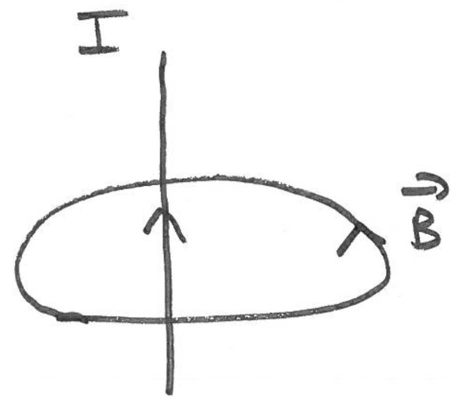
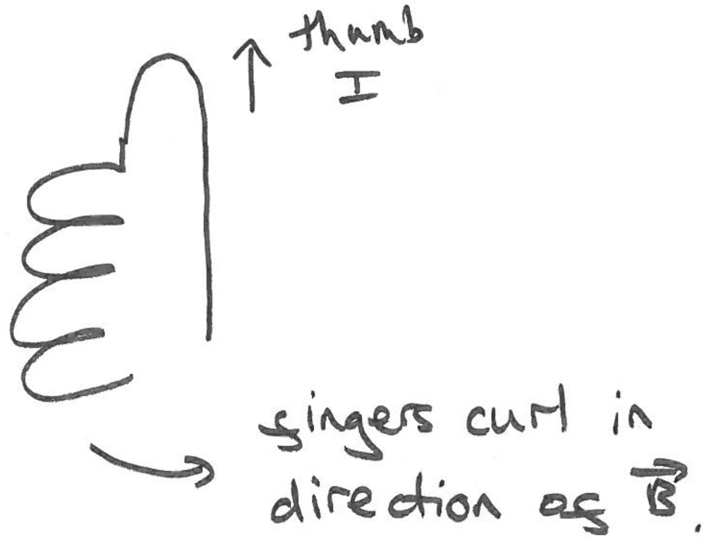
XII SOURCES OF MAGNETIC FIELDS

Why are there $\vec{B}$ fields if there are no magnetic monopoles to source them?

→ Moving electric charges / currents generate $\vec{B}$ fields ....



The direction $\vec{B}$ circulates is found experimentally & can be remembered by a right hand rule



Biot - Savart Law

This law is the equivalent of Coulomb's law. The minimum "generator" of a $\vec{B}$ field is a very short $\vec{\delta l}$ piece of current carrying wire

$|\vec{B}|$ falls as $1/r^2$

$|\vec{B}| \propto I\,|\vec{\delta l}|$ "strength of source"

$$d\vec{B} = \frac{\mu_0}{4\pi}\,\frac{1}{r^2}\,I\,d\vec{l} \times \hat{r}$$

constant of proportionality

μ_0 - permeability of free space

$= 4\pi \times 10^{-7}\ NA^{-2}$

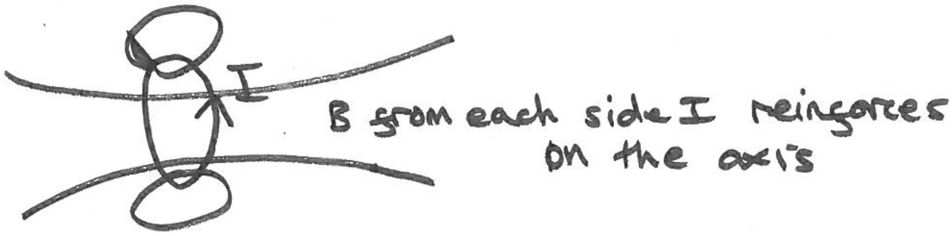
amp

at 90° into page.

the cross product is there to get the direction of $\vec{B}$ right. Note $\hat{r}$ does not contribute to magnitude.

