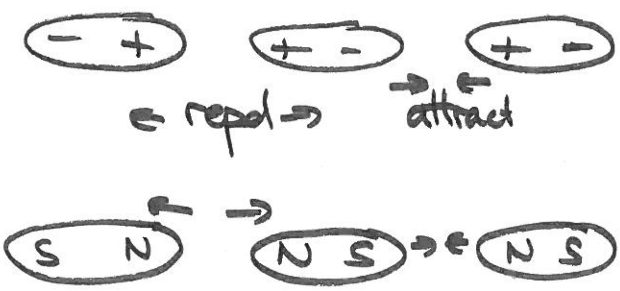
$$= \frac{q}{4\pi\epsilon_0}\left(\frac{1}{r^2} - \frac{1}{r^2}\left[1 + \frac{2d}{r} + \frac{d^2}{r^2}\right]^{-1/2}\right)$$

$$\simeq \frac{q}{4\pi\epsilon_0}\frac{2d}{r^3} \qquad \text{(binomial expansion)}$$

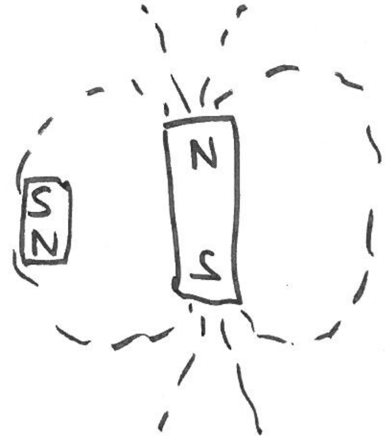
$$(1+x)^\delta = 1 + \delta x + \dots$$

Back to Magnets

Magnets attract & repel like electric dipoles



Magnets align in the $\vec{B}$ field produced by a bigger magnet



eg iron filings & a bar magnet – $\vec{B}$ field lines look just like $\vec{E}$ lines of an electric dipole

We will see that $B \sim 1/r^3$ on axis just like electric dipole too...

<u>IF</u> we could chop magnets in half & make "magnetic monopoles" they would interact like electric charges do.

<u>BUT</u> (oddly) magnetic monopoles do not exist in nature...

<u>IF</u> they did we could introduce monopoles & $\vec{B}$ as a precise <u>replica</u> of q & $\vec{E}$

eg Gauss' Law

$$\int_{\substack{\text{closed}\\\text{surface}}} \vec{B} \cdot d\vec{A} \quad \propto \quad \text{magnetic charge}$$

Magnetic Gauss' Law

Since there are no monopoles

$$\int_{\substack{\text{closed}\\\text{surface}}} \vec{B}.d\vec{A} = 0$$

NB/ $\vec{B}$ fields always come in loops since there are no "charges" for them to end on.

At this point one has to wonder why are there $\vec{B}$ fields if there are no sources for them?
The answer is that moving electric charges/currents generate $\vec{B}$ fields too. More soon....

First, since we can't define $\vec{B}$ by the force on a monopole we must use how they interact with electric charges.

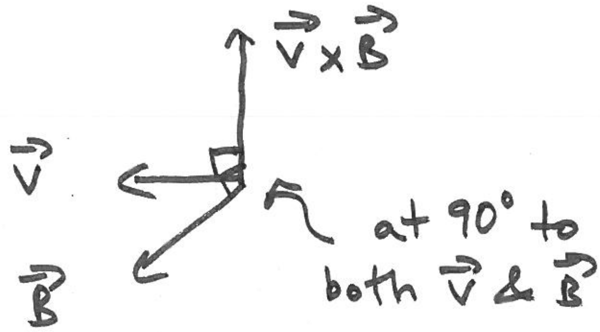
Note all of this is hinting at a unification
- $\vec{E}$ & $\vec{B}$ are aspects of a single phenomena....

