

That Tricky Differential

$$\frac{d}{dy} \left[\frac{y}{x(x^2+y^2)^{1/2}} + c \right] \quad \text{differentiates to zero.}$$

First note that "x" is just a "number" not a variable!
'constant

Write $f(y) = y$
 $g(y) = x^2 + y^2$

So $\frac{1}{x} \frac{d}{dy} \left[\frac{f(y)}{g(y)^{1/2}} \right]$

$$= \frac{1}{x} \left[\frac{1}{g(y)^{1/2}} \frac{d}{dy} f(y) + f(y) \frac{d}{dy} \left(\frac{1}{g(y)^{1/2}} \right) \right]$$

$$= \frac{1}{x} \left[\frac{1}{g(y)^{1/2}} \frac{d}{dy} f(y) + f(y) \frac{dg}{dy} \frac{d}{dg} \left(\frac{1}{g^{1/2}} \right) \right]$$

$$= \frac{1}{x} \left[\frac{1}{(x^2+y^2)^{1/2}} \cdot 1 + y \cdot 2y \cdot \frac{-1/2}{g^{3/2}} \right]$$

$$= \frac{1}{x(x^2+y^2)^{1/2}} - \frac{y^2}{x(x^2+y^2)^{3/2}}$$

$$= \frac{1}{(x^2+y^2)^{3/2}} \left[\frac{x^2+y^2}{x} - \frac{y^2}{x} \right] = \frac{x}{(x^2+y^2)^{3/2}}$$