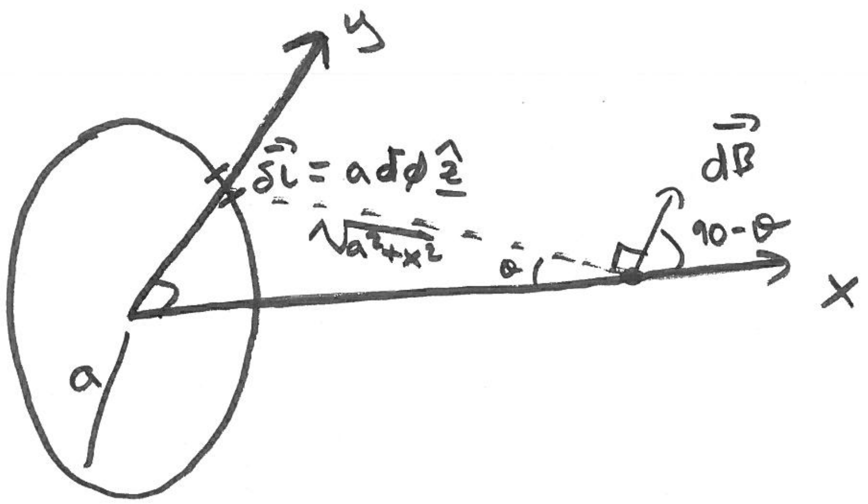
EG Circular Current Loop

A current loop generates a $\vec{B}$ field of this form



On axis $B_y = B_z = 0$ - elements of I on opposite sides of loop generate opposing components in these directions. So let's compute B_x on axis.

Consider the $\vec{\delta l}$ piece of wire cutting the x-y plane



Note $\vec{dB}$ is at $90°$ to $\vec{\delta l}$ so in the x, y plane. Then it is also at $90°$ to $\hat{r}$.....

$$\delta B_x = |\vec{\delta B}| \cos(\pi/2 - \theta)$$

$$= \delta B \sin\theta$$

$$= \delta B \frac{a}{\sqrt{a^2+x^2}}$$

Next use Biot-Savart law to compute $|\vec{\delta B}|$

$$\delta B = \frac{\mu_0}{4\pi} \frac{1}{\underset{\uparrow\, r^2}{(a^2+x^2)}} I \underbrace{a\,\delta\phi}_{\delta l}$$

$$\Rightarrow \quad \delta B_x = \frac{\mu_0}{4\pi} \frac{I a^2}{(a^2+x^2)^{3/2}} \delta\phi$$

Now by symmetry the contribution to $|\delta B_x|$ from all δl arcs round the circle will be equal so we can sum

$$B_x = \frac{\mu_0}{4\pi} \frac{I a^2}{(a^2+x^2)^{3/2}} \int_0^{2\pi} d\phi$$

$$= \frac{\mu_0}{2} \frac{I a^2}{(a^2+x^2)^{3/2}}$$

<u>Limits</u>: $\quad x \to 0 \qquad B_x = \mu_0 I / 2a$

$$x \gg a \qquad B_x \sim \frac{\mu_0 I \pi a^2}{2\pi x^3}$$

The latter is of the same form as the on axis $\vec{E}$ field from an electric dipole - we computed

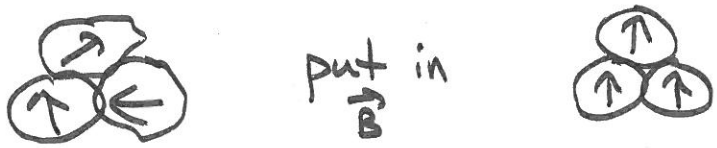
$$E \sim \frac{p}{2\pi\epsilon_0 r^3} \quad \leftarrow \text{dipole moment}$$

By analogy we can define the magnetic dipole moment $= I\pi a^2 = I \times \text{Area}$.

So what are magnets?

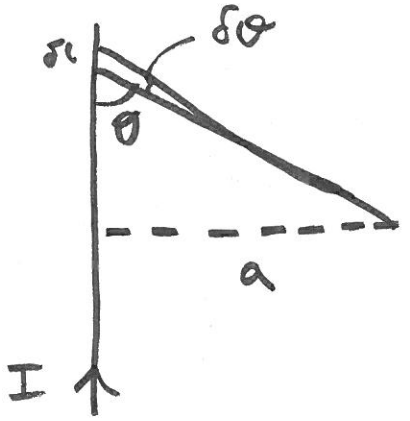
On the atomic scale atoms have orbiting & spinning electrons ... these currents induce $\vec{B}$. In ferromagnets interactions like to align spins in neighbouring atoms so the $\vec{B}$ fields reinforce.

Actually the alignment is normally in local domains. "Stroking" with a magnet aligns these domains & eg makes a nail into a magnet.

