NAGT

$$\mathcal{L}_{NAGT} = -\frac{1}{2} \operatorname{Tr} F^{\mu\nu} F_{\mu\nu} + \bar{\Psi} (i \not{D} - m \mathbb{I}) \Psi$$

GAUGE FIXING

<u>Warm Up</u> : consider the area integral $\int_0^L dx \int_0^L dy = L^2$

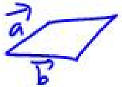
We can turn into a line integral by inserting $\delta(y)$

$$\int_0^L dx \int_0^L \delta(y)\, dy = \int_0^L dx = L$$

Move to polar (r, θ) coords

in general get some //ogram in the new coords

area = $\vec{a} \times \vec{b}$



The area changes by a "Jacobian" factor (a determinant)

$$J = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} \end{vmatrix} = \begin{vmatrix} \cos\theta & -r\sin\theta \\ r\sin\theta & r\cos\theta \end{vmatrix} = r$$

$$\iint dx\, dy \longrightarrow \int J\, d\theta\, dr = \int r\, dr\, d\theta$$

Now to get a line integral along x insert $\delta(\theta)$ (?)

$$\iint \delta(\theta)\, r\, dr\, d\theta = \int_0^L x\, dx = L^2/2$$

To correct this we need to change the normalization of δ-fn when change coords

$$\delta(y) \longrightarrow \frac{1}{J\big|_{\theta=\theta_0}} \delta(\theta-\theta_0)$$

NAGT

Gauge freedom $\overset{A^{\mu a}T^a}{A^\mu} \to \overset{1+T^a}{U} A_T^\mu \overset{1-T^a}{U^{-1}} + \frac{i}{g}(\partial_\mu U)U^{-1}$

For small ω^a $\quad U = 1 + i\omega^a T^a + \dots$

$$\delta A^a_\mu = f^{abc} A^b_\mu \omega^c - \frac{1}{g}(\partial^\mu \omega^a)$$

This is a redundancy in the description ($\vec{E}, \vec{B}$ are gauge inv)

The overcounting causes trouble...

$$S = \int d^4x \, F^{a\mu\nu} \overbrace{F^a_{\mu\nu}}^{\partial_\mu A^a_\nu - \partial_\nu A^a_\mu + \dots}$$

$$\partial_\mu A^a_\nu \partial^\mu A^{a\nu} - \partial^\mu A^a_\nu \partial^\nu A^{a\mu} - \partial^\nu A^{a\mu} \partial_\mu A^a_\nu$$

$$+ \partial^\nu A^{a\mu} \partial_\nu A^{a}_{\mu}$$

BUT $= \int d^4x \; \frac{1}{2} A^a_\mu \, O^{\mu\nu}_{ab} \, A^b_\nu$

$$\frac{1}{2}\left(g^{\mu\nu}\partial^2 - \partial^\mu\partial^\nu\right)\delta^a_b$$

The propagator is the inverse of O but there isn't an inverse - I can tell because there is a zero eigenvalue

$$O^{\mu\nu} \, \partial_\nu \omega(x) = 0$$

So we should restrict to solutions in some fixed gauge

$$D[A_\mu] \longrightarrow D[A_\mu^{\omega=0}] \, D[\omega] \, \delta(\omega)$$

<u>BUT</u> we normally fix a gauge by a condition like

$$\partial^\mu A_\mu = f(x)$$

so we put in (?) $\delta\left[\partial^\mu A_\mu - f\right]$ for $\delta(\omega)$

we can so long as we include a "functional determinant" to take care of change of variables... $J \leftarrow \ldots \det \frac{\delta(\partial^\mu A_\mu)}{\delta \omega}\Big|_{\omega=0}$

$$Z = \int D[A_\mu]\, \delta(\partial^\mu A_\mu - f)\, J\Big|_{\omega=0}\, e^{iS}$$

Now $\delta(\partial^\mu A^a_\mu) = f^{abc}\, \partial^\mu A^b_\mu\, \omega^c - \frac{1}{g}\partial^\mu \partial_\mu \omega^a$

~~Thus~~ $$\frac{\delta(\partial^\mu A^a_\mu)(x)}{\delta w^b(y)} = f^{abc}\,\partial^\mu A^c_\mu\,\delta^4(x-y) - \frac{1}{g}\partial^\mu\partial_\mu\,\delta^4(x-y)\,\delta^a_b$$

$$= -\frac{1}{g}\,\delta^4(x-y)\,\partial^\mu D^{ab}_\mu(x)$$

$$D^{ab}_\mu = \partial_\mu\delta^{ab} - g\underbrace{f^{abc}}_{\text{generators of adjoint rep.}} A_{c\mu}$$

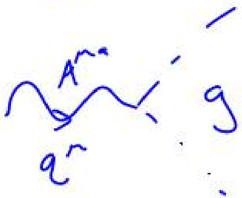
We want to take the determinant

$$\det M = \int \underbrace{D[\eta]\,D[\bar\eta]}_{\text{Grassmanian}}\, e^{-\int d^4x\ \bar\eta(x) M \eta(x)}$$

η & $\bar\eta$ are "Fadeev-Popov" ghosts - not physical but occur in loops in Feynman diagrams

They are Grassman scalars (they get a $-$ sign on loops)

$\frac{i}{p^2}$



$g f^{abc} q^\mu$

$\bar{\eta}\,\eta\;\partial A_i\; e^{iS}$

One final trick.. any choice of $f(x)$ gives the same answer... pick two, add & you'll get twice Z ... why not pick a smeared distribution of fs?

$$Z = \int D[f]\, e^{i\int d^4x \frac{1}{2(1-\xi)} f^2(x)} \int D[A_\mu]\, \delta(\partial \cdot A - f)\, J\, e^{iS}$$

do ξ integral...

$$Z = \int D[A_\mu]\, J\, e^{iS}\, e^{i\int d^4x \frac{1}{2(1-\xi)} (\partial^\mu A_\mu)^2}$$

$\hookrightarrow$ integrate by parts

$-A_\mu \partial^\mu \partial^\nu A_\nu$

Now $$O_{ab}^{\mu\nu} = i\delta_{ab}\left(g^{\mu\nu}\partial^2 + \frac{\xi}{1-\xi}\partial^\mu\partial^\nu\right)$$

The inverse is $$-\delta_{ab}\left(g_{\mu\nu} - \xi\frac{p_\mu p_\nu}{p^2}\right)\frac{i}{p^2}$$