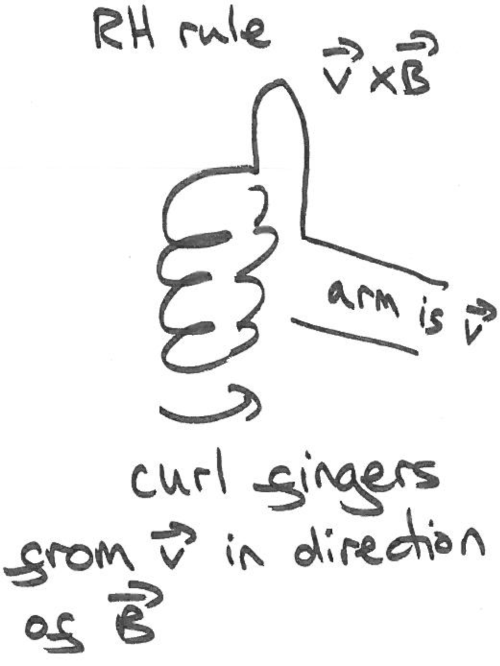
XI MAGNETIC FORCE ON ELECTRIC CHARGE

$\vec{B}$ is a vector field filling space that generates the force on charge q

$$\vec{F} = q\,\vec{v} \times \vec{B} \qquad (\text{cf } \vec{F} = q\vec{E})$$

- it only acts on moving charges
- $|\vec{F}| = qvB\sin\theta$
  ↑ angle between $\vec{v}$ & $\vec{B}$
- $\vec{F}$ is directed perpendicular to the plane in which $\vec{v}$ & $\vec{B}$ live





HISTORY: The derivation of this formula took from 1820 (Orsted's discovery of link between I & $\vec{B}$) to 1892 when Lorentz used Lagrangian methods....

Units

$$\vec{F} = q\vec{v} \times \vec{B}$$

$$|\vec{B}| \sim \frac{\text{Force}}{Q \text{ velocity}} \sim \frac{MLT^{-2}}{QMT^{-1}}$$

$$\sim MT^{-1}Q^{-1} \sim \underbrace{kg\,s^{-1}C^{-1}}_{T \text{ "Tesla"}}$$

Example $\vec{B}$ fields: Earth's $\vec{B}$ field $\sim 30\,\mu T$

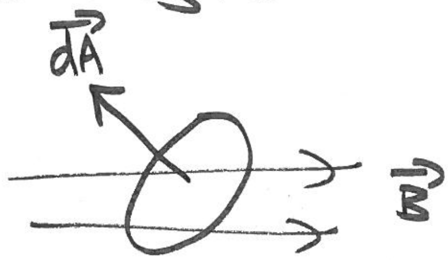
Fridge magnet $\sim 5\,mT$

Medical magnetic resonance $\sim 3T$

Magnetic Flux

As for $\vec{E}$ field flux we can define

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$



Φ_B has units $ML^2T^{-1}Q^{-1}$ or Wb "Webers"

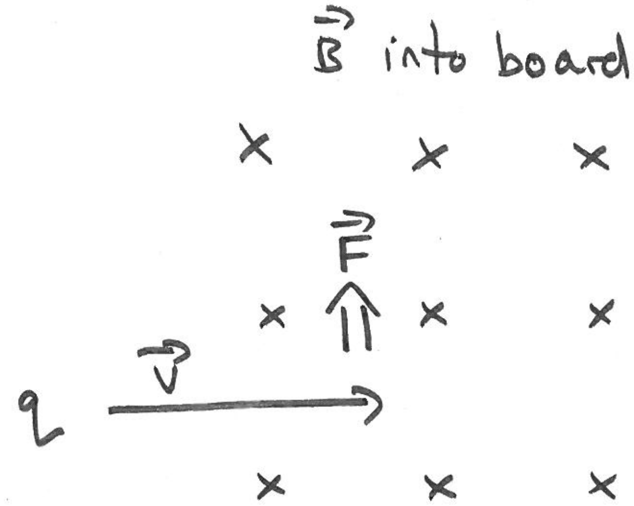
NB/ $\Phi_B \neq 0$ generally

$$\Phi_B^{\text{closed surface}} = 0$$

EXAMPLES

1) Motion in pure $\vec{B}$

First consider a charge moving perpendicular to $\vec{B}$



The force is always ⊥ to $\vec{B}$ so in plane of board.