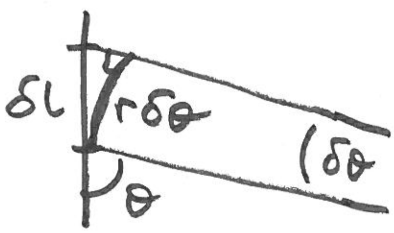


The $1/r^3$ fall off of $\vec{B}$ from the current loop matches that measured for a bar magnet.

eg ∞ Straight Wire



We will use angle θ between $\vec{\delta l}$ & $\hat{r}$ as our coordinate.



$$r = a/\sin\theta \qquad \delta l = \frac{r\,\delta\theta}{\sin\theta} = \frac{a\,\delta\theta}{\sin^2\theta}$$

Note all $\vec{\delta l}$ pieces generate $\vec{B}$ into page at the point a away considered - call it $\hat{z}$

Biot Savart: $\vec{\delta B} = \frac{\mu_0}{4\pi} \frac{1}{r^2} I \, \delta l \sin\theta \, \hat{z}$

$$= \frac{\mu_0}{4\pi} I \frac{\sin^2\theta}{a^2} \frac{a\,\delta\theta}{\sin^2\theta} \sin\theta \, \hat{z}$$

$$= \frac{\mu_0}{4\pi} I \frac{\delta\theta}{a} \sin\theta \, \hat{z}$$

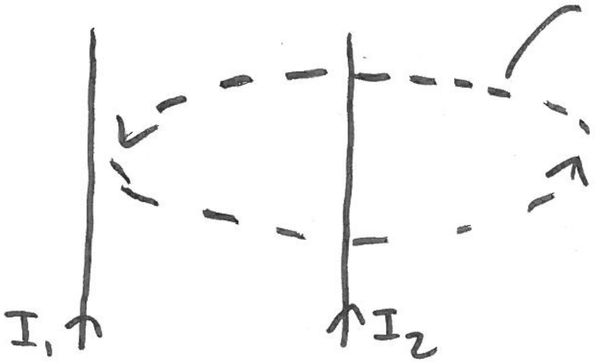
Now sum all components up & down wire

ie $0 < \theta < \pi$

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{I}{a} \underbrace{\int_0^\pi \sin\theta \, d\theta}_{[-\cos\theta]_0^\pi = 2} \hat{z}$$

$$= \frac{\mu_0 I}{2\pi a} \hat{z}$$

Force between two wires



$\vec{B}$ due to I_2

Wire with I_1 experiences force

$$\vec{F} = I \, \vec{l} \times \vec{B}$$

The resultant force is towards I_2....

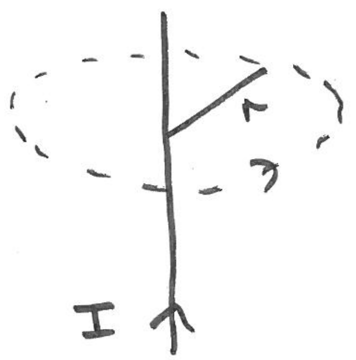
- Wires with like currents attract
- " " unlike " " repel

Ampère's Law

Coulomb's law $\longrightarrow$ Gauss' Law $\int \vec{E} . d\vec{A} = Q/\epsilon_0$

Biot - Savart law $\longrightarrow$?

Here's a motivational example ... (not a proof)

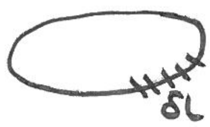


Let's perform the following line integral round the circular loop of radius r centred on the wire

$$\underset{\text{closed loop}}{\oint} \vec{B} . \vec{dl}$$

We've computed that $\vec{B} = \frac{\mu_0 I}{2\pi r} \underline{\hat{\theta}}$

Now $\oint \vec{B} . \vec{dl} = \sum_{\substack{\text{round} \\ \text{loop}}} \vec{B} . \vec{\delta l}$



$\vec{B}$ has constant magnitude round loop

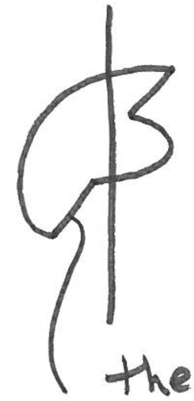
Any local $\vec{\delta l}$ is in $\underline{\hat{\theta}}$ like $\vec{B}$ $\Rightarrow \cos\theta = 1$

$$\oint \vec{B} . \vec{dl} = \frac{\mu_0 I}{2\pi r} \underbrace{\sum_{\substack{\text{round} \\ \text{loop}}} \delta L}_{2\pi r}$$

circumference of loop

$$= \mu_0 I$$

Now we can deform the path we did the line integral round eg two half circles linked