$\vec{F}$ is also always ⊥ to $\vec{v}$ — it causes circular motion.

Newton II $\quad F = ma$

$$qvB = mv^2/r$$

$\vec{v}\times\vec{B} = |\vec{v}||\vec{B}|\sin 90^\circ$
$= vB$

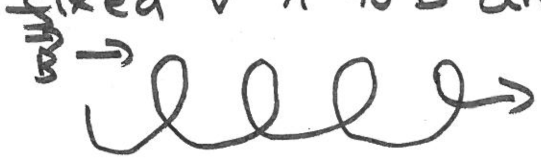
$$\Rightarrow \quad r = mv/qB$$

$$\text{Period, } T = \frac{2\pi r}{v} = \frac{2\pi m}{qB}$$

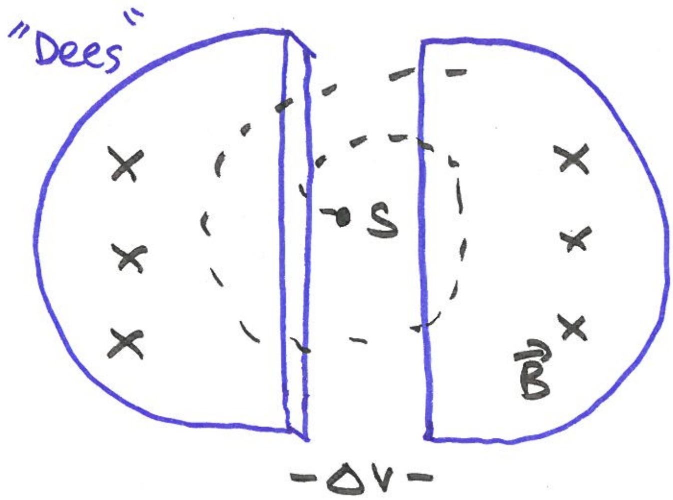
Note this is independent of $|\vec{v}|$.

If $\vec{v}$ is in the same direction as $\vec{B}$ $\quad \vec{v}\times\vec{B} = 0$ ($\sin 0^\circ = 0$) - the particle travels at constant $\vec{v}$.

If $\vec{v}$ has a component in $\vec{B}$ direction & one transverse then it travels at fixed $\vec{v}$ // to $\vec{B}$ and does circles transverse - a helix



2) Cyclotron - the earliest particle accelerator



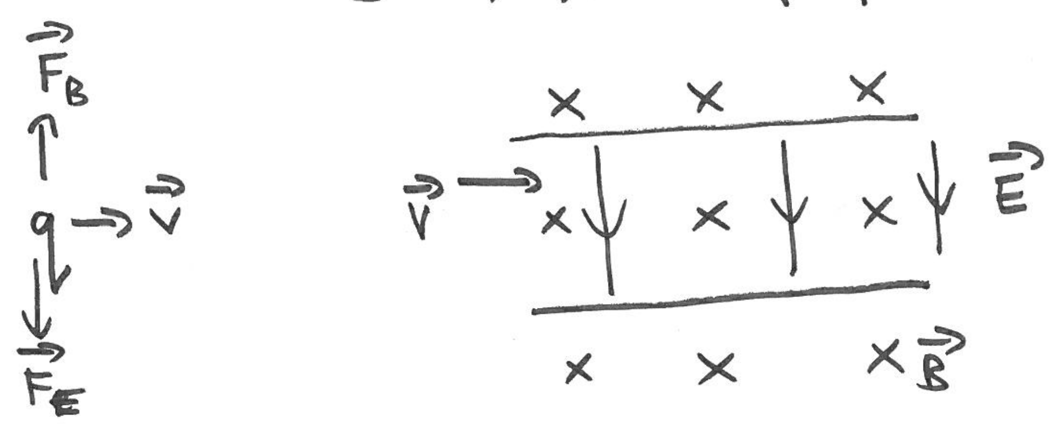
- Radioactive source generates beam of α or β (e^-)
- Perpendicular $\vec{B}$ gives rise to circular motion
- Surround by metal "Dees" (sardine tin!)
- Apply potential difference between 2 "dees" to accelerate charges