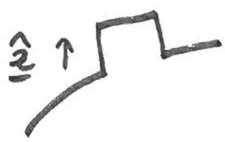


$$\oint \vec{B}.\vec{dl} = \frac{\mu_0 I}{2\pi r_1}\pi r_1 + \frac{\mu_0 I}{2\pi r_2}\pi r_2$$

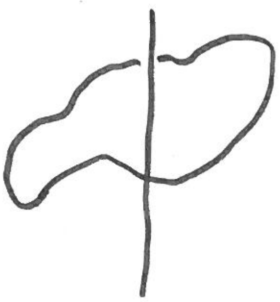
$$= \mu_0 I$$

these bits are in $\hat{r}$ not $\hat{\phi}$
so dot with $\vec{B}$ to zero

We can also hop up in $\hat{z}$ without changing the result. $\hat{z}$ pieces dot with $\vec{B}$ to zero... $\vec{B}$ is independent of z.



In this way we can deform our circle with infinitessimal shifts in $\hat{r}$ & $\hat{z}$ to trade the circle for <u>any</u> closed loop

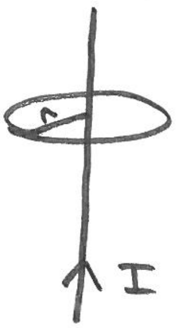


We still have

$$\boxed{\oint \vec{B}.\vec{dl} = \mu_0 I_{\substack{\text{enclosed}\\ \text{by loop}}}}$$

Ampère's law applies generally for steady currents eg <u>∞ wire</u>

We can reverse the derivation...



Symmetry: $\vec{B}$ must be in closed loops & here they must be circles

$|\vec{B}|$ can only depend on r

$$\vec{B} = B(r)\,\hat{\underline{\theta}}$$

Ampere's Law: choose a circular Amperian loop to reflect symmetries

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I$$

$$B(r)\,2\pi r = \mu_0 I$$

$$\vec{B} = \frac{\mu_0 I}{2\pi r}\,\hat{\underline{\theta}}$$

eg <u>inside the wire</u>

Amperian loop (same sym arguments as above)

assume constant current density $= I/\pi a^2$

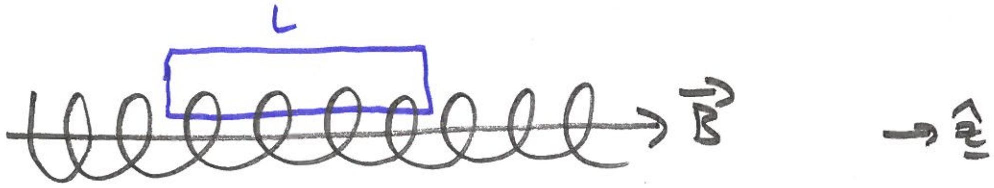
$$\oint \vec{B}\cdot d\vec{l} = \mu_0 \frac{I}{\pi a^2}\cdot \pi r^2$$

$$B(r)\,2\pi r = \mu_0 I \frac{r^2}{a^2}$$

$$\vec{B} = \mu_0 \frac{I r}{2\pi a^2}\,\hat{\underline{\theta}}$$

eg **Solenoid**

If you have many neighbouring current loops then they reinforce $\vec{B}$ along their axes - as $N \to \infty$ $\vec{B}_{inside} \gg \vec{B}_{outside}$ - we assume $\vec{B}$ is linear & restricted to the interior.



A sensible Ampèrian loop is the rectangle shown

$$\oint \vec{B} \cdot d\vec{l} = |\vec{B}|\, L$$ from interior // to $\vec{B}$ only

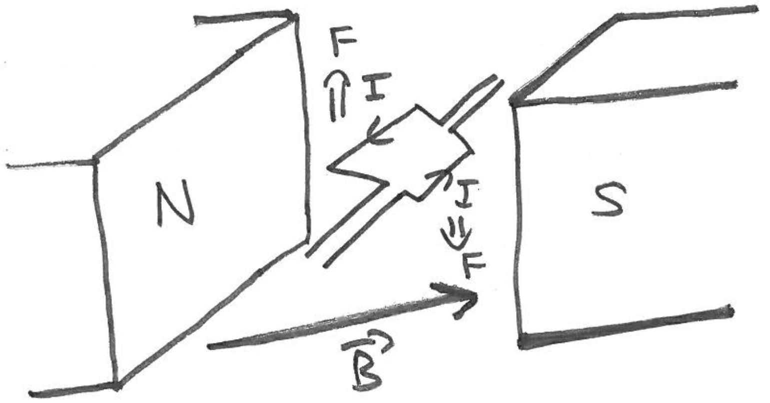
$$= \mu_0 I N$$ ← number of loops

$$\Rightarrow \quad \vec{B} = \mu_0 I n \, \hat{z}$$

↑ number of turns per unit length

XIII MAGNETIC INDUCTION

If we put a current through a wire loop that is between a magnet's poles....