Key point: period of circular motion unchanged by change in $\vec{v}$

- After ½ period switch ΔV to accelerate again - works for all pts in beam at all $\vec{v}$ since period independent of $\vec{v}$
- Eventually r = Radius "dee" & the beam is at maximum v $\left(r = {}^{mv}/_{qB}\right)$

3) Motion in perpendicular $\vec{E}$ & $\vec{B}$

The case of $\vec{v}, \vec{E}, \vec{B}$ all perpendicular is interesting



Since forces both in vertical plane but opposite sign we can have net force zero

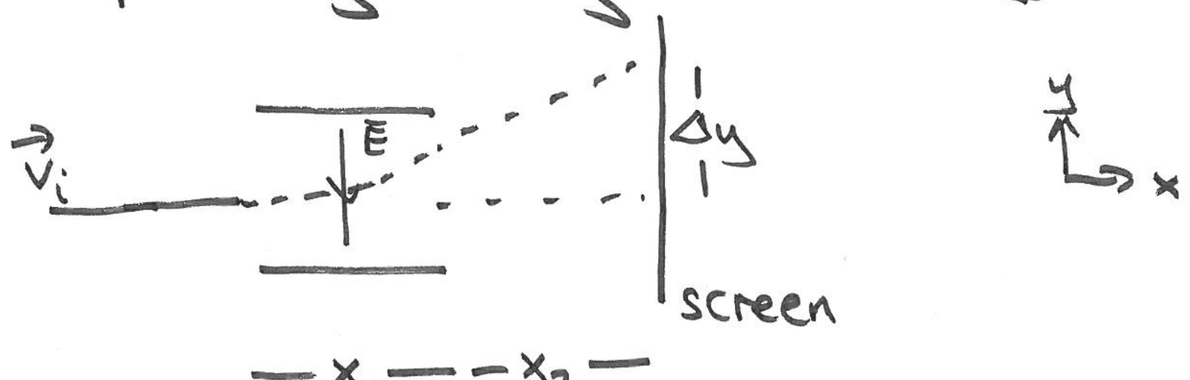
Force balance: $$qvB = qE$$

$$v = E/B$$

This set up can be used to pick a beam of specific $\vec{v}$. Any charges not on $v = E/B$ accelerate & hit the top or bottom plate.

JJ Thompson 1897

JJ used a velocity selector to make a beam of electrons of known $\vec{v}$. He then sent it between two plates generating $\vec{E}$ but no $\vec{B}$



The beam moves at constant v_i in x ... it accelerates from stationary in y (assume constant $\vec{E}$, so constant $a = qE/m$)

$$\Delta y = \overbrace{\frac{1}{2} a \underbrace{t_1^2}_{\left(\frac{x_1}{v_i}\right)^2}}^{\text{acceleration between plates}} + \overbrace{\underbrace{v_y^f}_{a \underbrace{t_1}_{(x_1/v_i)}} \underbrace{t_2}_{\left(\frac{x_2}{v_i}\right)}}^{\text{motion at const } v_y^f \text{ between plates \& screen}}$$

$$= a\left(\frac{1}{2}\frac{x_1^2}{v_i^2} + \frac{x_1 x_2}{v_i^2}\right)$$

$$\Delta y = \frac{q}{m}\,\frac{E x_1}{v_i}\left(\frac{1}{2}x_1 + x_2\right)$$

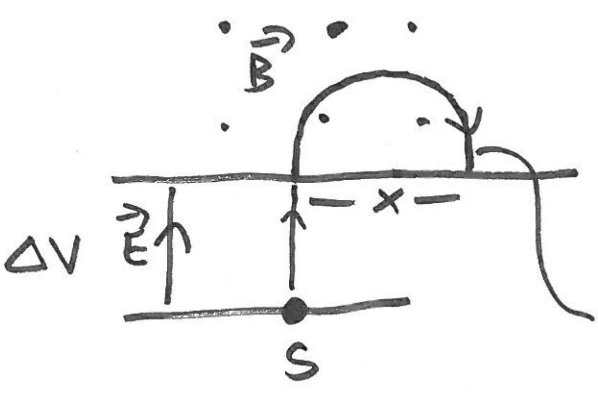
Since can measure everything else we learn q/m ratio for electron!

<u>Mass Spectrometer</u> - allows you to split a beam of mixed ions by q/m.

Accelerate beam with $\vec{E}$

$$\frac{1}{2}mv^2 = qV \qquad \text{conservation energy}$$

$$v = \sqrt{\frac{2qV}{m}}$$



circular motion in pure $\vec{B}$ zone

Position x after beam ~~hits~~ does semi-circle

$$r = \frac{mv}{qB}$$

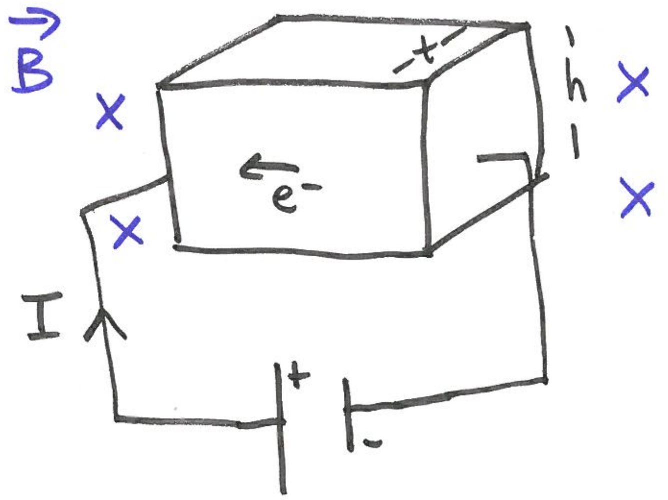
$$x = 2r = \frac{2m}{qB}\sqrt{\frac{2qV}{m}}$$

$$= \sqrt{\frac{8mV}{qB^2}} .$$

Different m/q arrive at different xs!

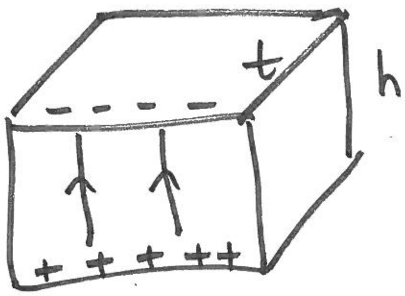
4) <u>Hall Effect</u>

Take a block of some conducting material and run a current through it



Apply a $\vec{B}$ field into board (direction of t)...

- The electrons moving in $\vec{B}$ experience a force upwards
- A net number of e^- will accumulate at the top
- The accumulation eventually stops when the e^- repel other e^-



There is a "Hall voltage" from top to bottom ie a $\vec{E}_H$ field

At equilibrium forces on e^- in current balance

$$qE_H = qvB$$

Now $J = nqv$ $\Rightarrow$ $E_H = Bv = \dfrac{BJ}{nq}$

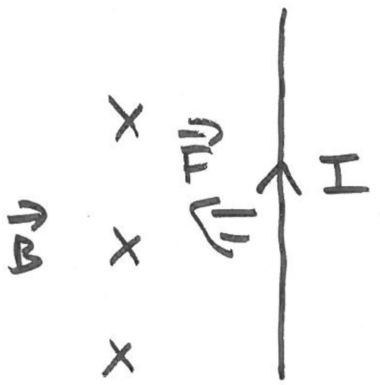
Area

Hall Voltage $$V_H = E_H h = \frac{BJh}{nq} \cdot \frac{t}{t}$$

$$= \frac{BI}{nqt}$$

$\dfrac{1}{nq}$ is "Hall Coefficient" $\rightarrow$ reveals nature & density of charge carriers.

5) Wire In $\vec{B}$ Field



The charges moving in $\vec{B}$ experience a force as shown

$$\vec{F} = q\,\vec{v} \times \vec{B}\ (\underbrace{n A}_{\text{number of charges}} L)$$

A: x-sectional area of wire

L: length wire

Using $I = qnAv$

$$\Rightarrow \quad \vec{F} = I\,(\vec{l} \times \vec{B})$$

here we've assumed I ($\vec{v}$) is in direction of the wire.