the moving charges experience a force

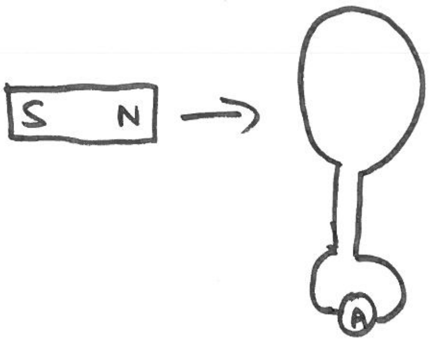
$$\vec{F} = q\vec{v} \times \vec{B}$$

in directions shown...

The loop spins due to the <u>torque</u>

Now if you switch off I ... but spin the loop by hand... you observe that a current flows.

eg

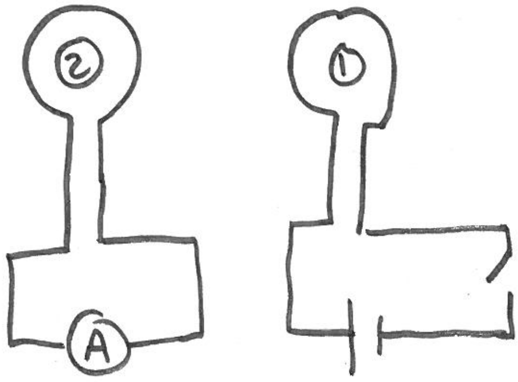


Relative motion of magnet & loop generates a current

Faster motion $\Rightarrow I \uparrow$

Direction of I depends on magnet orientation

eg



See I in 1 when current in 2 grows or falls

$\Rightarrow$ current/emf induced by changing $\vec{B}$

The key is the change in Magnetic Flux with wire loop

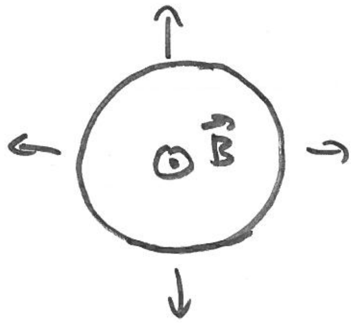
$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$

Faraday's Law

$$\underset{\text{induced}}{\text{emf}} = - \frac{d \Phi_B}{dt}$$

↑ rate of change of magnetic flux

eg thermal expansion of a loop



$dA/dt \neq 0$ (Φ_B can change due to A aswell as $\vec{B}$)

$\Rightarrow$ clockwise current

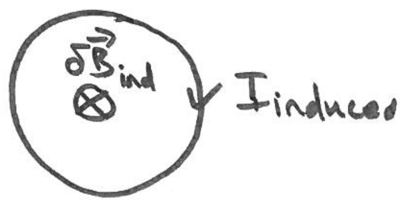
How do we know which direction for I?

Lenz's Law

The induced emf is in such a direction as to oppose the change that produces it.

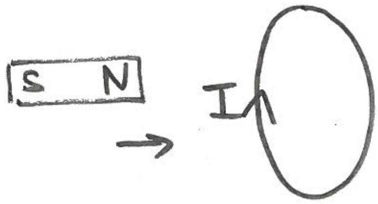
(the minus sign in Faraday's law encodes this but Lenz's law is best route to answer!)

thermal example:

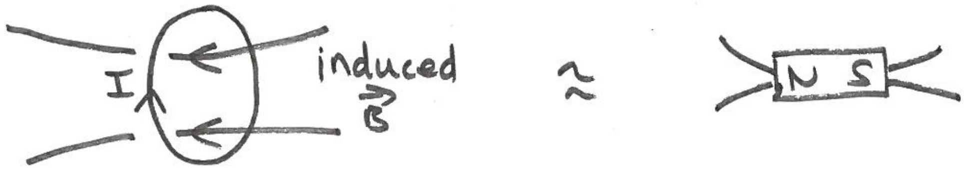


$\delta\vec{B}_{ind}$ opposite to external $\vec{B}$.

eg move a magnet towards a loop...

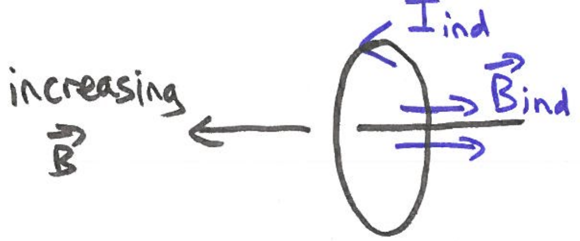


...so the wire gets hot - where does the energy come from?

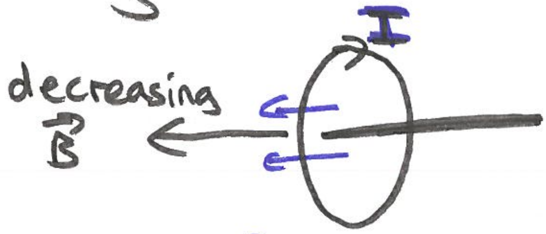


You have to put in energy to push the two north poles together....

eg the induced $\vec{B}$ does not always point in the opposite direction to the original $\vec{B}$

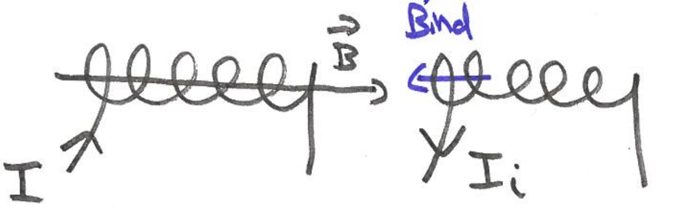


here it does oppose...



B_{ind} reinforces decreasing $\vec{B}$ opposing change.

eg



increasing

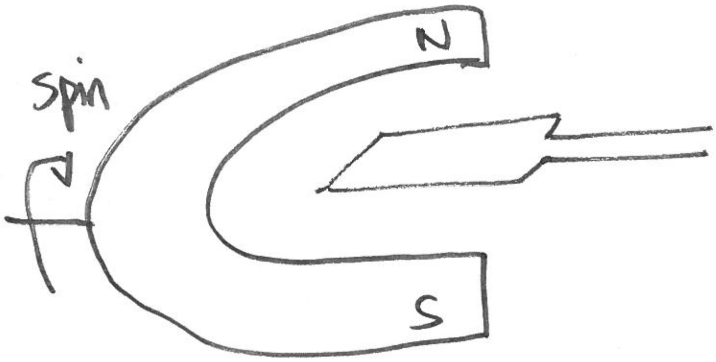
induced is opposite

decreasing

I_i in same direction

$\vec{B}_{ind}$ supports falling $\vec{B}$...

eg AC Power Generation



Generates an AC emf

$V_0 \sin \omega t$

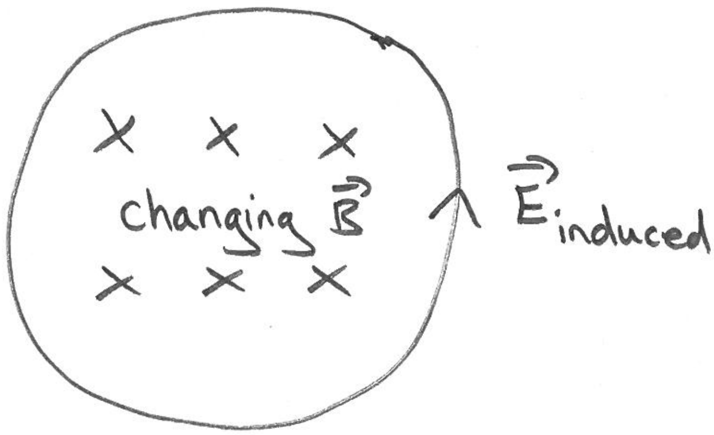
Use any power source to spin the magnet!

Induced Electric Fields

The current is caused by a potential difference round the loop

$$\Delta \phi = -\int \vec{E} \cdot d\vec{l}$$

ie we've really generated a circular $\vec{E}$ field



Faraday's law is

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$